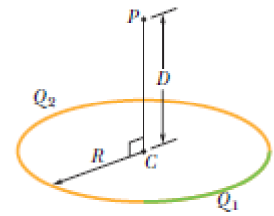
P9-24) plastic rod has been bent into a circle of radius $R$ =8.20 cm. It has a charge $Q1$ = +7.07 pC uniformly distributed along one quarter of its circumference and a charge $Q2$= 6$Q1$ uniformly distributed along the rest of the circumference With $V$= 0 at infinity, what is the electric potential at (a) the center $C$ of the circle and (b) point $P$, on the central axis of the circle at distance $D$ = 2.05 cm from the center?




**Solution:**

9. (a) All the charge is the same distance $R$ from $C$, so the electric potential at $C$ is

$$V=\frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{R}-\frac{6Q_1}{R}\right)=-\frac{5Q_1}{4\pi\varepsilon_0 R}=-\frac{5(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(7.07\times10^{-12}\mathrm{C})}{8.20\times10^{-2}\ \mathrm{m}}=-3.88\ \mathrm{V},$$

where the zero was taken to be at infinity.

(b) All the charge is the same distance from $P$. That distance is $\sqrt{R^2+D^2}$, so the electric potential at $P$ is

$$V=\frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{\sqrt{R^2+D^2}}-\frac{6Q_1}{\sqrt{R^2+D^2}}\right]=-\frac{5Q_1}{4\pi\varepsilon_0\sqrt{R^2+D^2}}$$

$$=-\frac{5(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(7.07\times10^{-12}\mathrm{C})}{\sqrt{(8.20\times10^{-2}\ \mathrm{m})^2+(2.05\times10^{-2}\ \mathrm{m})^2}}$$

$$=-3.76\ \mathrm{V}.$$

P11-24) An electron is placed in an *xy* plane where the electric potential depends on *x* and *y* as shown in (the potential does not depend on *z*).The scale of the vertical axis is set by *Vs* = 100 V. In unit-vector notation, what is the electric force on the electron?

11. The electric field (along some axis) is the (negative of the) derivative of the potential $V$ with respect to the corresponding coordinate. In this case, the derivatives can be read off of the graphs as slopes (since the graphs are of straight lines). Thus,

$$E_x = -\frac{\partial V}{\partial x} = -\left(\frac{-1000\text{ V}}{0.20\text{ m}}\right) = 5000\text{ V/m} = 5000\text{ N/C}$$

$$E_y = -\frac{\partial V}{\partial y} = -\left(\frac{600\text{ V}}{0.30\text{ m}}\right) = -2000\text{ V/m} = -2000\text{ N/C}.$$

The force on the electron is given by $\vec{F} = q\vec{E}$ where $q = -e$. The minus sign associated with the value of $q$ has the implication that $\vec{F}$ points in the opposite direction from $\vec{E}$ (which is to say that its angle is found by adding 180° to that of $\vec{E}$ ). With $e = 1.60 \times 10^{-19}$ C, we obtain

$$\vec{F} = (-1.60\times10^{-19}\text{ C})[(5000\text{ N/C})\hat{\text{i}} - (2000\text{ N/C})\hat{\text{j}}] = (-8.0\times10^{-16}\text{ N})\hat{\text{i}} + (3.2\times10^{-16}\text{ N})\hat{\text{j}}.$$

P17-23) What is the magnitude of the electric field at the point (1,-2,4) if the electric potential is given by *V* = 2.00*xyz*2, where *V* is in volts and *x*, *y*, and *z* are in meters?

**EXPRESS** With $V = 2.00xyz^2$, we apply Eq. 24–41 to calculate the *x*, *y*, and *z* components of the electric field:

$$E_x = -\frac{\partial V}{\partial x} = -2.00yz^2$$

$$E_y = -\frac{\partial V}{\partial y} = -2.00xz^2$$

$$E_z = -\frac{\partial V}{\partial z} = -4.00xyz$$

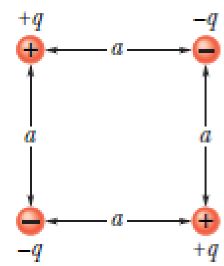
which, at $(x, y, z) = (-1.00\text{ m}, -2.00\text{ m}, 4.00\text{ m})$, gives

$$(E_x, E_y, E_z) = (+64.0\text{ V/m}, +32.0\text{ V/m}, -32.0\text{ V/m}).$$

**ANALYZE** The magnitude of the field is therefore

$$\left|\vec{E}\right| = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(64.0\text{ V/m})^2 + (+32.0\text{ V/m})^2 + (-32.0\text{ V/m})^2}$$
$$= 78.4\text{ V/m} = 78.4\text{ N/C}.$$

P33-24) The particle shown in the next figure each have charge of magnitude q = 5.00pC and were initially far apart. To form the square with edge length a =64.0 cm, (a) how much work must done by an external agent, (b) how work must be done by the electric force, and (c) what is the potential energy of the system?




**EXPRESS** We choose the zero of electric potential to be at infinity. The initial electric potential energy $U_i$ of the system before the particles are brought together is therefore zero. After the system is set up the final potential energy is

$$U_f = \frac{q^2}{4\pi\varepsilon_0}\left(-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}\right)=\frac{2q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right).$$

(a) Thus the amount of work required to set up the system is given by

$$W_{\text{app}} = \Delta U = U_f - U_i = U_f = \frac{2q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right)=\frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-12}\ \text{C})^2}{0.640\ \text{m}}\left(\frac{1}{\sqrt{2}}-2\right)$$
$$= -9.08\times10^{-13}\ \text{J}.$$

(b) The work done by the electric force is

$$W = -W_{\text{app}} = +9.08\times10^{-13}\ \text{J}.$$

(c) The potential energy of the system is the work required to set up the arrangement:

$$U = W_{\text{app}} = -9.08\times10^{-13}\ \text{J}.$$

P39-24)(a) What is the *escape speed* for an electron initially at rest on the surface of a sphere with a radius of 20.0 cm and a uniformly distributed charge of 1.6x $10^{-15}$ C? That is, what initial speed must the electron have in order to reach an infinite distance from the sphere and have zero kinetic energy when it gets there?(b) If the initial speed is twice the escape speed, what is the kinetic energy at infinity?

***Solution:*** The *escape speed* may be calculated from the requirement that the initial kinetic energy (of *launch*) be equal to the absolute value of the initial potential energy. Thus,

$$\frac{1}{2}mv^2 = \frac{qe}{4\pi\varepsilon_0 r} = \frac{kqe}{r}$$

where $m = 9.11 \times 10^{-31}$ kg, $e = 1.60 \times 10^{-19}$ C, $q = 10000e$, and $r = 0.20$ m. This yields

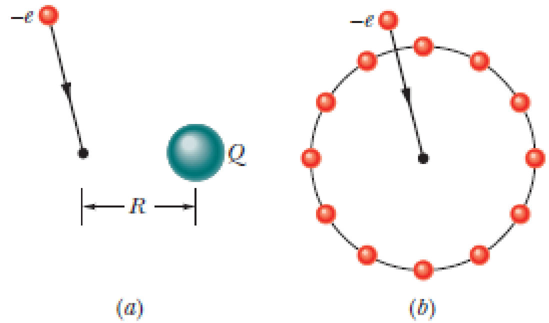
$$v_{\text{esc}} = \sqrt{\frac{2keq}{mr}} = \sqrt{\frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})(1.6\times10^{-15}\ \text{C})}{(9.11\times10^{-31}\ \text{kg})(0.20\ \text{m})}} = 5.0\times10^3\ \text{m/s}$$

(b) By energy conservation, $K_i + U_i = K_f + U_f$. At infinity, $U_f = 0$ and we have

$$K_f = K_i + U_i = \frac{1}{2}m(2v_{\text{esc}})^2 - \frac{keq}{r} = \frac{1}{2}m\left(2\sqrt{\frac{2keq}{r}}\right)^2 - \frac{keq}{r} = \frac{3keq}{r}$$

$$= \frac{3(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})(1.6\times10^{-15}\ \text{C})}{0.20\ \text{m}} = 3.5\times10^{-23}\ \text{J}$$

P44-24) In the next Fig, we move an electron from an infinite distance to a point at distance $R$ = 8.00 cm from a tiny charged ball. The move requires work $W$ = 5.32x10$^{-13}$ J by us. (a) What is the charge $Q$ on the ball? In Fig. *b*, the ball has been sliced up and the slices spread out so that an equal amount of charge is at the hour positions on a circular clock face of radius $R$ = 8.00 cm. Now the electron is brought from an infinite distance to the center of the circle. (b) With that addition of the electron to the system of 12 charged particles, what is the change in the electric potential energy of the system?




Solution:

44. (a) The work done results in a potential energy gain:

$$W = q\,\Delta V = (-e)\left(\frac{Q}{4\pi\varepsilon_0 R}\right) = +5.32 \times 10^{-13}\ \text{J}\,.$$

With $R$ = 0.0800 m, we find $Q = -2.96 \times 10^{-5}$ C.

(b) The work is the same, so the increase in the potential energy is $\Delta U = +5.32 \times 10^{-13}$ J.

P48-24) Two isolated, concentric, conducting spherical shells have radii $R_1$ = 0.500 m and $R_2$ = 1.00 m, uniform charges $q_1$ =+3.00μC and $q_2$ =+1.00μC, and negligible thicknesses. What is the magnitude of the electric field $E$ at radial distance (a) $r$ = 4.00 m, (b) $r$ = 0.700 m, and (c) $r$ =0.200 m? With $V$ = 0 at infinity, what is $V$ at (d) $r$ = 4.00 m, (e) $r$ _=1.00 m, (f) $r$ = 0.700 m, (g) $r$ = 0.500 m, (h) $r$ = 0.200 m, and (i) $r$ = 0? ( j) Sketch $E(r)$ and $V(r)$.

48. Since the charge distribution is spherically symmetric we may write

$$E(r)=\frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r},$$

where $q_{\text{enc}}$ is the charge enclosed in a sphere of radius $r$ centered at the origin.

(a) For $r$ = 4.00 m, $R_2$ = 1.00 m, and $R_1$ = 0.500 m, with $r > R_2 > R_1$ we have

$$E(r)=\frac{q_1+q_2}{4\pi\varepsilon_0 r^2}=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\text{C}+1.00\times10^{-6}\text{C})}{(4.00\ \text{m})^2}=2,25\times10^3\ \text{V/m}.$$

(b) For $R_2 > r$ = 0.700 m > $R_2$,

$$E(r)=\frac{q_1}{4\pi\varepsilon_0 r^2}=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\text{C})}{(0.700\ \text{m})^2}=5.50\times10^4\ \text{V/m}.$$

(c) For $R_2 > R_1 > r$, the enclosed charge is zero. Thus, $E$ = 0.

The electric potential may be obtained using Eq. 24-18:

$$V(r)-V(r')=\int_r^{r'} E(r)dr.$$

(d) For $r = 4.00$ m $> R_2 > R_1$, we have

$$V(r)=\frac{q_1+q_2}{4\pi\varepsilon_0 r}=\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\text{C}+1.00\times10^{-6}\text{C})}{(4.00\text{ m})}=8.99\times10^3\text{ V}.$$

(e) For $r = 1.00$ m $= R_2 > R_1$, we have

$$V(r)=\frac{q_1+q_2}{4\pi\varepsilon_0 r}=\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\text{C}+1.00\times10^{-6}\text{C})}{(1.00\text{ m})}=3.60\times10^4\text{ V}.$$

(f) For $R_2 > r = 0.700$ m $> R_2$,

$$V(r)=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r}+\frac{q_2}{R_2}\right)=(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{3.00\times10^{-6}\text{C}}{0.700\text{ m}}+\frac{1.00\times10^{-6}\text{C}}{1.00\text{ m}}\right)$$
$$=4.75\times10^4\text{ V}.$$

(g) For $R_2 > r = 0.500 \text{ m} = R_2$,

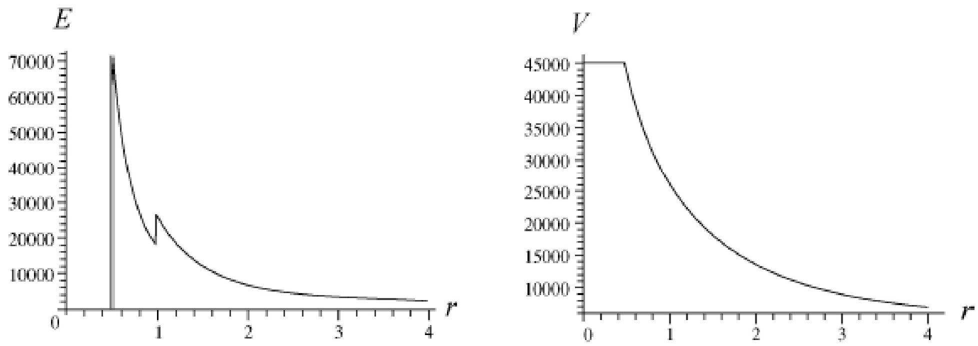
$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{R_2}\right) = (8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{3.00\times10^{-6}\text{C}}{0.500\text{ m}} + \frac{1.00\times10^{-6}\text{C}}{1.00\text{ m}}\right)$$
$$= 6.29\times10^4\text{ V}.$$

(h) For $R_2 > R_1 > r$,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1} + \frac{q_2}{R_2}\right) = (8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{3.00\times10^{-6}\text{C}}{0.500\text{ m}} + \frac{1.00\times10^{-6}\text{C}}{1.00\text{ m}}\right)$$
$$= 6.29\times10^4\text{ V}.$$

(i) At $r = 0$, the potential remains constant, $V = 6.29\times10^4$ V.

(j) The electric field and the potential as a function of $r$ are depicted below:



P59-24) The electric field in a region of space has the components *Ey = Ez* 0 and *Ex* =(4.00N/C)*x*. Point *A* is on the *y* axis at *y* 3.00 m, and point *B* is on the *x* axis at *x* 4.00 m.What is the potential difference *VB – VA?*

59. We connect $A$ to the origin with a line along the $y$ axis, along which there is no change of potential (Eq. 24-18: $\int \vec{E} \cdot d\vec{s} = 0$). Then, we connect the origin to $B$ with a line along the $x$ axis, along which the change in potential is

$$\Delta V = -\int_0^{x=4} \vec{E} \cdot d\vec{s} = -4.00\int_0^4 x^2\, dx = -4.00\left(\frac{4^3}{3}\right)$$

which yields $V_B - V_A = -85.3$ V.