Electric Potential

Electric Potential Energy

If a system of particles change It's configuration from an initial state i to a final state f, Then F does Work on The particles :

$$W_{\text{Electro-Static}} = \int_i^f q\vec{E}\cdot\vec{ds} = \int_i^f qE\cos\theta\, ds = -\Delta U$$

$$\boxed{W_E = -\Delta U}$$ → W_E is path independent since F_E is Conservative

- assuming That The particles are at ∞ as an initial positio

$U_\infty = 0$

so $W = -(U_f - U_i) = -U_f = \int_\infty^f qE\cdot ds$ (with $U_i = 0$)

Electric Potential

Def: It is the potential energy per unit charge at a point in an electric field

$$\Delta V = \frac{\Delta U}{q}$$

$$\Delta V = -\frac{W_E}{q}$$

$$V = -\frac{W_\infty}{q} \quad \text{if } U_i = 0$$

[J/C] and it's a scalar
↳ [Volts]

Note :-
equipotential surfac means $\Delta W = 0$ and $V_f = V_i$

for W_{applied} :-

$$\Delta V = \frac{W_a}{q}$$

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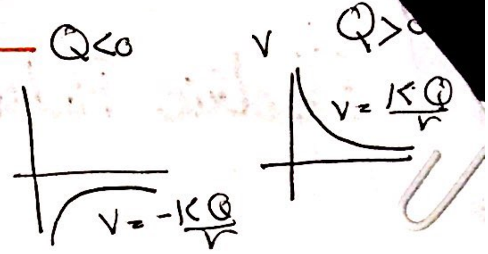
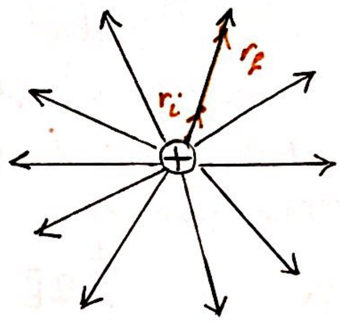
~~Potential Due to a Point charge~~



Potential Due to a Point charge

$$V_f - V_i = -\int_{r_i}^{r_f} \vec{E} \cdot \vec{ds}$$ → differential displacement

r_i → radial path



$$= -\int_{r_i}^{r_f} \vec{E} \cdot \vec{dr} \qquad \cos 0 = 1$$

$$= -\int_{r_i}^{r_f} E\,dr = -\int_{r_i}^{r_f} \frac{KQ}{r^2}\,dr$$

$$= -KQ\left[\frac{-1}{r}\right]_{r_i}^{r_f} = KQ\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

assuming that $r_i = \infty \Rightarrow V_f = \frac{KQ}{r_f}$ ~ • الجهد الناشئ عن شحنة نقطية

Potential due to an electric dipole

first:- V_A

$V_A = V_+ + V_-$ (scalar)

$$= \frac{KQ}{r_+} + \frac{K\bar{Q}}{r_-}$$

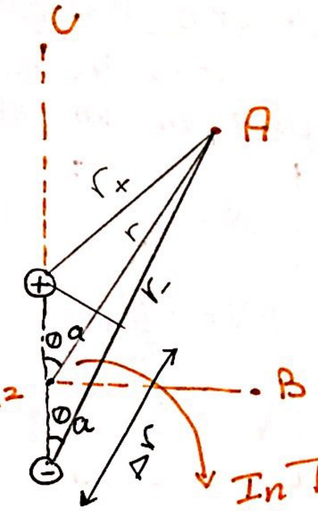
$$= KQ\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$$

$$= \frac{KQ\,(r_- - r_+)}{r_+ r_-}$$

assuming that $r \geq a$

Then : $r_- \cdot r_+ \approx r^2$

$\Rightarrow r_- - r_+ = \Delta r$



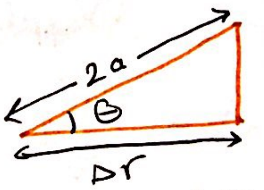
→ In This Triangle

Knowing That

$\Delta r = \cos\theta \; 2a$

$$V_A = \frac{KQ\,\Delta r}{r^2} = \frac{K\overbrace{Q\;2a}^{P}\cos\theta}{r^2}$$

$$= \frac{KP\cos\theta}{r^2}$$ → between Δr and $2a$



$$\cos\theta = \frac{\Delta r}{2a}$$

$$V_B = \frac{KQ\cos 90}{r^2} = 0$$

$V_C = KQ/r^2 \Rightarrow \cos 0 = 1$

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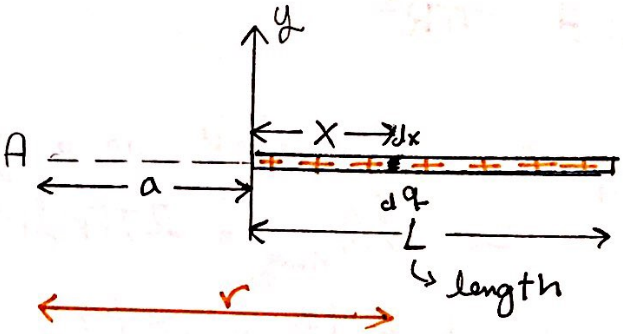
• Potential due to a continuous Charge distribution

Case 1: Potential due to a uniformly charged rod:-

$\lambda = \frac{q}{L} \Rightarrow dq = \lambda dx$



$V = \int \frac{K dq}{r}$

$V_A = K \int \frac{\lambda dx}{r}$

$= K\lambda \int_0^L \frac{dx}{a+x}$

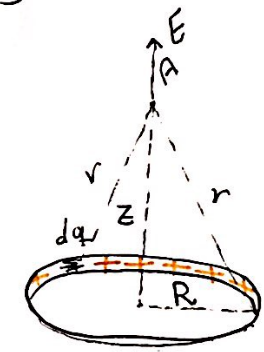
$= K\lambda \ln|a+x| \Big]_0^L$

$= K\lambda \ln|a+L| - \ln|a|$

$$V_A = \lambda K \ln\left|\frac{a+L}{a}\right|$$

Case 2: Potential due to a uniformly charged ring

$\lambda = \frac{q}{2\pi R}$



$V_A = \int \frac{K dq}{r}$

$V_A = K \int \frac{dq}{\sqrt{R^2 + z^2}}$ ← constant

$V_A = \frac{Kq}{\sqrt{R^2+z^2}}$ if you want to get E_z use $\frac{dV}{dE_z}$

$-\frac{dV}{dE_z} = \frac{-Kq \cdot 2z}{-2\sqrt{(R^2+z^2)^3}} = \frac{Kqz}{\sqrt{(R^2+z^2)^3}}$

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Case 3:- Potential due to an uniformly charged disk

a disk is a group of infinit Rings so:-

$dA_{ring} = 2\pi r\,dr$ $\qquad \sigma = \frac{q}{A}$

$dq_{ring} = \sigma\,dA$

$$V_{disk} = \int_0^R dv_{ring}$$

$$= \int_0^R \frac{K\,dq}{\sqrt{r^2+Z^2}}$$

$$= \int_0^R \frac{K\sigma\,2\pi r\,dr}{\sqrt{r^2+Z^2}} = K\sigma 2\pi \int_0^R \frac{r\,dr}{\sqrt{r^2+Z^2}}$$

بالتعويض :-

$$V_{disk} = 2K\sigma\pi\left[\sqrt{R^2+Z^2} - Z\right]$$

$$= \frac{2\sigma\pi\left[\sqrt{R^2+Z^2} - Z\right]}{4\pi\epsilon_0}$$

$$= \frac{\sigma\left(\sqrt{R^2+Z^2} - Z\right)}{2\epsilon_0}$$

Note: If you wanna find $E_z = -\frac{dV}{dZ}$

$$E = -\frac{dV}{dz} = -\frac{\sigma}{2\epsilon_0}\left(\frac{2Z}{2\sqrt{R^2+Z^2}} - 1\right)$$

$$= -\frac{\sigma}{2\epsilon_0}\left[\frac{Z}{\sqrt{R^2+Z^2}} - 1\right]$$

Result:- $\Delta V = -\int \vec{E}\cdot \vec{ds}$

if you want to get E then you differentiate to:-

- $X \rightarrow$ If you want E_x
- $y \rightarrow$ " " " E_y
- $Z \rightarrow$ " " " E_z

Note

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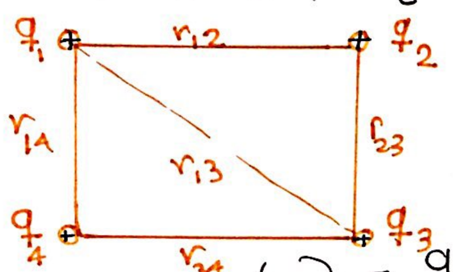
Electric Potential Energy (U) of a system of Point charges :-

⇒ U :- Work must be done by an external agent to assemble the Point charges

- بما أن الشغل يحتاج إلى قوة لكي يحصل فشغل احضار q_1 من اللانهاية إلى النقطة المطلوبة = 0 (لأنه لا يوجد شحنة اخرى)
- ولكن عندما احضرنا q_2 كانت q_1 موجودة وبالتالي هناك قوة متبادلة تبذل جهداً، يساوي $W_2 = q_2 \Delta V$

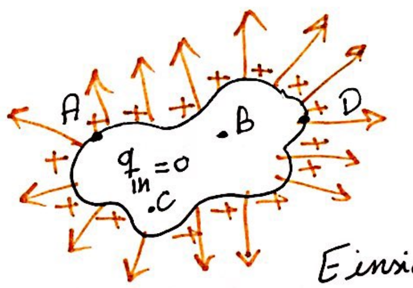


$$W_2 = q_2 \Delta V = q_2\left(\frac{Kq_1}{r_{12}}\right)$$

- وبالنسبة لـ q_3: $W_3 = q_3\left(\frac{Kq_1}{r_{13}} + \frac{Kq_2}{r_{23}}\right)$ ، $W_4 = q_4\left(\frac{Kq}{r_{14}} \ldots\right)$

- **Potential of a charged isolated**

q is distributed on the outer surface



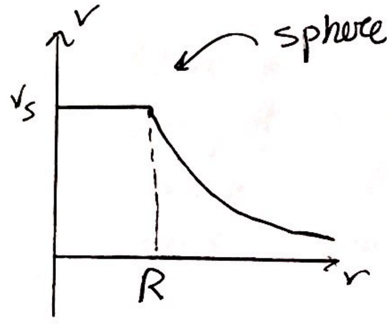
$E_{inside} = 0$

$$V_B = V_C = V_D = V_A$$

$$\Delta V = \int \vec{E} \cdot \vec{ds}, \quad \vec{E} = 0$$

$$\Delta V = 0$$

$$V_B - V_C = 0 \Rightarrow V_B = V_C = V_D = V_A$$



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