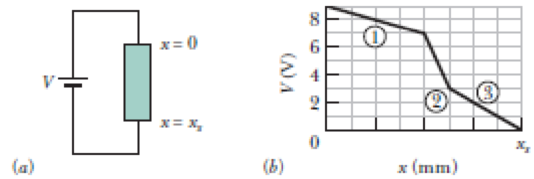
P4-26) In Next Fig. a 9.00 V battery is connected to a resistive strip that consists of three sections with the same cross-sectional areas but different conductivities. gives the electric potential $V(x)$ versus position x along the strip. The horizontal scale is set by x_s 8.00 mm. Section 3 has conductivity 3.00x 10^7.What is the conductivity of section (a) 1 and (b) 2?



4. The absolute values of the slopes (for the straight-line segments shown in the graph are equal to the respective electric field magnitudes. Thus, applying Eq. 26-5 and Eq. 26-13 to the three sections of the resistive strip, we have

$$J_1 = \frac{i}{A} = \sigma_1 E_1 = \sigma_1 (0.50 \times 10^3 \text{ V/m})$$

$$J_2 = \frac{i}{A} = \sigma_2 E_2 = \sigma_2 (4.0 \times 10^3 \text{ V/m})$$

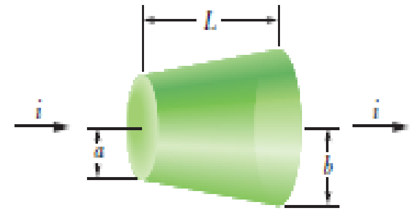
$$J_3 = \frac{i}{A} = \sigma_3 E_3 = \sigma_3 (1.0 \times 10^3 \text{ V/m}) \ .$$

We note that the current densities are the same since the values of i and A are the same (see the problem statement) in the three sections, so $J_1 = J_2 = J_3$.

(a) Thus we see that $\sigma_1 = 2\sigma_3 = 2\,(4.00 \times 10^7 (\Omega \cdot \text{m})^{-1}) = 8.00 \times 10^7 \ (\Omega \cdot \text{m})^{-1}$.

(b) Similarly, $\sigma_2 = \sigma_3/4 = (4.00 \times 10^7 (\Omega \cdot \text{m})^{-1})/4 = 1.00 \times 10^7 \ (\Omega \cdot \text{m})^{-1}$.

P5-26) In Fig. , current is set up through a truncated right circular cone of resistivity 731 , left radius a = 2.00 mm, right radius b = 2.30 mm, and length L = 1.94 cm. Assume that the current density is uniform across any cross section taken perpendicular to the length.What is the resistance of the cone?



5. The current i is shown in the figure entering the truncated cone at the left end and leaving at the right. This is our choice of positive x direction. We make the assumption that the current density J at each value of x may be found by taking the ratio i/A where $A = \pi r^2$ is the cone's cross-section area at that particular value of x.

The direction of $\vec{J}$ is identical to that shown in the figure for i (our $+x$ direction). Using Eq. 26-11, we then find an expression for the electric field at each value of x, and next find the potential difference V by integrating the field along the x axis, in accordance with the ideas of Chapter 25. Finally, the resistance of the cone is given by $R = V/i$. Thus,

$$J = \frac{i}{\pi r^2} = \frac{E}{\rho}$$

where we must deduce how r depends on x in order to proceed. We note that the radius increases linearly with x, so (with c_1 and c_2 to be determined later) we may write $r = c_1 + c_2 x$.

Choosing the origin at the left end of the truncated cone, the coefficient c_1 is chosen so that $r = a$ (when $x = 0$); therefore, $c_1 = a$. Also, the coefficient c_2 must be chosen so that (at the right end of the truncated cone) we have $r = b$ (when $x = L$); therefore, $c_2 = (b-a)/L$. Our expression, then, becomes

$$r = a + \left(\frac{b-a}{L}\right)x.$$

Substituting this into our previous statement and solving for the field, we find

$$E = \frac{i\rho}{\pi}\left(a + \frac{b-a}{L}x\right)^{-2}.$$

Consequently, the potential difference between the faces of the cone is

$$\begin{aligned} V &= -\int_0^L E\,dx = -\frac{i\rho}{\pi}\int_0^L \left(a + \frac{b-a}{L}x\right)^{-2} dx = \frac{i\rho}{\pi}\frac{L}{b-a}\left(a + \frac{b-a}{L}x\right)^{-1}\Bigg|_0^L \\ &= \frac{i\rho}{\pi}\frac{L}{b-a}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{i\rho}{\pi}\frac{L}{b-a}\frac{b-a}{ab} = \frac{i\rho L}{\pi ab}. \end{aligned}$$

The resistance is therefore

$$R = \frac{V}{i} = \frac{\rho L}{\pi ab} = \frac{(731\ \Omega\cdot\text{m})(3.50\times10^{-2}\ \text{m})}{\pi(1.70\times10^{-3}\ \text{m})(2.30\times10^{-3}\ \text{m})} = 2.08\times10^{6}\ \Omega$$

P25-26) Wire *C* and wire *D* are made from different materials and have length *LC* = *LD* = 1.0 m. The resistivity and diameter of wire *C* are 2.0x10^6 and 1.00 mm, and those of wire *D* are 1.0 x10^6 and 0.50 mm. The wires are joined as shown in Fig, and a current of 2.0 A is set up in them. What is the electric potential difference between (a) points 1 and 2 and (b) points 2 and 3? What is the rate at which energy is dissipated between (c) points 1 and 2 and (d) points 2 and 3?

ANALYZE (a) Using Eq. 26-16, we find the resistance of wire *C* to be

$$R_C = \rho_C \frac{L_C}{\pi r_C^2} = (2.0\times10^{-6}\,\Omega\cdot\text{m})\frac{1.0\text{ m}}{\pi(0.00100\text{ m})^2} = 0.637\,\Omega\,.$$

Thus, $\Delta V_{12} = iR_C = (2.0\text{ A})(0.637\,\Omega) = 1.3\text{V}.$

(b) Similarly, the resistance for wire *D* is

$$R_D = \rho_D \frac{L_D}{\pi r_D^2} = (1.0\times10^{-6}\,\Omega\cdot\text{m})\frac{1.0\text{ m}}{\pi(0.00050\text{ m})^2} = 1.27\,\Omega$$

and the potential difference is $\Delta V_{23} = iR_D = (2.0\text{ A})(1.27\,\Omega) = 2.546\text{V} \approx 2.5\text{V}$.

(c) The power dissipated between points 1 and 2 is

$$P_{12} = i^2 R_C = (2.0\text{ A})^2(0.637\,\Omega) = 2.5\text{W}.$$

(d) Similarly, the power dissipated between points 2 and 3 is

$$P_{23} = i^2 R_D = (2.0\text{ A})^2(1.27\,\Omega) = 5.1\text{ W}.$$

LEARN The results may be summarized in terms of the following ratios:

$$\frac{P_{23}}{P_{12}} = \frac{\Delta V_{23}}{\Delta V_{12}} = \frac{R_D}{R_C} = \frac{\rho_D}{\rho_C}\cdot\frac{L_D}{L_C}\cdot\left(\frac{r_C}{r_D}\right)^2 = \frac{1}{2}\cdot 1\cdot(2)^2 = 2.$$

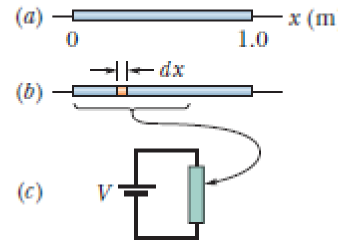
P32-26) The current-density magnitude in a certain circular wire is J = (2.75x 10^10 A/m^4)r^2, where r is the radial distance out to the wire's radius of 3.00 mm. The potential applied to the wire (end to end) is 80.0 V. How much energy is converted to thermal energy in 1.00 h?

32. Assuming the current is along the wire (not radial) we find the current from Eq. 26-4:

$$i = \int |\vec{J}|\, dA = \int_0^R kr^2\, 2\pi r dr = \frac{1}{2} k\pi R^4 = 3.50 \text{ A}$$

where $k = 2.75 \times 10^{10}$ A/m^4 and $R = 0.00300$ m. The rate of thermal energy generation is found from Eq. 26-26: $P = iV = 280$ W. Assuming a steady rate, the thermal energy generated in 40 s is $Q = P\Delta t = (280 \text{ J/s})(3600 \text{ s}) = 1.01 \times 10^6$ J.

P40-26) Figure shows a rod of resistive material. The resistance per unit length of the rod increases in the positive direction of the *x* axis. At any position *x* along the rod, the resistance *dR* of a narrow (differential) section of width *dx* is given by $dR = 5.00x\,dx$, where *dR* is in ohms and *x* is in meters. Figure shows such a narrow section. You are to slice off a length of the rod between $x = 0$ and some position $x = L$ and then connect that length to a battery with potential difference $V = 8.0$ V You want the current in the length to transfer energy to thermal energy at the rate of 180W.At what position $x = L$ should you cut the rod?



40. From $P = V^2/R$, we have

$$R = (8.0\text{ V})^2/(180\text{ W}) = 0.356\ \Omega.$$

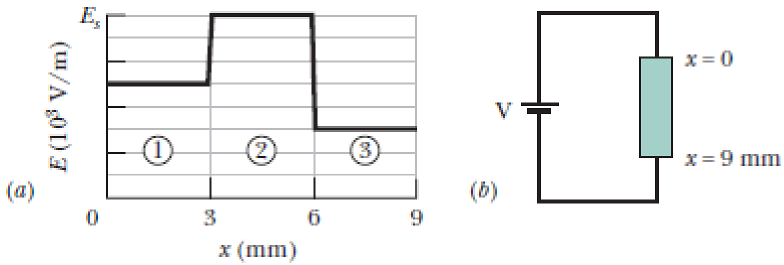
To meet the conditions of the problem statement, we must therefore set

$$\int_0^L 5.00x\,dx = 0.356\ \Omega$$

Thus,

$$\frac{5}{2}L^2 = 0.356 \quad \Rightarrow \quad L = 0.377\text{ m.}$$

P48-26) Figure gives the magnitude $E(x)$ of the electric fields that have been set up by a battery along a resistive rod of length 9.00 mm . The vertical scale is set by Es =8.00x10^3 V/m. The rod consists of three sections of the same material but with different radii. (The schematic diagram of Fig. does not indicate the different radii.) The radius of section 3 is 1.70 mm. What is the radius of (a) section 1 and (b) section 2?



48. (a) Since the material is the same, the resistivity ρ is the same, which implies (by Eq. 26-11) that the electric fields (in the various rods) are directly proportional to their current-densities. Thus, J_1: J_2: J_3 are in the ratio 2.5/4/1.5 (see Fig. 26-27). Now the currents in the rods must be the same (they are "in series") so

$$J_1 A_1 = J_3 A_3 \,,\ J_2 A_2 = J_3 A_3 \,.$$

Since $A = \pi r^2$, this leads (in view of the aforementioned ratios) to

$$4r_2^2 = 1.5r_3^2 \,,\ 2.5r_1^2 = 1.5r_3^2 \,.$$

Thus, with r_3 = 1.70 mm, the latter relation leads to $r_1 = (1.5/2.5)^{1/2}(1.70 \text{ mm}) = 1.32 \text{ mm}$.

(b) The $4r_2^2 = 1.5r_3^2$ relation leads to $r_2 = (1.5/4.0)^{1/2}(1.70 \text{ mm}) = 104 \text{ mm}$.