* Motion Along a Straight line:

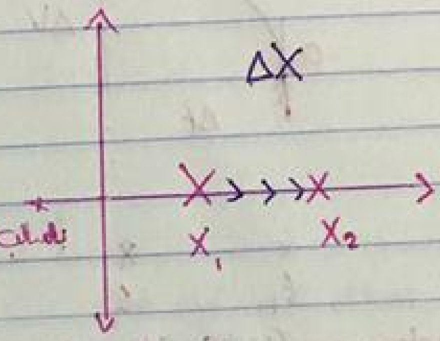
Chapter 2 :-

motion → Displacement = $X_2 - X_1$ ⇒ الموقع النهائي - الموقع الابتدائي

Chang in position $\Delta X = X_2 - X_1$

motion → time

position & displacement.



initial position = X_1 at t_1

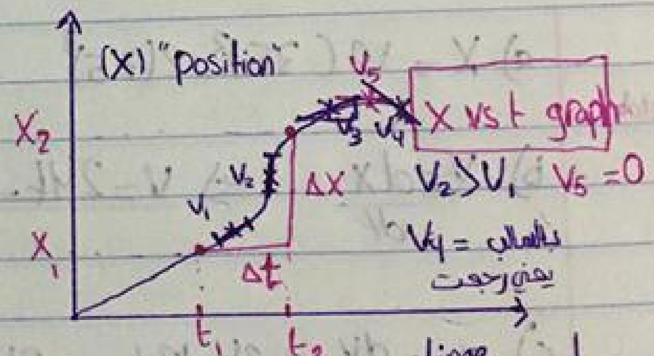
الموقع الابتدائي — الزمن

at t_2, find position = X_2

ΔX occurs during $\Delta t = t_2 - t_1$, at t_2 final position X_2

⇒ كمية متجهة Average Velocity = $\frac{\Delta X}{\Delta t}$ m/s "vector quantity"

Avarage Speed → Scaler quantity



"Savg"

⇒ Average Speed = $\frac{\text{distance}}{\Delta t}$.

$V_{avg} = \frac{\Delta X}{\Delta t}$

Instantaneous velocity: $= \lim_{\Delta t \to 0} \frac{\Delta X}{\Delta t}$ السرعة اللحظية

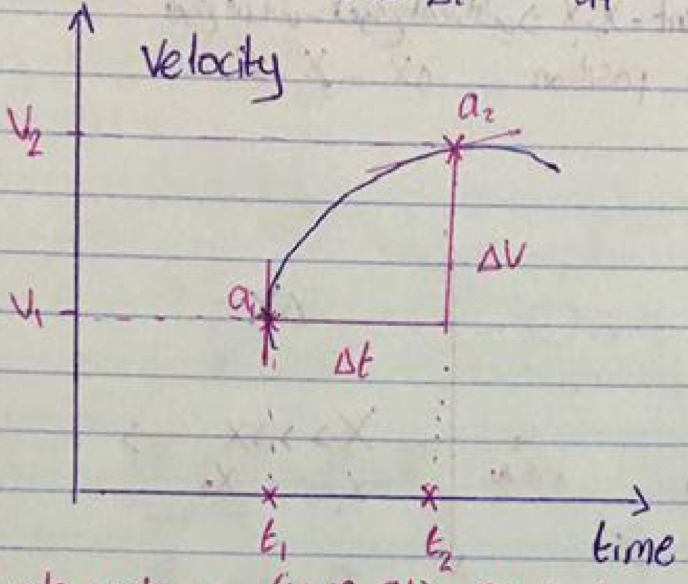
$V_{inst} = \frac{dx}{dt}$ (Slop at a certain point)

الميل

V_4 is (−)ve which mean it moves toward (−) X

* Avarage accelaration: متجهة $= \frac{\Delta V}{\Delta t}$ m/s^2 $\left(a_{avg} = \frac{\Delta V}{\Delta t}\right)$

تسارع لحظي $a_{instantenouse} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dv}{dt}$



$a_1 > a_2$
$V_1 < V_2$

⇒ Solve sample proplems page (16, 18, 21) Study it.

proplem 18 ⟹ $X = 12t^2 - 2t^3$, X in m, t in s

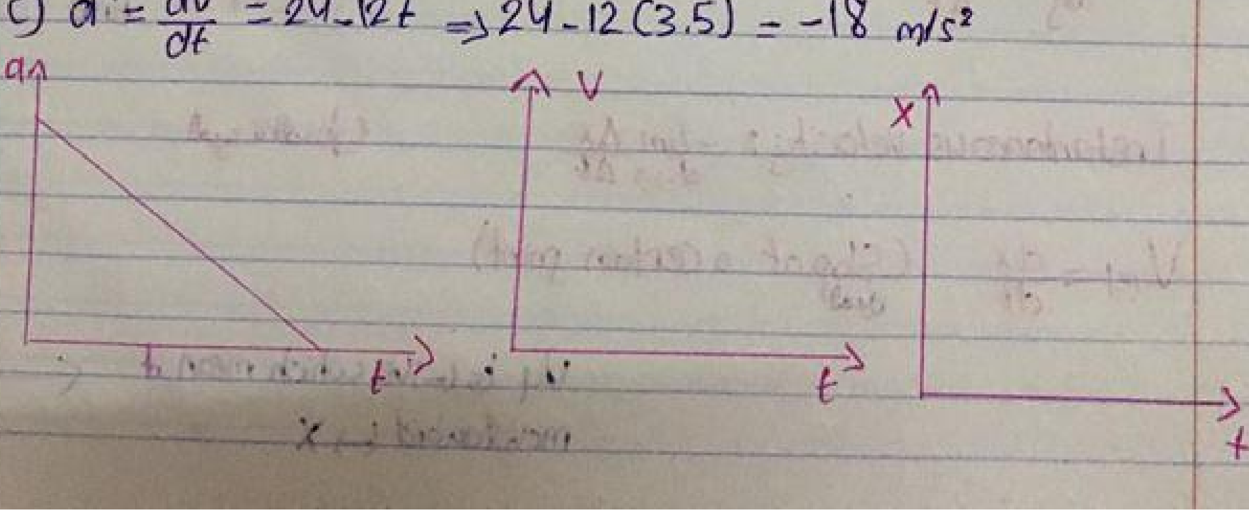
at $t = 3.5$ s find

a) $X = 12(3.5)^2 - 2(3.5)^3 = 61.25$ m

b) $V_{ins} = \frac{dx}{dt}$ | ⟹ $V = 24t - 6t^2$ ⟹ $24(3.5) - 6(3.5)^2$
$= +10.5$ m/s

c) $a = \frac{dv}{dt} = 24 - 12t$ ⟹ $24 - 12(3.5) = -18$ m/s²

sketch



good - d) X_{max} occurs at $V = \frac{dx}{dt} = 0$

- e) $24t - 6t^2 = 0 \Rightarrow 6t(4-t) = 0$

$4 - t = 0 \quad , \quad t = 4\ s$

$X_{max} = 12(4)^2 - 2(4)^3 = 64\ m$

(g) f) find V_{max}? g) time?

at V_{max}, $a = 0$

$a = 24 - 12t = 0 \quad t = 2s$

$V_{max} = 24(2) - 6(2)^2 = 24\ m/s$

do continue it at home.

h) when $t = 4 \quad a = 24 - 12 \times 4 = -24\ m/s^2$

i) $V_{avg} = \frac{\Delta X}{\Delta t} = \frac{54 - 0}{3 - 0} = 18\ m/s$

$x(3) = 12 \times (3)^2 - 2(3)^3 = 54$

$x(0) = 12(0)^2 - 2(0)^3 = 0$

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Q 81: $a = 5t$ at $t_1 = 2s$, $V_1 = +17 m/s$ at $t_2 = 4s$ find V_2:

$a = \frac{dv}{dt} \Rightarrow \int_{V_1}^{V_2} dV = \int_{t_1}^{t_2} dt \times a \Rightarrow V_2 - V_1 = \int_2^4 5t\,dt$

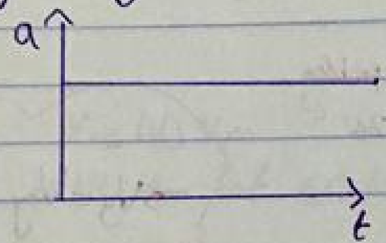
$V_2 - {+17} = \frac{5t^2}{2}\Big]_2^4$

$V_2 = 17 + 2.5(4)^2 - 2.5(2)^2$

$\therefore V_2 = +47 m/s$

Motion along a stright line with constant $\vec{a}$:

Constant $\vec{a}$

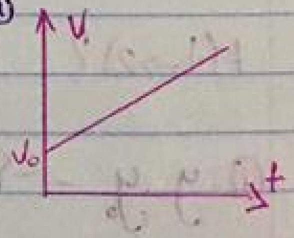


at t_1 velocity $= V_0$, position $= X_0$
at t_2 velocity $= V$, position $= X$

$a = \frac{dv}{dt} \Rightarrow \int_{V_0}^{V} dv = \int_{t_1}^{t_2} a\,dt \Rightarrow V - V_0 = a(t_2 - t_1)$, $t_2 - t_1 = t$

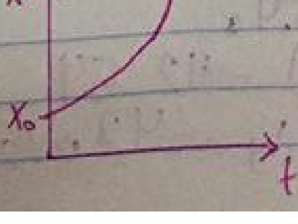
$V = V_0 + at$ ...①

Slop is @



from ① $\int_{X_0}^{X} dx = \int_{t_1}^{t_2} V\,dt \Rightarrow X - X_0 = \int_{t_1}^{t_2} (V_0 + at)\,dt$

$X - X_0 = V_0 t + \frac{at^2}{2} \Rightarrow X - X_0 = V_0 t + \frac{1}{2} a t^2$



$$V_{avg} = \frac{\Delta X}{\Delta t}$$

for constant a $\Rightarrow V_{avg} = \frac{V_0 + V}{2}$

important in Q42

$$\frac{\Delta X}{\Delta t} = \frac{V_0 + V}{2} \Rightarrow \boxed{X - X_0 = \left(\frac{V_0 + V}{2}\right) t}$$ from $V = V_0 + at$

$$X - X_0 = \frac{(V_0 + V)}{2} \times \left(\frac{V - V_0}{a}\right)$$

$$X - X_0 = \frac{V^2 - V_0^2}{2a}$$

$$\Rightarrow V^2 - V_0^2 = 2a(X - X_0)$$

Free fall:

solve problem 24-26

$a_y = 9.8$ m/s downward
$= -9.8$

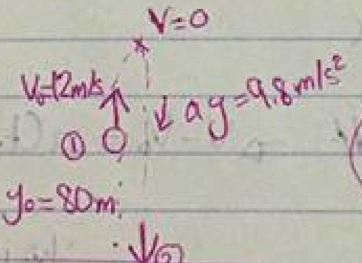
$$\Rightarrow V_y = V_{oy} + a_y t$$

$$\Rightarrow y - y_0 = V_{oy} t + \frac{1}{2} a_y t^2$$

$$\Rightarrow V_y^2 = V_{oy}^2 + 2a_y (y - y_0)$$

$$\Rightarrow y - y_0 = \left(\frac{V_{oy} + V_y}{2}\right) t$$

problem 49:
page 34



$t(1 \to 2)$? $V_2 = ??$

① $y - y_0 = -80 = V_{oy} t + \frac{1}{2} a_y t^2$

$$-80 = +12t + \frac{1}{2} \times -10t^2$$
$$5t^2 - 12t - 80 = 0$$

$t = 5.4$ s

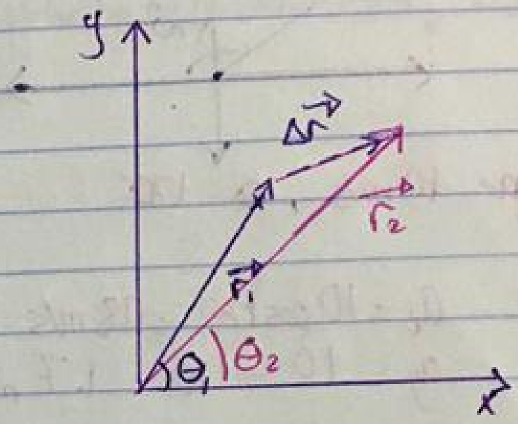
② $V_y = V_{oy} + a_y t \Rightarrow +12 + -10(5.4) = +12 - 54$

$V_y = -42$ m/s

Chapter 3: Vectors:

physical Quantities
- Scaler : magnitude → Speed, mass
- Vectores : magnitude and direction → position, displacement.

position is a vector quantity :.



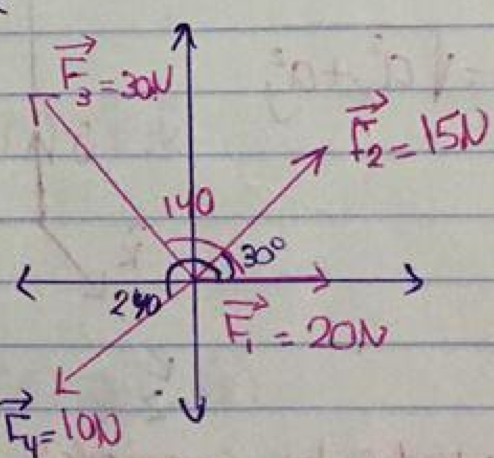
$\vec{r_1}$: magnitude $= r_1$
direction $= \theta_1$ ⇐ مع الاتجاه الموجب للسينات

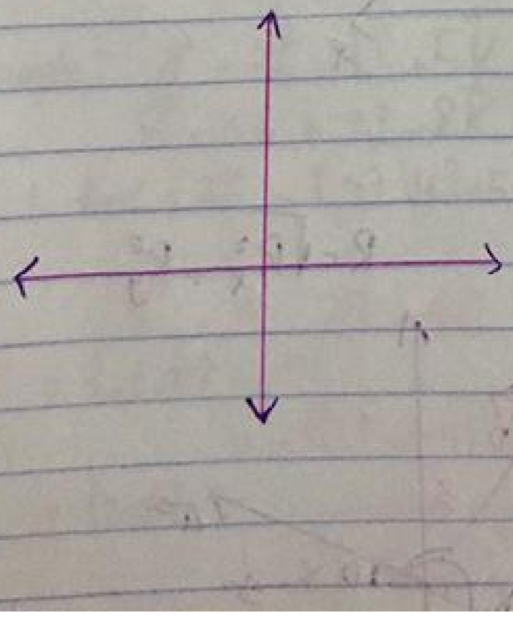
Displacement $\vec{\Delta r} = \vec{r_2} - \vec{r_1}$

→ Addition of vectors:

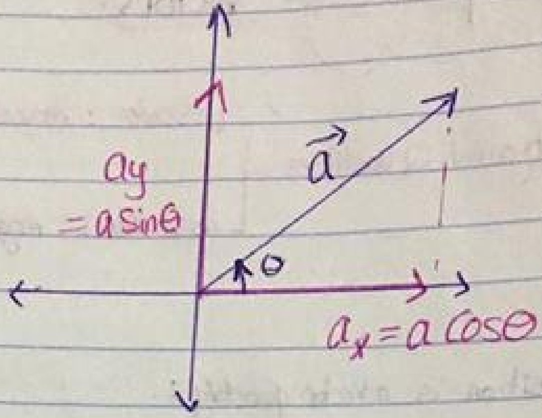
1) Graphical method : $\vec{r_2} = \vec{r_1} + \vec{\Delta r}$

example : find $\vec{F_1} + \vec{F_2} + \vec{F_4} + \vec{F_3} = R$

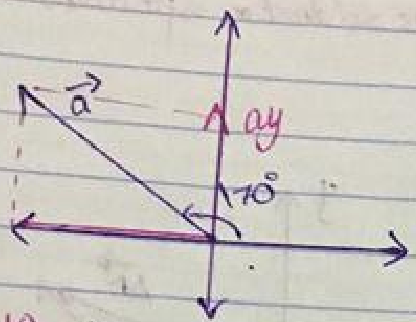




2) Componets of vectors:



example:



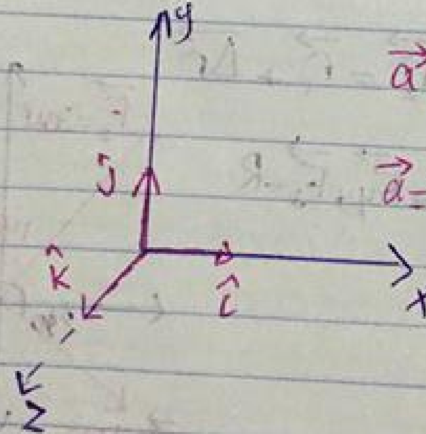
$a = 10\,m/s$, $\theta = 170°$

$a_x = 10\cos 170° = -9.8\,m/s$

$a_y = 10\sin 170° = 1.7\,m/s$

3) Unit vectors:-

$$a = \sqrt{a_x^2 + a_y^2}$$



$\vec{a} = a_x\hat{i} + a_y\hat{j}$

$\vec{a} = -9.8\hat{i} + 1.7\hat{j}$

4) Adding vectors by components:-

$$R = \sqrt{R_x^2 + R_y^2}$$

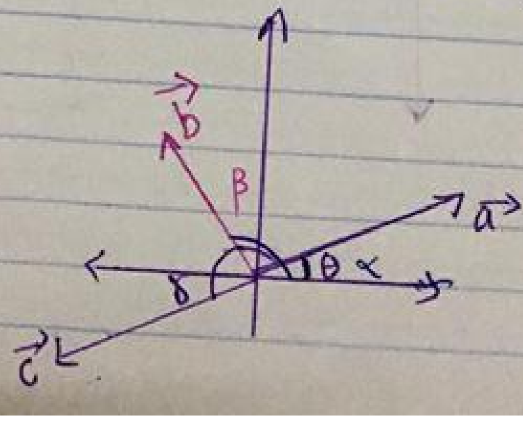
Find $\vec{a} + \vec{b} + \vec{c}$

$\vec{a} = a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j}$

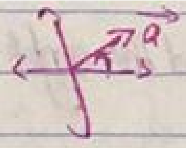
$\vec{b} = b\cos\beta\,\hat{i} + b\sin\beta\,\hat{j}$

$\vec{c}$

$\vec{R} = (R_x)\,\hat{i} + (R_y)\,\hat{j}$



$\vec{a} = 30\,m/s^2$, $30°$ counte clock wise $+x$



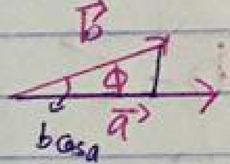
$\vec{a}\ 30\cos 30\,\hat{i} + 30\sin 30\,\hat{j}$

$\vec{a} = 26\hat{i} + 15\hat{j}$

* Vectors multiplication 1) multiply a vector by a constant $\rightarrow 2\vec{a}$, $-\vec{a}$

→ multiplying a vector by a vector:

* Scaler product (dot product)



$$\vec{a}.\vec{b} = ab\cos\phi$$

$$\vec{F}.\vec{\Delta r} = W = F\,\Delta r\cos\phi\ J$$

$$\hat{i}.\hat{j} = 0 = \hat{i}.\hat{k} = \hat{k}.\hat{j}$$

$$\hat{j}.\hat{j} = 1 \times 1\cos(0) = 1 = \hat{i}.\hat{i} = \hat{k}.\hat{k}$$

example: $\vec{a} = 3\hat{i} - 4\hat{j} + 5\hat{k}$
$\vec{b} = 6\hat{i} + 7\hat{j} - 8\hat{k}$

1. find $\vec{a}.\vec{b} = (3\hat{i} - 4\hat{j} + 5\hat{k}).(6\hat{i} + 7\hat{j} - 8\hat{k})$

$$\vec{a}.\vec{b} = 18 - 28 - 40 = -50$$

2. find a?

$$a = \sqrt{(3)^2 + (-4)^2 + (5)^2} = \sqrt{50} = 7.1$$

3. find b?

$$b = \sqrt{(6)^2 + (7)^2 + (8)^2} = \sqrt{149} = 12.3$$

3. find the angle between $\vec{a}$ & $\vec{b}$?

$$\vec{a}.\vec{b} = ab\cos\phi$$

$$-50 = 7.1 \times 12.3 \cos\phi$$

$$\cos\phi = -.57$$

$$\phi = 125^\circ$$

⇒ Vector product (cross product).

$$\vec{a} \times \vec{b} = \vec{c} \qquad = a \times b \times \sin\phi$$

$\vec{C}$ عمودي على a, b

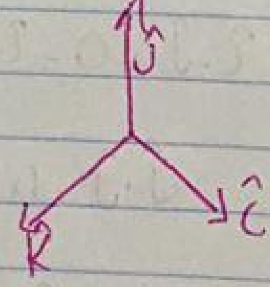
$$\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$$

متساويات بالمقدار متعاكسان بالاتجاه.

الخارج outward

الداخل inward.

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$



$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{i} = -\hat{k}$

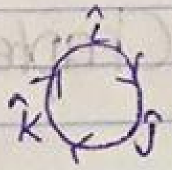
$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

$\hat{i} \times \hat{k} = -\hat{j}$

example: $\vec{a} = 3\hat{i} - 4\hat{j} + 5\hat{k}$
$\vec{b} = 6\hat{i} + 7\hat{j} - 8\hat{k}$



find $\vec{a} \times \vec{b}$:

$\vec{a} \times \vec{b} = (3\hat{i} - 4\hat{j} + 5\hat{k}) \times (6\hat{i} + 7\hat{j} - 8\hat{k})$

$12\hat{k} - 24(-\hat{j}) - 24(-\hat{k}) + 32\hat{i} + 30\hat{j} + 35(-\hat{i})$

$= -3\hat{i} + 54\hat{j} + 45\hat{k}$

Q38: $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$,

$\vec{B} = 3\hat{i} + 4\hat{j} + 2\hat{k}$

$\vec{C} = 7\hat{i} - 8\hat{j}$

$\hat{j} \rightarrow$ بالسالب

① $2\vec{A} \times \vec{B}$? $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 2 & 3 & -4 \\ 3 & +4 & +2 \end{vmatrix}$

$= \hat{i}(6 - -16) - \hat{j}(4 - 12) + \hat{k}(8 - 9)$

$2 \times (22\hat{i} + 8\hat{j} + 17\hat{k}) = 44\hat{i} + 16\hat{j} + 34\hat{k}$ ①

② $3\vec{C} = 21\hat{i} - 24\hat{j}$ ... ②

3) $3\vec{C} \cdot (2\vec{A} \times \vec{B}) \rightarrow$ ②.① $= 21 \times 44 - 24 \times 16 =$

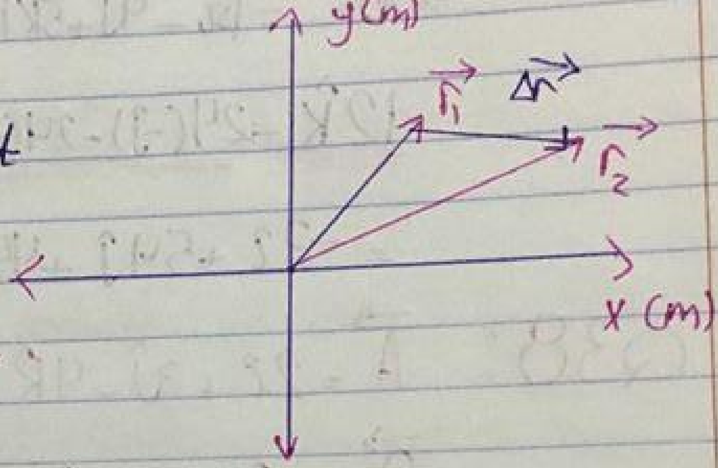
* Chapter 4: Motion in two Dimensions & 3 dimensions:

⇒ Positon & Displacement:

$$\vec{r_1} = x_1\hat{i} + y_1\hat{j} + Z_1\hat{k}$$

Δt

$$\vec{r_2} = x_2\hat{i} + y\hat{j} + Z_2\hat{k}$$



Displacement $\vec{\Delta r} = \vec{r_2} - \vec{r_1}$

$$\vec{\Delta r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

⇒ Avarage Velocity $= \vec{V_{avg}} = \dfrac{\vec{\Delta r}}{\Delta t} = \dfrac{\Delta x}{\Delta t}\hat{i} + \dfrac{\Delta y}{\Delta t}\hat{j} + \dfrac{\Delta Z}{\Delta t}\hat{k}$

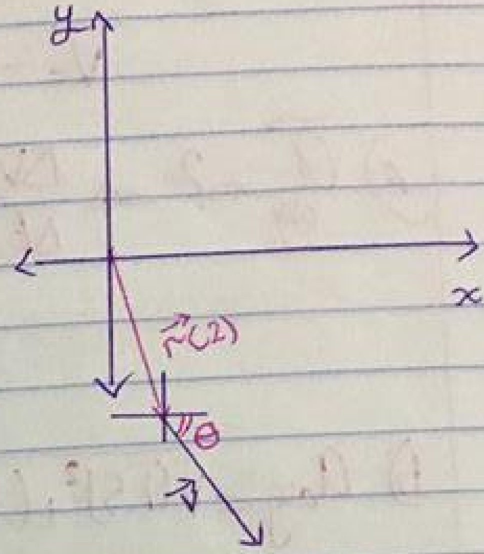
⇒ Instantaneous Velocity ⇒ $\vec{V}_{ins} = \dfrac{\vec{dr}}{dt} = \dfrac{dx}{dt}\hat{i} + \dfrac{dy}{dt}\hat{j} + \dfrac{dz}{dt}Z$

⇒ Avarage acceleration ⇒ $\vec{a}_{avg} = \dfrac{\vec{\Delta V}}{\Delta t}$

⇒ $a_{inst} = \dfrac{\vec{dv}}{dt}$

⇒ Solve Sample problem page (59, 62, 63)

Q11 :- $\vec{r} = (2t^3 - 5t)\hat{i} + (6 - 7t^4)\hat{j}$ m



a) at $t = 2s$ find $\vec{r}$

$\vec{r} = (2 \times 2^3 - 5(2))\hat{i} + (6 - 7(2)^4)\hat{j}$

$\vec{r} = 6\hat{i} - 106\hat{j}$ m

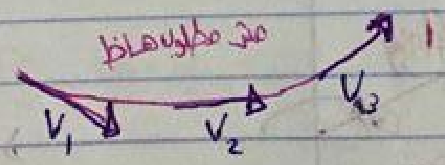
b) $\vec{V} = \frac{d\vec{r}}{dt} = (6t^2 - 5)\hat{i} - 28t^3\hat{j}$

→ find $\vec{V}$ at 2s $= (6 \times 2^2 - 5)\hat{i} - 28 \times 2^3\hat{j} = \underline{19\hat{i} - 224\hat{j}}$ m/s

c) $\vec{a} = \frac{dv}{dt} = 12t\,\hat{i} - 84t^2\hat{j}$ m/s

→ find $\vec{a}$ at $t = 2s$ → $\vec{a} = 24\hat{i} - 336\hat{j}$

d) path



find the direction of $\vec{v}$ at $t = 2s$

$\tan\theta = \frac{-336}{24}$ ⟹ $\theta = -85$ → V always of the tangent of the path

$\theta = -85$ clockwise with +x

Q14:

$\vec{V_0} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ m/s

$\Delta t = 4s$

$\vec{V} = -2\hat{i} - 2\hat{j} + 5\hat{k}$ m/s

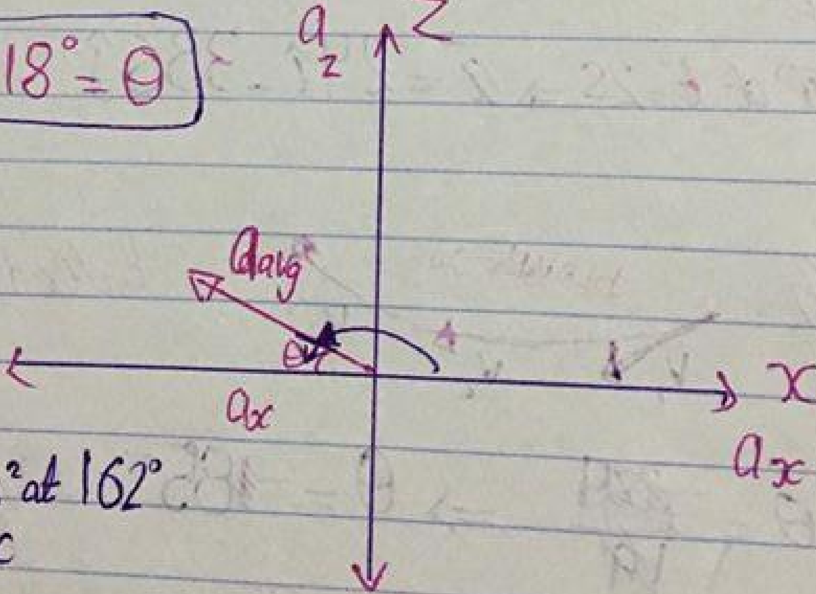
a) $\vec{a}_{avg} = ? \Rightarrow \frac{\Delta \vec{V}}{\Delta t} = \frac{\vec{V} - \vec{V_0}}{4} = \frac{-6\hat{i} + 2\hat{k}}{4}$

$\vec{a}_{avg} = -1.5\hat{i} + .5\hat{k}$ m/s

b) $a_{avg} = \sqrt{(1.5)^2 + (.5)^2} \Rightarrow a_{avg} = 1.58$ m/s²

c) the angle between $\vec{a}_{avg}$ & $+x$?

$\tan\theta = \frac{.5}{-1.5} \Rightarrow \boxed{-18° = \theta}$



$a_{avg} = 1.58$ m/s² at 162° with $+x$

* Motion in 2D:

projectile motion: المقذوفات

- it's a motion in two D → x-motion, horizntal.
  → y-motion, Vertical

→ Horizntal motion:

$V_{0x} = V_0 \cos\theta$, $a_x = 0$.

after a time = t

$V_x = V_{0x} = V_0 \cos\theta$

$x - x_0 = (V_0 \cos\theta_0) t$

→ Vertical motion.

$V_{0y} = V_0 \sin\theta_0$, $a_y = g$

after a time = t

$V_y = V_{0y} + a_y t$

$V_y = V_0 \sin\theta_0 - gt$

$y - y_0 = V_{0y} t + \frac{1}{2} a_y t^2 \Rightarrow y = V_0 \sin\theta_0 t + \frac{1}{2} \times -g t^2$

at $t \Rightarrow \vec{V} = V_x \hat{i} + V_y \hat{j}$

trajectory equation:

$$y = V_0 \sin\theta_0 t - \frac{1}{2} g t^2$$

$$x = V_0 \cos\theta_0 t$$

$$y = V_0 \sin\theta_0 \left(\frac{x}{V_0 \cos\theta_0}\right) - \frac{1}{2} g \frac{x^2}{V_0^2 \cos^2\theta}$$

$$y = (\tan\theta_0) x - \left(\frac{g}{2V_0^2 \cos^2\theta_0}\right) x^2$$

find the horizantal range:

$$R = V_x t = V_0 \cos\theta_0 \, t_{fligt}$$

to find it $\rightarrow y - y_0 = V_{0y} t + \frac{1}{2} a_y t^2$

$$0 = +V_0 \sin\theta_0 t_f + \frac{1}{2} x - g t_f^2$$

$$0 = t_f \left[V_0 \sin\theta_0 - \frac{1}{2} g t_f\right]$$

$$t_f = \frac{2V_0 \sin\theta_0}{g}$$

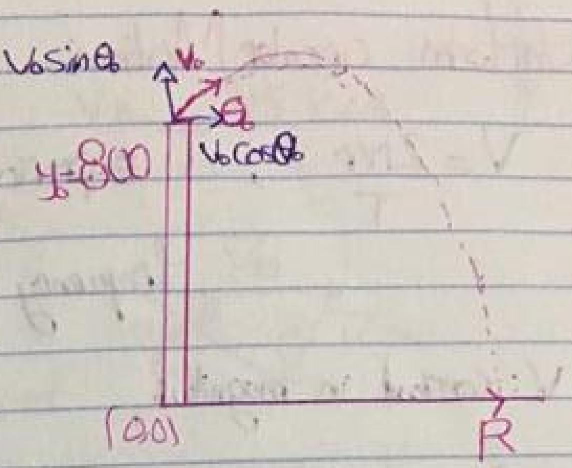
$$R = (V_0 \cos\theta_0)\left(\frac{2V_0 \sin\theta_0}{g}\right)$$

$$R = V_0^2 \frac{\sin 2\theta_0}{g}$$

the biggest (max) rang is when $2\theta_0 = 90°$

$$\boxed{\theta_0 = 45°}$$

example:



$V_0 = 20\ m/s$

$\theta = 53^\circ$

$t_f = ??$

$V_{0x} = 20\cos 53 = 12\ m/s$

$V_{0y} = 20\sin 53 = 16\ m/s$

$$y - y_0 = V_{0y}t + \frac{1}{2}a_y t^2$$

$$0 - 800 = +16\,t_f + \frac{1}{2}\times -10\,t_f^2$$

$$t_f = 8.8\,s$$

$$R = V_0\cos\theta_0\,t_f$$

$$R = 12\times 8.8 = 106\,m$$

at $t = 5s$ find $\vec{v}$ and $\vec{r}$

$V_x = V_{0x} = 12\ m/s$

$V_y = V_{0y} + a_y t \Rightarrow +16 + -10\times 5 = -34\ m/s$

$\vec{V_5} = 12\hat{i} - 34\hat{j}$

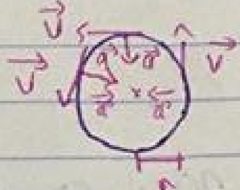
$X = (V_0\cos\theta)t \Rightarrow 12\times 5 = 60m$

$y - y_0 = V_{0y}t + \frac{1}{2}a_y t^2 \Rightarrow y - 800 = +16\times 5 + \frac{1}{2} -10(5)^2,\ \boxed{y = 7{,}55}$

Uniform circular Motion:

$V = \frac{2\pi r}{T}$, T = preriodic time

frequency $= \frac{1}{T}$



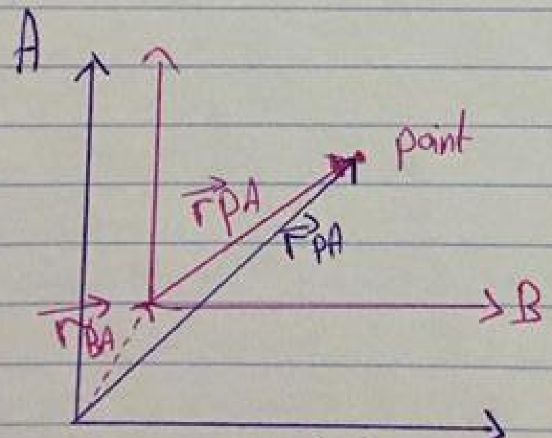
V : constant in magnitue

$\vec{V}$: is charging $\Rightarrow$ acceleration.

$a = \frac{V^2}{r}$ centrepital acceleration.

$\vec{a} \perp \vec{V}$ always

Relative Motion:



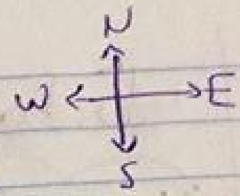
observer A at rest
observer B moves
at con $\vec{V}_{BA}$

$\left(\vec{r}_{pA} = \vec{r}_{pB} + \vec{r}_{BA}\right) \frac{d}{dt}$

$\frac{d}{dt}\left(\vec{V}_{pA} = \vec{V}_{pB} + \underbrace{\vec{V}_{BA}}_{con}\right)$

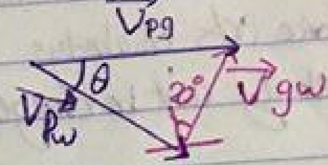
$\vec{a}_{pA} = \vec{a}_{pB}$

Page 75 Sample problem..



$V_{pw} = 215$ Km/h

$\vec{V}_{pg} \Rightarrow V$ with plane with respect to the ground.



$\vec{V}_{pw} \Rightarrow V$ with plane w.r.t wind

$\vec{V}_{gw} = V$ wind w.r.t g

$$\vec{V}_{pg} = \vec{V}_{pw} + \vec{V}_{wg}$$

$$V_{pg}\hat{i} = 215\cos\hat{i} + 215\sin\theta(-\hat{j}) + 65\cos 20\hat{j} + 65\sin 20\hat{i}$$

$$V_{pg}\hat{i} = (215\cos\theta + \cancel{2}65\sin 20)\hat{i} + (65\cos 20 - 215\sin\theta)\hat{j}$$

$$V_{pg} = 215\cos\theta + \underbrace{65\sin 20}_{22} \quad x \ldots (1)$$

$$y \Rightarrow 0 = 61 - 215\sin\theta \Rightarrow \sin\theta = \frac{61}{215} \Rightarrow \theta \simeq 16.5°$$

from (1) $V_{pg} = 215\cos 16.5 + 22 = 228$ Km/h

5: Force & motion.

* Newton's first law of motion.

لقوة لتغير حالة الجسم

محصلة القوى = 0 ، الجسم ساكن أو متحرك بسرعة ثابتة "اتجاهاً"

→ if no net force acts on a body's the body's velocity can't be change

We understand → 1- $\vec{F}_{net} = 0$ → at rest → $\vec{V} = 0$

$\vec{V} = 0$, $V =$ cont

2- the body can't change its state.

3. This law called "the law of inertia".

Newton's 2nd law:

$\vec{F}_{net}$ causes the change in $\vec{V}$.

$\vec{F}_{net}$ causes the change in body's state.

→ $\vec{F}_{net} = m\vec{a}$, $\vec{a} = \frac{\Delta \vec{V}}{\Delta t}$, $\frac{Kg.m}{s^2}$

2nd law

We understand :- 1 mass resist the effect of external force. "inertial mass"

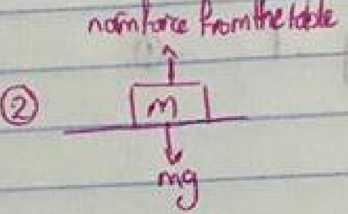
2- $F = Kg.m/s^2$ = Newton.

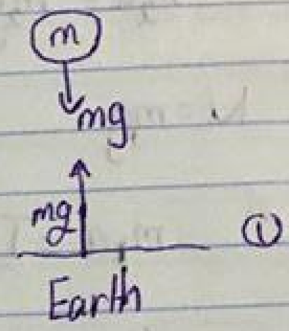
3. if $\vec{F}_{net}$ is constant, mass constant → $\vec{a}$ is constant.

$\vec{F}_{net} = \dfrac{d\vec{P}}{dt}$, $\vec{P} = m\vec{v}$ (momentum)   كمية التحرك

Newton's third law:

⇒ For every action there's a reaction equals in magnitued and opposite in direction.





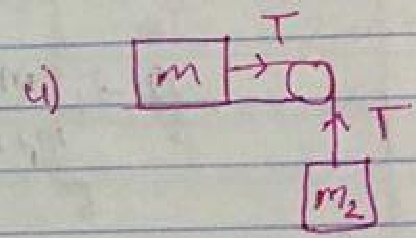
Forces:

1_ weight = mg ⇒ $\vec{W} = m\vec{g}$

2. normal force.

3. friction force   قوة الاحتكاك

4. Tension force   الشد



5. spring force = $-Kx$

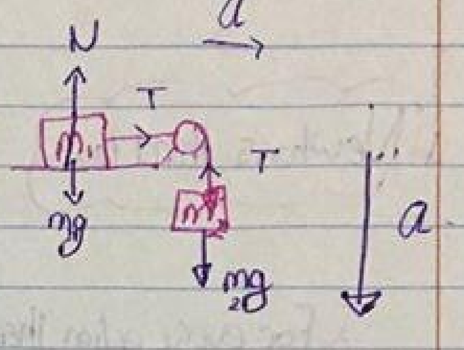
6. electric force

7. magnatic force

Applications :

* Sample problem page 100:

[1] frictionless . لا يوجد احتكاك



$m_1 = 3.3$ Kg , $m_2 = 2.1$ Kg. a? T?

$\vec{F}_{net} = m\vec{a}$

$m_2 a = m_2 g - T$ ...①

$N = m_1 g$

$m_1 a = T$ ...②

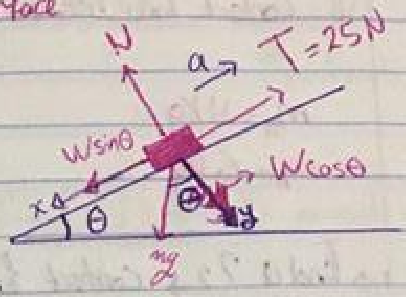
①② $(m_1 + m_2) a = m_2 g$

$a = \frac{m_2}{m_1 + m_2} g = \boxed{3.8\, m/s^2}$

from 2 $T = m_1 a \Rightarrow \boxed{T = 13N}$

Sample problem page 102

2 inclined plane   السطح أملس , frictionless surface



$\theta = 30°$ N?

$m = 5\ kg$ a?

no motion along the axis ⊥ surface

Normal force => عمودية على السطح

T => موازية للسطح

$\Rightarrow N - W\cos\theta = 0$

$N = W\cos\theta = \boxed{42N}$

المركبة القريبة من الزاوية cosθ

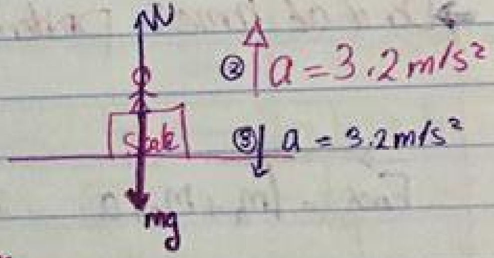
المركبة البعيدة عن الزاوية sinθ

$\Rightarrow W\sin\theta = 24.5\ N$

$ma = T - mg\sin\theta \Rightarrow \boxed{a = 0.1\ m/s^2}$

3 elevator motion.

Sample problem/page 103



* find the reading of the scale of the following cases.

1- the elevator at rest. V=0 $N - mg = 0 \Rightarrow N = mg \Rightarrow \underline{708N}$ = scale reading

2. the elevator moves upward with acceleration = $3.2\ m/s^2$. $ma = N - mg \Rightarrow N = mg + ma$

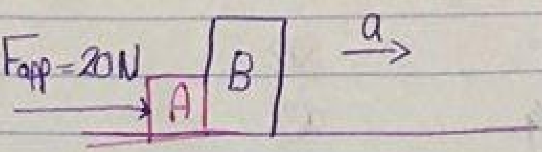
→ the scale reading = 939N $\underline{N = 939\ N}$

3. elevator moves downward with acceleration = $3.2\ m/s^2$. $ma = mg - N \Rightarrow N = mg - ma$

$\underline{N = 477N}$

4 Contact force between two contacted masses.

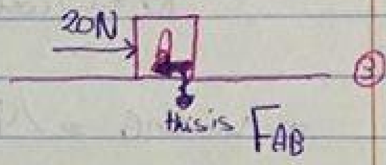
$m_a = 4\,kg$
$m_b = 6\,kg$



1→ find a ? 2→ contact force between them.

$\vec{F}_{net} = m\vec{a} \Rightarrow 20 = (m_a+m_b)a$

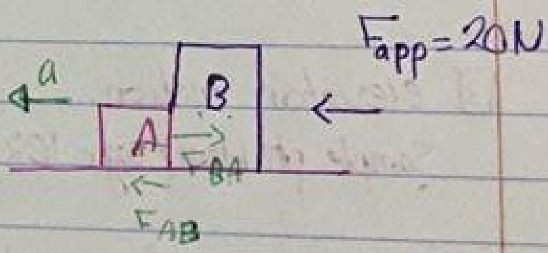
$\boxed{a = 2\,m/s^2}$



for B → $F_{BA} = m_B a \Rightarrow \underline{12N}$

$m_A a = 20 - F_{AB} \Rightarrow F_{AB} = 20 - m_A a = \underline{12N}$

→ Do it at home: problem: → find a?
→ find F_{AB}?



$\vec{F}_{net} = (m_A+m_B)a = 2\,m/s$

$F_{\cancel{B}AB} = a m_A = 2\times 4 = 8\,N$

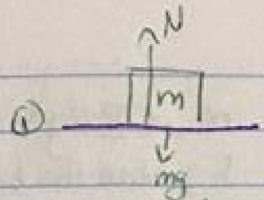
$a m_B = 20 - F_{BA}$

$\underset{-20}{12} = \underset{-20}{20} - F_{BA} \Rightarrow F_{BA} = 8$

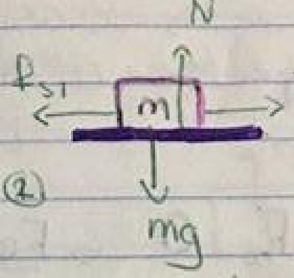
Chapter 6 :-

Friction Force:

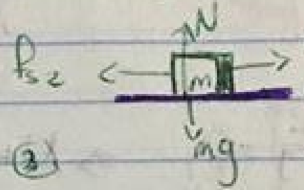
f_s = Static friction force



① no motion
at no friction

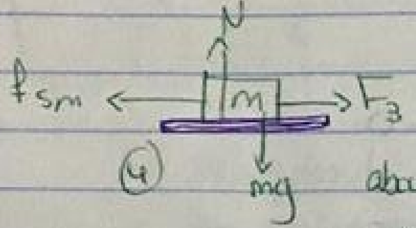


② no motion



③ no motion

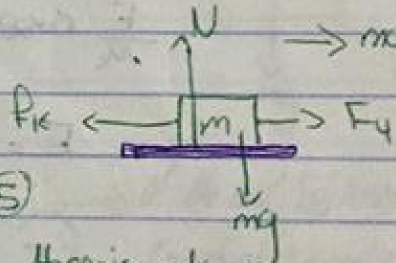
$F_2 = f_{s2}$



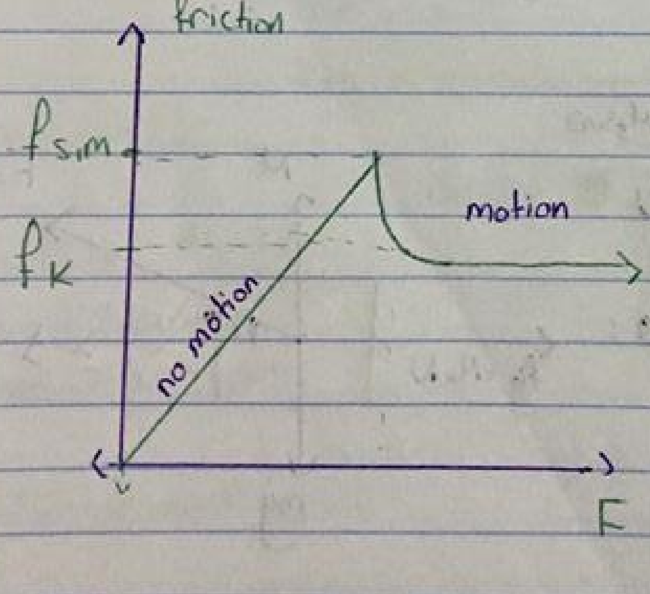
④ about to move

④ $f_{sm} = \mu_s N$, μ_s = coefficient of static friction معامل الاحتكاك السكوني

⑤ $f_K = \mu_K N$, μ_K = coefficient of Kinetic friction معامل الاحتكاك الحركي



⑤ there is motion



$\mu_s > \mu_K$ always

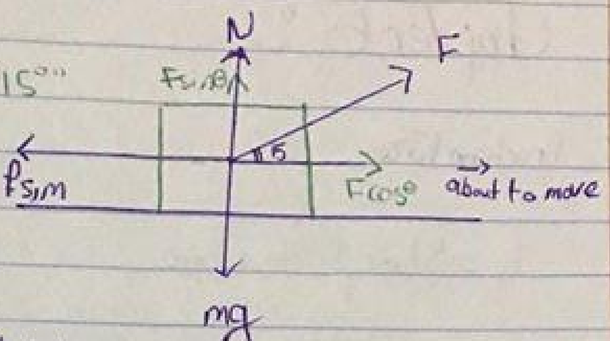
⟸ لا توجد وحدة قياس لمعامل الاحتكاك

6-11) $m = 68 kg$

F; above the horizontal "15°"

$\mu_s = 0.65$

$\mu_k = 0.35$



a) $\sum F_x = 0 \Rightarrow \frac{F\cos\theta}{\mu_s} = \frac{\mu_s N}{\mu_s} \quad ...①$

$\sum F_y = 0 \Rightarrow N + F\sin\theta - mg = 0$

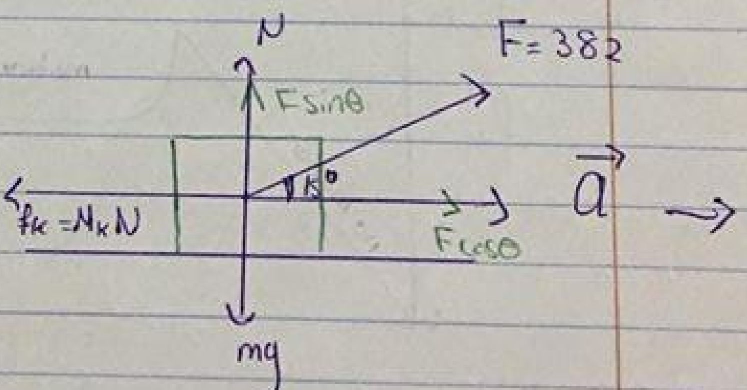
$N = mg - F\sin\theta \quad ②$

② in ① $\quad F\cos\theta = \mu_s(mg - F\sin\theta)$

$\frac{F}{\mu_s}\cos\theta + F\sin\theta = mg$

$F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}$

$F = 328 N$



b) $F = 328$, $\theta = 15°$ find a:

$\sum F_x = ma_x$

$F\cos\theta - \mu_k N = ma_x \quad ...①$

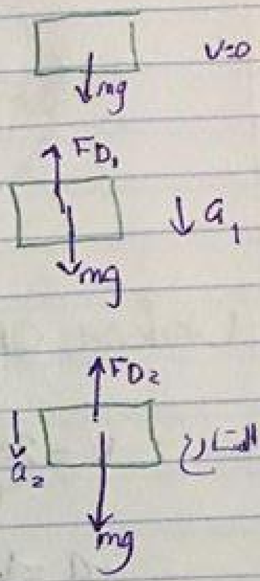
$\sum F_y = 0 \Rightarrow N + F\sin\theta - mg = 0 \quad ...②$

$a_x = 1.3 m/s^2$

6-14) The drag force and terminal speed :-

Drag force => a force from fluid medium.

=> Drag force causes the velocity.



$F_D \propto V^2$

مقاومة الهواء

$F_D = \frac{1}{2} C \rho A V^2$

$a_1 > a_2$

السرعة تزداد التسارع يقل

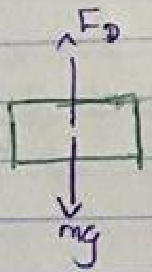
$0 < C < 1$

$\rho \equiv$ medium density (air)

$A \equiv$ cross section area for $\boxed{m}$

when $mg = F_D$ $\boxed{m}$ moves at a constant speed

$V_{terminal}$ أكبرسرعة $= \sqrt{\frac{2mg}{C \rho A}}$

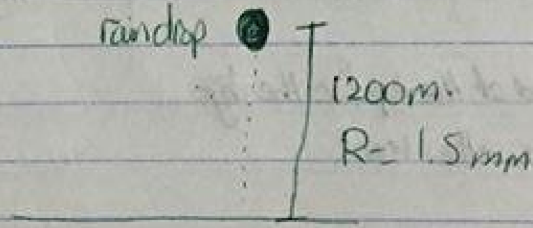


$a = 0$

$F_D = mg$

=> at the end $\boxed{m}$ moves at a constant speed = Terminal speed.

Sample / p123:



$C = 0.6$

$\rho_{air} = 1.2 \ kg/m^2$

$\rho_{water} = 1000 \ kg/m^3$

a) find the terminal speed.

$m_{rain\,drop} = \frac{\rho_{water} 4 \pi R^3}{3}$

$A = \pi R^2$

$V = \sqrt{\frac{2mg}{C \rho_a A}}$

$= 7.4 \ m/s \Rightarrow 27 \ Km/h$

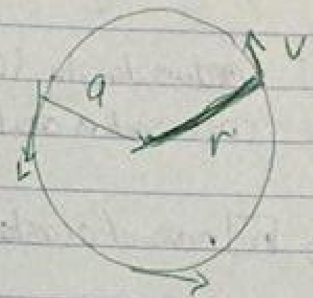
b) find V in the absence of F_D

$$F_D = 0$$

$$V_y^2 = V_y^2 + 2a_y \Delta y$$

$$V_y = 153 \text{ m/s} = 550 \text{ km/s}$$

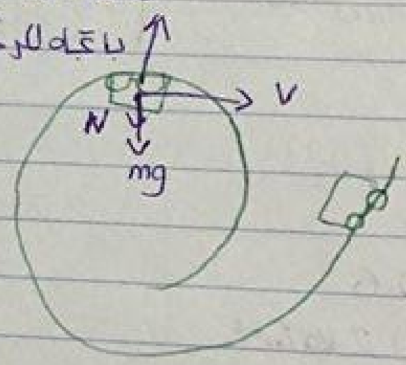
Uniform circular motion:



$$V = \frac{2\pi r}{T}$$

$$a = \frac{V^2}{r}$$

$F = \frac{mV^2}{r}$ centrepital ~~acceleration~~ force



Sample problem page 125:

find min Speed at the top for the ~~bys~~ bicycle to remain in the track

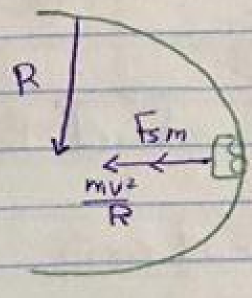
$$\vec{F}_{net} = mg + N = ma \rightarrow \frac{\cancel{m}V^2}{R}$$

$$mg + N = \frac{mV^2}{R} \Rightarrow mg + \cancel{N}^{0} = \frac{mV_{min}^2}{R}$$

$$V_{min} = \sqrt{Rg} \qquad R = 2.7 \text{ m}$$

$$V_{min} = 5.1 \text{ m/s}$$

Cars in flat turn:

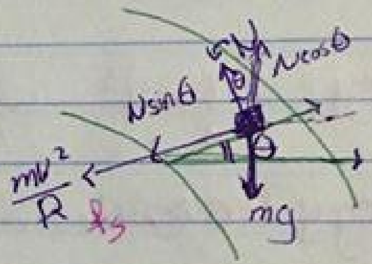


$$\frac{mV^2}{R} = \mu_s N$$

$$\frac{mV^2}{R} = \mu_s mg$$

$$V = \sqrt{\mu_s Rg}$$

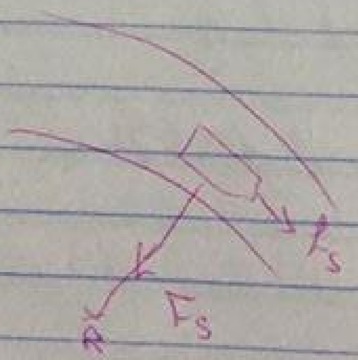
Cars in a banked turn:



$$N\cos\theta = mg \quad \ldots ①$$

$$N\sin\theta = \frac{mV^2}{R} \quad ②$$

$$\frac{②}{①} \Rightarrow \tan\theta = \frac{V^2}{Rg} \Rightarrow V = \sqrt{Rg\tan\theta}$$

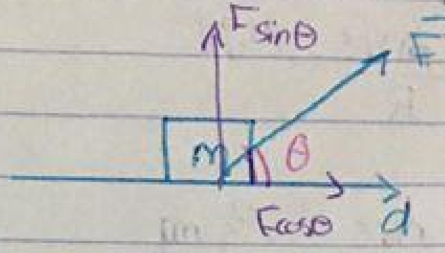


$f_s = ma$

$F_s = m\frac{V^2}{r}$

7. Kinetic Energy & Work

Work $= \vec{F} \cdot \vec{d}$. For any force



$W = F\,d\cos\theta$

N.m = Joul

$W = (F\cos\theta)\,d$

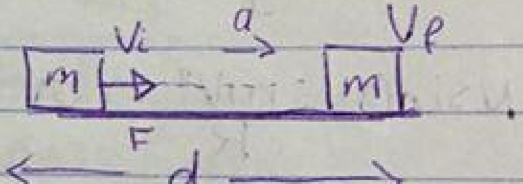
[illegible]

d = displacement

work → (+) تزيد طاقة حركية , work done by the force increases K.E
↳ (−) تسحب طاقة حركية , ,, ,, ,, ,, decreasing K.E

* Kinetic energy $= \frac{1}{2}mV^2$ ⇒ Joul

Work - Energy Theorem:



$V_f^2 = V_i^2 + 2a\Delta x$

$V_f^2 = V_i^2 + 2ad \Rightarrow V_f^2 - V_i^2 = 2a(d)$

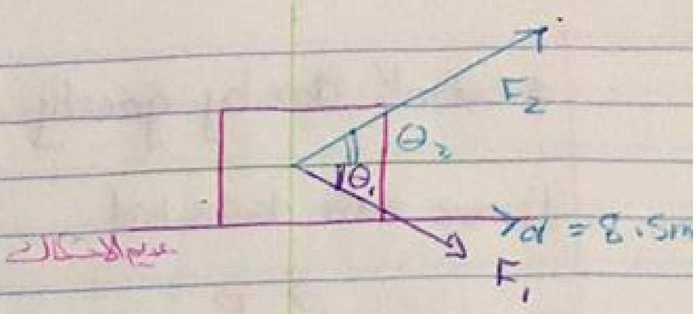
$\frac{1}{2}mV_f^2 - \frac{1}{2}mV_i^2 = ma(d)$

$\boxed{W = \Delta K}$

work done by resultant force

[illegible]

* Sample problem page 145:



$m = 225\,kg$
$F_1 = 12N$
$\theta_1 = 30°$ under horizontal.
$F_2 = 10N$
$\theta_2 = 40°$

الزاويتين أقل من 90: الشغل موجب

→ find work done by ⇒ F_1? F_2? N? w?

$W_N = N d \cos 90 = 0$

$W_g = mg d \cos 90 = 0$

$W_{F_1} = F_1 d \cos 30 \Rightarrow W_{F_1} = +88.33\,J$

$W_{F_2} = F_2 d \cos 40 \Rightarrow W_{F_2} = +65.11\,J$

$W_{net\,force} = W_g + W_N + W_{F_1} + W_{F_2}$

$= 0 + 0 + 88.33 + 65.11 = 153.44\,J$

→ find V_f?

$W_{net\,force} = K_f - K_i \Rightarrow 153.44 = \frac{1}{2}(225)V_f^2$

$\boxed{V_f = 1.12\ m/s}$

$F_{net} = ma$

يمكننا ايجاد
التسارع من هذه العلاقة

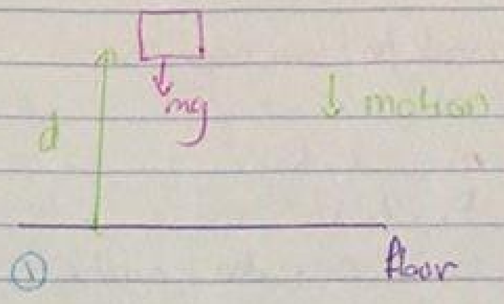
+ Work done by gravity:

1) mass moves downward

$W_g = \vec{F} \cdot \vec{d}$

$mg\,d\cos\theta$

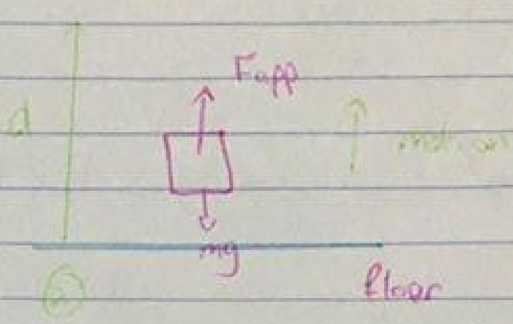
$\boxed{+mgd}$



2) mass moves upward

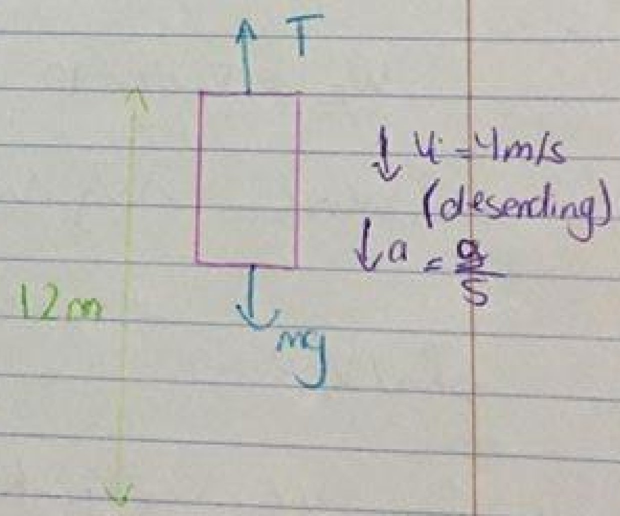
$W_g = \vec{F} \cdot \vec{d}$

$= mg(d)\cos(180) = \boxed{-mgd}$



Sample problem page 148:

$m = 500$ kg
$d = 12$ m



a) W_g?

$W_g = mg\,d(\cos 0)$

$W_g = +5.88 \times 10^4$ J

b) W_T?

from Newton's second law $\Rightarrow$ $ma = mg - T$

$W_T = (T)(d)(\cos 180)$

$\boxed{-4.7 \times 10^4 \text{ J}}$

$T = mg - ma$

$T = mg - \frac{mg}{5} \Rightarrow \boxed{\frac{4mg}{5} = T}$

$\boxed{= 3920 \text{ N}}$

o find K_f ?

$W_{net\,force} = K_f - K_i$

$W_g + W_T = K_f - \frac{1}{2}(500)(4)^2$

$5.88 \times 10^4 - 4.7 \times 10^4 = K_f - 4000$

$K_f = 1.58 \times 10^4$ Joul

Sample / p.146

$\vec{F} = 2\hat{i} - 6\hat{j}$ N

$\vec{d} = -3\hat{i}$ m

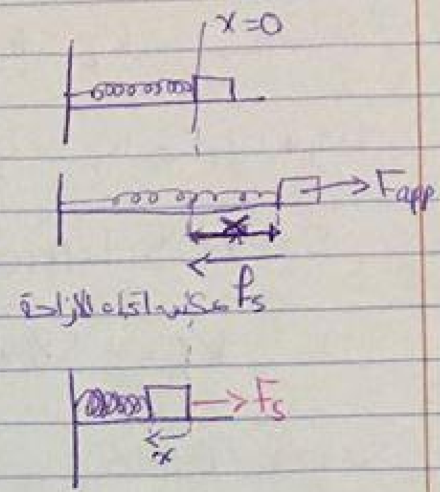
find W_F?

$W_F = \vec{F} \cdot \vec{d} \Rightarrow (2\hat{i} - 6\hat{j}) \cdot (-3\hat{i})$

$\boxed{W_F = -6J}$

* Work done by variable force :

I Spring force قوة المرونة



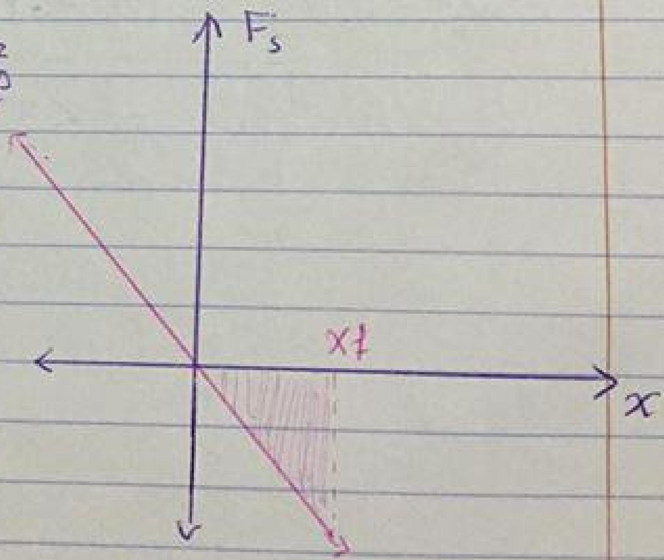
$F_s = -Kx$ "Hook's law"

$$W_s = \int_{x_i}^{x_f} F_x \, dx$$

$$W_s = -K \int_{x_i}^{x_f} x \, dx$$

$$W_s = -K \frac{x^2}{2} \Big]_{x_i}^{x_f} \Rightarrow W_s = \frac{-K}{2} \left[x_f^2 - x_i^2 \right]$$

$$W_s = \frac{1}{2} K x_i^2 - \frac{1}{2} K x_f^2$$



note the following :-

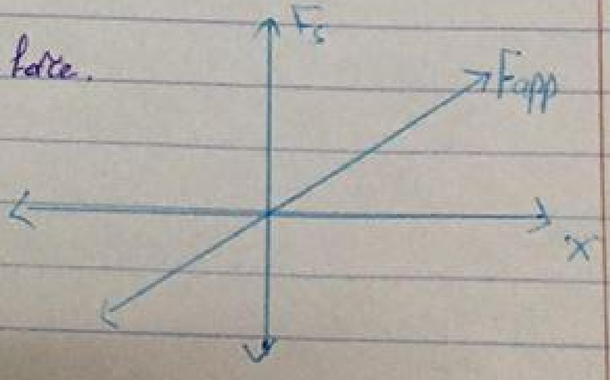
* if $x_i = 0 \Rightarrow W_s = -\frac{1}{2} K x_f^2$

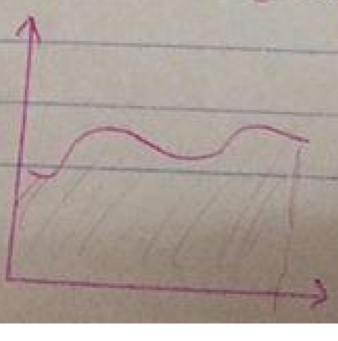
$\Rightarrow$ Work = the area under the graph

$x_i = 1, x_f = 1, W_s = 0$

* Work done by one dimensional variable force.

= the area under F_s Vs x curve





*Solve sample problem page 151 + 154.

38, 39 solve it

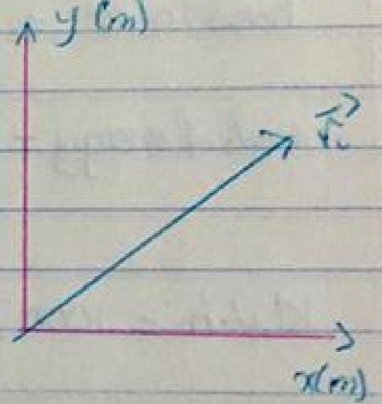
*Worke done by three dimentional variable force.

$$W = \int \vec{F} \cdot \vec{dr} \Rightarrow F = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$\vec{dr} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$W = \int \vec{F} \cdot \vec{dr} = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

⇒ Sample problem page 155.



$$\vec{F} = (3x^2\hat{i} + 4\hat{j}) \text{ N} \qquad W = ??$$

$$(\vec{r_i}) = (2,3) m \quad , \quad \vec{r_f} \ (3,0) m$$

$$W = \int_{x_i}^{x_f} F_x \, dx + \int_{y_i}^{y_f} F_y \, dy$$

ع الاعلى الى الاسفل

$$= \int_{2}^{3} 3x^2 dx + \int_{3}^{0} 4 dy$$

$$x^3 \Big]_2^3 + y \Big]_3^0 \Rightarrow W = 7 \text{ Joul.}$$

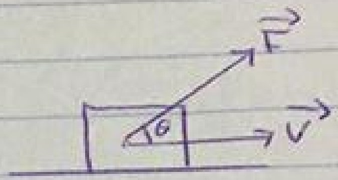
Power: the time rate of doing work.

$$P_{avg} = \frac{\Delta W}{\Delta t} \qquad J/s = watt$$

$$P_{instant} = \frac{dW}{dt} \qquad watt$$

$$P_{inst} = \frac{d}{dt}(\vec{F}\cdot\vec{r}) \ , \quad \vec{r} = \text{displacement}$$

$$P_{inst} = \vec{F}\cdot\vec{V} = FV\cos\theta$$

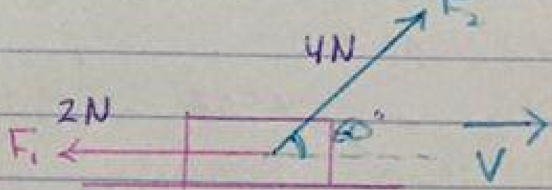


horse power $\Rightarrow$ hp = 746 watt

Units of energy = Joul = power . time
kW . h

$$kW.h = 1000\frac{J}{s} * 3600s = 3.6\times10^{6}J$$
$$3.6MJ$$

sample problem page 156:



find P_{inst} for each force.

في لحظة → Certain instant

3 = V

$$P_1 = \vec{F}.\vec{V} = F.V\cos\theta = (2)(3)\cos 180° = -6\,watt$$

$$P_2 = (4)(3)\cos 60 = +6\,watt$$

$$P_{net} = P_1 + P_2 = 0 \qquad \text{الجسم يمشي بسرعة ثابتة}$$

[7-41] $m = 3\,kg$. $x = 3t - 4t^2 + t^3$ $W?$ $t_1 = 0s \rightarrow t_2 = 4s$

$W_{net} = K_f - K_i = \frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2$

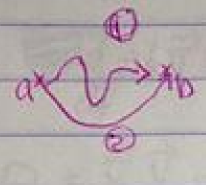
$V_x = \frac{dx}{dt} = 3 - 8t - 3t^2$

$V_x(0) = 3\,m/s$, $V_x(4) = 3 - 32 + 48 = 19\,m/s$

8- Potential energy & conservation of energy:

Forces:

1- conservative forces → ① work done by F_{cons} between 2 points don't depend on the path.

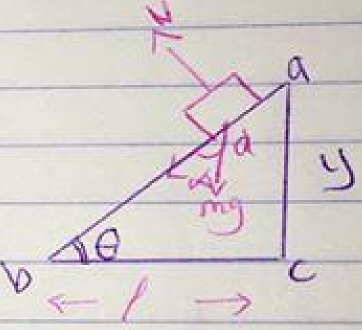


$$W_{cons1} = W_{cons2}$$

② work done by F_{cons} around a closed path = 0

$$W_{\substack{a\to a \\ cons}} = 0$$

(eg) → mg , -K



Force of gravity (mg)

⇒ Work done by gravity $a \to c$ مباشرة

$$= mg \cdot y \cos\theta = mgy$$

$$Wg(a\to b\to c) = Wg_{(a\to b)} + Wg_{(b\to c)}$$

$$mgd\cos\alpha + mg\,l\cos 90$$

$$wg(a\to b\to c) = mg\sin\theta\, d + 0$$

$$mg\frac{dy}{d} = mg\,y$$

$$W_{g(a\to c)} = Wg(a\to b\to c) = mgy$$

$$Wg(a\to b\to c\to a) = 0 = mgy + -mgy$$

$W_{cons} = -\Delta U$, U = potential energy

$$\int F_{x\,cons}\, dx = -\Delta U$$

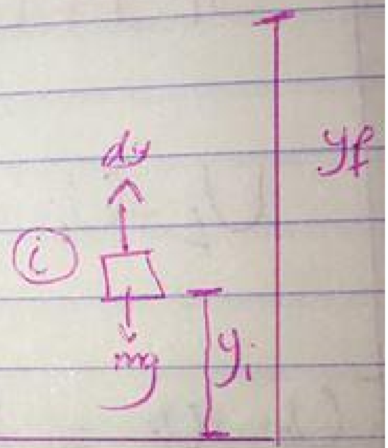
$$\Delta U = -\int_{x_i}^{x_f} F(x)_{cons}\, dx$$

$$U_f - U_i = -\int_{x_i}^{x_f} F_{cons}^{(x)}\, d(x)$$

Gravitational potential energy:

$$U_f - U_i = -\int_{y_i}^{y_f} mg\, dy \cos(180)$$

$$= mg\int_{y_i}^{y_f} dy$$

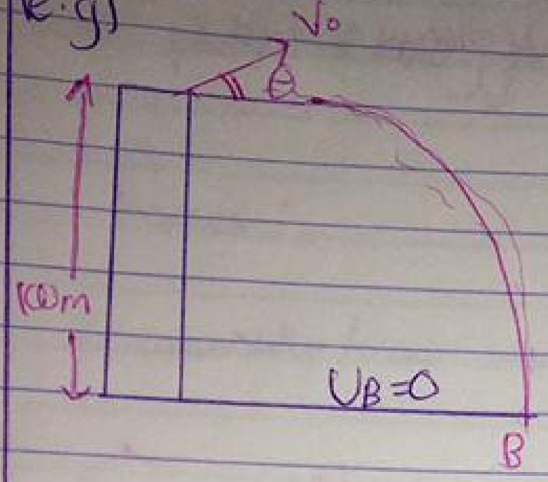


$$U_f - U_i = mg\, y_f - mg\, y_i$$

let $y_i = 0$, $U_i = 0$

$$U_{final} = mg\, y_f$$

(e.g)



$m = 2\,Kg$ $\quad V_0 = 50\,m/s$

$\theta = 37°$

$\Rightarrow$ find $W_g(A \to B)$?

$W_g = -\Delta U = -[U_B - U_A]$

$U = mgy$

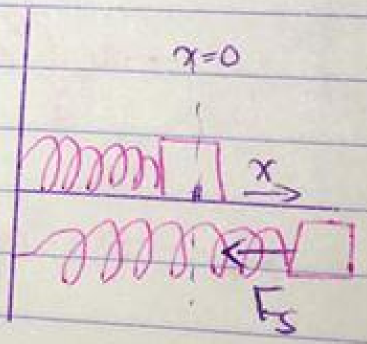
$W_g(A \to B) = -[0 - 100mg]$

$+2000\,J$

$U_A = mg\,(100)$

SPRING Potential energy:

$$U_f - U_i = -\int_{x_i}^{x_f} -Kx\,dx = K\frac{x^2}{2}\Big]_{x}^{x_f}$$



$$U_f - U_i = \frac{1}{2}Kx_f^2 - \frac{1}{2}Kx_i^2$$

Let $U_i = 0$ at $x_i = 0$

$$U_f = \frac{1}{2}Kx_f^2\ J$$

*MECHANICAL ENERGY:

$\Rightarrow$ mechanical energy $= \frac{1}{2} m V^2 + U$

conservation of mechanical energy:

the only force acting on the system is F_{cons}

$W_{\text{done by } F_{cons}} = -\Delta U$

$W_{\text{done by } F_{cons}} = \Delta K$

$\Delta K = -\Delta U \Rightarrow \Delta K + \Delta U = 0$

$\Rightarrow K + U \Rightarrow$ constant

$(K+U)_i = (K+U)_2$

$W = \int \vec{F} \cdot \vec{dr} = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$ (work done by any force)

$W_{net} = \Delta K$ (work done by $\sum \vec{F}$)

$W = -\Delta U$ [work done by conservative force.

$$F_{cons} \begin{cases} \to mg \Rightarrow U = mgy \text{ Joul} \\ \to -Kx \Rightarrow U = \frac{1}{2} K x^2 \text{ Joul} \end{cases}$$

$(K+U)_1 = (K+U)_2$ "$F_{conservative}$ is acting only

⇒ FINDING CONSERVATIVE FORCE FROM U:

$W_{con} = -\Delta U$

$F_{con}\,dx = -du \Rightarrow \boxed{F_{cons} = \frac{-du}{dx}}$

__example:__ $20\,kg = m$, a conservative force force act on m

$F = -3x - 5x^2\ N$

take $U=0$ at $x=0$ الظروف الابتدائية

① find potential , U as a function of x ? $\underline{U(x)}$

$$\Delta U = -\int_{x}^{x} F_x\,dx \Rightarrow U(x) - U(0) = -\int_0^x (-3x - 5x^2)\,dx$$

$$U(x) = \left[\frac{3}{2}x^2 + \frac{5}{3}x^3\right]$$

$$U(x) = 8x^2 + 2x^4\ J$$

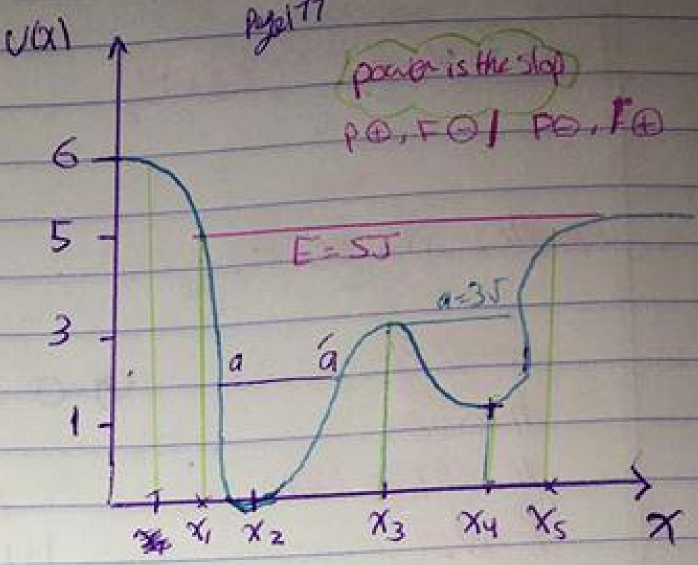
find $F_{cons}(x)$? $F_{cons} = \frac{-du}{dx} = -16x - 8x^3\ N$

find the equilibrium points? at equilibrium points $F_{cons} = 0$

$$0 = -8x(2 + x^2)$$

Only one equilibrium point $\Rightarrow x = 0$

→ READING POTENTIAL ENERGY CURVE:



$\boxed{m}$ moves along the x-axsis

you can find F_{ex} in any time by using

$$F_x = \frac{-du}{dx}$$

$$F_x = -\text{slop}$$

⇒ Conside the following 4 cases:

equilibrium point   اللي بكون فيه الجسم ساكن

1) let $E = 5J$, $K + U = 5$ at any point

at x_1, $K = 0$ is the turning point because $K = 0$ $F \neq 0$, $F(x_1) \Rightarrow ⊕$

at x_2, $K = 5J$ $K = 5 - U$

at x_3, $K = 2J$

at x_4, $K = 4J$

At x_5 $K = 0$ neutral equilibrium point

2) let $E = 1J$ $(a, \acute{a})$ are turning points at x_2, $K = 1J$

at $x_4 \Rightarrow U = 1J$, $E = 1J$, $K = 0$, $F = 0$ Stable equilibrium point

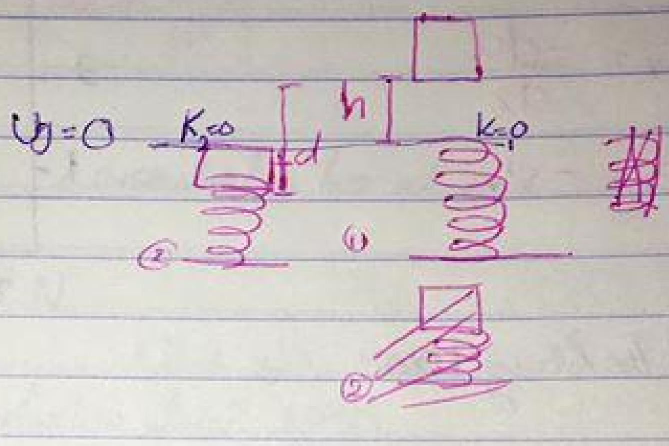
3) let $E = 3J$, at x_3, $K = 0$, $F = 0$ unstable equilibrium point

F_{cons} → U طاقة وضع
→ E ⇒ mechanical is conserved

$$K_1 + U_1 = K_2 + U_2$$

example: Problem 24:

$m = 2\,kg$
$h = 50\,cm$
$K = 1960\,N/m$
find d



$$(K+U)_1 = (K+U)_2$$

$$0 + mg(h+d) = 0 + \frac{1}{2}Kd^2$$

$$9.8 + 19.6d = 980d^2$$
$$980d^2 - 19.6d - 9.8 = 0$$
$$100d^2 - 2d - 1 = 0$$

Work done by nonconservative forces:

F_{cons} & F_{non} act on m, moves

$$W_{net} = W_{cons} + W_{non}$$

$$\Delta K = -\Delta U + W_{non} \Rightarrow W_{non} = \Delta K + \Delta U$$

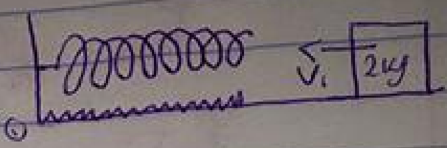
$$W_{non} = \Delta E$$

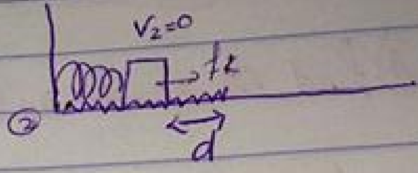
Sample problem page 186:

$V_1 = 4\, m/s$

$f_K = 15N$

$K = 10000 N/m$





$$W_{non} = \Delta E = E_2 - E_1$$

$$\Delta E = (K_2 + U_2) - (K_1 + U_1)$$

$$W_{non} = (0 + \tfrac{1}{2}Kd^2) - (\tfrac{1}{2}mV^2 + 0)$$

$$W_{non} = f_K d \cos 180 \rightarrow -15d = \tfrac{1}{2} \times 10000 \times d^2 - 16$$

$$5000d^2 + 15d - 16 = 0 \quad , \boxed{d = 55\,cm}$$

example:

$m = 0.2\,kg$, A cons force act on m, where

at $x=0$, $V_0 = 9\,m/s$

$$U(x) = x^2 - 4x - 5J$$

a) find Velocity at $x = 1m$

$$(K+U)_0 = (K+U)_1 \; = \left(\tfrac{1}{2}(0.2)(9)^2 + -5\right) = \tfrac{1}{2}(0.2)V_1^2 + (1-4-5$$

$$3.1 = \cancel{10} .1\, V_1^2 + -8$$

b) find the turning points? at the turning point K=0, E=U

$$3.1 = x^2 - 4x - 5 \quad , x_1 = 5.5\,m$$

$$x_2 = -1.5\,m$$

c) find the conservative force?

$$F_{cons} = \frac{-du}{dx} \Rightarrow F_{cons}$$

$$F_{cons} = -2x+4 \, N$$

unstable equilibrium

d) find the equilibrium points?

at equilibrium point $V=0$

$$0 = -2x+4 \quad \boxed{x = 2m}$$

$E = 31 J$ at any point

* problem (104)

$$F_{cons} = -3x - 5x^2$$

act on $m = 20kg$, at $x=0$, $U=0$

a) find $U(x)$? find U at $x=2m$

$$U_f - U_i = -\int_{x_i}^{x_f} F_{cons}\, dx \Rightarrow U(x) - U_0 = -\int_0^x (-3x-5x^2)\,dx$$

$$U(x) - 0 = \frac{3x^2}{2} + \frac{5x^3}{3}$$

$$U(x) = \frac{5}{3}x^3 + \frac{3}{2}x^2$$

$$U(2) = 19J$$

b) at $x = 5m$, $\boxed{m}$ move with velocity $= -4$ m/s. find the velocity at $x=0$

$$(K+U)_5 = (K+U)_0 \Rightarrow K_5 = \frac{1}{2}(20)(-4)^2 = 160J$$

$$U_5 = \frac{5}{3}(5)^3 + \frac{3}{2}(5)^2$$

$$U(5) = 246J$$

$$(K+U)_5 = 406J$$

$U_0 = 0$, $K_0 = \frac{1}{2} m(20) V_0^2$

$= 10 V_0^2$

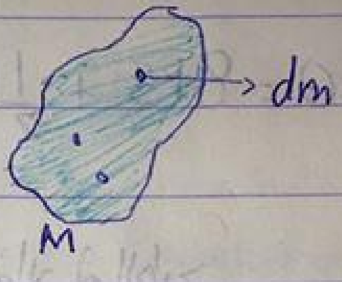
$(K+U)_S = (K+U)_0 = 406 = 10 V_0^2$, $\boxed{V_0 = 6.4 \text{ m/s}}$

Ch

Chapter 9

* center of mass for a rigid body.

$\Rightarrow x_{cm} = \frac{1}{M} \int x \, dm$



$\Rightarrow y_{cm} = \frac{1}{M} \int y \, dm$

Example: uniform rod of → length = L
→ mass = M

find: ① linear mass density? λ?

$$\lambda = \frac{m}{L} \ \text{kg/m}$$

② find the coordinate of the center of the mass?

$x_{cm} = \frac{1}{m} \int x \, dm \quad , \ dm = \lambda dx$

$$x_{cm} = \frac{1}{m} \int_0^L x \lambda dx = \frac{\lambda}{m} \int_0^L x dx$$

$$x_{cm} = \frac{\lambda}{m} \frac{x^2}{2} \Big|_0^l \Rightarrow x_{cm} = \frac{\lambda}{m} \frac{L^2}{2}$$

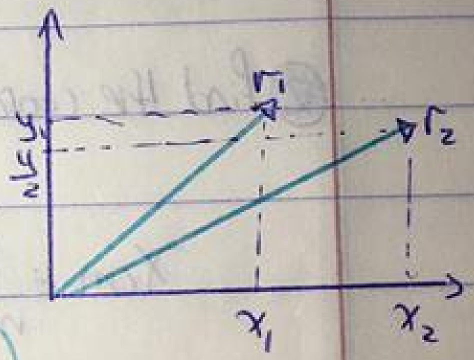
$$x_{cm} = \frac{L}{2}$$

* center of mass & linear momentam:

⇒ center of mass for a set of point masses in a point

1. All forces acting on it
2. The mass of the system concentrated on it.

⇒ The center of mass for a system of point masses:



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$x_{cm} = \frac{\sum mx}{M}, \quad M = \sum M$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$y_{cm} = \frac{\sum my}{M}$$

$$Z_{cm} = \frac{\sum mz}{M}$$

Example: A nonuniform rod of length $= L$. linear mass density λ is not constant, given by $\lambda = \alpha x$,

α is constant

① find the mass of rod.

$$M = \int dm = \int_0^L \lambda dx = \int_0^L (\alpha x) dx$$

$$= \alpha \frac{x^2}{2}\Big|_0^L \Rightarrow \boxed{M = \frac{\alpha}{2} L^2}$$

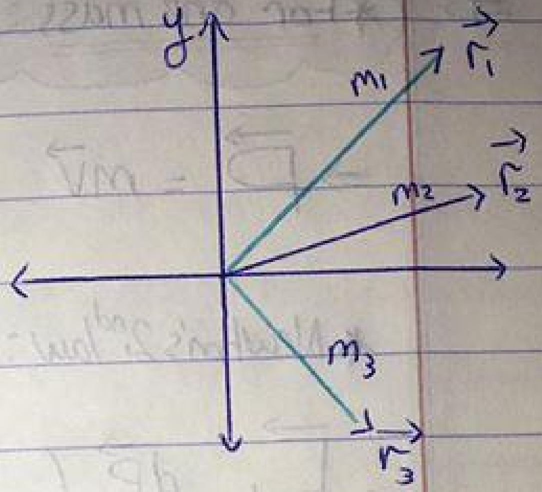
② find the coordinate of the center of mass.

$$x_{cm} = \frac{1}{m} \int x\, dm = \frac{1}{m} \int x\, dx (\lambda)$$

$$x = \frac{1}{m} \int_0^L x (\alpha x) dx = \frac{\alpha}{m} \int_0^L x^2 dx$$

$$\frac{\alpha}{m} \cdot \frac{L^3}{3} \Rightarrow \frac{\alpha}{\frac{\alpha L}{2}} * \frac{L^3}{3} = \boxed{\frac{2L}{3}}$$

*Newton's 2nd law for a system of particals:



$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots}{M}$$

$$\vec{V}_{cm} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2 + \dots}{M}$$

$$M\vec{V}_{cm} = m_1\vec{v}_1 + m_2\vec{V}_2 + \dots$$

$$P_{cm} = \vec{P}_1 + \vec{P}_2 + \dots$$

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \dots}{M}$$

$$M\vec{a}_{cm} = \vec{F}_1 + \vec{F}_2 + \dots$$

$$M\vec{a}_{cm} = \vec{F}_{ext}$$

→ Study example page 209

Linear Momentum :

* For one mass :

$\Rightarrow \vec{P} = m\vec{V}$ Kgm/s

* Newton's 2^{nd} law :

$\Rightarrow \vec{F}_{net} = \frac{d\vec{P}}{dt}$ (net force acting on m is the time rate of charging momentum)

* For constant mass : $m\vec{a} = \vec{F}_{net} = \frac{dm\vec{V}}{dt}$

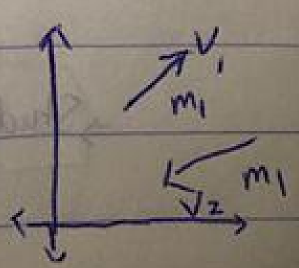
* linear momentum for a system of particals

$\vec{P} = m_1\vec{V}_1 + m_2\vec{V}_2 = M\,\vec{V}_{cm}$

مجموع الكتل ← M

محصلة السرعة ← $\vec{V}_{cm}$

$\vec{F}_{net} = \frac{d\vec{P}}{dt} \Rightarrow \boxed{\vec{F}_{net} = \frac{d}{dt}(M\vec{V}_{cm})}$



* Impulse & linear momentum:

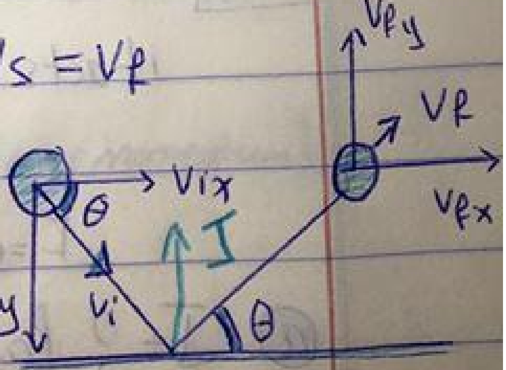
$$\vec{F}_{net} = \frac{d\vec{P}}{dt} \Rightarrow \int_{t_i}^{t_f} \vec{F}_{net}\, dt = \int_{P_i}^{P_f} d\vec{P}$$

$$\int_{t_i}^{t_f} F_{net}\, dt = \vec{P}_f - \vec{P}_i$$

$$\Rightarrow \text{الدفع}\ J = \int_{t_i}^{t_f} F dt \ (N.S)$$

$$\Delta P = \int \vec{F}_{net} dt$$

$\boxed{\text{problem 38}}$ $m = 300g$, $v_i = 8m/s = V_f$



① $\vec{J}$ on the ball?

$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = \vec{J}$

$\vec{P}_i = m\vec{V}_i = .3\,(8\cos 30\hat{i} - 8\sin 30\hat{j})$

$= 2.1\hat{i} - 1.2\hat{j}$ kgm/s

$\vec{P_f} = m\vec{V_f} = 2.1\hat{i} + 1.2\hat{j}$ kg m/s

$$J = \vec{P_f} - \vec{P_i} = \boxed{2.4\hat{j}} \text{ kgm/s}$$

(2) contact time = 10 s, find F on the wall?

$$J_{on\,ball} = 2.4\hat{j} = \vec{F}_{\substack{on \\ ball}} \Delta t$$

$$\vec{F}_{on\,ball} = \frac{2.4\hat{j}}{10\times10^{-3}} = 240\hat{j}\ N$$

$$\boxed{\vec{F}_{on\,wall} = -240\hat{j}\ N}$$

Q $\boxed{36}$ $m = 0.25$ kg, $\vec{F} = (12 - 3t^2)\hat{i}$ N

it acts on m from $t_1 = 0$ to t when $\vec{F} = 0$

$$\vec{F} = 0,\ 12 - 3t^2 = 0 \quad \boxed{t = 2s}$$

(a) J = ? from $t_1 = .75$ s to $t_2 = 1.25$

$$\vec{J} = \int \vec{F}\,dt \Rightarrow J_x = \int_{.75}^{1.25} (12 - 3t^2)\,dt$$

$$J_x = 12t - t^3 \Big|_{.75}^{1.25} = \boxed{75 \text{ N/s}}$$

(b) $\Delta\vec{P}$ from $t_1 = 0 \rightarrow t_f = 2\,s$

$$(\Delta P)_x = \int_0^2 F\,dt = 12t - t^3 \Big|_0^2 = 16\ N.s$$

$\Rightarrow J = \int \vec{F}\,dt$ = Area under the curve of (F vs t)

* Conservation of linear momentum:

$\vec{F}_{net} = \frac{d\vec{P}}{dt}$ if $\sum F_{net} = 0$ النظام معزول

$\frac{d\vec{p}}{dt} = 0 \Rightarrow \Delta\vec{P} = 0$ $\vec{p}$ constant

$\vec{P}_i = \vec{P}_f$ (conservation of linear momentum)

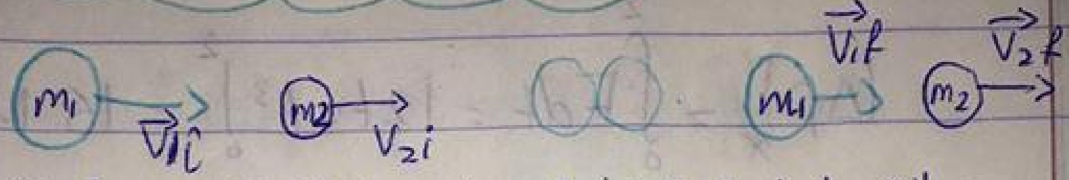
$F_{ext} = 0$

$\Rightarrow$ collisions + explosions + rocket motion in free Space

تصادمات

sample problem page 216, ~~217~~ 217

* Collisions in one dimension:



⇐ للتسهيل في الحل نضع الجسم المجهول بالاتجاه الموجب للسينات .

$$\vec{P}_b = \vec{P}_a \Rightarrow m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i} = m_1 V_{1f} + m_2 V_{2f} \ldots (1)$$

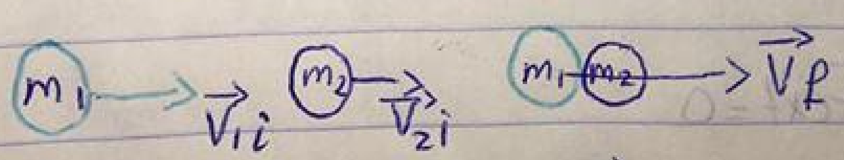
(1) Forelastic collisions. تصادم مرن

$$. K_b = K_a \Rightarrow \frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2 . (2)$$

(2) Inelastic collisions. تصادم غير مرن .

$K_b \neq K_a$ "Just use equation (1)"

(3) Completely Inelastic collisions. تصادم عديم المرونة



$$m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i} = (m_1 + m_2) \vec{V}_f$$

$$K_b \neq K_a$$

"سرعة مركز الكتلة ثابت"

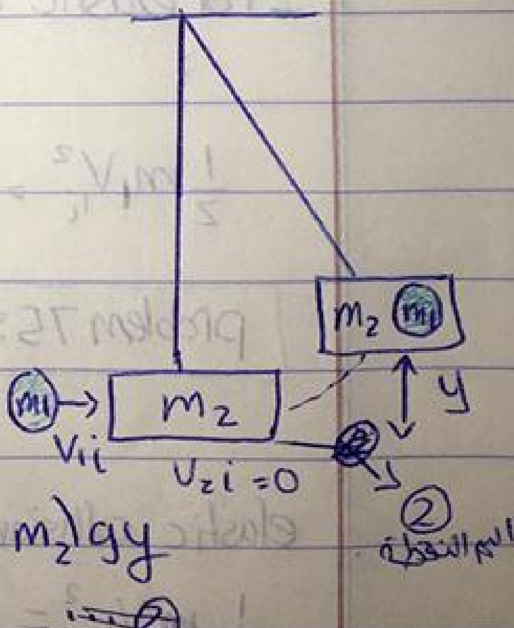
$$\vec{P_b} = \vec{P_a}$$

$$\frac{m_1 \vec{V_{1i}} + m_2 \vec{V_{2i}}}{m_1 + m_2} = \frac{m_1 \vec{V_{1f}} + m_2 \vec{V_{2f}}}{m_1 + m_2}$$

سرعة مركز الكتلة بعد = سرعة مركز الكتلة قبل.

=> Solve sample problem page 200

$\boxed{\text{problem 49}}$ bollistic pandulum البندول القذفي



$$\vec{P_b} = \vec{P_a}$$

$$m_1 \vec{V_{1i}} = (m_1 + m_2) \vec{V_f} \quad \ldots ①$$

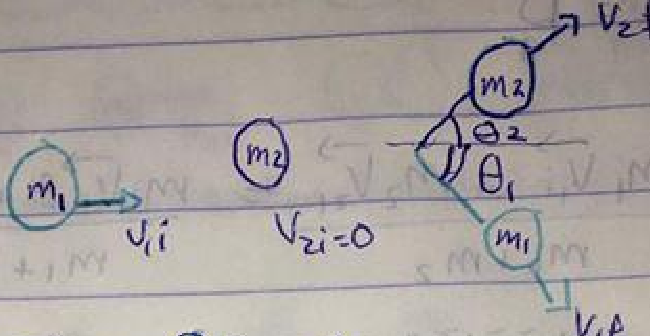
$$(K+U)_2 = (K+U)_f$$

$$\frac{1}{2}(m_1 + m_2) \vec{V_f} + 0 = 0 + (m_1 + m_2) g y$$

$$V_f^2 = 2gy \quad \ldots ②$$

from ① & ② we can find V_i

* Collisions in two Dimentions:



$\vec{P_b} = \vec{P_a} \Rightarrow$ 1 $P_{bx} = P_{ax}$

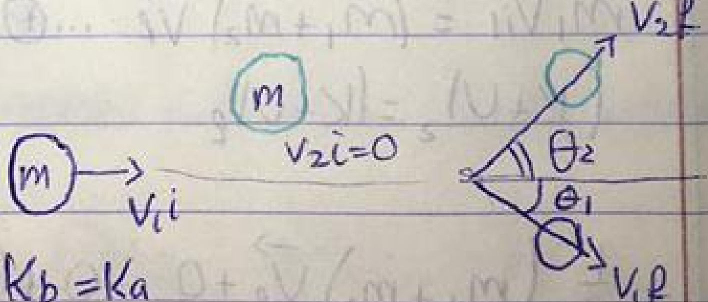
$m_1 V_i = m_1 V_1 f \cos\theta_1 + m_2 V_2 f \cos\theta_2 \ldots$ 1

2 $P_{by} = P_{ay}$, $-m V_1 f \sin\theta + m_2 V_2 \sin\theta_2 = 0$ 2

- For elastic collision $\Rightarrow K_b = K_a$

$$\frac{1}{2} m_1 V_{1i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2$$

$\boxed{\text{problem 75:}}$



elastic collision $\Rightarrow K_b = K_a$

$$\frac{1}{2} m V_{1i}^2 = \frac{1}{2} m V_{1f}^2 + \frac{1}{2} m V_{2f}^2$$

$(500)^2 = (V_1 f)^2 + (V_2 f)^2$ angle is 90 , $\theta_1 = 60$ given

$\theta_2 = 30^\circ$

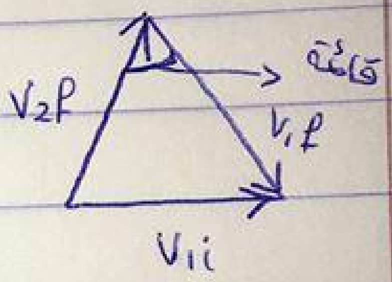
$$P_{bx} = P_{ax} \Rightarrow m(500) = m(V_2f\cos 30 + V_1f\cos 60)$$

$$500 = .87\,V_{f_2} + .5\,V_1f \ldots 2$$

$$P_{by} = P_{ay} \Rightarrow m(V_2f\sin 30 - V_1f\sin 60) = 0$$

$$.5V_2f = .87\,V_1f \ldots 3$$

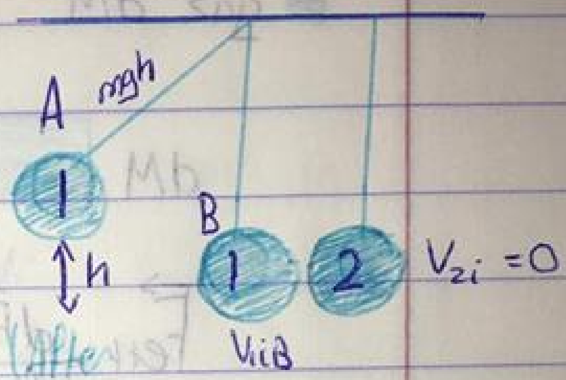
the same mass => angle 90



$$\vec{V_1f} + \vec{V_2f} = \vec{V_1i} \ldots 1$$

Chapter 9: "ELASTIC COLLISION"

*Sample problem page 223



$m_1 = 30\,g$

$m_2 = 75\,g$

$h = 8\,cm$

elastic collision, V_1f? V_2f? (After the collision)

$\Rightarrow (K+U)_A = (K+U)_B$, $V_i = \sqrt{2gh} = 1.252\ m/s$

$\Rightarrow \vec{P_b} = \vec{P_a} \Rightarrow m_1 V_i + \cancel{m_2 V_{2i}} = m_1 V_{1f} + m_2 V_{2f}$

$(.03)(1.252) = 0.03 V_{1f} + 0.075 V_{2f}$

$0.03756 = 0.03 V_1 f + 0.075 V_2 f$

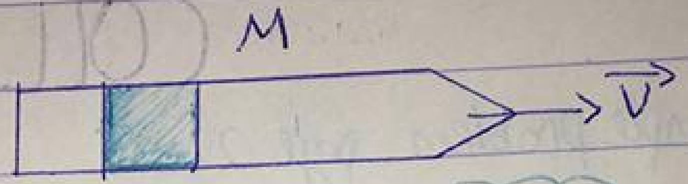
$3756 = 3 V_1 f + 7.5 V_2 f \quad \ldots ①$

$K_b = K_a$ or $\vec{V_{1i}} - \cancel{\vec{V_{2i}}}^{\,0} = -(\vec{V_{1f}} - \vec{V_{2f}})$

$* \ 1.252 = \vec{V_2}f - V_1 f \ \ldots ②$ from ① & ②

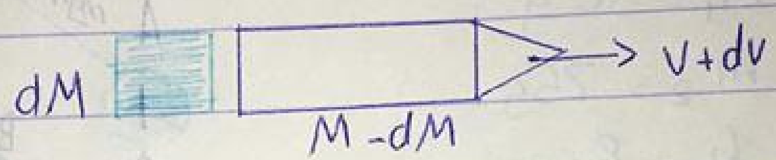
$\boxed{V_2f = 0.72\ m/s}$, $\boxed{V_1f = -0.54\ m/s}$

System with Varying mass (A Rocket)



gaz "dM"

isolated



$$\vec{F}_{ext} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(m\vec{V}) = M\frac{dv}{dt} + V\frac{d\vec{m}}{dt}$$

* In deep space $\vec{F}_{ext} = 0$

$$\Rightarrow \frac{M d\vec{v}}{dt} + V_{rel}\frac{d\vec{M}}{dt} = 0 \Rightarrow M\frac{d\vec{v}}{dt} + V_{rel}\frac{d\vec{M}}{dt}$$

$$\boxed{-V_{rel}\frac{d\vec{M}}{dt} = M\frac{d\vec{v}}{dt}}$$

$$\Rightarrow V_{rel}\frac{dM}{dt} = \text{Thrust (Newton)}$$

مقدار استهلاك الوقود "الغاز"

$$\text{Thurst} = M\frac{d\vec{v}}{dt} = M\vec{a} \quad \text{(Newton)}$$

$$-V_{rel}\frac{dM}{dt} = M\frac{d\vec{v}}{dt}$$

$$-V_{rel}\,dM = M\,d\vec{v}$$

$$\int_{m_i}^{m_f} -V_{rel}\frac{dM}{M} = \int_{V_i}^{V_f} d\vec{v}$$

$$-V_{rel}\ln M\Big|_{m_i}^{m_f} = V_f - V_i$$

$$\Rightarrow V_f - V_i = (V_{rel})\left(\ln\left(\frac{m_i}{m_f}\right)\right)$$

P79: $F_{ext} = 0$, $V_i = 0$, $M_i = 2.55 \times 10^5$ kg

$M_{fuel} = 1.8 \times 10^5$ kg, $\Delta t = 250$ sec, $\frac{dM}{dt} = 480$ kg/s

$V_{rel} = 3.27$ km/s

a) Thrust? $\text{Thrust} = V_{rel}\frac{dM}{dt} = (3.27 \times 10^3)(480)$

$= 1.57 \times 10^6$ N

b) $M_f = M_i - \Delta t\left(\frac{dM}{dt}\right) = 2.55 \times 10^5 - 250(480)$

$= 1.35 \times 10^5$ kg

© $V_f - V_i = V_{rel} \ln\left(\frac{M_i}{M_F}\right)$

$V_f = 3.27 \times 10^3 \ln\left(\frac{2.55 \times 10^5}{1.35 \times 10^5}\right)$

$2.1 \times 10^3 \, m/s \Rightarrow \boxed{2.1 \, Km/s}$

Sample problem page 220

$M_i = 850 Kg$, $R = 2.3 \, kg/s$, $V_{rel} = 2800 \, m/s$

a) $\text{Thrust} = V_{rel} \frac{dM}{dt} = (2800)(2.3)$

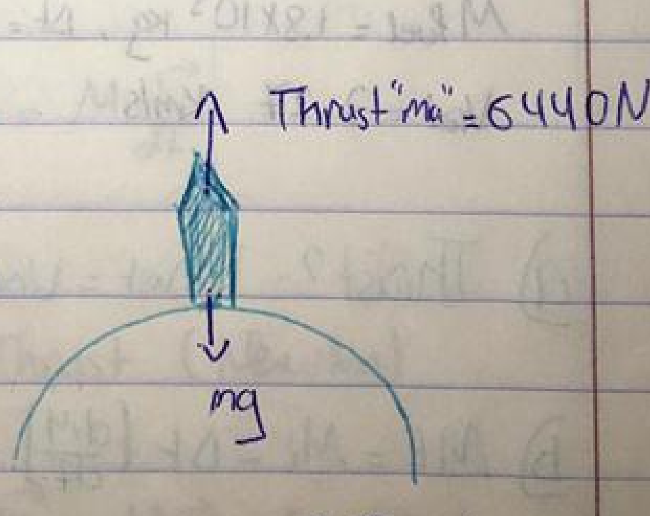
$= \boxed{6440 \, N}$

b) a_i?? $\text{Thrust} \; M_i \vec{a}_i$

$\Rightarrow \vec{a}_i = \frac{6440}{850}$

$\Rightarrow 7.6 \, m/s$

لن ينطلق الصاروخ إذا ←
كان على الأرض لكن في
الفضاء يمكن أن ينطلق



mg > Thrust

$\Rightarrow$ Rocket will not fired

Chapter 10 → Rotation:

The rotational Variable

عكس عقارب الساعة +

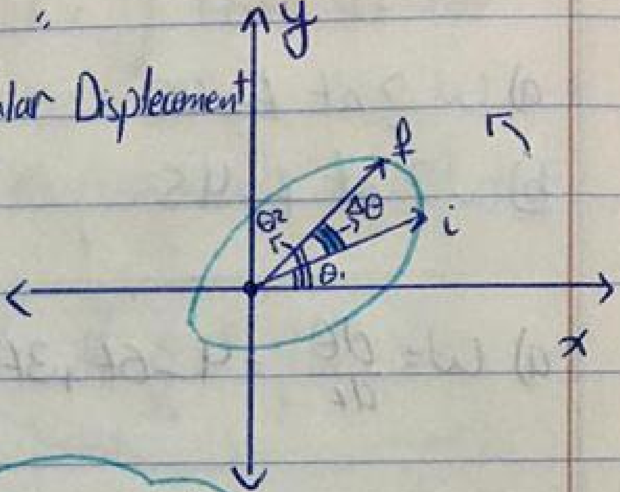
مع عقارب الساعة −

θ_1 = initial angular position

θ_2 = final " "

$\Delta\theta = \theta_2 - \theta_1$ ⇒ Angular Displacement

⇒ **(Rad)**



$2\pi = 360°$

- Average angular Velocity $= \frac{\Delta\theta}{\Delta t}$

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t} \quad (Rad/s$$

لكل أنواع الحركة

$$* \quad \omega_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Average angular acceleration $= \frac{\Delta\omega}{\Delta t}$

$$\alpha_{avg} = \frac{\Delta\omega}{\Delta t} \quad (Rad/s^2)$$

$$\alpha_{inst} = \frac{d\omega}{dt}$$

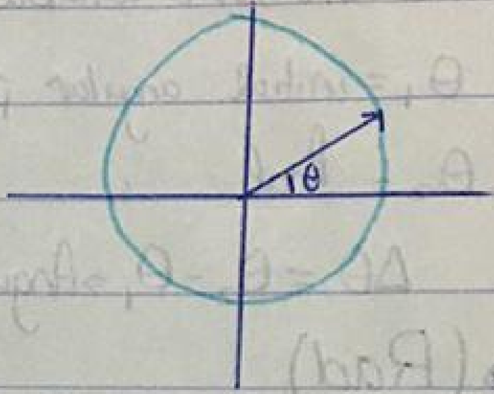
→ Solve:

→ page 244

→ 246

Q6

$\theta = 4t - 3t^2 + t^3$



a) ω ? at $t = 2s$

b) ω ? at $t = 4s$

a) $\omega = \frac{d\theta}{dt} = 4 - 6t + 3t^2$

$\omega(2) = 4$ rad/s

b) $\omega(4) = 4 - 24 + 48 = 28$ rad/s

c) $\alpha_{ave} = \frac{\Delta\omega}{\Delta t} = \frac{\omega(4) - \omega(2)}{2} = \frac{28-4}{2} = 12 \text{ rad/s}^2$

d) $\alpha_{inst} = \frac{d\omega}{dt} = -6 + 6t \text{ rad/s}^2$

$\alpha(2) = -6 + 12 = 6 \text{ rad/s}^2$

e) $\alpha(4) = -6 + 24 = 18 \text{ rad/s}^2$

Q8

$\alpha = 6t^4 - 4t^2$, at $t = 0$ $\begin{cases} \omega_0 = (+)2.5 \text{ rad/s} \\ +1.5 \text{ rad} \end{cases}$

عكس عقارب الساعة

a) $\omega(t)$? $\quad \alpha = \frac{d\omega}{dt} \Rightarrow d\omega = \alpha\, dt$

$$\int d\omega = \int \alpha\, dt \Rightarrow \omega = \int (6t^4 - 4t^2)\, dt$$

$$\omega = \frac{6}{5}t^5 - \frac{4}{3}t^3 + C$$

$$C = 2.5 \Rightarrow \omega(t) = \frac{6}{5}t^5 - \frac{4}{3}t^3 + 2.5$$

b) $\theta(t)$? $\quad \omega = \frac{d\theta}{dt} \Rightarrow d\theta = \omega\, dt$

$$\int d\theta = \int \omega\, dt \Rightarrow \theta = \int \left(\frac{6}{5}t^5 - \frac{4}{3}t^3 + 2.5\right) dt$$

$$\theta = 2.5t + \frac{t^6}{5} + \frac{-t^4}{3} + C' \quad , \; C' = 1.5$$

$$\theta = 1.5 + 2.5t + \frac{t^6}{5} - \frac{t^4}{3} \text{ rad}$$

Motion with constant Angular Acceleration:

α = constant

$\omega = \omega_0 + \alpha t$

$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

$\Delta\theta = \left(\frac{\omega_0 + \omega}{2}\right) t$

$\frac{\omega_0 + \omega}{2} = \omega_{avg}$

for constant α

Q10 $\omega_0 = 0$, α constant

in $t = 5s$, $\Delta\theta = 20\,rad$

a) α? $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ ($\omega_0 t = 0$)

$\alpha = \frac{2\Delta\theta}{t^2} = \frac{2(20)}{25} = \frac{40}{25} = 1.6\ rad/s^2$

b) ω_{avg}? $t = 0 \rightarrow 5$ sec

$\omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{20}{5} = 4\ rad/s$

c) $\omega_{avg} = \frac{\omega_0 + \omega}{2}$ (for constant α)

$4 = \frac{0 + \omega}{2} \Rightarrow \omega = 8\ rad/s$

d) $\Delta\theta$? in the next 5 sec.

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

ω_5

$$\Delta\theta = 8 \times 5 + \frac{1}{2}(1.6)(5)^2 =$$

ويمكن ايجاد θ بعد 5 ثوان و θ بعد 10 ثوان ثم الطرح

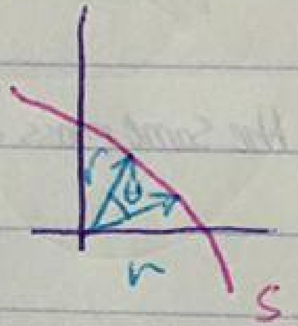
* Angular Quantities $\rightarrow \Delta\theta$ rad

$\rightarrow \omega = \frac{d\theta}{dt}$

$\rightarrow \alpha = \frac{d\omega}{dt}$ rad/s²

* Relating linear & Angular Variables:

$S = r\theta$, $\theta \equiv$ rad



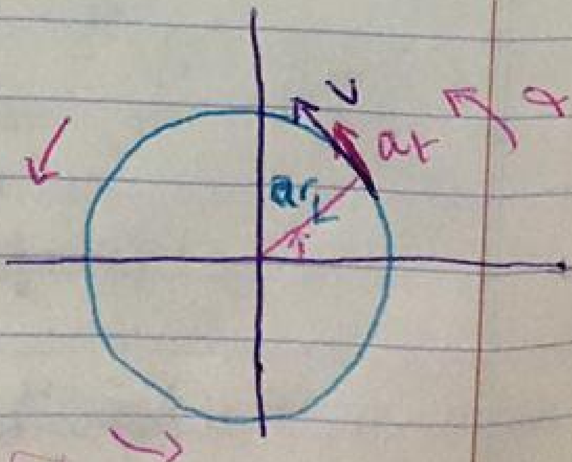
$\frac{ds}{dt} = r\frac{d\theta}{dt} \rightarrow \omega$

$V = r\omega$

$V = r\omega$

$\frac{dV}{dt} = r\frac{d\omega}{dt}$

التسارع $a_t = \alpha r$
على المماس



على ← $V = \omega r$
المماس

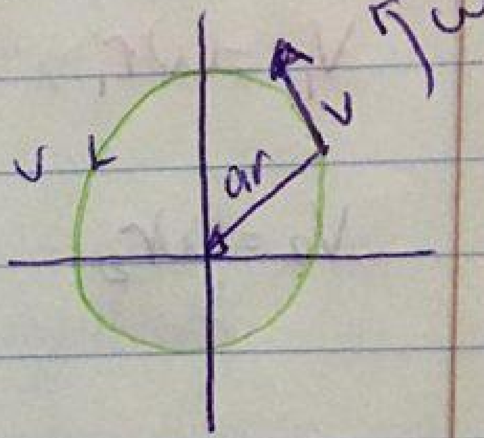
$a_r = \frac{V^2}{r}$

$\boxed{\omega}$ increasing

α is positive

$a_r = \omega^2 r$

Remember! uniform circular motion:



$$v = \frac{2\pi r}{T}$$

$$a_r = \frac{v^2}{r}$$

$$\omega = \frac{2\pi}{T}$$

$$v = \omega r$$

$$\alpha = 0$$

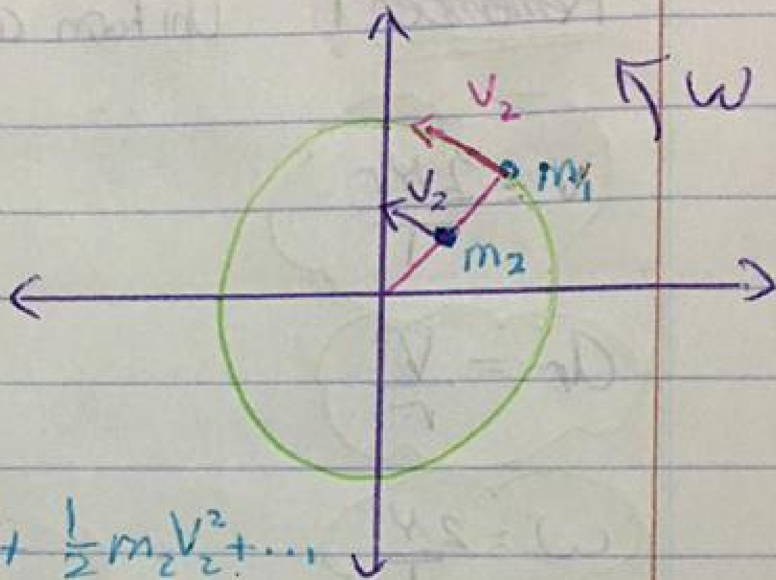
$$a_t = 0$$

v constant, ω constant

* Rotational kinetic energy:

$V_1 = \omega r_1$

$V_2 = \omega r_2$



$K = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 + \dots$

$K = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \dots$

$K = \frac{1}{2} \left[m_1 r_1^2 + m_2 r_2^2 + \dots \right] \omega^2$

$K = \frac{1}{2} \left(\sum m r^2 \right) \omega^2$

number of points

$K_{rot} = \frac{1}{2} I \omega^2$

$I = \sum m r^2 \quad kg \cdot m^2$

$I = \sum mr^2$ rotational inertia moment of Inertia

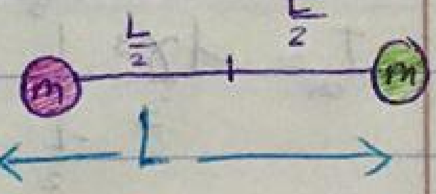
الكتلة وكيفية توزيعها تؤثر
في مقاومة الجسم للقوى الخارجية

$I = \int r^2 dm$ for rigid body.

$I = \sum mr^2$ for point masses.

Sample Problem page 256 :-

+ two equal masses sperated by L, rod is negligable mass



$I_{com} = m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{2}$ kg.m حول المركز

find I around the left mass :

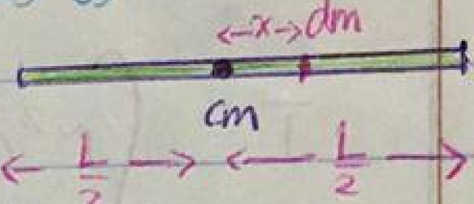
$I_{axe\ end} = m(0) + mL^2 = mL^2$

*Sample problem page 256, 257:

*Uniform rod $\begin{cases} \rightarrow \text{length} = L \\ \rightarrow \text{mass} = M \end{cases}$

$\lambda = \frac{M}{L}$ kg/m is constant

حول مركز الكتلة



$$I = \int r^2 dm$$

$$I = \int x^2 dm \qquad dm = \lambda dx$$

$$I_{cm} = \int_{-\frac{L}{2}}^{+\frac{L}{2}} x^2 (\lambda dx)$$

$$I_{cm} = \lambda \frac{x^3}{3} \Big|_{-\frac{L}{2}}^{+\frac{L}{2}} = \frac{M}{3L} \left(\left(\frac{+L}{2}\right)^3 - \left(\frac{-L}{2}\right)^3 \right)$$

$$I_{cm} = \frac{M}{3L} \left(\frac{2L^3}{8} \right)$$

$$\boxed{I_{cm} = \frac{ML^2}{12}}$$