Physics

Measurement

- There is Unit and There is standard

Exp length $\underline{L}$ → standard

$\underline{m, Km \dots}$ → Unit

Unit → International system of units (si)

Unit → Changing units : Chain-link Conversion → Multiply The original Measurement by a conversion factor.

- International system = metric system

- Base quantities :- + Temp + electrical current

- Length : (Meter) : The Distance Traveled by light During a precisely specified Time interval 1/299 792458 second

- Mass : (Kg) : platinium-iridium standard cylinder
(Atomic Mass) : atome Carbon -12

- Time : (second) : oscillations of light emitted by an atomic (cesium-133)
ارتجاج/تذبذب

- Density : $\rho = \frac{M}{V}$

Problems to revise :-

simple problem : p 7

p53 $= 1$ parsic $= 2.0626 \times 10^5$ au $= 3.261$

Revise

Chap2 P9 p30

giga : 10^9
Mega : 10^6
Kilo : 10^3
Centi : 10^{-2} c
milli : 10^{-3} m
Micro : 10^{-6} μ
nano : 10^{-9} n
pico : 10^{-12} p

femto
pico 10^{-12}
Angstarm 10^{-10}
10^{-9}
nan
Micro 10^{-8}

Motion in a straight lines

- **Displacement** $= \Delta X = X_2 - X_1$ vector quantity
  - Magnitude $= |\Delta X|$
  - has 2 features: → direction, → Magnitude

- **Distance Traveled**: measured along the actual path

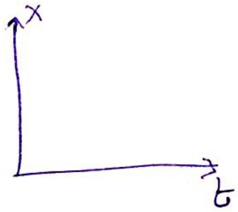
- **Average velocity & Average speed**

(vector) velocity $= \dfrac{\Delta X}{\Delta t}$ → pos → V+, neg → V−; Δt → always +

↳ speed$_{avg}$ = $\dfrac{\text{distance}}{\Delta t}$ (scalares) $= \dfrac{d_1 + d_2}{t_1 + t_2}$

↳ no algebric sign

velocity in a graph = The slope of the line That connects 2 points (x_1, t_1 and x_2, t_2)



V.I.N

our bodies reacts to accelerations but not to velocities

- **Instantaneous velocity and speed**

$V_{Inst} = \dfrac{dx}{dt}$ The slope of $x(t)$ curve at that point

sign of $\vec{V}$ and $\vec{a}$ the same → acceleration

Physics of Motion

- Kinematics: describes change in motion
- Dynamics: outhion The effect

Motion → Translation, Vibaration, Rotation

vector • **Average Acceleration**

$acc = \dfrac{\Delta V}{\Delta t}$ = slope

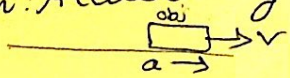
$acc_{Inst} = \dfrac{dV}{dt}$ = slope at that point

$\Delta X \rightarrow V \rightarrow acc$ اشتقاق

$acc \rightarrow V \rightarrow \Delta X$ تكامل

Acceleration

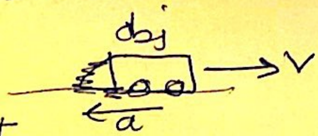
a +
v +
Speed : Increases
Motion : Accelerating



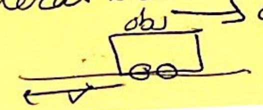
a -
v -
Speed: Increases
Motion : Acceleration

Deceleration

a −
v +
Speed : decreases
Deceleration

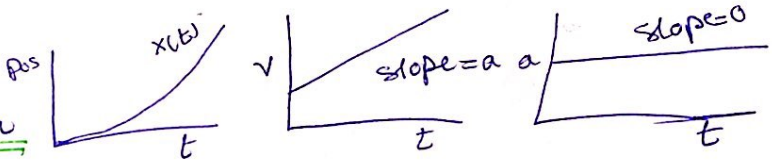


a +
v −
Speed : decreases
Deceleration.



* THe Humain body has an awarness of acceleration But not The velocity

Constant acceleration



$\bar{a} = \frac{V_2 - V_1}{t_2 - 0}$

$V_2 - V_1 = at_2$

$\boxed{V_2 = V_1 + at_2}$ 1 we us This if we don't have $\Delta X (X - X_0)$

$\bar{V} = \frac{V_1 + V_2}{2}$

$= \frac{V_1 + V_1 + at}{2}$

$\boxed{\bar{V} = V_1 + a(\frac{t}{2})}$

$\Delta X = \bar{V} t$

$= \frac{1}{2}(V_1 + V_1 + at)t$

$\boxed{\Delta X = V_1 t + \frac{1}{2} a t^2}$ 3 ~ ~ ~ V_2

Const acc

$\boxed{\bar{V} = \frac{V_1 + V_2}{2}}$ 2

ΔV a-t ~ ~ ~

ΔX : يساوي مساحة تحت

V-t

$= \Delta V \cdot t$

$\Delta X = \bar{V} t = \left(\frac{V_1 + V_2}{2}\right)\left(\frac{V_2 - V_1}{a}\right)$

$\Delta X = \frac{V_1^2 - V_2^2}{2a}$

$\boxed{V_2^2 = V_1^2 + 2a\Delta X}$ 4 ~ ~ ~ ~ ~ ~ ~ t

$\frac{V_1 + V_2}{2}$

Variable account

Acceleration $\xrightarrow{\int a(t)}$ velocity $V(t)$ $\xrightarrow{\int v(t)}$ position $X(t)$

$\frac{dv}{dt} = a(t)$

vectores

Ex: displacment
- force
- accelaration
- velocity

- has a magnitude and a direction

- Adding vectors

$\vec{a} + \vec{b} = \vec{C}$

the result should be from tail to head

→ properities:

- Commutative $\vec{a}+\vec{b} = \vec{b} + \vec{a}$
- associative $(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$

- $\vec{b}$ and $-\vec{b}$

$-\vec{b}$ has the same magnitude as $\vec{b}$ but differs in direction (opposite)

- Components of a vector

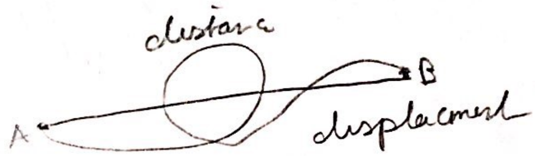
$\vec{a} = \vec{a}_x + \vec{a}_y$

$= a\cos\theta \ + a\sin\theta$

$a = \sqrt{(a_x)^2 + (a_y)^2}$ $\quad \tan\theta = \frac{a_y}{a_x}$

scalars

- Mass
- density
- time
- energy
- Temperature
- distance : length
- of the curved line



- has no direction

- Adding scalars

$a + b = c$

Unit vectors

$a = a_x\hat{i} + a_y\hat{j}$ → vector components

scalar components

Adding vectors by Components

$r = \vec{a} + \vec{b}$

$r = r_x + r_y + r_z$

$r = (a_x + b_x) + (a_y + b_y) + (a_z + b_z)$

Multiplying vectors

vector X scalar

= new vector

scalar > 0 : same direction

scalar < 0 : opposite direction

Vector . Vector

= scalar product

$= V_1 \times V_2 \times \cos\theta$

$\theta = 0 \Rightarrow V_1 \cdot V_2 \Rightarrow Max$

Vector X vector

= vector product

$= V_1 \times V_2 \times \sin\theta$

$\theta = 90$

$V_1 \times V_2 \Rightarrow Max$

θ: الزاوية بينهم (الصغرى)

Motion in 2 and 3 Dimensions

- Position and Displacement

If the position vector changes from $\vec{r_1}$ to $\vec{r_2}$ during a certain time interval Then the displacement $\Delta\vec{r}$ during that time interval is:-

$$\Delta\vec{r} = \vec{r_2} - \vec{r_1}$$

$$\Delta\vec{r} = \underbrace{(x_2 - x_1)}_{\Delta x}\hat{i} + \underbrace{(y_2 - y_1)}_{\Delta y}\hat{j} + \underbrace{(z_2 - z_1)}_{\Delta z}\hat{k}$$

In 2D: When an object reaches Max position in one direction Then it's velocity in this direction = 0 component

- Average velocity & Instantaneous velocity

If a particle moves through a displacement $\Delta\vec{r}$ in a time interval Δt then it's average velocity $\vec{V}_{avg}$ is

$$\vec{V}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

V is the same direction as r

$$\vec{V}_{avg} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}K$$

$$V_{Instant} = \frac{d\vec{r}}{dt}$$

V_{avg} takes the tangent's (at r) direction

- Average Accelaration & instantaneous Acceleration

$$a_{avg} = \frac{\vec{V}_2 - \vec{V}_1}{\Delta t} = \frac{\Delta\vec{V}}{\Delta t}$$

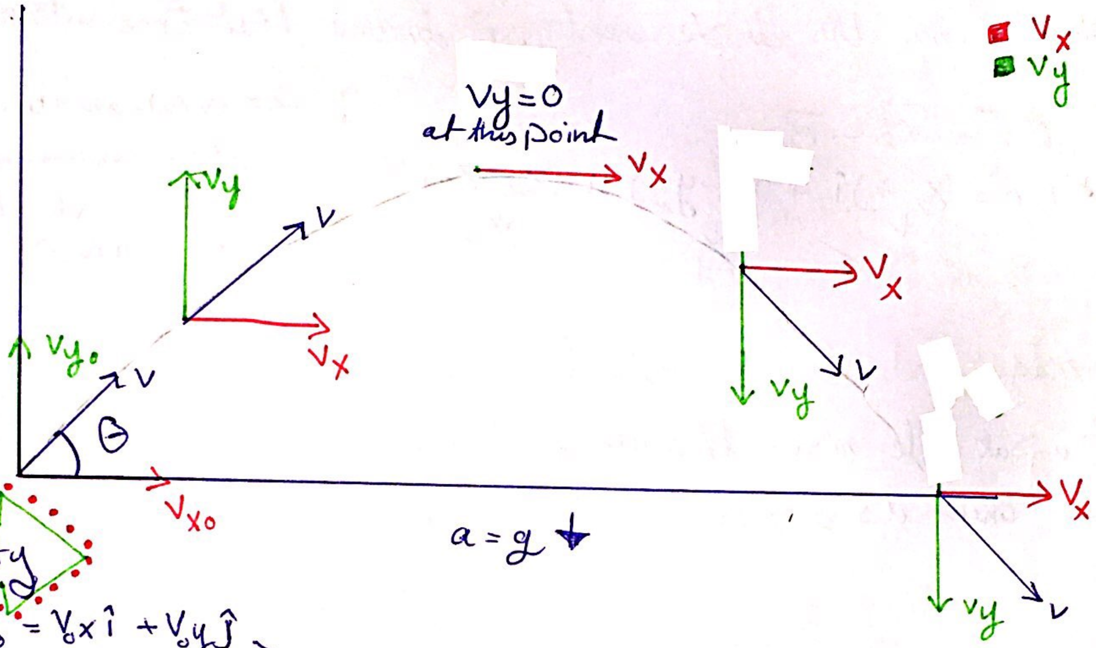
$$\vec{a}_{Instant} = \frac{d\vec{V}}{dt}$$

If velocity changes in Magnitude or direction or Both The particle Must have $\vec{a}$

* if the velocity changes in direction or magnitude → particle Must have an accelaration

2D motion

Horizontal and vertical motions are independent of each other



Velocity

$V_0 = V_{0x}\hat{i} + V_{0y}\hat{j}$

$\boxed{V_{0x} = V_0 \cos\theta^\circ}$ Constant

$\boxed{V_{0y} = V_0 \sin\theta^\circ}$ Variable

$V = \sqrt{V_x^2 + V_y^2}$

$V_x = V_{0x}$

- $V_y = V_{0y} - gt$ or • $V_y^2 = \underbrace{V_{0y}^2}_{= V_0 \sin\theta} - 2g(y - y_0)$

Displacement

$X = V_0 t \cos\theta$

$y = V_0 t \sin\theta - \frac{1}{2} g t^2$

$\Delta r = \sqrt{X^2 + y^2}$

* throught the path $a_x = 0$, $a_y = g$
* path is parabolic
* air has a large effect on R or the path in general

• the Horizontal Range

$R = V_{0x} t_f \quad \text{---} \; ①$

$\Delta y = V_{0y} t - \frac{1}{2} g t^2$

$= 0$ لأنه انطلق من مستوى للمستوى نفسه

$0 = V_{0y} t - \frac{1}{2} g t^2$

$t_f = 2 V_{0y} / g$

نعوضه في ① ونستنتج

$$\boxed{R = \frac{V_0^2 \sin 2\theta}{g}}$$

$R_{Max} = V_0^2 / g$ when $\theta = 45°$

Range: نقطتين على نفس المستوى

• the height

$$h = \frac{V_0^2 \sin^2(\theta)}{2g}$$

• Relation between R and h

$\frac{h}{R} = \frac{\tan\theta}{4} \Rightarrow \boxed{h = \frac{R \tan\theta}{4}}$

• The equation of the path

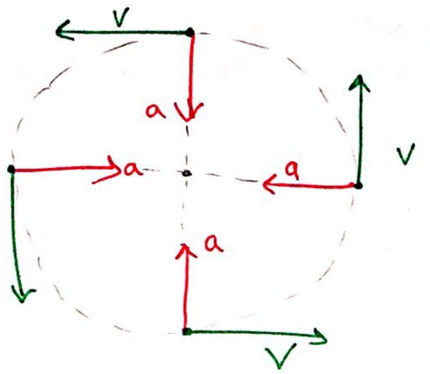
$$\boxed{y = \tan\theta \cdot X - \frac{g \cdot X^2}{2 V_0^2 \cos^2\theta}}$$

Uniform Circular Motion

- a particle travelling around a circle in a constant speed
- accelaration : Centripetal (uniform circular Motion)

$\vec{a}_r = \frac{V^2}{r}\hat{r}$ — V → speed of the particle, r → radius, $\hat{r}$: vector

$T = \frac{2\pi r}{V}$ T: period
$\pi = 3.14$



$a_t = 0$

$a_{tangent} = \frac{dv}{dt}$ (non-uniform circular Motion)

- V is not Constant
- so accelerating with a tangental accelaration

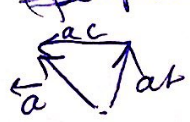
$a_t \neq 0$

$a_r = \frac{V^2}{2\pi r}$

- Net force : صافي القوة

$\sum \vec{F} = m\vec{a}$

$\vec{a} = \vec{a}_t + \vec{a}_c$ ~~(peripecti culare)~~



$F = |\sum \vec{F}| = |m\vec{a}|$
$= m|\vec{a}|$
$= m\sqrt{at^2 + ac^2}$

Motion with Constant Accelaration

$$r = x\hat{i} + y\hat{j}$$

$$v = v_x\hat{i} + v_y\hat{j}$$

$$v_f = v_{xf}\hat{i} + v_{yf}\hat{j} \longrightarrow \underline{v_f = v_i + at}$$

$$r_f = x_f\hat{i} + y_f\hat{j} \longrightarrow \underline{r_f = r_i + v_i t + \tfrac{1}{2}at^2}$$

Constant

UPLOADED BY AHMAD T JUNDI

- Difference between Motion in 1D, 2D and 3D

$\boxed{1D}$: The Motion is in a straight line

$\boxed{2D}$: The Motion is in a curved path but in a single plane

$\boxed{3D}$: The Motion is throughout the space / not in a plane but in a complete space

Ex : a paper moving freely in the air

Note : • speed is the Magnitude of velocity

• if we have $\Delta\vec{V} \neq 0$ The There is a $\begin{cases} \text{Change in speed} \\ \text{and/or} \\ \text{Change in direction} \end{cases}$

Force and Motion - 1

- force Causes accelaration and it's a vector / it's a Motion that can Change Motion / push or pull / ↑ or ↓ in speed.
- Newtonian mechanics : 3 laws of motion

- **Newton's first law** : law of Inertia (القصور)

"if no force acts on a body, the body's velocity Cannot change : that is: the body Cannot accelarate"

→ if the body is at rest than it stays at rest and if its moving it continuous to move with the same velocity (same magnitude & same direction

- If $\vec{F}_{net} = 0$ → velocity doesn't change → No accelaration *_*

If Mass ↑ → Inertia ↑

- Inertia : * خاصية من خواص الجسم التي تقيس مدى قدرته على مقاومة التغير

Mass is the measure of Inertia

- If the forces are balanced there will be 2 Cases :-

* object at Rest
$V = 0$ m/s
↓
stays at Rest

* object in uniform motion
($V \neq 0$ m/s)
↓
stays in Motion same velocity & direction

- Inertial & non-inertial frames

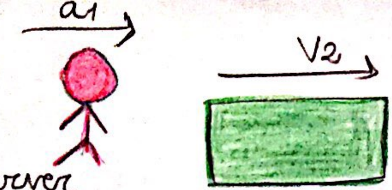
At rest   at rest

: it's an inertial frame
Bcz :- observer's Relative acceleration = 0

$\vec{V_1}$ $\vec{V_2}$

: it's an inertial frame Bcz, Relative acceleration = 0 and Relative velocity is uniform $= V_2 - V_1$

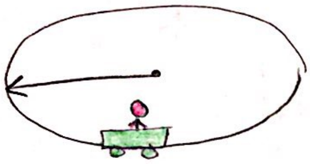
small uniform   higher uniform velocity → No Net force acting



- Observer Moving withe acceleration

- No acceleration
- V cste

- Oberver's frame is non-inertial fr as person has a Relative Acceleration with a moviing object. The person would see things differently than someone standing stationary on the Ground



- it's a non-inertial frame: observer inside a car would feel the car as stationary, He would feel a centrifugal force throwing him outside the ground
But in an inertial frame: a person standing outside the car on the ground would see the car whirring around

• Newton's second law

"The net force on a body is equal to the product of the body's Mass & it's acceleration"

$$\vec{F}_{net} = m\vec{a}$$

• Some particular forces

The Gravitational force $\vec{F}_g$

- Pull toward the center of Earth
- In free-fall

$F_g = mg$

Friction : الاحتكاك

- opposite of the intended Motion
- it's a resistance

Tension $\vec{T}$

- when a cord is attached to a body and pulled taut the cord pulls on the body with a force $\vec{T}$ directed away from the body
- always a pull
- in the direction of the rope

Weight W

- Magnitude of the Net force required to prevent the body from falling freely as measured by someone on the Ground

- $W = |F_g|$
- $W \neq m$
- m is Cste → W is not
  ↳ if g changes W changens

The Normal force $\vec{F_N}$

مهم : يجب ان يكون الجسم على السطح ليكون هناك FN

- when a body presses against a surface, the surface (even a ~~son~~ seemingly rigid one) deforms and pushes on the body with a Normal force (⊥ to the surface)

$\vec{F_N}$ of a body $= m(g + a_{\text{of the body}})$ if $a = 0$ then $\vec{F_N} = mg$

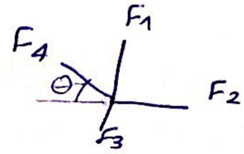
• **Newton's Third law**   force pairs

" When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction "

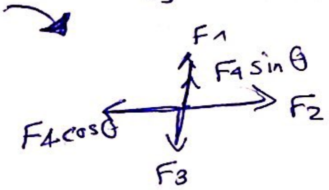
* There has to be 2 bodies

• Notes for doing Exercises

Steps:
1) Draw a free body diagram →



2) Break forces into components →
3) Redraw Free body diagram
4) Sum the forces
5) " " " again



Force and Motion - II

* a frictional force is the vector sum of many forces acting between the surface atomes of one body and those of another body

- **Friction**
  - Kinetic friction : Resistive force ⇒ f_k
    → opposite to the Motion
    $f_k = \mu_k F_N$ — μ_k: coefficient of kinatic fric, F_N: Normal force
  - Static friction : $f_{s,max} = \mu_s F_N$ — μ_s: Coefficient of static friction
    it's a Resistive force
    - It's direction and It's strength can change

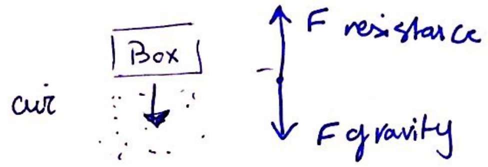
* an object moves when the strength that pushing is greater then the Maximum of static friction $f_{s,max}$

* usually . $|f_k| < |f_{s,max}|$

* f_s and f_k is always parallel to the surface and opposed to the attempted sliding
friction is independant of surface area and velocity

Drag force - Resistive force $\vec{D}$

* when objects move via fluids (liquids and gases) the Molecul create a resistance force known as Drag force
  - depends on velocity of object



$$\boxed{D = \frac{1}{2} C \rho A v^2}$$

C: The drag coefficient
ρ: the air density
A: effective cross-sectional area of the body
V: velocity

$$mg = \frac{1}{2} C \rho A v_t^2$$

if $D = mg \Rightarrow a = 0 \Rightarrow$ the body falls at a constant speed called
. Terminal speed

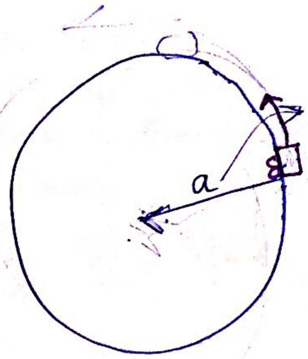
• Uniform circular Motion

$F = m\frac{v^2}{R}$ (a)

$\mu_k F_N = m\frac{v^2}{R}$

- speed is constants
- |a| is constant and F is constant too

* Directions of a and F are changing



centripetal accele… ⇒ caused by centripetal Force which is :- frictional force

- When $F_{centripetal}$ is bigger than f Then it slide off to the outside of the curve

- If a person is strapped into their seat belt on a ferris wheel Then at the Top

$F_{belt\ up} = 0$

$F_{belt\ down} = 0$

$F_g = mg$

$\boxed{F = ma}$

$F_g = mg$

Ch7 Kinetic Energy and Work

- **Constant Force**

$W = \vec{F} \cdot \vec{d} = F \cos\theta \, d$ Joul (N.m)

↳ to use It F should be cste in Mag and direction + object like particle

- **Work can be**
  - → **Positive**: Force adds energy to the system ($0 < \theta < 90°$)
  - → **Zero**: ⇒ ($\theta = 90°$) ⇒ Force does not add energy
  - → **negative**: ⇒ Force decreases the Energy of the system ($\theta > 90$)

- **Variable Force**

$$W = \int_{r_i}^{r_f} \vec{F} \cdot \vec{dr} = \int_{r_i}^{r_f} F \cos\theta \, dr = \int_{x_i}^{x_f} F_x \, dx + \int_{y_i}^{y_f} F_y \, dy + \int_{z_i}^{z_f} F_z \, dz$$

- Remark! the Area under the curve F_x vs x is (=) W

↳ $W = \int_{x_i}^{x_f} F_x \, dx$

- **Work-Energy Theorem**

$W_{net} = \Delta K$ → Kinetic Energ :- Units → Joul ; → 1KW.h = Kilowatt . hour = 3.6 MJ

↳ used by Electrical companies

W_{net} ↳ Work done by $\vec{F}_{net}$

- **Work Done by Gravity**

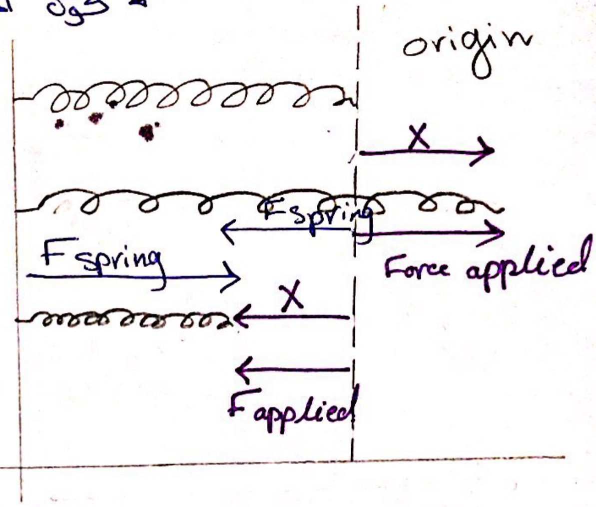
$W_g = (\vec{mg}) \cdot d$ (Constant Force

$= \vec{mg} \cos\theta \, d$

- In Circular Motion $W = 0$ Bcz θ between V and F is 90 so ⇒ $\cos 90 = 0$

Work Done by a spring force (variable force)

قوة المرونة في النابض (قانون هوك)



* $F_S = -k\,x$ (Hook's law)

↳ elastic constant

which measures the stiffness of the spring so:

$k\uparrow$ → the material is stiffer
→ spring is stronger

x^+ ⇒ stretched to the right
x^- ⇒ " " " left

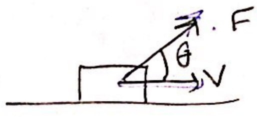
→ $W_S = \int_{x_i}^{x_f} F_x\,dx = \int_{x_i}^{x_f} -kx\,dx = -k\int_{x_i}^{x_f} x\,dx$

→ $W_S = -\frac{k}{2}(x_f^2 - x_i^2)$ if $x_i = 0 \Rightarrow W = -\frac{k}{2}x_f^2$

Power القدرة scalar [Watt]

$P_{avg} = \frac{\Delta W}{\Delta t}$ $\frac{J}{S} = \text{Watt}$

$P_{instantinuous} = \vec{F}\cdot\vec{V}$



1hp : howrse power = $\boxed{746}$ watt ∴ قدرة الحصان

Potential Energy & Conservation of Energy

Forces

- non Conservative
  Ex: friction

- Conservative
  Ex: mg / F_{spring}
  proprietes :-
  1- Path independant
  2- $W_{F\,cons} = 0$ when around a closed loope
  3- $W_{F\,cons} = -\Delta U$

- Conservative forces

Gravitational Potentiel Energy

$U_g = mg\,\Delta y$

when $y_i = 0$:-
$U_g = mgy$ Joul

Spring Potential energy

$U_s = \frac{1}{2}k(x_f^2 - x_i^2)$

when $x_i = 0$
$U_s = \frac{1}{2}kx^2$ Joul

- Conservation of Mechanical Energy

$E_{mec} = K + U$ (when F acting is conserfative)

$W_{net} = W_{cons}$

$(K+U)_1 = (K+U)_2$

Remark ⚠ :-

when the system consists of Conservative and non-conservative forces :-

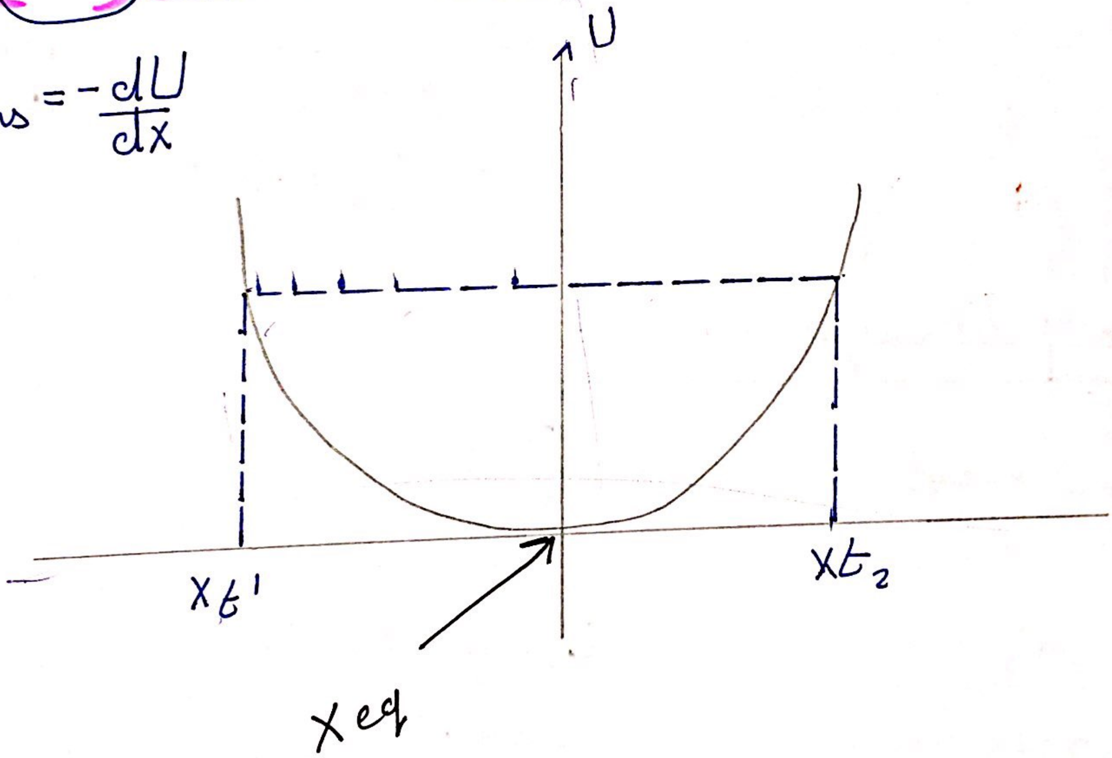
$$W_{cons} + W_{noncons} = \Delta K$$

$$-\Delta U + W_{non\,cons} = \Delta K$$

$$\boxed{W_{non\,cons} = \Delta E} = \Delta U + \Delta K$$

Reading a Potential Energy Curve

$$F_{cons} = -\frac{dU}{dx}$$



X_{t_1}, X_{t_2} :- Turning Points $\rightarrow K\,0$ $\rightarrow E=U$

X_{eq} :- equilibriam Point $\rightarrow U=0 \rightarrow E=K$ $\rightarrow E0$

Center of Mass

- The center of mass of a system of particles is the point that moves as though :-
  1- all of the system's mass were concentrated there
  2- all external forces were applied there

- **for a discrete system**

1 Dimen → the location of com :- $X_{com} = \dfrac{m_1 X_1 + m_2 X_2}{m_1 + m_2}$

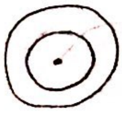
3 Dime → the location of com :- $\vec{r}_{com} = x_{com}\,\hat{i} + y_{com}\,\hat{j} + z_{com}\,\hat{k}$

- **for a continuous system** (solid body) uniform objects

3 Dimen → $\vec{r}_{com} = \dfrac{\int \vec{r}\, dm}{m_{tot}}$

- A uniform object has a uniform <u>density</u> or <u>mass</u> per unit volum which is:- $\rho = \dfrac{M}{V}$ ↷ mass, ↳ volume

Ps:- the center of Mass does not necessarly lies within the object : exp: a doughnut



com

- If an external force acts on a Com

$$\boxed{\vec{F}_{net} = M\,\vec{a}_{com}}$$

→ M: the total Mass of the system (constant)
→ $\vec{a}_{com}$: acceleration of Com (not the particles)
↳ $\vec{F}_{net}$: net force of all external forces that acts on system
→ internal forces are not included

linear Momentum

↳ of a particle

→ • $\vec{P} = m\vec{V}$ of a particle

Mass, velocity

• $F_{net} = \frac{d\vec{P}}{dt}$ Newton's 2nd law in terms of momentum

" The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force "

external

P changes when there is a net external force only

↳ of a system of particles

• $\vec{P} = \vec{P_1} + \vec{P_2}$ of particle 1, of particle 2

• $\vec{P} = M\vec{V}_{com}$

• $\vec{F}_{net} = \frac{d\vec{P}}{dt}$

Collision

brief collision :- in a small duration

P is conserved

$\vec{P}_i = \vec{P}_f$

and impulse

$J = \Delta P$

• in a single collision :- $\Delta P = J$

• in series of collisions :- n is the number of collisions

$J = -n\Delta P$

Ps: the minus sign indicates that J and ΔP has opposite Directions

$F_{avg} = \frac{n}{\Delta t} m \Delta V$

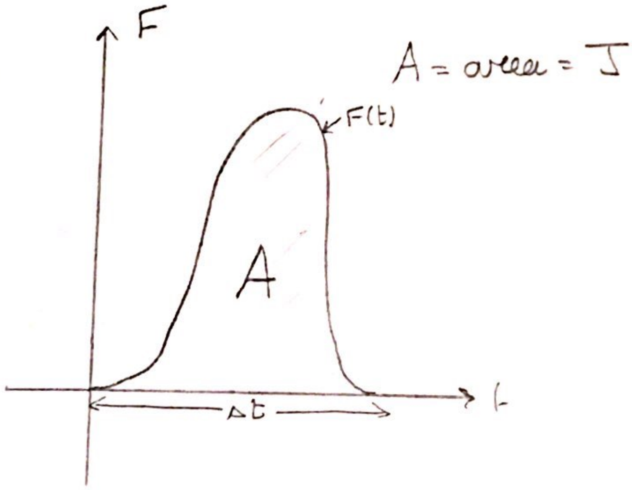
$= -\frac{\Delta m}{\Delta t} \Delta V \quad (\Delta m = nm)$

impulse

$$J = \int_{t_i}^{t_f} \vec{F}(t)\,dt$$

$$\Delta P = J$$

$$J = F_{avg}\,\Delta t$$



Kinetic Energy in collisions

elastic collision

K.E conserved

$$K.E_i = K.E_f$$

↳ we use this equation

$$(v_{1i} - v_{2i}) = -(\vec{v}_{1f} - \vec{v}_{2f})$$

Completely inelastic collision

- greatest loss of K.E
- the bodies stick together

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

inelastic collision

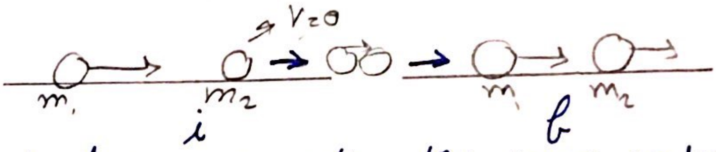
- K.E is not conserved

$$K.E_f \neq K.E_i$$

P.S :- $\vec{V}_{com}$ is constant before and after a collision because $\Sigma F_{net\,ext} = 0$

problems with a Projectile and an object :-

- The object (Target) is Stationary :- $V_{2i} = 0$
  - If $m_1 > m_2 \Rightarrow m_1$ moves forward
  - if $m_1 < m_2 \Rightarrow m_1$ bounces (يرتد للخلف)
  - if $m_1 = m_2 \Rightarrow$ body 2 stops after collision and body 1 moves with the same velocity as body 1



- The object (Target) is moving :- $V_{2i} \neq 0$

$$(m_1 v_1 + m_2 v_2)_i = (m_1 v_1 + m_2 v_2)_f$$

$$\left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2\right)_f = \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2\right)_f$$

Systems with Varying Mass : A Rocket

to find a :- $R \, v_{rel} = M a_{Total}$ $\Rightarrow$ $T = M a_{Total}$

R ← positive mass rate $= \frac{dM}{dt}$

T ↳ Thrust

to find v :- $V_f - V_i = V_{rel} \ln \frac{M_i}{M_f}$

How to solve Problems

Center of Mass

Those are The main ideas

1- If the problem is about a system of Particles we use

$X_{com} = \frac{x_1 m_1 + x_2 m_2}{(m_1 + m_2)}$ to find the position of COM

2- If the system is a uniform body then it's probably in the center

3- in some question X_{com} doesn't change cause there is no horizontal or vertical Force So we put $X_{com} = 0$

Example : P17 Page 231

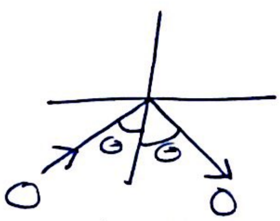
→ Dog moves But $X_{com} = 0$

4- If 2 bodies are projected the com will be moving in a projectile motion So we use equation of Constant acceleration to find V_f for each body and then we can find V_{com} and a_{com}

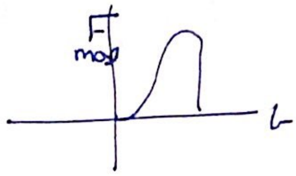
5 Linear Momentum

1- If the problem is about finding ΔP Then we find V_f and V_i

2- in these problems you should pay attention to θ's and Directions, The main trick is in it



3- in Problems with such plots remember that



Area under (F, t) curve $= J = \Delta P$

4- in explosions :-

$\Delta P = 0$

$P_i = P_f$

$\Sigma m(v) = m_1 v_1 + m_2 v_2 \dots$

5- sometimes you need to use $(K+U)_f = (K+U)_i$ specially if the question is talking about ~~a~~ h or when the question is about releasing a ball

Or if the question is talking about ~~a~~ spring

Then $K = U$

$\frac{1}{2} m v^2 = \frac{k x^2}{2}$

6- in Rockets problems :

you use : $v_f - v_i = v_{rel} \ln \frac{M_i}{M_f}$

or $R \, v_{rel} = M_{Tdt}$

$\frac{dM}{dt}$

Rotation

a Rigid body :- a body that can rotates without any change in its shape

a fixed axis :- the rotation occurs about an axis that does not more

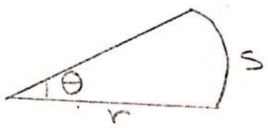
Pure rotation: angular motion

Angular Position

θ: is measured relative to the positive x-axis

$\theta = \frac{s}{r}$ → s: length of the circular arc; r: radius of the circle

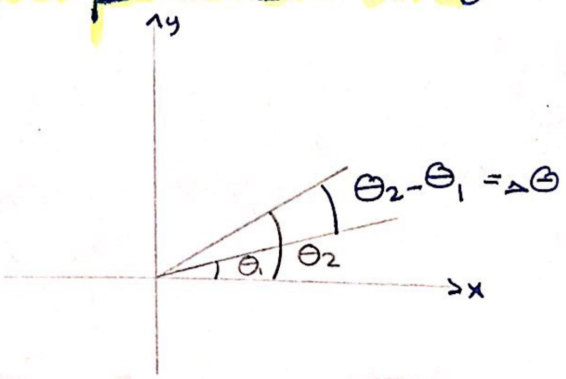
θ in radians



$$1 \text{ revolution} = 360° = \frac{2\pi r}{r} = 2\pi \text{ rad}$$

Angular displacement

$$\Delta\theta = \theta_2 - \theta_1$$



- $\Delta\Theta > 0 \Rightarrow$ If the movement is counterclockwise عكس عقارب الساعة

- $\Delta\Theta < 0 \Rightarrow$ If the movement is clockwise مع عقارب الساعة

Angular velocity

ω average:-

$$\omega_{avg} = \frac{\Theta_2 - \Theta_1}{t_2 - t_1} = \frac{\Delta\Theta}{\Delta t}$$

unit: rad/s
or rev/s

ω instantaneous

$$\omega_{inst} = \frac{d\Theta}{dt}$$

Magnitude of ω is called the angular speed

$\omega > 0 \Rightarrow$ if the movement is counterclock wise
$\omega < 0 \Rightarrow$ if the movement is clock wise

Angular acceleration

- If ω is not constant then the body han an angular acceleration

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

unit: rad/s^2
or rev/s^2

$$\alpha_{inst} = \frac{d\omega}{dt}$$

①
②

Are Angular Quantities vectors ?

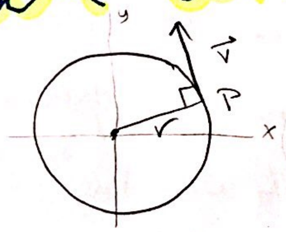
- Yes it is but we don't need to use vector notation Because we only have 2 cases : Counterclockwise and we use the plus sign (+) and for clockwise we use minus sign (−)

• Rotation with Constant Angular acceleration

- when α is constant you can use these three equations

1- $\omega = \omega_0 + \alpha t$
2- $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
3- $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

• Relating the linear and Angular variables



The position :- $S = \theta r$

The speed :- $\frac{ds}{dt} = \frac{d\theta}{dt} r$

$V = \omega r$

θ: in radians
ω in rad/s

If ω is constant $\longrightarrow$ uniform circular Motion

Then $T = \frac{2\pi r}{V} = \frac{2\pi r}{\omega r}$

$= \frac{2\pi}{\omega}$

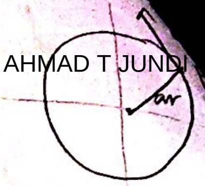
- linear speed is always tangental to the circular Path

The acceleration:

$$\frac{dv}{dt} = \frac{d\omega}{dt} r$$

$a_t = \alpha r$ tangental acceleration

α in rad/s^2



$a_r = \frac{V^2}{r} = \omega^2 r$ radial acceleration

Kinetic Energy of Rotation

$$K = \frac{1}{2} I \omega^2$$

ω^2 → ω is the same for all points

I → rotational inertia $I = \sum m_i r_i^2$

I is smaller → rotation is easier

- If we have a system of particles then we can calculate I by calculating rotational inertia for each particle in the system:

if we had a system of 2 particles Then:

$$I = I_1 + I_2$$
$$= m_1 r_1 + m_2 r_2$$

Knowing that m is the mass and r is perpendicular distance between the particle and the rotation axis

If we have a continuous system (نظام متصل يعني انه هناك عدد لانهائي من الاجسام الصغيرة (النقاط))

we use integration

$I = \int r^2 dm$ or we use **The parallel axis Theory**

This Theory is used when you have I_{com} (inertia for the Center of Mass) and h(which is the perpendicular distance between The given access and the access passing through the Com)

in one Condition: The given axis and the axis passing through Com should be **Parallel**

Note:-

I :
دائماً تكون
من
ثابت x الكتلة
x البعد تربيع

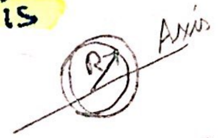
$\Rightarrow I = I_{com} + Mh^2$

↳ Mass of the body

I for continuous systems :-

- **Hoop about central axis**

$I = MR^2$

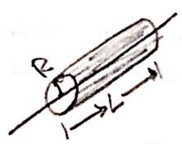


- **Annular cylinder (ring)**

$I = \frac{1}{2} M (R_1^2 + R_2^2)$

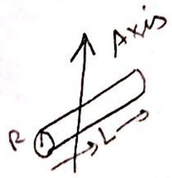
- **Solid cylinder (disk)**

$I = \frac{1}{2} MR^2$



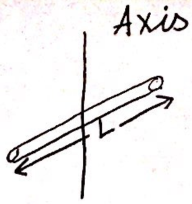
- **Solid cylinder (disk)**

$I = \frac{1}{4} MR^2 + \frac{1}{12} ML^2$



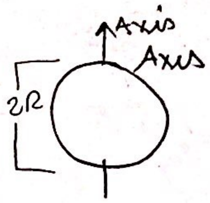
Thin rod about axis

$I = \frac{1}{12} ML^2$



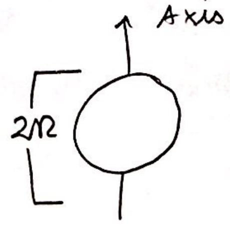
Solid sphere

$I = \frac{2}{5} MR^2$



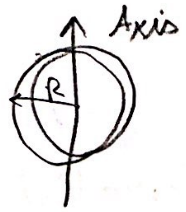
Thin spherical shell

$I = \frac{2}{3} MR^2$



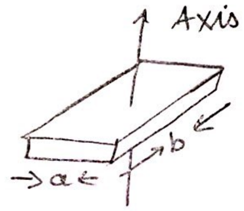
Hoop

$I = \frac{1}{2} MR^2$



Slab

$I = \frac{1}{12} M(a^2 + b^2)$



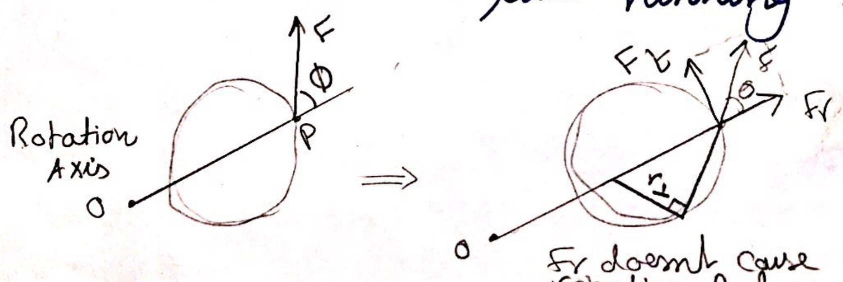
Torque (عزم الدوران)

- is a product of 2 factors (F and r)

$\tau = F r \sin\theta$

or $= F_t r$ — where F_t is the tangential component of $\vec{F}$

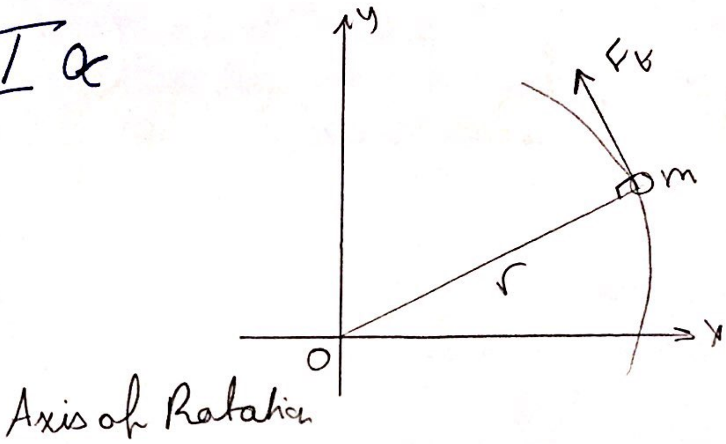
or $= r_\perp F$ — where $r_\perp$ is the moment arm (distance between axis of rotation and the extended line running through $\vec{F}$ (Fig 1)



Fr doesn't cause rotation But Ft does

Newton's second law for Rotation

$\tau_{net} = I\alpha$



Work & Rotational Kinetic Energy

$\xrightarrow{} $ Work

$W = \Delta K$

$= \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$

→ Angular velocity

or $W = \tau(\theta_f - \theta_i)$

and P (Power) $= \tau\omega$

How to salve Problems

1- If you had $\Theta(t)$ you can find angular velo (ω) , and α By differentiation

2- If the question gives you number of revalutions It equa Θ and you ~~can~~ can turn it into radians by multiplying it (X 2π)

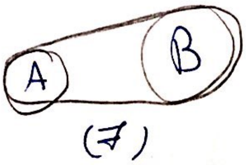
3- If the question asks for "average" acceleration or vel you use $\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$, $\omega_{avg} = \frac{\Delta\Theta}{\Delta t}$

4- always pay attention to units because it helps you ~~to~~ to know if you're steps are right or wrong

5- Θmax can be found using $\Delta\Theta = \Theta_{max} - \Theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$ if α is constant

6- If you're given V (speed) you can turn it into angular velocity using $\omega = \frac{V}{r}$

7- $V_A = V_B$ (careful V not ω)

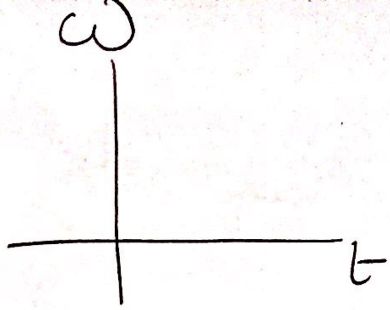


(7)

if the question tells you that the belt does not slip

In these graphs

α is the Slope



9- ~~In~~ questions that has particles and rods with mass

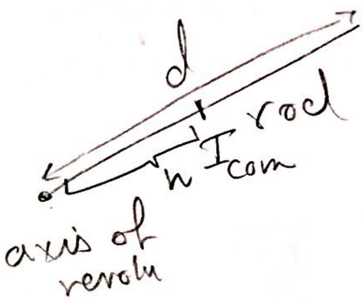
$$I = I_{particl_1} + I_{particle_2} + I_{rod_1} + I_{rod_2}$$

$I_{particle} = md^2$

I_{rod} : (you have to use The parallel axis Theory)

meaning :- $I_{rod} = I_{com} + Mh^2$

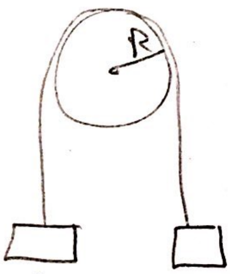
$$= \frac{1}{12} Md^2 + M\left(\frac{d}{2}\right)^2$$



10- When the question asks for the Torque pay attention to θ (It's not always the given angle)

11- In such Questions

a_t for the pulley = a of the box



12- If you're given F as a function you can find τ and then α using

$\tau = Fr$

and $\alpha = \frac{\tau}{I}$

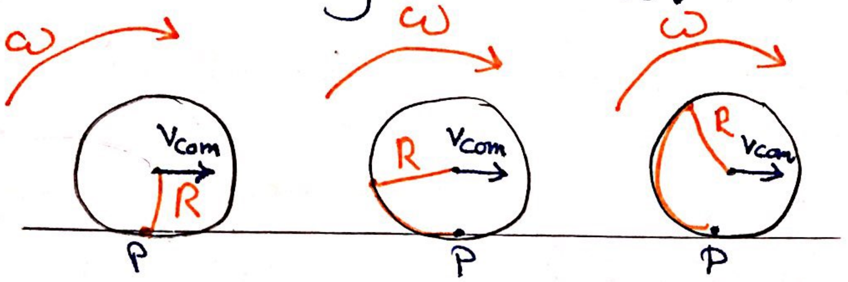
13- you can use the ~~Theory~~ Conservation of Energy Theory In a lot of questions Just pay attention !

Chap 11: Rolling, Torque and Angular Momentum ①

11-2: Rolling as Translation and Rotation Combined

Roll smoothly :- roll without slipping or bouncing

com



$S = R\theta$

$V_{com} = R\omega$

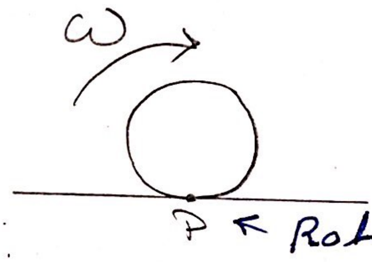
- when the object roll smoothly $V_{com} = R\omega$
- when V_{com} and ω are constants $\Rightarrow$ friction force $= 0$

11-3 The Kinetic Energy of Rolling

$K_{rolling} = \frac{1}{2} I_p \omega^2$

where $I_p = I_{com} + Mh^2$

$= I_{com} + MR^2$



← Rotation axis

$K_{rolling} = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M \underbrace{(R\omega)^2}_{(V_{com})^2}$

$$\boxed{K_{rolling} = \underbrace{\frac{1}{2} I_{com}\omega^2}_{K \text{ rotating around com}} + \underbrace{\frac{1}{2} M (V_{com})^2}_{K \text{ Translation of com}}}$$ Impor

11-4: The forces of Rolling

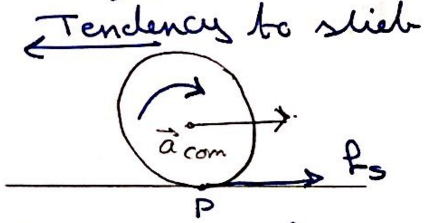
friction & Rolling

- If an object is Rolling at a constant speed Then it doesn't slide at the point P and frictional force = 0

- But If an object is Rotating in a variable speed (There is a net force acting on it) The net force causes acceleration $\vec{a}_{com}$ and An angular acceleration α which means the object tends to slide at P $\Rightarrow$ frictional force $\neq 0$ to resist the sliding

Smooth Rolling $\rightarrow$ the wheel does not slide $\rightarrow$ the force is a static frictional force f_s $\rightarrow$

$$\boxed{\vec{a}_{com} = \alpha R}$$



Rolling down a Ramp (an inclined plane)

- here: f_s is necessary to prevent sliding

$\tau_{net} = I\alpha$ $\quad$ τ for mg & N $= 0$ cause $\theta = 0$

$\tau_o = I\alpha$

$f_s R = I\alpha$

$$f_s = \frac{I\left(\frac{a_{com}}{R}\right)}{R} = \frac{I\, a_{com}}{R^2} \quad \text{.....} ①$$

The object is sliding: $M\vec{a}_{com} = Mg\sin\theta - f_s$ .. ②

$$\boxed{1 \text{ in } 2} \Rightarrow M\vec{a}_{com} = Mg\sin\theta - \frac{I\, a_{com}}{R^2}$$

lastely : $\boxed{a_{cm} = \dfrac{g \sin\theta}{1 + \dfrac{I}{MR^2}}}$

• where I depends on The geometry

11-7 Angular Momentum

• It's a vector quantity

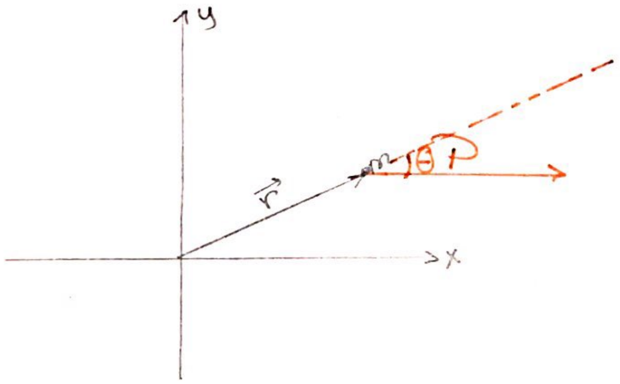
$$l = \vec{r} \times \vec{P}$$

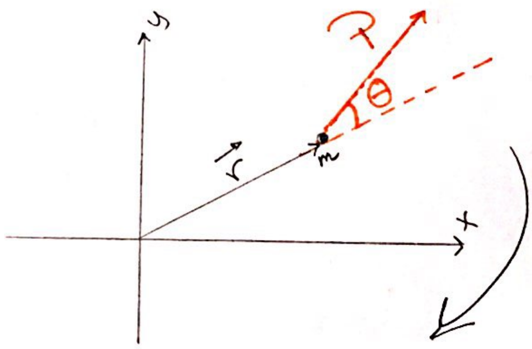
$\vec{r}$ → Position of The particle

$\vec{P}$ → كمية التحرك الخطية

$$\boxed{l = r m v \sin\theta} \qquad [l] = Kg \cdot m^2/s$$

θ → The smallest angle between r and P





11-8: Newton's second law in Angular form

$$\vec{\tau}_{net} = \frac{d\vec{l}}{dt} \quad \text{for a single particl}$$

$$\vec{F} \times \vec{r} = \frac{d\vec{l}}{dt}$$

11-9: The Angular Momentum of a system of Particles

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} \quad \text{(system of particles)}$$

where $\vec{L} = \vec{l}_1 + \vec{l}_2 + \vec{l}_3 - - - - \vec{l}_n$

11-10: The Angular Momentum of a Rigid body Rotating about a fixed axes

$$\vec{L} = I\vec{\omega}$$

11-11 Conservation of Angular Momentum

$$\boxed{L_i = L_f}$$

when $\tau_{net} = 0 \rightarrow \frac{d\vec{L}}{dt} = 0 \rightarrow \vec{L} = Cste$

L_i: net angular momentum at some initial time t_i

L_f: net angular momentum at some later time t_f

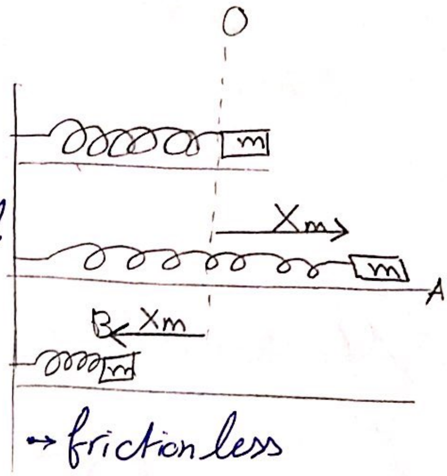
Remark:- The system should be ~~to~~ isolated

$$I_i \omega_i = I_f \omega_f$$

Chap 15: oscillations

- **Simple Harmonic Motion** :- الحركة التوافقية البسيطة

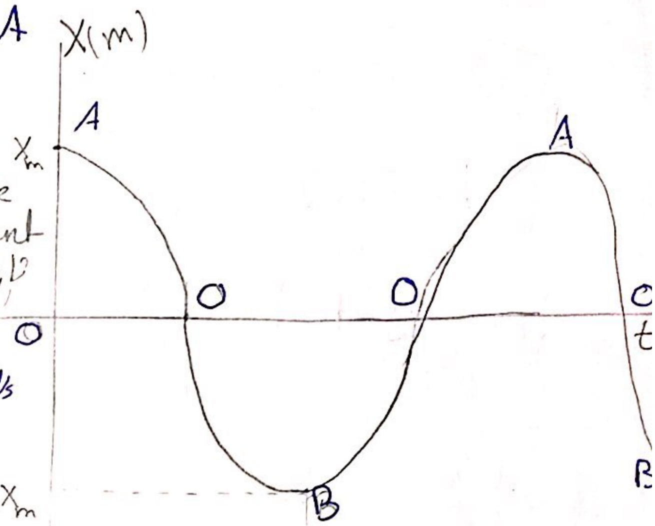
is the motion of a particle (m) Back and forth about the Origin



X_m is the maximum displacement = Amplitude

time $(A \to O \to B \to O \to A)$
↳ time $(A \to A)$ is The Periodic time T) الزمن الدوري

T = Periodic Time = $t_{A \to A}$



$$X(t) = X_m \cos(\omega t + \phi)$$

phase Constant ثابت الطور (rad)

ω = Angular frequency

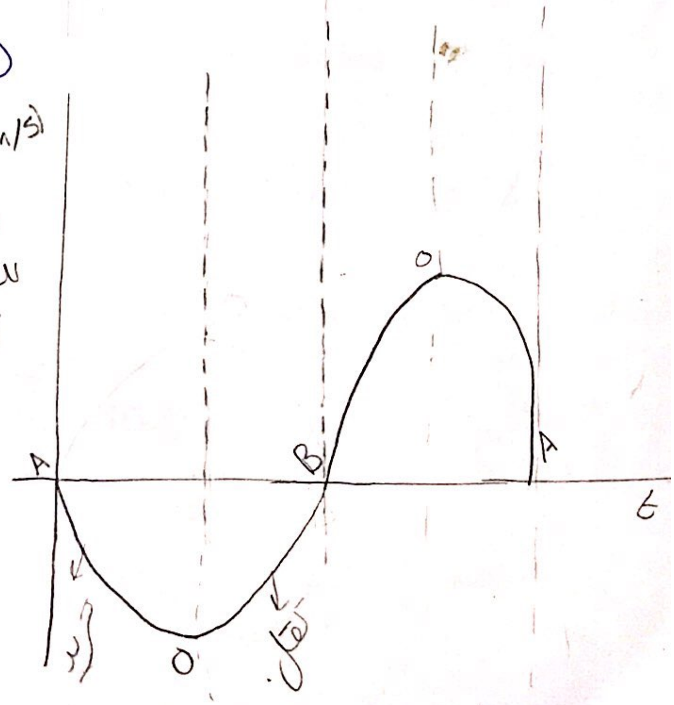
$$= \frac{2\pi}{T} \text{ rad/s} = 2\pi f \text{ rad/s}$$

- frequency $= \frac{1}{T}$ cycle/s = Hz

$$V(t) = \frac{dx}{dt} = X_m(-\sin(\omega t + \phi))\,\omega$$

$$= -\omega X_m(\sin(\omega t + \phi))$$

السالب للاتجاه فقط



$$a(t) = \frac{dv}{dt} = -\omega^2 X_m \cos(\omega t + \phi)$$

$$a_{max} = \omega^2 X_m \quad m/s^2$$

$$a_{max} = -\omega^2 X(t)$$

I Simple harmonic Motion

f, T, ω are Constants

$x(t)$ is changing
$v(t)$ is changing
$a(t)$ is changing
X_m (Amplitude is constant)

Potential Energy $= \frac{1}{2}Kx^2$ is changing
Kinetic Energy is changing ($\frac{1}{2}mv^2$)

→ Mechanical Energy is constant

→ force is changing ($-Kx$)

The Force law for SHM

$F = -Kx$ (conservative force)

$$m\frac{d^2x}{dt^2} + Kx = 0$$

$$\boxed{\frac{d^2x}{dt^2} + \frac{K}{m}x = 0} \quad \text{---- (1)}$$

from the equation: $\omega^2 = \frac{K}{m}$, $\omega = \sqrt{\frac{K}{m}}$

$x(t) = X_m\cos(\omega t + \phi)$ is the solution of equation (1)

$\omega = \frac{2\pi}{T}$ rad/s

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$ (s)

$\omega = 2\pi f$ rad/s

$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$ (Hz)

Energy in SHM

$$K(t) = \frac{1}{2} m v^2 = \frac{1}{2} m \left[-\omega x_m \sin(\omega t + \phi)\right]^2$$

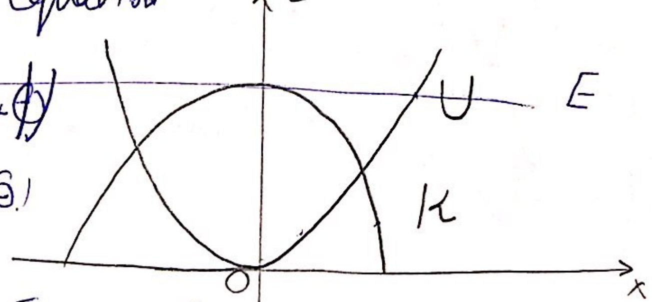
$$= \frac{1}{2} m \omega^2 x_m^2 \sin^2(\omega t + \phi)$$

$$U(t) = \frac{1}{2} K x^2 = \frac{1}{2} K \left[x_m \cos(\omega t + \phi)\right] = \frac{1}{2} K x_m^2 \cos^2(\omega t + \phi)$$

Put $\omega^2 = \frac{K}{m}$ in $K(t)$ equation

$$K(t) = \frac{1}{2} K x_m^2 \sin^2(\omega t + \phi)$$

$$U(t) = \frac{1}{2} K (x_m)^2 \cos^2(\omega t + \phi)$$

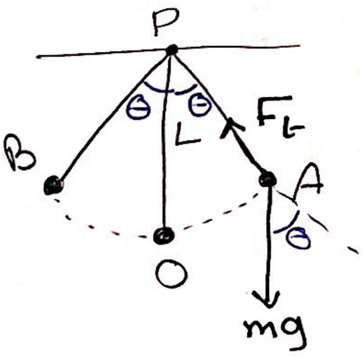


$$K(t) + U(t) = E(t) = \frac{1}{2} K x_m^2 \left[\underbrace{\left(\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)\right)}_{1}\right]$$

$$= \frac{1}{2} K x_m^2$$

$\Longrightarrow$ E is Constants $= \frac{1}{2} K x_m^2$

• simpl penelulum : البندل البسيط



$\vec{\tau} = \vec{r} \times \vec{F}$

$\theta = 0 \underset{Bcz}{\Rightarrow} \sin\theta = 0 \underset{Bcz}{\Rightarrow} \theta = 180$

$\vec{\tau} = F_t(L)\sin 180 + \bar{m}g(L)\sin\theta$

$\vec{\tau}_{net} = 0 - mgL\sin\theta = -mgL\sin\theta$

But $\vec{\tau}_{net} = I\alpha$

$\underline{I}\alpha = -mgL\sin\theta$

$\alpha = \frac{(-)mgL\sin\theta}{I}$ L: length.

$\alpha = \frac{-mgL\sin\theta}{mL^2}$

$\alpha = \frac{g\sin\theta}{L}$

$\frac{d^2\theta}{dt^2} + \frac{g\,mL}{mL^2}\sin\theta = 0$

$\frac{d^2\theta}{dt^2} + \frac{g\sin\theta}{L} = 0$ • it's not a simple Harmonic Motion

This should be θ

But $\sin\Theta \approx \Theta$ when $\Theta < 15$ (1)

$10^{0} = 0.174$ rad

$\sin 10 = 0.174$

$$\frac{d^2\Theta}{dt^2} + \frac{g}{l}\Theta = 0 \Rightarrow \text{SHM}$$

$\frac{g}{l} \longrightarrow \omega^2 = \frac{g}{l}$

$$\Theta(t) = \Theta_m \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{g}{l}} \quad , \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

$$, \quad T = \frac{2\pi}{\sqrt{l/g}}$$