22 ELECTRIC POTENTIAL

EXERCISES

Section 22.1 Electric Potential Difference

15. **INTERPRET** For this problem, we are to find the work required to move 50 μC of charge through a potential difference of 12 V. Recall that the SI units of potential difference in volts is in J/C, so it represents the potential energy difference per unit charge.

DEVELOP The potential difference in volts is the negative of the work required per unit charge in moving a positive charge from point *A* to point *B*. Mathematically, this may be expressed as

$$W_{AB} = -qV_{AB}$$

(see also Equation 22.1a). For this problem, the potential difference is −12 V because we are moving against the potential difference (or against the electric field, so from high potential to low potential), so we can solve for W_{AB}.

EVALUATE Inserting the given values gives

$$W = -q\Delta V = (50\ \mu\text{C})(-12\ \text{V}) = 600\ \mu\text{J}$$

ASSESS Work is required to increase the potential energy of a charge.

16. **INTERPRET** This problem deals with the energy gained by an electron as it moves through a potential difference ΔV.

DEVELOP We assume that the electron is initially at rest. When released from the negative plate, it moves toward the positive plate, and the kinetic energy gained is $W = -\Delta U = -q\Delta V$.

EVALUATE As the electron moves from the negative side to the positive side (i.e., against the direction of the electric field), the *kinetic* energy it gains is

$$\Delta K = -(-e)\ \Delta V = 120\ \text{eV} = (1.60\times10^{-19}\ \text{C})(120\ \text{V}) = 1.9\times10^{-17}\ \text{J}$$

ASSESS Moving a negative charge through a positive potential difference is like going downhill; potential energy decreases. However, the kinetic energy of the electron is increased.

17. **INTERPRET** This problem involves calculating a potential difference per unit charge between two points given the work required to move a given charge between the two points.

DEVELOP The work done by an external agent against the electric field is the potential energy change, $\Delta U_{AB} = 45\ \text{J} = q\ \Delta V_{AB}$.

EVALUATE Solving for ΔV_{AB} and inserting the given values gives

$$\Delta V_{AB} = (45\ \text{J})/(15\ \text{mC}) = 3.0\ \text{kV}.$$

ASSESS Note that the work done *by* the electric field is the negative of the potential difference between two points.

18. **INTERPRET** This problem is an exercise in converting units.

DEVELOP By definition, 1 V = 1 J/C and 1J =1 N·m. Use these relationships to find the relationship between 1 V/m and 1 J/C.

EVALUATE Combining the two expressions gives

$$1\text{ V} = \frac{1\text{ J}}{1\text{ C}} = \frac{1\text{ N}\cdot\text{m}}{1\text{ C}}$$

$$\frac{1\text{ V}}{1\text{ m}} = \frac{1\text{ N}}{1\text{ C}}$$

Thus, 1 V/m = 1 N/C.

ASSESS These are the units for the electric field strength.

19. **INTERPRET** This problem is an exercise in calculating the potential difference between two points, given their separation and the magnitude of the uniform electric field between the two points.

DEVELOP Apply Equation 22.1b, which applies for a uniform electric field.

$$\Delta V = -\vec{E}\cdot\vec{r}$$

Because we are moving parallel to the electric field, the dot product gives cos(0°) = 1. Also, because we are only interested in the magnitude of the potential difference, we can omit the negative sign.

EVALUATE The magnitude of the potential difference is

$$|\Delta V| = Er = (650\text{ N/C})(1.4\text{ m}) = 910\text{ V}$$

ASSESS If this involves a charge moving with (against) the electric field, the potential will decrease (increase).

20. **INTERPRET** This problem is about the work done by a 9.0-V battery to move the given (positive) charge from the positive terminal to the negative terminal. The battery is thus moving a positive charge in the direction of the electric field, so the potential difference is negative.

DEVELOP The work done by the battery is equal to the kinetic energy gained by the charge, and is given by $W_{AB} = q\Delta V_{AB}$ (see discussion in Section 22.1). Because we are moving a positive charge in the direction of the electric field, the potential difference will be negative (see Equation 22.1a), so $\Delta V_{AB} = -9.0$ V.

EVALUATE Substituting the values given, we have

$$W_{AB} = -q\Delta V_{AB} = -(3.1\text{ C})(-9.0\text{V}) = 28\text{ J}$$

ASSESS The charges gain kinetic energy as it moves toward the negative plate (in the direction of the electric field). The battery is needed to maintain the potential difference between the plates.

21. **INTERPRET** We are to find the energy gained by the three given charged particles as they move through a 100-V potential difference.

DEVELOP The energy gained is $q\Delta V$ (see Example 22.1). For the proton, alpha particle, and singly ionized He atom, $q = e$, $2e$, e, respectively.

EVALUATE For proton and the ionized He atom, the energy gained is

$$\Delta K = q\Delta V = e(100\text{ V}) = 100\text{ eV} = (1.60\times10^{-19}\text{ C})(100\text{ V}) = 1.6\times10^{-17}\text{ J}$$

For the alpha particle, the charge is twice that of the other particles, so the energy gained is twice: 3.2×10^{-17} J.

ASSESS Note that the velocity of each particle is different at the output because each has a different mass.

22. **INTERPRET** The problem involves the work done on an ion in moving across the potential difference of a cell membrane.

DEVELOP The work done on the ion in the electric field of the cell membrane is equal to the increase in the internal energy: $W_{AB} = \Delta U_{AB} = q\Delta V_{AB}$, where we have used the definition of the electric potential difference from Equation 22.1a.

EVALUATE A singly-charged potassium ions has charge $+e$, so the work needed to move it through the cell membrane is

$$W_{AB} = e\Delta V_{AB} = (1.6\times10^{-19}\text{C})(80\text{ mV}) = 1.3\times10^{-20}\text{J}$$

ASSESS This is a small amount of work, since we're dealing with a single fundamental charge. Recall that moving a positive charge across a potential difference is like moving it up a an electric potential "hill."

Section 22.2 Calculating Potential Difference

23. **INTERPRET** In this problem, we are given a uniform electric field and asked to calculate the potential difference between two points.

DEVELOP For a uniform field, the potential difference between two points *a* and *b* is given by Equation 22.1b:

$$\Delta V_{AB} = V_B - V_A = -\vec{E}\cdot\Delta\vec{r}$$

where $\Delta\vec{r}$ is a vector from *a* to *b*.

EVALUATE With $\Delta\vec{r} = \vec{r}_B - \vec{r}_A = y\hat{j}$, we obtain

$$V(y) - V(0) = V(y) = -\vec{E}\cdot\Delta\vec{r} = -\left(E_0\hat{j}\right)\cdot\left(y\hat{j}\right) = -E_0 y$$

$$V(y) = -E_0 y$$

ASSESS The electric potential decreases in the direction of the electric field. In other words, electric field lines always point in the direction of decreasing potential.

24. **INTERPRET** This problem involves finding the potential due to a point charge.

DEVELOP The potential of the proton, at the position of the electron (both of which may be regarded as point-charge atomic constituents) is (Equation 22.3) $V = ke/a_0$ where a_0 is the Bohr radius.

EVALUATE Inserting the values given, we find

$$V = \frac{\left(9.0\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)}{5.29\times10^{-11}\ \text{m}} = 27\ \text{V}$$

ASSESS The energy of an electron in a classical, circular orbit about a stationary proton is one half its potential energy, or $\frac{1}{2}U = \frac{1}{2}(-e)V = -13.6\ \text{eV}.$ The excellent agreement with the ionization energy of hydrogen was one of the successes of the Bohr model.

25. **INTERPRET** We are asked to find the charge on a sphere, given the potential at its surface. Because the charge distribution is spherically symmetric, we will use the equation for the potential of a point charge.

DEVELOP Equation 22.3 gives the potential for a point charge as

$$V(r) = \frac{kq}{r}$$

Since any spherically symmetric charge distribution looks like a point charge from outside the distribution, we can solve this for *q*. The potential at the surface of the sphere is $V = 4.8$ kV and the radius is $r = 0.10$ m.

EVALUATE Inserting the given quantities yields

$$q = \frac{Vr}{k} = \frac{\left(4.8\ \text{N}\cdot\text{m/C}\right)\left(0.10\ \text{m}\right)}{\left(9.0\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)} = 53\ \text{nC}$$

ASSESS The key is to recognize that spherically symmetric charge distributions look like point charges from the outside. This is the same as for gravitational potentials. Note also that the units in the result cancel to give coulombs.

26. **INTERPRET** This problem resembles the previous one in that it deals with the potential at the surface of a sphere. We are to find the maximum potential that a 5.0-cm diameter sphere can withstand given the 3-MV/m maximum electric field of air.

DEVELOP For an isolated metal sphere, the electric field at the surface is that of a point charge at a distance *R*:

$$kQ/R^2 = V/R$$

Thus, $V/R < 3$ MV/m gives the condition for *V* for a 5.0-cm sphere.

EVALUATE Inserting the numbers, we find $V < (3\ \text{MV/m})(0.025\ \text{m}) = 75\ \text{kV}$.

ASSESS This is quite a high voltage (compare to the 120 or 240 V that is typical of household circuits).

27. INTERPRET This problem is about the electric potential of a spherically symmetric charge distribution. We are to find the potential at the surface of a charged conducting sphere and the kinetic energy (or speed) of a proton accelerated from the surface to infinity by the sphere's potential.

DEVELOP Since the electric field outside the spherical charge distribution is the same as that of a point charge, the electric potential outside the metal sphere $(r \geq R)$ is given by Equation 22.3:

$$V(r) = \frac{kQ}{r}$$

Note that we have taken the zero of the potential to be at infinity.

EVALUATE **(a)** An isolated metal sphere has a uniform surface charge density, so the potential at its surface is

$$V(R) = \frac{kQ}{R} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.86\ \mu\text{C})}{0.035\ \text{m}/2} = 440\ \text{kV}$$

(b) The work done by the repulsive electrostatic field (the negative of the change in the proton's potential energy) equals the proton's kinetic energy at infinity:

$$W_{AB} = -qV_{AB} = -e\left[V_\infty - V(R)\right] = eV(R) = \frac{1}{2}mv^2$$

Thus, the speed of the proton far from the sphere is

$$v = \sqrt{\frac{2eV(R)}{m}} = \sqrt{\frac{2(1.60\times10^{-19}\ \text{C})(442\ \text{kV})}{1.67\times10^{-27}\ \text{kg}}} = 9.2\times10^6\ \text{m/s}$$

ASSESS As the proton moves away from the metal sphere, its potential energy decreases. However, by energy conservation, its kinetic energy increases. Also, notice that the result of part (a) is given to two significant figures, but that three significant figures are used when we insert that result into part (b) because it serves as an intermediate result for part (b).

Section 22.3 Potential Difference and the Electric Field

28. INTERPRET We are given the potential difference between two plates a given distance apart and are required to find the strength of the electric field between the plates.

DEVELOP Assuming the dimensions of the plate are much, much greater than their 2.5-cm separation, we can assume the electric field between this is uniform (see Example 21.6). For a uniform field, Equation 22.9 can be written as $\Delta V = -E\Delta x$, where the x axis is in the direction of the field (V decreases in the direction of the field).

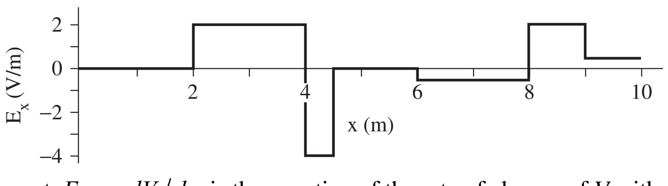
EVALUATE Thus, $E = |\Delta V/\Delta x| = (1\ \text{V})/(0.025\ \text{m}) = 40\ \text{V/m}$.

ASSESS This result may be found by a simple dimensional analysis. The electric field can be expressed in units of V/m, so given a voltage and a distance, the electric field is simply the voltage change per unit length.

29. INTERPRET This problem involves calculating the electric field from the electric potential (or voltage).

DEVELOP Given electric potential $V(x)$, the x component of the electric field may be obtained as $E_x = -dV/dx$ (see Equation 22.9). Use this equation to estimate E_x for the seven straight-line segments shown in Fig. 22.20.

EVALUATE Using the equation above, we find $E_x = 0$ for $x = 0$ to 2 m. Similarly, for x = 2 to 4 m, $E_x = -(-2\ \text{V} - 2\ \text{V})/(4\ \text{m} - 2\ \text{m}) = 2\ \text{V/m}$. The field strength in other regions can be calculated in a similar manner. The result is sketched below.

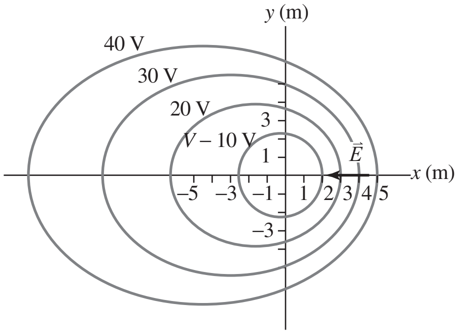


ASSESS The field component $E_x = -dV/dx$ is the negative of the rate of change of V with respect to x. The negative sign means that if we move in the direction of increasing potential, then we're moving against the electric field.

30. **INTERPRET** This problem requires us to interpret an equipotential map, which shows lines of constant electric potential. From this plot, we are to find the region where the electric field is the strongest and its magnitude and direction.

DEVELOP As explained in Section 22.3 (see discussion accompanying Figure 22.15), the electric field is everywhere perpendicular to the lines of equipotential and is strongest where the equipotential lines are most closely spaced. The electric field direction is always from regions of high potential to regions of low potential. The magnitude of the electric field may be found as per Problem 22.29: by dividing the potential difference by the distance between equipotential lines.

EVALUATE See figure below. **(a)** The equipotentials in Fig. 22.21 are most closely spaced along the x axis between $x = 2$ m and $x = 5$ m. **(b)** The potential decreases in the direction of the electric field which, for the region $2\text{ m} \le x \le 5\text{ m}$, is in the negative x direction. **(c)** The potential drops by 10 V/m, which is the field strength in this region.



ASSESS Reading an equipotential plot is analogous to reading a topological map. Where the equipotential lines are most closely spaced is the region of most rapidly changing electric potential, just like a topological map where the most closely spaced lines indicate the regions of most rapidly changing gravitational potential (i.e., the steepest inclines).

31. **INTERPRET** This problem is about calculating electric field given the electric potential.
DEVELOP Given the electric potential V, the corresponding electric field is (see Equation 22.9)

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

Thus, taking the partial derivatives of V allows us to get the field components.
EVALUATE **(a)** Direct substitution gives the voltage at $(x, y, z) = (1\text{ m}, 1\text{ m}, 1\text{ m})$:

$$V(x,y,z) = 2xy - 3zx + 5y^2 = (2\text{ Vm}^{-2})(1\text{ m})(1\text{ m}) - (3\text{ Vm}^{-2})(1\text{ m})(1\text{ m}) + (5\text{ Vm}^{-2})(1\text{ m})^2 = 4\text{ V}$$

(b) Use of Equation 22.9 gives the components of the electric field:

$$E_x = -\frac{\partial V}{\partial x} = -2y + 3z$$

$$E_y = -\frac{\partial V}{\partial y} = -2x - 10y$$

$$E_z = -\frac{\partial V}{\partial z} = 3x$$

At $(x, y, z) = (1\text{ m}, 1\text{ m}, 1\text{ m})$, we obtain $E_x = 1$ V/m, $E_y = -12$ V/m and $E_z = 3$ V/m.
ASSESS Electric field is strong in the region where the potential changes rapidly. At (1 m, 1 m, 1 m), the potential changes most rapidly in the direction of the electric field

$$\vec{E} = (\hat{i} - 12\hat{j} + 3\hat{k})\text{V/m}$$

Section 22.4 Charged Conductors

32. **INTERPRET** Given a sphere's size, we are to find the maximum potential (i.e., voltage) at its surface before dielectric breakdown of air occurs. We can use the result of Problem 22.26 to address this problem.

DEVELOP As per Problem 22.26, $V/R < 3$ MV/m gives the maximum limit for V.

EVALUATE (a) Inserting the given quantities gives

$$V_{\max} = RE_{\max} = (2.30\text{ m})(3.0\text{ MV/m}) = 6.9\text{ MV}$$

(b) Dielectric breakdown in the air occurs if the field at the surface $E = \sigma/\varepsilon_0$ exceeds 3×10^6 V/m. Therefore, the charge (for a uniformly charged sphere) must not be greater than

$$q = 4\pi R^2\sigma = 4\pi\epsilon_0 ER^2 = 4\pi\left[8.85\times10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2)\right](3.0\times10^6\text{ V/m})(2.30\text{ m})^2 = 1.8\text{ mC}$$

ASSESS This gives an appreciation of the amount of charge represented by 1 C. If we have some 10^{-3} C on a conducting sphere 2.3 m in diameter, we risk to engender dielectric breakdown of the air!

33. **INTERPRET** This problem is about finding the minimum potential that leads to a dielectric breakdown in air.

DEVELOP We shall treat the field from the central electrode as if it were from an isolated sphere, for which Equation 20.3 gives the electric field to be $E = kq/R^2$ and Equation 22.3 gives the potential to be $V = kq/R$. Combining these two expressions gives $V = RE$.

EVALUATE Breakdown of air occurs at a field strength of $E = 3\times10^6$ V/m. Therefore, dielectric breakdown in air would occur for potentials exceeding

$$V = RE = (1.0\times10^{-3}\text{ m})(3\times10^6\text{ V/m}) = 3\text{ kV}$$

ASSESS The result means that if we attempt to raise the potential of the electrode in air above 3 kV, then the surrounding air would become ionized and conductive; the extra added charge would leak into the air, resulting in plug sparks.

34. **INTERPRET** This problem requires us to find the potential and electric field strength at the surface of two isolated conducting spheres with the given size and charge. Note that the charge distribution is spherically symmetric.

DEVELOP Outside a spherically symmetric charge distribution, it may be considered as a point charge with all the charge concentrated at that point. Thus, we can apply Equation 22.3 $V = kq/R$ to find the potential at the surface of the two spheres. In addition, we can use Equation 20.3, $E = kq/R^2$, which gives the electric field strength of a point charge, to find the electric field strength at the surface of the two spheres.

EVALUATE (a) Applying Equation 22.3 to each sphere, we see that the

$$V_1 = \frac{kQ}{R}$$

$$V_2 = \frac{k(3Q)}{3R} = \frac{kQ}{R} = V_1$$

so the potential at the surface of each sphere is the same.

(b) Applying Equation 20.3 to each sphere gives

$$E_1 = \frac{kQ}{R^2}$$

$$E_2 = \frac{k(3Q)}{(3R)^2} = \frac{kQ}{3R^2} = \frac{E_1}{3}$$

so the electric field strength at the surface of the small sphere is 3 times that at the surface of the large sphere.

ASSESS At the surface of a charged, conducting sphere, you can also use Equation 21.8, $E = \sigma/\varepsilon_0$, but this gives the same result because $k = 1/(4\pi\varepsilon_0)$ and $\sigma = q/(4\pi R^2)$.

PROBLEMS

35. **INTERPRET** This problem is about finding the electric field strength, given the potential difference between two points a given distance apart. We are also given the orientation of the electric field with respect to the line joining the two points.

DEVELOP Since the field $\vec{E}$ is uniform, Equation 22.1b, $\Delta V_{AB} = -\vec{E}\cdot\Delta\vec{r}$, can be used to relate $\vec{E}$ to the potential difference ΔV_{AB}. Since the path *AB* is parallel to $\vec{E}$, the angle between $\vec{E}$ and $\Delta\vec{r}$ is 0°. Because cos(0°) = 1 the dot product reduces to

$$\Delta V_{AB} = E\Delta r$$

where *E* is the field strength, and Δr is the separation between points *A* and *B*.

EVALUATE The field strength is

$$E = \frac{\Delta V_{AB}}{\Delta r} = \frac{840\text{ V}}{0.15\text{ m}} = 5.6\text{ kV/m}$$

ASSESS Since $dV = -\vec{E}\cdot d\vec{r}$ the potential always decreases in the direction of the electric field. Note that the angle between $\vec{E}$ and $\Delta\vec{r}$ is 180° if the two are antiparallel.

36. **INTERPRET** This problem is about finding the potential difference between two points, given the electric field strength.

DEVELOP Since the field $\vec{E}$ is uniform, Equation 22.1b, $\Delta V = -\vec{E}\cdot\Delta\vec{r}$, can be used to relate $\vec{E}$ to the potential difference ΔV across the membrane. For a uniform electric field normal to the membrane, we have

$$|\Delta V| = E\Delta r$$

where *E* is the field strength, and Δr is the membrane thickness.

EVALUATE Using the equation above, the potential difference is

$$|\Delta V| = E\Delta r = (8.0\text{ MV/m})(10\times10^{-9}\text{ m}) = 80\text{ mV}$$

ASSESS Since $dV = -\vec{E}\cdot d\vec{r}$ the potential always decreases in the direction of the electric field. Note that we have no information in this problem regarding the direction of the electric field, so we cannot put a sign on the potential difference.

37. **INTERPRET** This problem involves finding the potential difference between two points (i.e., the terminals of the battery) given the work done on each elementary charge that moves between these points.

DEVELOP From the discussion in Section 22.1, we know that the work done by the electric field on each charge between two points is the potential difference between the same two points. Thus, $|W/q| = \Delta V$.

EVALUATE Inserting the given quantities gives

$$\frac{W}{q} = \frac{7.2\times10^{-19}\text{ J}}{1.6\times10^{-19}\text{ C}} = 4.5\text{ V}$$

ASSESS The energy imparted per electron is 4.5 eV.

38. **INTERPRET** This problem requires us to find the charge of a particle that gains the given amount of energy due to its passage through the given potential difference.

DEVELOP The work done on a charge that traverses a potential difference is

$$W_{AB} = -q\Delta V_{AB}$$

From Equation 22.1b, we see that the potential difference always decreases in the direction of the electric field, so

$$\Delta V_{AB} = V_B - V_A < 0$$
$$\Delta V_{AB} = -2500\text{ V}$$

EVALUATE Solving the expression for potential difference for the charge *q* gives

$$q = -\frac{W_{AB}}{\Delta V_{AB}} = -\frac{1.6\times10^{-15}\text{ J}}{-2500\text{ V}} = -\frac{1.0\times10^{4}\text{ eV}}{-2500\text{ V}} = 4.0e$$

ASSESS The ion is positively charged.

39. **INTERPRET** This problem involves finding the potential difference between two conducting plates separated by a distance d and having opposite charge densities.

DEVELOP We first calculate the electric field between the plates. Using the result obtained in Example 21.6 for one sheet of charge, and applying the superposition principle, the electric field strength between the plates is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \sigma/(2\epsilon_0)\hat{i} + [-\sigma/(2\epsilon_0)](-\hat{i}) = (\sigma/\epsilon_0)\hat{i}$$

where $\hat{i}$ is directed from the positive plate to the negative plate. Once E is known, we can use Equation 22.1b to calculate V.

EVALUATE Equation 22.1b gives

$$V = V_+ - V_- = -\vec{E}\cdot\Delta\vec{r} = -\left(\frac{\sigma}{\epsilon_0}\right)(-d) = \frac{\sigma d}{\epsilon_0}$$

ASSESS The displacement from the negative to the positive plate is opposite to the field direction. In other words, the potential always decreases in the direction of the electric field.

40. **INTERPRET** This problem involves the work-energy theorem (Equation 6.14), which we can use to find the potential energy difference given the change in kinetic energy. Once we know the potential energy difference, we can divide it by the charge to find the potential difference.

DEVELOP The work-energy theorem (Equation 6.14) says that the change in an objects kinetic energy is equal to the work done on the object, or $\Delta K_{AB} = W_{AB}$. We also know from the discussion preceding Equation 22.1a that the work done on the charge by the electric field is the negative of the potential energy change, or

$$\Delta U_{AB} = -W_{AB} = -\Delta K_{AB}$$

For this problem, the kinetic energy of the electron decreases, so $\Delta K_{AB} = -m_e v^2/2$.

EVALUATE Using Equation 22.1b, $\Delta V_{AB} = \Delta U_{AB}/q$, we find

$$\Delta V_{AB} = -\frac{\Delta K_{AB}}{q} = -\frac{-m_e v_A^2}{-2e} = \frac{(9.11\times10^{-31}\text{ kg})(6.5\times10^6\text{ m/s})^2}{2(-1.6\times10^{-19}\text{ C})} = -120\text{ V}$$

ASSESS There potential difference is negative because we are dealing with an electron, which has negative charge. In other words, to stop an electron, a negative potential difference must be applied.

41. **INTERPRET** The problem asks for the charge of the particle which has been accelerated through a potential difference. We can find the magnitude of this potential difference given the information of the speed acquired upon traversing the potential difference by the mass with the given charge.

DEVELOP The speed acquired by a charge q, starting from rest at point A and moving through a potential difference of V can be found using the work-energy theorem (see Problem 22.40). The result is

$$\Delta K_{AB} = q\Delta V_{AB} \quad \Rightarrow \quad \frac{1}{2}mv^2 = q\,\Delta V_{AB} \quad \Rightarrow \quad v = \sqrt{\frac{2q\Delta V_{AB}}{m}}$$

This is the work-energy theorem for the electric force. A positive charge is accelerated in the direction of decreasing potential (i.e., increasing electric field). If we have two masses moving through the same potential difference, the ratio of their speeds would be

$$\frac{v_2}{v_1} = \sqrt{\frac{2q_2V/m_2}{2q_1V/m_1}} = \sqrt{\frac{q_2}{q_1}\frac{m_1}{m_2}}$$

EVALUATE If the second object acquires twice the speed of the first object $(v_2/v_1 = 2)$, moving through the same potential difference we find its charge from the equation above to be

$$q_2 = \left(\frac{m_2}{m_1}\right)\left(\frac{v_2}{v_1}\right)^2 q_1 = \left(\frac{2\text{g}}{5\text{g}}\right)(2)^2(3.8\ \mu\text{C}) = 6.1\ \mu\text{C}$$

ASSESS The speed of the particle moving through a potential difference is proportional to the square root of its charge, and inversely proportional to the square root of its mass.

42. **INTERPRET** This problem involves finding the point charge that gives rise to the given potential difference between two points on a radial line. We are also to find the distance between the point charge and on of the potential points.

DEVELOP Equation 22.3 gives the potential due to a point charge with respect to the potential an infinite distance from the point charge: $V = kQ/r$. We are given the potential at points A and B so dividing the two gives

$$\left.\begin{aligned} V_A &= \frac{kQ}{r_A} \\ V_B &= \frac{kQ}{r_B} \end{aligned}\right\} \frac{V_A}{V_B} = \frac{r_B}{r_A}$$

Given this and that $r_B - r_A = 20$ cm, we can solve for r_A and, knowing r_A, we can find Q.

EVALUATE Solving for r_A gives

$$r_A = r_B \frac{V_B}{V_A} = \left(r_A + 20\text{ cm}\right)\frac{V_B}{V_A}$$

$$r_A = \frac{(20\text{ cm})V_B}{V_A - V_B} = \frac{(20\text{ cm})(130\text{ V})}{280\text{ V} - 130\text{ V}} = 17\text{ cm}$$

The charge Q is

$$Q = \frac{V_A r_A}{k} = \frac{(280\text{ V})(17.3\text{ cm})}{\left(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)} = 5.4\text{ nC}$$

ASSESS The charge is positive, so the electric field points away from the charge and the potential decreases in the direction of the electric field, in agreement with the data.

43. **INTERPRET** The positively charged proton is attracted to the negatively charged sphere via Coulomb interaction. Work must be done to pull the proton away from the sphere.

DEVELOP From the work-energy theorem (Equation 6.14), the work done by the electric field when a proton escapes from the surface to an infinite distance, equals the change in kinetic energy, or

$$W_{\text{surf},\infty} = -e\left(\overbrace{V_\infty}^{=0} - V_{\text{surf}}\right) = eV_{\text{surf}} = \overbrace{K_\infty}^{=0} - K_{\text{surf}} = -\frac{1}{2}mv_{\text{surf}}^2$$

where we have assumed zero kinetic energy for the proton at infinity and that the sphere is stationary.

EVALUATE For a uniformly charged sphere with a total charge $-Q$, $V_{\text{surf}} = -kQ/R$ (see Equation 22.3). Inserting this into the expression above and solving for v_{surf} gives

$$v_{\text{surf}} = \sqrt{\frac{-2eV_{\text{surf}}}{m}} = \sqrt{\frac{2keQ}{mR}}$$

ASSESS The escape speed of a proton from the electric field of the charged sphere in this problem is analogous to the escape speed of a rocket from the Earth's gravitational field.

44. **INTERPRET** The cyclotron accelerates particles using a potential difference. We want to know how many times a proton must pass through this difference to achieve the desired energy.

DEVELOP In each pass, each proton gains kinetic energy: $\Delta U_{AB} = e\Delta V_{AB}$.

EVALUATE (a) The number of passes needed to reach $E_\text{f} = 1.2\times10^{-11}\text{ J}$ is

$$N = \frac{E_\text{f}}{e\Delta V_{AB}} = \frac{1.2\times10^{-11}\text{ J}}{\left(1.6\times10^{-19}\text{ C}\right)(15\text{ kV})} = 5000$$

(b) In terms of electronvolts, the final energy of the protons is

$$E_\text{f} = 1.2\times10^{-11}\text{ J}\left[\frac{1\text{ eV}}{1.6\times10^{-19}\text{ J}}\right] = 75\text{ MeV}$$

ASSESS The units in part (a) work out because $1\text{ V} = 1\text{ J/C}$, but for particles it's much simpler to use electronvolts. The proton gains 15 keV for each pass through the cyclotron, and the number of passes is simply: $N = 75\text{ MeV}/15\text{ keV} = 5000$.

45. **INTERPRET** For this problem, we are to find the potential at the center of a hollow spherical shell that carries a uniform charge density on its surface.

DEVELOP From Gauss's law, we know that the electric field inside the shell is zero, because there is no charge in the shell. Because the electric field is constant, we can apply Equation 22.1b, which shows that the change in the potential between any two points within the sphere must be zero. Consider the boundary condition for the electric potential (at the surface of the sphere) to find the electric potential everywhere in the sphere.

EVALUATE At the surface of the sphere, the electric potential must be (see Equation 22.3)

$$V = \frac{kQ}{R}$$

Thus, this must be the potential everywhere inside the sphere, since we have argued above that the potential inside the sphere must be constant.

ASSESS The potential at the surface of the sphere is measured with respect to infinity, which is to say a distance $r \gg R$ from the sphere.

46. **INTERPRET** The problem is about the potential difference between the center of a charged sphere and a point on its surface.

DEVELOP From Equation 22.1a, we see that the potential difference from point A to point B is given by

$$\Delta V_{AB} = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{r}$$

As shown in Example 21.1, the electric field inside a uniformly charged sphere is $E = kQr/R^3$ and is radially symmetric.

EVALUATE The integration above gives

$$V(R) - V(0) = -\int_0^R \left(\frac{kQr}{R^3}\right) dr = -\frac{kQr^2}{2R^3}\bigg|_0^R = -\frac{kQ}{2R}$$

ASSESS The potential is higher at the center if Q is positive.

47. **INTERPRET** For this problem, we are given the electric field as a function of position, and we are to find the electric potential as a function of position. We are also given the electric potential at a given point, so we will define our electric potential with respect to this point.

DEVELOP Apply Equation 22.1a,

$$\Delta V_{AB} = -\int_A^B \vec{E} \cdot d\vec{r}$$

where $\vec{E} = ax(\hat{i})$ and $d\vec{r} = dx(\hat{i})$. Furthermore, we take point A to be x = 0, so $V_A = V(x = 0) = 0$, and point B to be an arbitrary point x.

EVALUATE Evaluating the integral gives

$$\Delta V_{AB}(x) = -\int_A^B ax' dx' (\hat{i} \cdot \hat{i}) = -\int_0^x ax' dx' = -\frac{a}{2}x^2$$

so

$$V(x) = V(0) + \Delta V_{AB}(x) = -\frac{a}{2}x^2$$

ASSESS This potential increases quadratically with position, whereas the electric field is linear in position.

48. **INTERPRET** You need to check if the charge on a coaxial cable will create an unsafe potential difference.

DEVELOP You can assume the cable is long compared to its diameter. Due to the cylindrical symmetry, the electric field should be radial and therefore amenable to Gauss's law. In Equation 22.4, this fact was used to find the potential difference around a long wire:

$$\Delta V_{AB} = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_A}{r_B}\right)$$

where λ was the charge density per unit length and r_A was the radius of the wire. In the case of a coaxial cable, the wire is surrounded by a thin conductor in the shape of a cylindrical shell. By Gauss's law, this outer shield has no effect on the field in between the conductors, so we can use Equation 22.4.

EVALUATE For the given charge density, the voltage difference between the two conductors is

$$\Delta V_{AB} = \left|\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_A}{r_B}\right)\right| = \frac{62\text{ nC/m}}{2\pi\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}\left|\ln\left(\frac{2.0\text{ mm}}{1.6\text{ cm}}\right)\right| = 2.3\text{ kV}$$

This is 300 V over the safe limit, so the cable won't work.

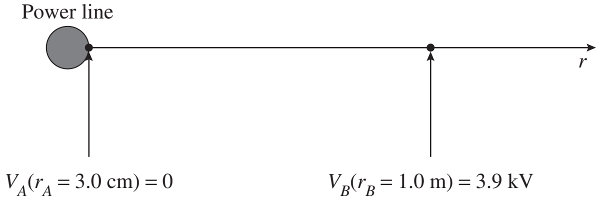
ASSESS We took the absolute value of the potential difference because we were only concerned with the magnitude (i.e. 2.3 kV is just as unsafe as –2.3kV). It is somewhat striking that the answer doesn't depend on the charge lying on the outer conductor. But this is only true when the cable maintains the cylindrical symmetry. If the cable were bent, for instance, this answer would no longer be valid.

49. **INTERPRET** This problem involves finding the charge density on a power line given the potential difference over a given distance away. To use Gauss's law for geometries with line symmetry, we will assume that the power line is much, much longer than 1.0 m.

DEVELOP The electric potential around an object with line symmetry (such as our power line) is derived in Example 22.4. The result is

$$\Delta V_{AB} = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_A}{r_B}\right)$$

From the problem statement, make a sketch showing the location of the given voltages (see figure below). From this sketch, we see that r_A = 3.0 cm, r_B = 1.0 m, and $\Delta V_{AB} = V_B - V_A$ = +3.9 kV for this problem.



EVALUATE Solving the expression above for the line charge density λ gives

$$\lambda = \frac{2\pi\epsilon_0\Delta V_{AB}}{\ln\left(r_A/r_B\right)} = \frac{2\pi\left(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(3.9\text{ kV}\right)}{\ln\left(0.030\text{ m}/1.0\text{ m}\right)} = -52\text{ nC/m}$$

ASSESS Thus, the wire carries excess negative charge.

50. **INTERPRET** This problem is about the electric potential at a point due to a system of charges. The principle of superposition will be useful for this problem.

DEVELOP The electric potential at a point P due to a collection of charges is given by the superposition of the potential from each point charge (i.e., Equation 22.5):

$$V_P = \sum_i \frac{kq_i}{r_i}$$

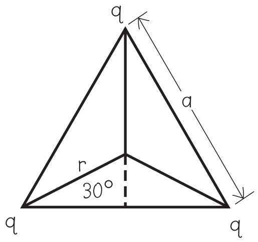
Use this formula to compute the electric potential at the center of a triangle.

EVALUATE The center is an equidistance r from each vertex, with

$$r = \frac{a}{2\cos\left(30°\right)} = \frac{a}{\sqrt{3}}$$

Since each charge contributes equally to the potential, the potential at the center is

$$V = \frac{3kq}{r} = \frac{3\sqrt{3}kq}{a}$$



ASSESS Electric potential is a scalar, so there is no need to consider angles, vector components, or unit vectors.

51. **INTERPRET** This problem involves using the superposition principle to find two points on a line joining the two given charges where the electric potential is zero.

DEVELOP Using the superposition principle in the form of Equation 22.5, the potential at the x-axis is

$$V(x) = \sum_i \frac{kq_i}{x_i} = \frac{kQ}{|x|} + \frac{k(-3Q)}{|x-a|}$$

Set this expression equal to zero and solve for the position x.

EVALUATE The potential is zero when $3|x| = |x-a|$. For $x < 0$, this implies $-3x = a - x$, or $x = -a/2$. For $0 < x < a$, the condition is $3x = a - x$, or $x = a/4$. For $x > a$, there are no solutions.

ASSESS The same results follow from the quadratic $8x^2 + 2ax - a^2 = 0$, which results from the square of the above condition.

52. **INTERPRET** This problem involves finding the electric potential everywhere in a plane that contains two identical point charges. This problem involves the principle of superposition.

DEVELOP The electric potential at a point P due to a collection of charges is given by Equation 22.5:

$$V_P = \sum_i \frac{kq_i}{r_i}$$

Consider a point $P(x, y)$. The distance from P to $(0, a)$ is $r_+ = \sqrt{(x-a)^2 + y^2}$. Similarly, the distance from P to $(0, -a)$ is $r_- = \sqrt{(x+a)^2 + y^2}$.

EVALUATE **(a)** Inserting r_+ and r_- into the summation gives

$$V_P = \frac{kq}{r_+} + \frac{kq}{r_-} = kq\left(\frac{1}{\sqrt{(x-a)^2 + y^2}} + \frac{1}{\sqrt{(x+a)^2 + y^2}}\right)$$

(b) If $r = \sqrt{x^2 + y^2} \gg a$, then a can be neglected relative to x or y, so

$$V_P = kq\left(\frac{1}{\sqrt{(x-a)^2 + y^2}} + \frac{1}{\sqrt{(x+a)^2 + y^2}}\right) \approx kq\left(\frac{1}{\sqrt{x^2 + y^2}} + \frac{1}{\sqrt{x^2 + y^2}}\right) = \frac{2kq}{r}$$

which is the potential of a point charge of magnitude $2q$.

ASSESS At a distance much greater than the separation of two charges q_1 and q_2, the electric potential is like that due to one single point charge $q_1 + q_2$. Note that electric potential is a scalar, so there is no need to consider angles, vector components, or unit vectors.

53. **INTERPRET** We're asked to find the electric potential around an electric dipole.

DEVELOP Equation 22.6 gives the potential from a dipole at a distance, r, much larger than the charge separation: $V(r, \theta) = kp\cos\theta/r^2$, where θ is the angle from the dipole axis.

EVALUATE (a) For $\theta = 0°$,

$$V(r,\theta) = \frac{\left(9.0\times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(2.9\ \text{nC}\cdot\text{m})\cos 0°}{(10\ \text{cm})^2} = 2.6\ \text{kV}$$

(b) For $\theta = 45^\circ$,

$$V(r,\theta) = \frac{\left(9.0\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(2.9\ \text{nC}\cdot\text{m})\cos 45^\circ}{(10\ \text{cm})^2} = 1.8\ \text{kV}$$

(c) For $\theta = 90^\circ$,

$$V(r,\theta) = \frac{\left(9.0\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(2.9\ \text{nC}\cdot\text{m})\cos 90^\circ}{(10\ \text{cm})^2} = 0$$

ASSESS The results seem reasonable. It should be made clear that these values assume the potential is zero at infinity. This is an arbitrary choice, since the only physical quantity is the potential difference between two points.

54. **INTERPRET** The problem is about finding the electric potential due to two continuous charge distributions, both of which have circular symmetry.

DEVELOP From Example 22.6, we see that the electric potential at the center of a charged ring (i.e., $x = 0$) of radius a is

$$V = \frac{kQ}{a}$$

The radius a can be found from the relation $L = 2\pi a$, where L is the length of the rod. For the second part, we can use the derivation of Example 22.6 again, the radius of the semicircle is larger because $L = \pi a$. The integration stays the same because the distance to the center of the semicircle is the same (= a) for all points on the semicircle and we integrate over the length L.

EVALUATE **(a)** The radius of the circle is $a = L/2\pi$. Therefore, the potential at the center of a uniformly charged ring is

$$V = \frac{kQ}{a} = \frac{2\pi kQ}{L} = \frac{2\pi\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(3.2\ \text{nC})}{0.20\ \text{m}} = 910\ \text{V}$$

to two significant figures.

(b) The radius is now $a' = L/\pi$, so the potential is

$$V' = \frac{1}{2}V = \frac{905\ \text{V}}{2} = 450\ \text{V}$$

to two significant figures.

ASSESS Electric potential is a scalar, so there is no need to consider angles, vector components, or unit vectors.

55. **INTERPRET** This problem involves finding the electric potential at the center of a non-uniform circular charge distribution. Because the charge is non-uniform, we will need to integrate over it to find the potential.

DEVELOP Note that the result of Example 22.6 does not depend on the ring being uniformly charged. For a point on the axis of the ring, the geometrical factors are the same, and $\int_{\text{ring}} dq = Q_{\text{tot}}$ for any arbitrary charge distribution, so

$$V = \frac{kQ_{\text{tot}}}{\sqrt{x^2 + a^2}}$$

still holds.

EVALUATE Thus, at the center (i.e., $x = 0$) of a ring of total charge $Q_{\text{tot}} = 3Q - Q = 2Q$, and radius $a = R$, the potential is $V = 2kQ/R$.

ASSESS This integration was simple, because the charge was all located the same distance from the point of interest (i.e., from the center of the circle).

56. **INTERPRET** This problem is about the electric potential due to a charged ring, which is a circularly symmetric, continuous charge distribution.

DEVELOP From Example 22.6, we see that the electric potential at the center of a charged ring of radius a is

$$V(x=0)=\frac{kQ}{a}$$

At a distance *x* along the ring axis from the center of the ring, the potential is

$$V(x)=\frac{kQ}{\sqrt{x^2+a^2}}$$

These two equations allow us to determine the radius *a* and the total charge *Q*.

EVALUATE Substituting the values given in the problem statement yields

$$V(0)=\frac{kQ}{a}=45\text{ kV},\quad \text{and}\quad V(0.15\text{ m})=\frac{kQ}{\sqrt{(0.15\text{ m})^2+a^2}}=33\text{ kV}$$

Thus, we find

$$\frac{33\text{ kV}}{45\text{ kV}}=\frac{a}{\sqrt{(0.15\text{ m})^2+a^2}}\quad\Rightarrow\quad a=(0.15\text{ m})\left(\frac{33}{45}\right)\frac{1}{\sqrt{1-(33/45)^2}}=0.16\text{ m}$$

The charge is

$$Q=\frac{V(0)a}{k}=\frac{(45\text{ kV})(0.162\text{ m})}{9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}=810\text{ nC}$$

to two significant figures.

ASSESS In this problem, we are given two conditions, which allow us to solve for two unknowns—the radius and the charge of the ring. Note that the electric potential is the greatest at the center of the ring and falls off as *x* increases. When $x \gg a$, the potential resembles that of a point charge: $V(x)\approx kQ/x$.

57. **INTERPRET** This problem involves a circularly symmetric, uniform charge distribution for which we are to find an expression for the electric potential at arbitrary points along its axis.

DEVELOP The annulus can be considered to be composed of thin rings of radius r $(a\le r\le b)$ and charge $dq=2\pi\sigma\, r dr$ (see Example 22.7 and Figs. 22.12 and 12.13). The contribution from a ring to the electric potential on the axis, a distance *x* from the center, is $dV=kdq\big/\sqrt{x^2+r^2}$ (see Example 22.6), which we can integrate from *r* = *a* to *r* = *b* to find the potential *V*.

EVALUATE The potential from the whole annulus is:

$$V(x)=\int dV=2\pi\sigma k\int_a^b\frac{r\,dr}{\sqrt{x^2+r^2}}=2\pi k\sigma\left|\sqrt{x^2+r^2}\right|_a^b=2\pi k\sigma\left(\sqrt{x^2+b^2}-\sqrt{x^2+a^2}\right)$$

ASSESS This reduces to the potential on the axis of a uniformly charged disk if $a\to0$.

58. **INTERPRET** We are to find the electric field given the electric potential as a function of position and sketch some field and equipotential lines.

DEVELOP Apply Equation 22.9 to find the electric field:

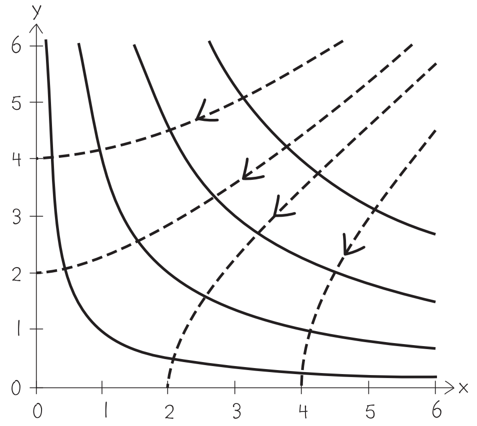
$$\vec{E}=-\left(\frac{\partial V}{\partial x}\hat{i}+\frac{\partial V}{\partial y}\hat{j}+\frac{\partial V}{\partial z}\hat{k}\right)$$

For this problem the potential *V* is a function of *x* an *y* only, so the third term in the expression above is zero. To draw the equipotential lines, first draw the electric field lines, then draw the equipotential lines so that they are everywhere perpendicular to the electric field lines.

EVALUATE **(a)** Using *V*(*x*, *y*) = *axy*, the electric field is

$$\vec{E}=-\left(\frac{\partial(axy)}{\partial x}\hat{i}+\frac{\partial(axy)}{\partial y}\hat{j}+\overbrace{\frac{\partial(axy)}{\partial z}\hat{k}}^{=0}\right)=-ay\hat{i}-ax\hat{j}$$

(**b**) See sketch below. The field lines (dashed) are perpendicular to the equipotentials (solid) in the direction of decreasing potential (arrows for $a > 0$ in this case). These equipotentials and field lines are confocal hyperbolas, proportional to xy and $\frac{1}{2}\left(x^2 - y^2\right)$ respectively, and are sketched only for x and y in the first quadrant.



ASSESS Notice that the field lines and the equipotential lines are everywhere mutually perpendicular.

59. **INTERPRET** In this problem we are to use the expression for the electric dipole potential to find the electric field at a point on the perpendicular bisector of the dipole.

DEVELOP The dipole potential is given by Equation 22.6:

$$V(r,\theta) = \frac{kp\cos\theta}{r^2}$$

Using Equation 22.9, the general expressions for the r and θ components of the electric fields are

$$E_r = -\frac{\partial V}{\partial r} = \frac{2kp\cos\theta}{r^3}$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{kp\sin\theta}{r^3}$$

EVALUATE On the bisecting plane, $\theta = 90°$, which yields $E_r = 0$ and $E_\theta = kp/r^3$, or $\vec{E} = E_\theta\hat{\theta} = \left(kp/r^3\right)\hat{\theta}$. To compare with Equation 20.6a, we take the origin at the center of the dipole, the dipole moment along the x axis ($\vec{p} = p\hat{i}$), and the y axis up in Fig. 22.10, so $\hat{\theta} = -\hat{i}\sin(90°) + \hat{j}\cos(90°) = -\hat{i}$ and $r = \sqrt{0^2 + y^2} = y$ on the bisecting plane. This leads to $\vec{E} = -\left(kp/y^3\right)\hat{i}$.

ASSESS Instead of using polar coordinates, one could first express V in terms of x and y (using $x = r\cos\theta$ and $y = r\sin\theta$):

$$V(x,y) = \frac{kpx}{\left(x^2 + y^2\right)^{3/2}}$$

and then differentiate, $E_x = -\partial V/\partial x$ and $E_y = -\partial V/\partial y$. The result is the same.

60. **INTERPRET** We are to derive an expression for the electric field on the axis of a charged ring given the potential on the axis. This geometry has circular symmetry.

DEVELOP On the axis of a uniformly charged ring (the x axis), $V = kQ\big/\sqrt{x^2 + a^2}$ (Equation 22.8). Because of the circular symmetry all contributions to the electric field cancel except for those along the x axis (because any contribution in the $\hat{j}$ or $\hat{k}$ direction will be cancelled by an equal but opposite contribution from the other side of the ring). Thus, we can apply the x-component of Equation 22.9 to find the electric field.

EVALUATE The electric field on the axis of the ring is

$$\vec{E} = -\frac{dV}{dx}\hat{i} = \frac{kQx}{\left(x^2 + a^2\right)^{-3/2}}\left(\hat{i}\right)$$

which is the result of Example 20.6.

ASSESS In general, one needs to know the potential in a 3-dimensional region in order to calculate the field from its partial derivatives.

61. **INTERPRET** We are given the electric potential and asked to find the corresponding electric field.

DEVELOP We first note that the potential $V(r) = -V_0 r/R$ depends only on *r*. This implies that the electric field is spherically symmetric and points in the radial direction. The field can be calculated using Equation 22.9,

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

In spherical coordinates, this is

$$\vec{E} = -\left[\left(\frac{\partial V}{\partial r}\right)\hat{r} + \frac{1}{r\sin\theta}\left(\frac{\partial V}{\partial \phi}\right)\hat{\phi} + \frac{1}{r}\left(\frac{\partial V}{\partial \theta}\right)\hat{\theta}\right]$$

Because the potential depends only on r, the second two terms in this expression will give zero.

EVALUATE The electric field is

$$\vec{E} = -\frac{dV}{dr}\hat{r} = \frac{V_0}{R}\hat{r}$$

where $\hat{r}$ is a unit vector that points radially outward.

ASSESS The electric field is uniform, but the potential is linear in *r*. The difference of one power in *r* is because the potential is an integral of the field over distance.

62. **INTERPRET** For this problem, we are to find the potential of two isolated metal spheres carrying the given charge. After we connect them with a wire, we are to find the new potential of each, and find how much charge moved between them to reach equilibrium.

DEVELOP Because the spheres are far apart (approximately isolated), we can use Equation 22.3 $V = kq/r$ to find their potentials with respect to the same zero potential at infinity. When connected by a thin wire, the spheres reach electrostatic equilibrium and the same potential, so $V = kQ_1'/R_1 = kQ_2'/R_2$. Since the radii are equal, the charges must be equal, so $Q_1' = Q_2'$ and we can solve this given conservation of charge (i.e. $Q_1 + Q_2 = Q_1' + Q_2' = 2Q_1'$). Finding the charge that transferred is then a simple problem of finding the difference $Q_1 - Q_1'$.

EVALUATE (a) The initial electric potential on sphere 1 is

$$V_1 = \frac{kQ_1}{R_1} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(38\ \text{nC})}{0.010\ \text{m}} = 34\ \text{kV}$$

The initial electric potential on sphere 2 is

$$V_2 = \frac{kQ_2}{R_2} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-10\ \text{nC})}{0.010\ \text{m}} = -9.0\ \text{kV}$$

(b) The total charge is $38\ \text{nC} - 10\ \text{nC} = 28\ \text{nC} = Q_1' + Q_2' = 2Q_1'$ (if we assume that the wire is so thin that it has a negligible charge), so $Q_1' = Q_2' = 14$ nC. Then $V' = k(14\ \text{nC})/(0.010\ \text{m}) = 13$ kV.

(c) In this process, the first sphere loses 38 − 14 = 24 nC to the second.

ASSESS Conservation of charge proved useful in this problem, and is a concept that is used throughout physics.

63. **INTERPRET** We are given two charge-carrying conducting spheres, and we want to find the electric potential and electric field at various points.

DEVELOP Since the spheres are separated by a distance that is over an order of magnitude greater than the radii of the spheres, we can consider them to be isolated spheres. Thus, their charge distributions are essentially spherical and we can apply Equation 22.3 to find the potential.

EVALUATE **(a)** Using the result obtained in Example 22.3, at the surface of either sphere, the potential is

$$V(R) = \frac{kq}{R} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.2\times10^{-7}\ \text{C})}{0.025\ \text{m}} = 43\ \text{kV}$$

(b) From Equation 22.9 (or Equation 21.3), the electric field at the surface of each sphere is

$$E(R) = \frac{kq}{R^2} = \frac{V(R)}{R} = \frac{43.2\ \text{kV}}{0.025\ \text{m}} = 1.7\times10^6\ \text{N/C}$$

(c) Midway between the spheres, the potential from each one is the same, so we apply the principle of superposition and sum the two potentials to find

$$V_{\text{mid-pt.}} = \frac{2kq}{r} = \frac{2\left(9.0\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.2\times10^{-7}\ \text{C}\right)}{4.0\ \text{m}} = 540\ \text{V}$$

(d) Since the spheres are at the same potential, the difference is zero.

ASSESS In this problem, the two conducting spheres can be treated as being isolated because they are far apart ($r \gg R$) and the superposition principle applies. If they were brought close to each other, then the charge distribution would no longer be spherical.

64. **INTERPRET** This problem involves a pair of concentric spheres, the inner one solid and the outer hollow. Both are conducting and carry the given charge. We are to find the potential between the spheres. The principle of superposition will be of use for this problem.

DEVELOP From Problem 22.45, we learned that the potential inside a conducting spherical shell is constant and the same as the potential at the surface of the sphere, which we can find using Equation 22.3. The potential difference between the two spheres is the difference in the potential of the each sphere individually.

EVALUATE (**a**) The potential at the surface of the outer spheres is

$$V_1 = \frac{kQ_1}{R_1}$$

and the potential due to the inner sphere is

$$V_2 = \frac{kQ_2}{R_2}$$

The potential difference between the spheres is

$$\Delta V = V_2 - V_1 = k\left(\frac{Q_2}{R_2} - \frac{Q_1}{R_1}\right) = \left(9.0\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{75\ \text{nC}}{0.020\ \text{m}} - \frac{-75\ \text{nC}}{0.10\ \text{m}}\right) = 41\ \text{kV}$$

(**b**) Adding more charge to the outer shell will change V_1, but this will become the new constant potential inside the sphere to which is added the potential of the inner sphere. Thus, the potential difference will not change. Only the potential difference outside the outer sphere will change with respect to the potential at infinity.

ASSESS Adding the charge to the outer sphere raises the potential of the entire space inside it, so the potential of the inner sphere simply "floats" up on that of the outer sphere. Therefore, the potential difference does not change.

65. **INTERPRET** This problem gives the electric field of a spherically symmetric charge distribution (i.e., it only depends on *r*, not on θ or ϕ), and we are to find the difference in electric potential between the sphere center and its outer edge.

DEVELOP Apply Equation 22.1a,

$$\Delta V_{AB} = -\int \vec{E}(r)\cdot d\vec{r}$$

where $\vec{E}(r) = E_0\left(r/R\right)^2 \hat{r}$, *A* is the sphere center, and *B* is the outer surface of the sphere.

EVALUATE Evaluating the integral gives

$$\Delta V_{AB} = V_B - V_A = -\int_0^R \vec{E}(r)\hat{r}\cdot d\vec{r} = -E_0\int_0^R \left(r/R\right)^2 dr = -\frac{E_0 R}{3}$$

ASSESS The outer surface of the sphere (point *B*) is thus at a lower potential than the inner surface, which is normal because the potential decreases in the direction of the electric field, which in this case points radially outward.

66. **INTERPRET** This problem gives an electric potential function from which we are to get several pieces of information. First, we must find the point of zero potential on the *x* axis. Second, we are to find the electric field function. Finally, we are to find the positions on the *x* axis where the electric field is zero.

DEVELOP We first note that the potential can be rewritten as

$$V(x) = x(x+3)(1-x)$$

Since the potential V is independent of y and z, the electric field has only an x component, which can be computed using Equation 22.9: $E_x = -\partial V/\partial x$.

EVALUATE **(a)** From the equation above, we see that $V(x) = 0$ at $x = 0$, 1, and 3 m.

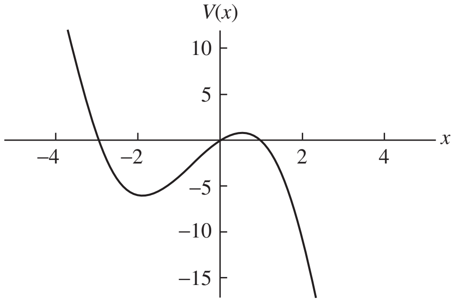
(b) Differentiating V with respect to x, we find

$$E_x = -\frac{dV}{dx} = 3x^2 + 4x - 3$$

(c) Solving the quadratic equation, the solutions for $E_x = 0$ are

$$x = \frac{-4 \pm \sqrt{16 - 4(3)(-3)}}{2(3)} = 0.535 \text{ m and } -1.87 \text{ m}$$

ASSESS The component of the electric field is the negative of the rate of change of the potential. So the points where $E_x = 0$ are where the slope of $V(x)$ vanishes (see figure below).



67. **INTERPRET** This problem concerns a conducting sphere surrounded by a concentric conducting shell. We are given the charge on each and are to find the electric potential at the sphere's surface.

DEVELOP Recall from Example 21.1 that the electric field outside a spherically symmetric charge distribution is that of a point charge with all the charge Q at that point (i.e., $\vec{E}(r) = kQ/r^2$). Use this result in Equation 22.1a to find the potential of the inner sphere with respect to the potential at infinity. This gives

$$V_{\text{sphere}} = \Delta V_{AB} = -\int_{A=R_1}^{B=\infty} \vec{E}(r)\cdot d\vec{r} = -\int_{R_1}^{R_2} \frac{kQ_1}{r^2}dr - \int_{R_2}^{\infty} \frac{k(Q_1+Q_2)}{r^2}dr$$

$$= -kQ_1\left(\frac{1}{R_2} - \frac{1}{R_1}\right) + \frac{k(Q_1+Q_2)}{R_2}$$

EVALUATE (a) Inserting $Q_1 = 60$ nC and $Q_2 = -60$ nC into the expression above gives

$$V_{\text{sphere}} = -kQ_1\left(\frac{1}{R_2} - \frac{1}{R_1}\right) = -\left(9.0\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)(60 \text{ nC})\left(\frac{1}{0.15 \text{ m}} - \frac{1}{0.050 \text{ m}}\right) = 7.2 \text{ kV}$$

(b) Inserting $Q_1 = 60$ nC and $Q_2 = 60$ nC into the expression above gives

$$V_{\text{sphere}} = -kQ_1\left(\frac{1}{R_2} - \frac{1}{R_1}\right) + \frac{k(Q_1+Q_2)}{R_2} = -\left(9.0\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)(60 \text{ nC})\left(\frac{-1}{15 \text{ cm}} - \frac{1}{5.0 \text{ cm}}\right) = 14 \text{ kV}$$

ASSESS As discussed in Problem 22.64, the potential inside a conducting shell is constant and is equal to the potential at the surface of the shell (with respect to the potential at infinity). Thus, the potential of any object inside the shell "floats" on the potential of the shell.

68. **INTERPRET** This problem deals with the electric field of a charged disk. We want to show that the expression derived in Example 22.8 has the correct asymptotic behavior.

DEVELOP From Example 22.8, the electric field on the axis ($x > 0$) of a charged disk of radius a is

$$E_x = \frac{2kQ}{a^2}\left(1 - \frac{x}{\sqrt{x^2+a^2}}\right)$$

In the limit where $x \gg a$, the quantity $x/\sqrt{x^2+a^2}$ may be approximated as

$$\frac{x}{\sqrt{x^2+a^2}} = \left[1+(a/x)^2\right]^{-1/2} \approx 1 - \frac{1}{2}\frac{a^2}{x^2} + \cdots$$

EVALUATE Inserting the above approximation into the exact expression, we obtain

$$E_x = \frac{2kQ}{a^2}\left(1 - \frac{x}{\sqrt{x^2+a^2}}\right) \approx \frac{2kQ}{a^2}\left(1 - 1 + \frac{1}{2}\frac{a^2}{x^2}\right) = \frac{kQ}{x^2}$$

which is like that of a point charge.

ASSESS As always, a finite-size charge distribution looks like a point charge at a large distance.

69. **INTERPRET** We are given the potential of a disk on its axis at two distances from the disk and are to find the disk radius and its total charge.

DEVELOP Combining the given data with the potential in Example 22.7, we find

$$150\text{ V} = \frac{2kQ}{a^2}\left(\sqrt{(5.0\text{ cm})^2 + a^2} - 5.0\text{ cm}\right)$$

$$110\text{ V} = \frac{2kQ}{a^2}\left(\sqrt{(10\text{ cm})^2 + a^2} - 10\text{ cm}\right)$$

Taking the ratio of these two equations eliminates the charge, so we can solve for the radius a, following which we can solve for the charge Q.

EVALUATE The ratio of these two expressions gives

$$\left(\frac{150}{110}\right) = \frac{\sqrt{1+(a/5.0\text{ cm})^2} - 1}{\sqrt{4+(a/5.0\text{ cm})^2 - 2}}$$

Several lines of algebra to remove the square roots finally yields

$$a = (5.0\text{ cm})\frac{\sqrt{105\times 209}}{52} = 14\text{ cm}$$

We can now solve for Q from either of the first two equations, which gives

$$Q = \frac{(110\text{ V})a^2}{2k\left(\sqrt{(10\text{ cm})^2 + a^2} - 10\text{ cm}\right)}$$

$$= \frac{(110\text{ V})(0.142\text{ m})^2}{2(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\left(\sqrt{(0.10\text{ m})^2 + (0.142\text{ m})^2} - 0.10\text{ m}\right)} = 1.7\text{ nC}$$

ASSESS The units for the expression for charge is $\text{V}\cdot\text{C}^2/(\text{N}\cdot\text{m}) = (\text{N/C})\cdot\text{m}\cdot\text{C}^2/(\text{N}\cdot\text{m}) = \text{C}$.

70. **INTERPRET** This problem is about the work done by the electric field in separating the thorium nucleus and the alpha particle.

DEVELOP The work done by the Coulomb repulsion as the thorium nucleus and the alpha particle separate is

$$W = \Delta U_{R\to\infty} = q_\alpha \Delta V_{\text{Th},\text{R}\to\infty} = q_\alpha\left(\frac{kq_{\text{Th}}}{R}\right) = \frac{kq_\alpha q_{\text{Th}}}{R}$$

where $R = 7.4$ fm is the initial separation. (The final separation is essentially infinity.) However, the work-energy theorem requires that this equal the change in the kinetic energy of the two particles:

$$W = \frac{1}{2}m_\alpha v_\alpha^2 + \frac{1}{2}m_{\text{Th}}v_{\text{Th}}^2$$

Note that the two particles start from rest. The conservation of momentum (under the same assumptions) requires also that $0 = m_\alpha v_\alpha + m_{\text{Th}} v_{\text{Th}}$ (since the total momentum is zero initially), so that v_α and v_{Th} can be determined.

EVALUATE Combining the two expressions, and noting that $|v_{\text{Th}}| = (4/234)\,|v_\alpha|$ gives

$$\frac{k(2e)(90e)}{R} = \frac{1}{2}\left(234\text{ u } v_{\text{Th}}^2 + 4\text{ u } v_\alpha^2\right) = \frac{1}{2}\left(\frac{16}{234}\text{ u } + 4\text{ u}\right) v_\alpha^2$$

The speeds are $v_\alpha = 4.1\times10^7$ m/s and $v_{\text{Th}} = 7.0\times10^5$ m/s, where we have used $1\text{ u} = 1.66\times10^{-27}$ kg.

ASSESS Since $m_{\text{Th}} \gg m_\alpha$, we expect $v_\alpha \gg v_{\text{Th}}$ which is what we found.

71. **INTERPRET** This problem involves a disk with a circularly symmetric charge distribution. We are to find an expression for the potential on the disk axis, the electric field on the disk axis, and show that the electric field decays as $1/x^2$ for $x \gg a$ (a is the disk radius).

DEVELOP For part (a), use the integral expression for voltage of Example 22.7,

$$V(x) = \int_0^a \frac{k\,dq}{\sqrt{x^2 + r^2}}$$

For this problem, $dq = 2\pi\sigma\, r dr;$ with $\sigma = \sigma_0 r/a$, which gives

$$V(x) = \frac{2\pi k\sigma_0}{a}\int_0^a \frac{r^2 dr}{\sqrt{x^2 + r^2}}$$

The electric field is the spatial derivative of the potential (see Equation 22.9).

EVALUATE **(a)** Reference to standard integral tables gives

$$V(x) = \frac{2\pi k\sigma_0}{a}\int_0^a \frac{r^2\,dr}{\sqrt{x^2 + r^2}} = \frac{2\pi k\sigma_0}{a}\left[\frac{a}{2}\sqrt{x^2 + a^2} - \frac{x^2}{2}\ln\left(\frac{a + \sqrt{x^2 + a^2}}{x}\right)\right]$$

$$= \pi k\sigma_0 a\left[\sqrt{1 + (x/a)^2} - (x/a)^2 \ln\left(a/x + \sqrt{1 + (a/x)^2}\right)\right]$$

(b) As in Example 22.8, $E_x = -dV/dx$ results in

$$E_x = \pi k\sigma_0\left[\frac{2x}{a}\ln\left(\frac{a + \sqrt{x^2 + a^2}}{x}\right) - \frac{x}{\sqrt{x^2 + a^2}} + \frac{x^2}{a}\left(\frac{x}{a + \sqrt{x^2 + a^2}}\right)\left(\frac{1}{\sqrt{x^2 + a^2}} - \frac{a + \sqrt{x^2 + a^2}}{x^2}\right)\right]$$

$$= \frac{2\pi k\sigma_0 x}{a}\left[\ln\left(\frac{a}{x} + \sqrt{1 + \left(\frac{a}{x}\right)^2}\right) - \frac{a}{x\sqrt{1 + (a/x)^2}}\right]$$

(c) The logarithm has to be expanded carefully, up to order $(a/x)^3$ to evaluate E_x for $x \gg a$. Thus,

$$\ln\left(\frac{a}{x} + \sqrt{1 + \left(\frac{a}{x}\right)^2}\right) = \ln\left(1 + \frac{a}{x} + \frac{a^2}{2x^2} + \cdots\right) \approx \left(\frac{a}{x} + \frac{a^2}{2x^2}\right) - \frac{1}{2}\left(\frac{a}{x} + \frac{a^2}{2x^2}\right)^2 + \frac{1}{3}\left(\frac{a}{x} + \frac{a^2}{2x^2}\right)^3 + \cdots \approx \frac{a}{x} - \frac{a^3}{6x^3}$$

Also,

$$\frac{a}{x}\left(1 + \frac{a^2}{x^2}\right)^{-1/2} \approx \frac{a}{x}\left(1 - \frac{a^2}{2x^2}\right) = \frac{a}{x} - \frac{a^3}{2x^3}$$

Then,

$$E_x \approx \frac{2\pi k\sigma_0 x}{a}\left[\frac{a}{x} - \frac{a^3}{6x^3} - \frac{a}{x} + \frac{a^3}{2x^3}\right] = \frac{2\pi k\sigma_0 a^2}{3x^2}$$

which is the field for a point charge with

$$Q = \frac{2\pi\sigma_0}{a}\int_0^a r^2 dr = 2\pi\sigma_0 a^2/3$$

ASSESS Checking the limits is a good manner to test the validity of expressions. If we were to have found that our expression for the electric field did not reduce at large distances to that of a point charge, then we could be sure that the expression is wrong. Of course, the correct asymptotic behavior does not guarantee that the expression is valid!

72. **INTERPRET** The cylinder is a continuous charge distribution, and we want to find the potential at its center on the axis.

DEVELOP Following the hint given in the problem, we consider the cylinder to be composed of rings of radius a, width dx, and charge $dq = \left[q/(2a)\right]dx.$ The potential at the center of the cylinder (which we take as the origin, with x axis along the cylinder axis) due to a ring at $x\ (-a \le x \le a)$ is (see Example 22.6)

$$dV_{\text{cyl}} = \frac{kdq}{\sqrt{x^2+a^2}} = \frac{kqdx}{2a\sqrt{x^2+a^2}}$$

EVALUATE The potential at the center is given by integrating the expression above, which gives

$$V_{\text{cyl}} = \int_{-a}^{a}\left(\frac{kq}{2a}\right)\frac{dx}{\sqrt{x^2+a^2}} = \frac{kq}{2a}\ln\left(\frac{a+\sqrt{2}a}{-a+\sqrt{2}a}\right) = \frac{kq}{a}\ln\left(1+\sqrt{2}\right) = 0.881\frac{kq}{a}$$

ASSESS The result can be compared with that at the center of a charged ring of radius a: $V_{\text{ring}} = kq/a$. For the cylinder, the charge elements generally are farther away from the center compared to the ring, so we expect $V_{\text{cyl}} < V_{\text{ring}}$, which is what we find.

73. **INTERPRET** We need to find the potential due to a line charge with a non-constant charge density. The problem is one-dimensional: the line and the point of interest are all on the x axis. We will use the integral expression for potential. We must also show that the result reduces to that of a point charge for distances much, much larger than the length of the line charge.

DEVELOP Consider the line charge to be a superposition of many point charges. The potential for each point charge is given by Equation 22.3, $dV = kdq/r$. Integrate this expression over the length of the line charge to find the total potential:

$$V(x) = \int_{-L/2}^{L/2}\frac{kdq}{r}$$

with $dq = \lambda\, dx' = \lambda_0 (x'/L)^2\, dx'$ and $r = x - x'$.

EVALUATE Evaluating the integral gives

$$\begin{aligned} V &= \int_{-L/2}^{L/2}\frac{kdq}{r} = \int_{-L/2}^{L/2}\frac{k\lambda_0}{x-x'}\left(\frac{x'}{L}\right)^2 dx' = \frac{k\lambda_0}{L^2}\int_{-L/2}^{L/2}\frac{x'^2}{x-x'}dx' \\ &= \frac{k\lambda_0}{L^2}\left[-\frac{x'^2}{2} + x'x - x^2\ln(x-x')\right]_{-L/2}^{L/2} \\ &= -\frac{k\lambda_0}{L^2}\left[Lx + x^2\ln\left(\frac{2x-L}{2x+L}\right)\right] \end{aligned}$$

For $x \gg L,$ we rearrange slightly and use the approximation $\ln(1+\xi) \approx \xi$ for small ξ

$$\begin{aligned} V &= -\frac{k\lambda_0}{L^2}x^2\left[\frac{L}{x} + \ln\left(1-\frac{L}{2x}\right) - \ln\left(1+\frac{L}{2x}\right)\right] \\ &\approx -\frac{k\lambda_0}{L^2}x^2\left[\frac{L}{x} + \left(-\frac{L}{2x} - \frac{1}{2}\left(-\frac{L}{2x}\right)^2 + \frac{1}{3}\left(-\frac{L}{2x}\right)^3\right) - \left(\frac{L}{2x} - \frac{1}{2}\left(\frac{L}{2x}\right)^2 + \frac{1}{3}\left(\frac{L}{2x}\right)^3\right)\right] \\ &\approx -\frac{k\lambda_0}{L^2}x^2\left[\frac{L}{x} - \frac{L}{x} - \frac{L^2}{8x^2} + \frac{L^2}{8x^2} - \frac{L^3}{24x^3} - \frac{L^3}{24x^3}\right] = \frac{k\lambda_0}{L^2}x^2\frac{L^3}{12x^3} = \frac{k\lambda_0 L}{12x} \end{aligned}$$

The total charge on the rod is

$$q = \int dq = \int_{-L/2}^{L/2}\lambda_0\left(\frac{x'}{L}\right)^2 dx' = \frac{\lambda_0}{L^2}\left[\frac{x'^3}{3}\right]_{-L/2}^{L/2} = \frac{\lambda_0}{3L^2}\left[\frac{2L^3}{8}\right] = \frac{\lambda_0 L}{12}$$

so in the limit of $x \gg L$,

$$V = \frac{k\lambda_0 L}{12x} = \frac{kq}{x}$$

ASSESS At large distances, the potential looks like that of a point charge, as expected.

74. **INTERPRET** We need to find the potential due to a line of charge with a non-constant charge density. The problem is one-dimensional: the line and the point of interest are all on the *x* axis. We will use the integral expression for potential. We must also show that the result reduces to that of a point charge for distances much, much larger than the length of the line charge.

DEVELOP Use the same strategy as for the previous problem. Start with

$$V(x) = \int_{-L/2}^{L/2} \frac{kdq}{r}$$

where $dq = \lambda\, dx' = \lambda_0 (x'/L)\, dx'$ and $r = x - x'$. We must also check to see that our expression reduces to the expected result for $x \gg L$. Because the total charge is zero for this charge distribution, we expect that the potential at large *x* goes as the potential for a dipole: $V \propto 1/x^2$

EVALUATE

$$V = \int_{-L/2}^{L/2} \frac{k\lambda_0}{x - x'}\left(\frac{x'}{L}\right) dx' = \frac{k\lambda_0}{L}\Big[-x - x\ln(x - x')\Big]_{-L/2}^{L/2}$$
$$= -\frac{k\lambda_0}{L}\left[L + x\ln\left(\frac{2x - L}{2x + L}\right)\right]$$

For $x \gg L$, we rearrange slightly and use the approximation $\ln(1+\xi) \approx \xi$ for small ξ:

$$V = -\frac{k\lambda_0}{L}x\left[\frac{L}{x} + \ln\left(1 - \frac{L}{2x}\right) - \ln\left(1 + \frac{L}{2x}\right)\right]$$
$$\approx -\frac{k\lambda_0}{L}x\left[\frac{L}{x} + \left(-\frac{L}{2x} - \frac{1}{2}\left(-\frac{L}{2x}\right)^2 + \frac{1}{3}\left(-\frac{L}{2x}\right)^3\right) - \left(\frac{L}{2x} - \frac{1}{2}\left(\frac{L}{2x}\right)^2 + \frac{1}{3}\left(\frac{L}{2x}\right)^3\right)\right]$$
$$= -\frac{k\lambda_0}{L}x\left[\frac{L}{x} - \frac{L}{x} - \frac{L^2}{8x^2} + \frac{L^2}{8x^2} - \frac{L^3}{24x^3} - \frac{L^3}{24x^3}\right] = \frac{k\lambda_0}{L}x\frac{L^3}{12x^3} = \frac{k\lambda_0 L^2}{12x^2}$$

ASSESS The potential at large distances goes as $1/x^2$, as predicted.

75. **INTERPRET** You need to find the minimum possible wire diameter that won't be susceptible to breakdown, which is when the air gets ionized by the electric field near the wire.

DEVELOP You're told to neglect any charge distributions on the ground. Therefore, the electric field around the wire can be found with Gauss's law: $E = \lambda/2\pi\epsilon_0 r$ (Equation 21.6). The maximum field, which will be at the outer surface of the wire at radius r_A, needs to be at most 25% of the breakdown field in air, 3MV/m. The potential difference for a long wire was given in Equation 22.4: $\Delta V_{AB} = \lambda/2\pi\epsilon_0 \ln(r_A/r_B)$. You know that there will be 115 kV between the wire's outer surface and the ground below $(r_B = 60\text{ m})$. You can combine the field and potential equations to solve for the radius r_A.

EVALUATE Combining the above information gives

$$r_A \ln(r_A/r_B) = \frac{\Delta V_{AB}}{E} \quad \rightarrow \quad r_A \ln(r_A/60\text{m}) = \frac{-115\text{ kV}}{3\text{ MV/m}} = -0.0383\text{ m}$$

where a negative sign has been added to the potential difference to be consistent with the fact that $r_A < r_B$. One way to solve this equation is with Newton's method. Let $y = r_A \ln(r_A/60) + 0.0383$ and let $r_{A,0}$ be your best guess for the root of *y*. Then you use the derivative of *y*, which in this case is $y' = \ln(r_A/60) + 1$, to find a better guess:

$$r_A = r_{A,0} - \frac{y(r_{A,0})}{y'(r_{A,0})} = r_{A,0} - \frac{r_{A,0}\ln(r_{A,0}/60) + 0.0383}{\ln(r_{A,0}/60) + 1}$$

This process can be repeated several times with the r_A of one iteration becoming the $r_{A,0}$ of the next iteration. A good first guess for the radius might be 1 cm. The table below shows how quickly Newton's method converges on the root.

$r_{A,0}$	$y(r_A)$	$y'(r_A)$	r_A
0.01	–0.048695147	–7.699514748	0.003675556
0.003675556	0.002645651	–8.700395344	0.00397964
0.00397964	1.22454E-05	–8.62090841	0.003981061
0.003981061	2.53465E-10	–8.620551548	0.003981061
0.003981061	0	–8.620551541	0.003981061

The minimum radius is 4.0 mm, so the minimum diameter is 8.0 mm.

ASSESS An 8-mm wire would likely be too fragile for a transmission line, but this at least gives you the lower limit on what you could use.

76. INTERPRET We are asked to analyze an electrocardiograph showing equipotentials in a human torso.

DEVELOP Let's first imagine what each of the answer possibilities might look like. A charged sheet would vary linearly with distance from the sheet (see Example 22.2), so the equipotentials would be straight lines parallel to the sheet. A dipole would have a "hill" and a "hole" in the electric potential separated by a bisector with $V = 0$ (see Figures 22.11 and 22.16). A point charge would have concentric circles around it as equipotentials, as would a charged sphere (see Figure 22.15).

EVALUATE The electrocardiograph shows a zero potential line running from the bottom left to the upper right. Above this line, there appears to be a sharp peak in the potential, whereas below the line we see a sharp valley. This is in rough agreement with the electric potential of a dipole.

The answer is (b).

ASSESS Recall Problems 20.80 through 20.83, where we explored how heart muscles gain a dipole moment when the heart contracts.

77. INTERPRET We are asked to analyze an electrocardiograph showing equipotentials in a human torso.

DEVELOP From the $V = 0$ and the $V = 0.5$ mV equipotentials, we can surmise that the potential rises to a peak in the upper left corner of the heart, and drops to a valley in the lower right corner of the heart.

EVALUATE Since electric field lines point downhill, the electric field in the heart must point from the upper left to the lower right.

The answer is (a).

ASSESS Between the two charges of the dipole, the electric field is parallel to the dipole moment (see Figure 22.16).

78. INTERPRET We are asked to analyze an electrocardiograph showing equipotentials in a human torso.

DEVELOP The electric field will point downhill with respect to the electric potential, and it will be stronger where the hill is steeper. The electric potential is steepest where the equipotential lines are closer together.

EVALUATE Of all the points, *C* has the most equipotential lines crammed together, so the electric field will be the strongest there.

The answer is (c).

ASSESS The point C is right between the two charges of the dipole, so there is a large contribution from both charges to the field.

79. INTERPRET We are asked to analyze an electrocardiograph showing equipotentials in a human torso.

DEVELOP The equipotential line at point A has $V = 0.2$ mV. This line is approximately parallel and midway between the two surrounding equipotentials at 0.1 mV and 0.3 mV. Therefore, we can assume that the electric potential is approximately linear in this region with regard to the distance x from the $V = 0.2$ mV equipotential, i.e.,

$$V = 0.2\ \text{mV} + 0.1\ \text{mV}\left(\frac{x}{x_0}\right)$$

where x_0 is the distance between the equipotential lines. Assuming the torso is about 30 cm across, we estimate that $x_0 \approx 3$ cm.

EVALUATE The electric field is the derivative of the electric potential (Equation 22.9), so at the point A the field should be roughly:

$$\left|\vec{E}\right| = \frac{\partial}{\partial x}V \approx \frac{0.1\ \text{mV}}{3\ \text{cm}} \approx 3\ \text{mV/m} = 3\ \text{mN/C}$$

The closest answer is (b).

ASSESS The units work out because $1\ \text{V} = 1\ \text{J/C}$. You can arrive at a similar answer by taking the derivative in radial components of the dipole potential in Equation 22.6:

$$\left|\vec{E}\right| = \left|\frac{\partial}{\partial r}V + \frac{1}{r}\frac{\partial}{\partial \theta}V\right| = V(r,\theta)\left(\frac{2+\tan\theta}{r}\right)$$

If we assume point A is located about 15 cm from the dipole center and at an angle of about 60° from the dipole axis, then the magnitude of the electric field at point A is about $E \approx (0.2\ \text{mV})(25\ \text{m}^{-1}) = 5\ \text{mN/C}$.