Birzeit University
Physics Department
Physics 111

Experiment No.9

RC circuit

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Abstract:

1)The aim of the experiment is to find the time constant τ of an RC circuit and the value of its capacitor.
2)The method used is by measuring the voltage on the capacitor at certain moments in a charging and a discharging circuits.
3)**The main result is**:

$$C = 21 \pm 1 \mu F$$

Theory:

Lets consider the series RC circuit shown in figure 1, consisting of a capacitor C and a resistance R connected in series to a voltage source ε , at t=0 the capacitor is uncharged that is, $Q_0 = 0$, when the circuit is closed then the capacitor will start charging.

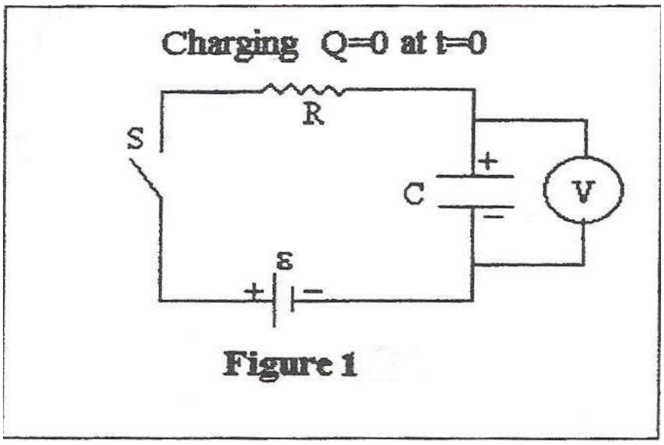


Figure 1

Here are the two states , charging and discharging , we will treat at this experiment:

A. Charging:

From Kirchhoff's second rule , which implies that the sum of all voltage drops over a closed group is zero ($\sum_{i=1}^{n} V_i = Zero$) we find that ,in our circuit, :

$$\varepsilon - IR - \frac{Q}{C} = 0 \Rightarrow I = \frac{\varepsilon}{R} - \frac{Q}{RC} \Rightarrow \frac{\partial Q}{\partial t} = \frac{\varepsilon}{R} - \frac{Q}{RC}$$

$$\Rightarrow \frac{\partial Q}{\frac{\varepsilon}{R} - \frac{Q}{RC}} = \partial t \qquad ;(\text{as } I = \frac{\partial Q}{\partial t})$$

Integrating both sides:

$$\int_{Q=0}^{Q(t)} \frac{1}{\frac{\varepsilon}{R}-\frac{Q}{RC}}\partial Q = \int_{0}^{t}\partial t \Rightarrow -RCLn(\frac{(\varepsilon/R)-(Q/RC)}{\varepsilon/R}) = t$$

$$Ln(\frac{(\varepsilon/R)-(Q/RC)}{\varepsilon/R}) = \frac{-t}{RC}$$

Exponentiating both sides , we get :

$$\frac{(\varepsilon\ /\ R)-(Q\ /\ RC\ )}{\varepsilon\ /\ R} = e^{\frac{-t}{RC}} \Rightarrow 1-\frac{Q}{\varepsilon C} = e^{\frac{-t}{RC}}$$

$$\Rightarrow Q(t) = \varepsilon C(1-e^{\frac{-t}{RC}})$$

and as the voltage on the capacitor has the relation (V=Q/C) with the charge on it , thus:

$$V(t) = \varepsilon(1-e^{\frac{-1}{RC}})$$

At the time t=RC ,which is known as the time constant τ of the circuit the voltage on the capacitor is:

$$V(t=RC) = \varepsilon(1-e^{-1}) = \varepsilon(1-\frac{1}{e}) = 0.63\varepsilon \qquad \text{E1}$$

We see from equation E1 that the time constant τ is the time needed for the potential difference on the capacitor V(t) to reach 0.63 of the maximum voltage ε .

Note : τ can be found from the graph of the charging curve as that of figure 3 by drawing a parallel line to the t-axis passes through 0.63 ε volts on the v-axis , then by drawing a parallel line to the v-axis , from the point where the first line cuts the curve of V vs. t , and the point where it cuts the t-axis is τ as shown in figure 2.

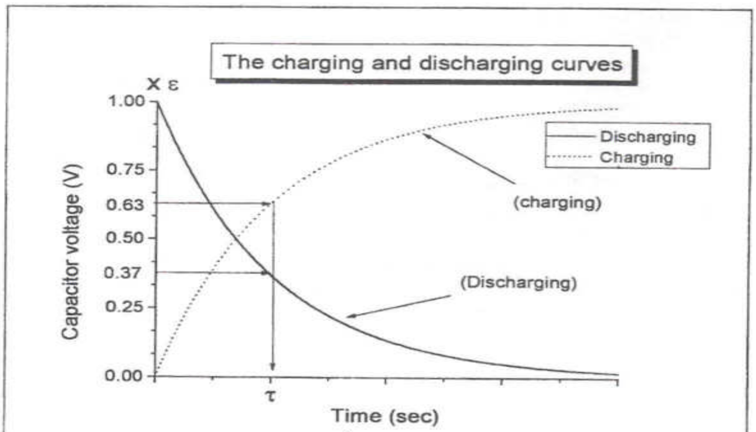


Figure 2

B. Dicharging:

After the capacitor has been charged , and if we removed the power supply , and a resistance R is connected ,by series, to this circuit as shown in figure 3 , the capacitor has an initial potential difference of ε and an initial charge of $Q_0 = C\varepsilon$ .Then the capacitor will start discharging through the resistance . Back again to Kirchhoff's second rule ;

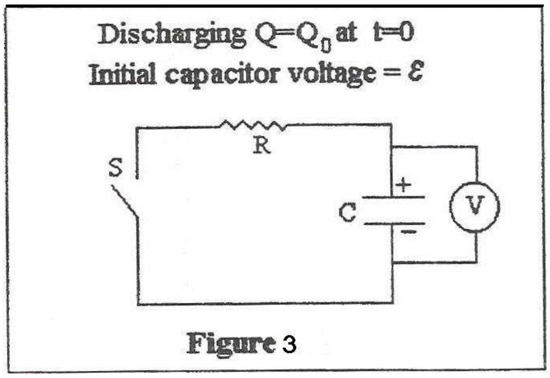


Figure 3

$$-IR - \frac{Q}{C} = 0 \Rightarrow -\frac{\partial Q}{\partial t}R - \frac{Q}{C} = 0 \Rightarrow R\frac{\partial Q}{\partial t} = -\frac{Q}{C}$$

Dividing both sides by RQ;

$$\frac{\partial Q}{Q} = -\frac{1}{RC}\partial t$$

Integrating both sides;

$$\int_{Q=0}^{Q(t)} \frac{\partial Q}{Q} = -\frac{1}{RC}\int_0^t \partial t \Rightarrow LnQ(t) - LnQ_0 = -\frac{1}{RC}t$$

$$\Rightarrow Ln(\frac{Q(t)}{Q_0}) = -\frac{1}{RC}t$$

Exponentiating both sides , we get ;

$$\frac{Q(t)}{Q_0} = e^{-\frac{1}{RC}t} \Rightarrow Q(t) = Q_0 e^{-\frac{1}{RC}t} \Rightarrow Q(t) = \varepsilon C e^{-\frac{1}{RC}t}$$

And as the voltage on the capacitor V =Q/C ;

$$V(t) = \frac{Q(t)}{C} = \varepsilon e^{-\frac{t}{RC}}$$

And at time t=τ ;

$$V(t = RC) = \varepsilon e^{-1} = \frac{\varepsilon}{e} = 0.37\varepsilon$$

E2

From the discharging curve as that shown in figure 2 we can find τ , but this time the first line cuts the v-axis at v=0.37ε volts (see figure 2).

A third way for finding τ is from the linear equation we get from taking the natural logarithm of both sides of equation E2 ;

$$Ln\{v(t)\} = Ln\,\varepsilon e^{-\frac{t}{RC}} \Rightarrow Ln\{v(t)\} = Ln\,\varepsilon - \frac{t}{RC} \qquad \text{E3}$$

Equation E3 gives a linear equation with the y-intercept Ln(ε) and with a slope $m = -1/RC = -1/\tau$.
So $\tau = -1/m$
The value of the unknown capacitor can be found using this equation :

$$C = \frac{\bar{\tau}}{R}$$

The uncertainty in C ΔC is found by taking the partial derivative of the later equation , thus;

$$\Delta C = \left|\frac{\Delta\bar{\tau}}{R}\right| + \left|-\frac{\bar{\tau}\Delta R}{R^2}\right| \Rightarrow \Delta C = C\left(\frac{\Delta\bar{\tau}}{\bar{\tau}} + \frac{\Delta R}{R}\right)$$

The uncertainty in $\bar{\tau}$, $\Delta\bar{\tau} = \sigma_m(\tau)$,the standard deviation of the mean value of the measurements of τ .The resistance R is measured either by the ohmmeter or using the color code on it, and ΔR , the uncertainty in R, is to be estimated from the multi-meter (switched to as an ohmmeter) or from the color code.

Procedure:

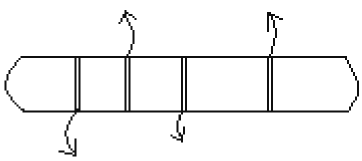
1.We connected the resistance and the capacitor in series and connected the multi-meter in parallel to the capacitor (the anode to the anode and the cathode to the cathode), and the wire which was connected to the positive part of the resistance was connected to the anode of the power supply, and the cathode of the capacitor was connected to the cathode of the power supply.

2.We connected the two terminals of the capacitor to each other by a wire so as to discharge it from any charge , and we kept the them connected .
3.We switched the multi-meter on and switched it to work as a voltmeter and we chose the suitable scale that included the minimum and maximum voltage which was about 5 Volts.
4.My partner removed the mentioned wire at the same time he started the stop watch.
5.My partner counted to five (a number per second was counted) and when he said “five” I read the voltmeter and wrote the measured value down.
6.We repeated step five until I had 3 same successive measured values.
7.We removed the wire connected to the cathode and connected it with that connected to the anode so as to take the power supply away from the circuit. At this moment my partner started the stop watch again (after it was calibrated).
8.We repeated step 5 until the time equaled that of the charging one.
9.We measured the value of the resistance twice : from the color code and using the multi-meter which was switched to work as an ohmmeter.
10.We took the value of the capacitor which was written on it.

Data:

$C = 22\mu F$

(From the color code)



$R = (100 \pm 5)\times 10^4\ \Omega$

(From the multi-meter)[1]

$R = 994 \pm 3\ \Omega$

Time (sec)	Charging $V_{Capacitor}$ (volts)	Discharging $V_{Capacitor}$ (volts)
0	0.00	4.83
5	1.04	3.75
10	1.84	2.96
15	2.47	2.37
20	2.99	1.88

[1] In this measurement the multi-meter was switched to work as an ohmmeter.

25	3.34	1.49
30	3.63	1.18
35	3.90	0.93
40	4.09	0.75
45	4.25	0.60
50	4.37	0.47
55	4.47	0.37
60	4.54	0.30
65	4.60	0.24
70	4.65	0.19
75	4.69	0.15
80	4.72	0.12
85	4.75	0.09
90	4.77	0.08
95	4.78	0.06
100	4.79	0.05
105	4.80	0.04
110	4.81	0.03
115	4.82	0.02
120	4.83	0.02
125	4.83	0.01
130	4.83	0.01

Calculations:

1)From the linear graph paper:

*a) The time constant from the charging curve (τ_c) :

$\tau_c = 20.56$ Sec

*b) The time constant from the discharging curve (τ_d) :

$\tau_d = 21.48$ Sec

2)From the semi-log graph ;

$$Slope = \frac{Ln0.23 - Ln2.96}{65-10} = -0.0462\ Sec^{-1}$$

$\tau_s = 21.60$ Sec (the time constant from the semi-log graph)

$\bar{\tau} = 21.2125$ Sec

$$\Delta\tau = \frac{\sigma_s}{\sqrt{N}} = \frac{0.5721}{\sqrt{3}} = 0.33 Sec$$

Resistance

$R = (100 \pm 5) \times 10^4\ \Omega$

$$C = \frac{\tau}{R} = \frac{21.2}{10^6} = 21.2\times10^{-6}F$$

$$\Delta C = C(\frac{\Delta\bar{\tau}}{\bar{\tau}} + \frac{\Delta R}{R})$$

$$\Delta C = C(\frac{\Delta\bar{\tau}}{\bar{\tau}} + \frac{\Delta R}{R}) = 21.2\times10^{-6}(\frac{0.33}{21.2} + \frac{5\times10^4}{100\times10^4})$$

$$= 21.2\times10^{-6}(0.016 + 0.05) = 1.39\times10^{-6}F$$

$$C = 21\pm1\mu F$$

Results & conclusion:

$$C = 21\pm1\mu F$$

1.The range of measured value of C is from 20 to 22 μF and so the manufacturer's stated value of C written on the capacitor lies within the range.

2.Increasing the resistance R affects both the charging and discharging processes by increasing the time needed for charging and discharging. Notice that $\tau = RC$

3.The unit of $\tau = RC$ is seconds :

$$[\tau] = [RC] = \Omega\times F = \frac{Votls}{A}\times\frac{colom}{Volts} = \frac{colom}{colom/\sec} = \sec$$

Notice that:

* $R = \frac{V}{I}$
* $C = \frac{Q}{V}$
* $I = \frac{\partial Q}{\partial t}$

4.A systematic error I expect in this experiment is that when my partner finished his periodic counting and I read the value of the voltage it was not the certain value , as sometimes the value changed after its ,relatively, long steady just after he began a new counting. Much more digits are to be taken for the voltage.