GAUSS' LAW

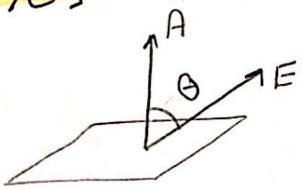
- Gauss' law considers a hypothetical (imaginary) closed surface enclosing the charge distribution called

→ Gaussian Surface

<u>Electric flux</u> :- (Φ) (scalar quantity)

$\Phi = \vec{E}.\vec{A}$ [Nm^2/C]

$= (E)(A) \cos\theta$



$\vec{A}$: . area vector
. magnitude = A
. perpendicular to the surface and dircted away from the interior

To calculate Φ for The whole surface:

$\Phi = \int \vec{E}.d\vec{A}$ Nm^2/C

θ: The angle between $\vec{A}$ and $\vec{E}$

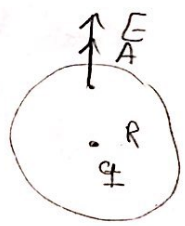
- The electric flux Φ through a Gaussian surface is Proportional to the net number of electric field lines passing through that surface

Gauss' law :- Electric flux from a closed surface = $\frac{q_{en}}{\epsilon_0}$

$$\Rightarrow \oint \vec{E}.d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

التدفق من خلال سطح مغلق

Gauss' law and columb's law :-



$\theta = 0$

$$\boxed{\oint \vec{E}.d\vec{A} = \frac{q}{\epsilon_0}}$$ Gauss' law

$$\oint E dA = \frac{q}{\epsilon_0}$$

constant

$$E \int dA = \frac{q}{\epsilon_0} \Rightarrow EA = \frac{q}{\epsilon_0} \Rightarrow \boxed{E = \frac{q}{4\pi\epsilon_0 (r^2)}}$$

$4\pi r^2$ (sphere)

columb's law

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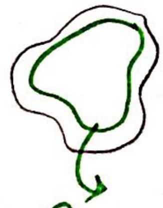
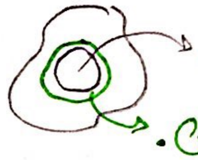
• A charged isolated Conductor

• If an excess Charge is placed on an isolated conductor It will move to the surface $\Rightarrow \vec{E}_{internal} = 0$



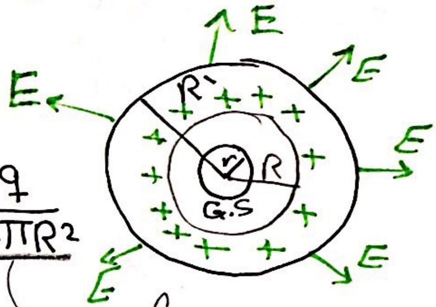
• An isolated Conductor with a Cavity

• stays the same and q will move to the surfac



Application on Gauss' law

1- spherical symmetry



A) spherical shell - Radius R, charge q, $\sigma = \frac{q}{4\pi R^2}$

surface area

→ E inside the sphere:-

$$\oint E \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

since q inside $= 0 \rightarrow E = 0$ at $r \leq R$

→ E outside the sphere:-

$$\oint \underset{\text{Constant}}{\underbrace{E}} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \qquad \theta = 0$$

$$= E \oint dA = \frac{q_{enc}}{\epsilon_0} \Longrightarrow EA = \frac{q_{enc}}{\epsilon_0} \qquad R` \geq R$$

$$E = \frac{q_{enc}}{4\pi\epsilon_0 R`^2}$$

$\Rightarrow$ same as E for a point charge at the center of the sphere

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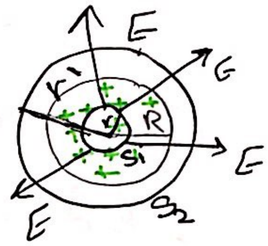
B) Non-Conducting solid Sphere

Solid Sphere : Radius R / charge $= q$ / $\rho = \frac{q}{\text{Volume}} = \frac{q}{\frac{4\pi R^3}{3}}$ c/m³

→ E inside the sphere



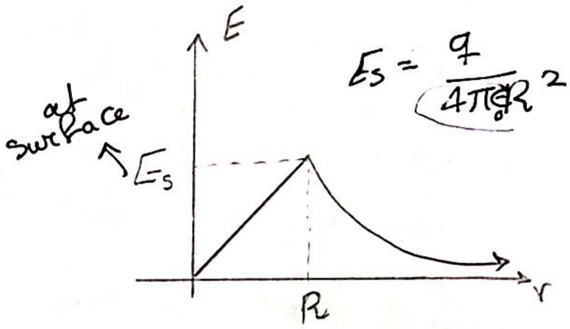
$$\oint_{S_1} E \cdot \vec{dA} = \frac{q_{enc}}{\epsilon_0}$$

$$\epsilon_0 E \oint \vec{dA} = \rho \left(\frac{4}{3}\pi r^3\right)$$

$$E \epsilon_0 A = \rho \frac{4}{3}\pi r^3$$

$$E = \frac{\rho \frac{4}{3}\pi r^3}{\epsilon_0 4\pi r^2} = \frac{\rho r}{3\epsilon_0} \qquad r \leq R$$

$$= \frac{r q}{4\pi r^3 \epsilon_0} = \frac{q}{4\pi\epsilon_0 r^2}$$



→ E outside the sphere

$$\oint_{S_2} E \cdot \vec{dA} = \frac{q_{enc}}{\epsilon_0}$$

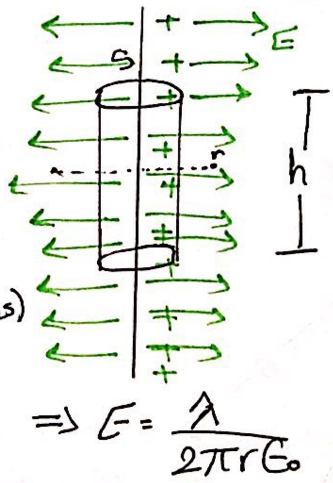
$$E \cos 0 \oint_{S_2} dA = \frac{q}{\epsilon_0}$$

$$E 4\pi r^2 = \frac{q}{\epsilon_0} \Rightarrow E = \frac{q}{4\pi\epsilon_0 r^2} \qquad r \geq R$$

2- Cylinder symmetry

Charged infinite Wire :-

$$\lambda = \text{constant} \quad c/m = \frac{q}{h}$$



→ E near the rod (At distance r)

$$\oint_S \vec{E} \cdot \vec{dA} = \frac{q_{enc}}{\epsilon_0}$$

$$E \cos 0 \oint_S \vec{dA} = \frac{q}{\epsilon_0} \Longrightarrow E A = \frac{\lambda h}{\epsilon_0} \Rightarrow E = \frac{\lambda}{2\pi r \epsilon_0}$$

$q = \lambda h$

$A = 2\pi r h$ (Area of sides)

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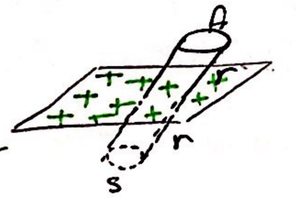
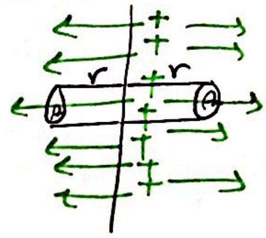
3- Planner symmetry -

Charged infinite nonconducting sheet (plane)



in another view



$\sigma = \frac{q}{A}$

$\oint E \cdot dA = \frac{q_{enc}}{\epsilon_0}$

$EA + EA = \frac{q}{\epsilon_0}$

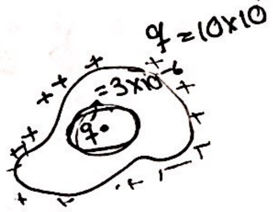
$2EA = \frac{q}{\epsilon_0}$

$E = \frac{\sigma A}{2A\epsilon_0} = \frac{\sigma}{2\epsilon_0}$ (constant)

- Φ is a scalar Quantity

How to solve Problems?

- when you have a charge inside a cube, Φ_{total} = flux resulting from E.A for each face (6 faces)
- Solve 39 (a new idea)
- If you have a conductor with a cavity :



Example : Problem 21 :-

- q inside the cavity $= 3\times10^{-6}$ / $q_{net} = 10\times10^{-6}$
  point

→ on the cavity wall $q = q$

- and q on the outer surface $= q_{net}$

→ q = net charge + point charge $= (10 + 3)\times10^{-6} = 13\times10^{-6}$

- Make sure to know $\vec{E}$ for each symmetry (you have to know the way to get $\vec{E}$)

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