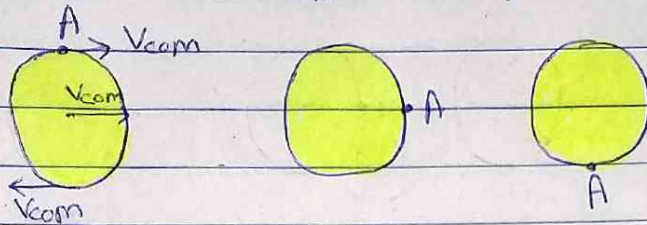


## Rolling

→ When a wheel of Radius  $R$  rolling then  
 $\leftarrow v_{com} = \omega R \rightarrow$



→ Forces and kinetic energy of rolling

$$K = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2$$

↓  
from the rotation  
motion

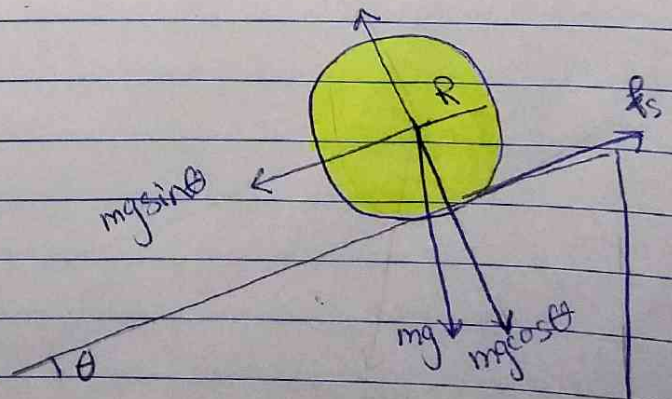
↓  
from the linear  
motion

→  $a_{com} = \alpha R$

• If the wheel rolls smoothly down a ramp of angle  $\theta$  its acceleration along an X-axis extending up the ramp is

→  $a_{com, x} = \frac{-g \sin \theta}{1 + \left( \frac{I_{com}}{MR^2} \right)}$

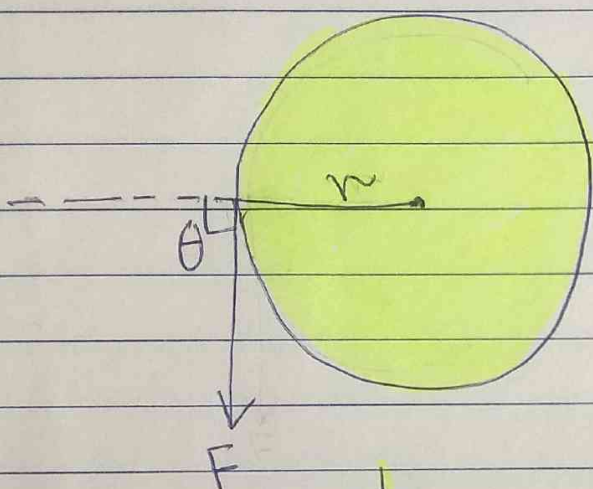
→  $f_s = \frac{-I_{com} a_{com, x}}{R^2}$



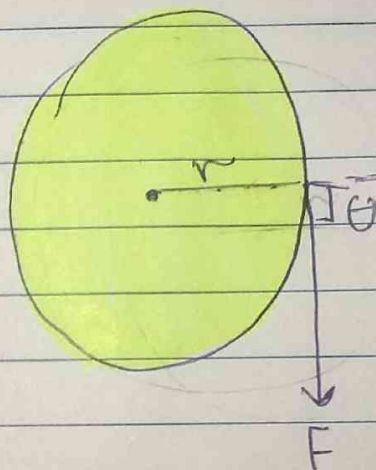
→ Torque revisited:

$$\vec{\tau} = \vec{F} \times \vec{r} \rightarrow |\tau| = |F||r| \sin\theta$$

- The direction of  $(\tau)$  is given by the right-hand rule for cross product.



$(\tau)$  is  $(+\hat{k})$   
 (counter clockwise)  
 (out of the page)



$(\tau)$  is  $(-\hat{k})$   
 (with clock wise)  
 (on the page)

the fingers are in the direction of  $r$

→ Angular momentum:

$$\vec{l} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

$$l = m r v \sin\theta \quad \theta: \text{between } r \text{ and } v$$

- The direction of  $(l)$  is given by the right-hand rule for cross product.

$$\vec{l} = I \vec{\omega}$$

the fingers are in the direction of  $v$

→ Newton's second law in angular form:-

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}, \text{ look like } \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

Note:

$$\frac{d(\vec{r})}{dt} = \vec{v}$$

$$m\vec{r} \times \vec{v} = \vec{L}$$

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$

→ look to the sample 11.04

→ Conservation of angular momentum:-

• The angular momentum  $\vec{L}$  of a system remains constant if the external torque acting on the system is zero;

$$\vec{L} = \text{a constant (isolated system)}$$

$$\vec{L}_i = \vec{L}_f$$