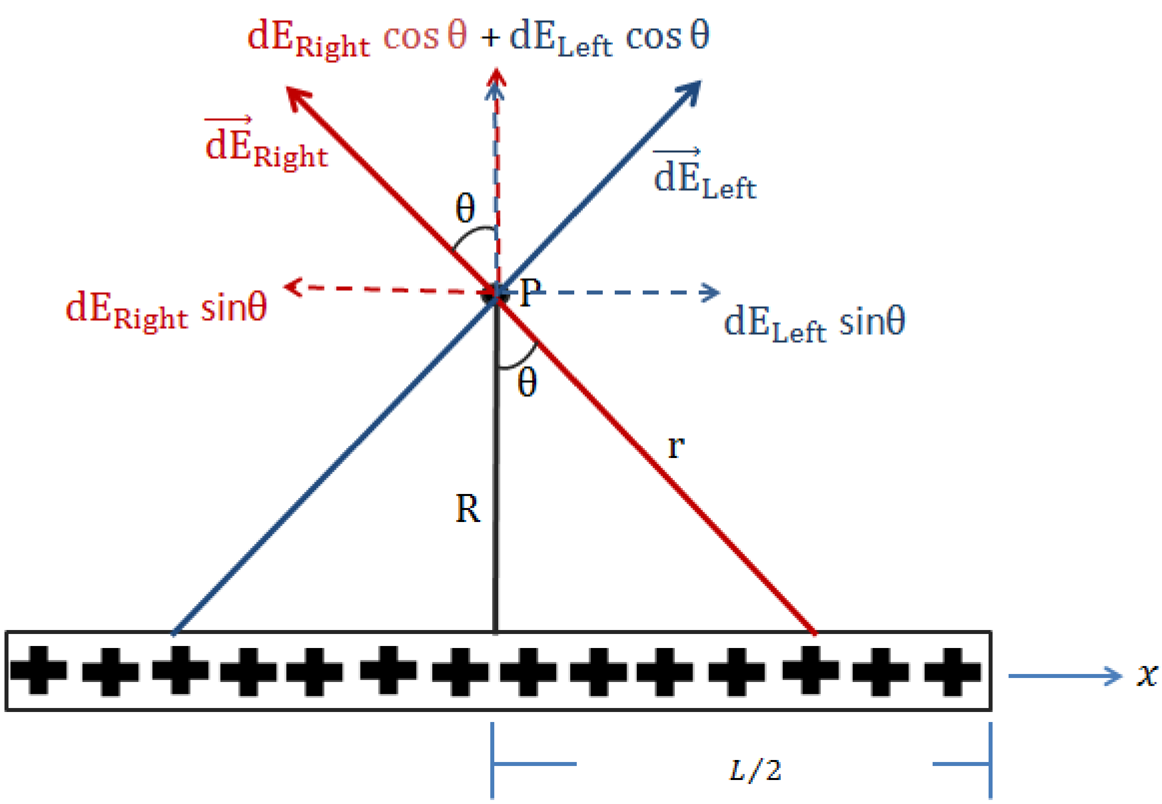
Problem 22-10: Positive charge (q = 9.25 pC = 9.25×10^{-12} C) is spread uniformly along a thin non-conducting rod of length (L = 16.0 cm = 0.16 m). What is the electric field produced at point P, at distance (R = 6.00 cm = 0.06 m) from the rod along its perpendicular bisector?

$$\overrightarrow{dE} = \frac{k\,dq}{r^2}\hat{r}$$



$dE_{Left} = dE_{Right}$ (both have same magnitude)

$dE_x = -dE_{Right}\, sin\theta\, \hat{\imath} + dE_{Left}\, sin\theta\, \hat{\imath} = zero.$ (by symmetric)

$$\vec{E} = \int dE_y = \int dE\cos\theta\hat{\jmath} = \int \frac{k\,dq}{r^2}\cos\theta\hat{\jmath}$$

Using q = λ x; linear charge density. dq = λ dx

$r^2 = (R^2 + x^2)$ and $\cos\theta = \frac{R}{r} = \frac{R}{(R^2+x^2)^{1/2}}$

$$\vec{E} = \int \frac{k\,dq}{r^2}\,cos\,\theta\hat{j} = \int_{-L/2}^{L/2} \frac{k\,\lambda\,\mathrm{dx}}{(R^2+x^2)}\,\frac{R}{(R^2+x^2)^{1/2}}\,\hat{j}$$

$$= k\,\lambda\,\mathrm{R}\,\hat{j}\int_{-L/2}^{L/2} \frac{dx}{(R^2+x^2)^{3/2}} = 2\,k\,\lambda\,\mathrm{R}\,\hat{j}\int_{0}^{L/2} \frac{dx}{(R^2+x^2)^{3/2}}$$

$$\begin{Bmatrix} take\,\tan\theta = \frac{x}{R}, x = R\tan\theta \;\rightarrow\rightarrow \mathrm{dx} = \mathrm{R}\,sec^2\theta\,d\theta \\ (R^2+x^2)^{3/2} = (R^2+R^2tan^2\theta)^{3/2} = (R^2sec^2\theta)^{3/2} = R^3sec^3\theta \end{Bmatrix}$$

$$\vec{E} = 2\,k\,\lambda\,\mathrm{R}\,\hat{j}\int \frac{\mathrm{R}\,sec^2\theta\,d\theta}{R^3sec^3\theta} = \frac{2\,k\,\lambda}{\mathrm{R}}\hat{j}\int \frac{d\theta}{\sec\theta} = \frac{2\,k\,\lambda}{\mathrm{R}}\hat{j}\int \cos\theta\,d\theta$$

$$= \frac{2\,k\,\lambda}{\mathrm{R}}\hat{j}\sin\theta$$

Use $\sin\theta = \frac{x}{(R^2+x^2)^{1/2}}$

$$\Rightarrow \vec{E} = \left[\frac{2\,k\,\lambda}{\mathrm{R}}\frac{x}{(R^2+x^2)^{1/2}}\;\hat{j}\right]_0^{L/2} = \frac{k}{\mathrm{R}}\frac{\lambda\,L}{(R^2+(L/2)^2)^{1/2}}\;\hat{j}$$

Substituted q = 9.25 pC = 9.25×10^{-12} C = λ L, L = 16.0 cm = 0.16 m and R = 6.00 cm = 0.06 m.

$$\boxed{\vec{E} = 13.86\,N/C\;\hat{j}}$$