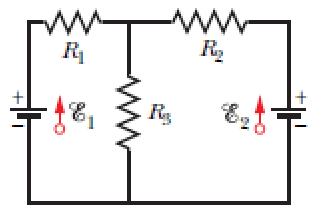
P2-27) In Fig. the ideal batteries have emfs ε_1 = 12.0 V and ε_2 = 0.500 ε_1, and the resistances are each 4.00 Ω.What is the current in (a) resistance 2 and (b) resistance 3?



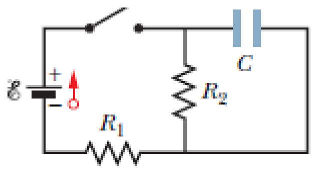
2. Using the junction rule ($i_3 = i_1 + i_2$) we write two loop rule equations:

$$12.0 \text{ V} - i_1R_1 - (i_1 + i_2)\, R_3 = 0$$
$$5.00 \text{ V} - i_2R_2 - (i_1 + i_2)\, R_3 = 0.$$

(a) Solving, we find i_2 = -0.167 A , and

(b) $i_3 = i_1 + i_2$ = 1.41 A (downward, as was assumed in writing the equations as we did).

P9-27)Fig, $R1$= 10.0 k Ω, $R2$ = 15.0 k Ω, C _=0.400 mF, and the ideal battery has emf ε_1 =20.0 V. First, the switch is closed a long time so that the steady state is reached Then the switch is opened at time t = 0. For resistor 2 at t = 4.00 ms. What are (a) the current, (b) the rate at which the current is changing (c) the rate at which the dissipation rate is changing.



9. (a) In the steady state situation, the capacitor voltage will equal the voltage across R_2 = 15 kΩ:

$$V_0 = R_2 \frac{\varepsilon}{R_1 + R_2} = (15.0\ \text{k}\Omega)\left(\frac{20.0\text{V}}{10.0\ \text{k}\Omega + 15.0\ \text{k}\Omega}\right) = 12.0\text{V}.$$

Now, multiplying Eq. 27-39 by the capacitance leads to $V = V_0 e^{-t/RC}$ describing the voltage across the capacitor (and across R_2 = 15.0 kΩ) after the switch is opened (at t = 0). Thus, with t = 0.00400 s, we obtain

$$V = (12)e^{-0.004/(15000)(0.4\times10^{-6})} = 6.16\,\text{V}.$$

Therefore, using Ohm's law, the current through R_2 is 6.16/15000 = 4.11 × 10^{-4} A.

(b) The charge in the capacitor is $q(t) = CV(t) = CV_0 e^{-t/\tau}$, where

$$\tau = R_2 C = (15.0\times10^3\,\Omega)(0.400\times10^{-6}\,\text{F}) = 6.00\times10^{-6}\,\text{s}$$

The current is

$$i(t) = \frac{dq(t)}{dt} = -\frac{CV_0}{\tau} e^{-t/\tau},$$

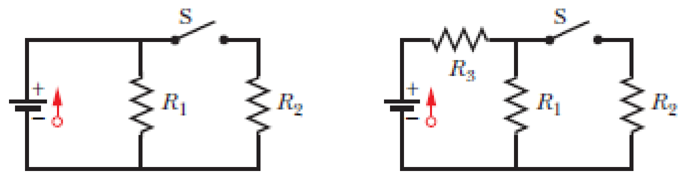
so the rate of change of the current at t = 4.00 ms is

$$\frac{di(t)}{dt} = \frac{d^2q(t)}{dt^2} = \frac{CV_0}{\tau^2} e^{-t/\tau} = \frac{(0.400\times10^{-6}\ \text{s})(12.0\ \text{V})}{(6.00\times10^{-3}\ \text{s})^2} e^{-(4.00\times10^{-3}\,\text{s})/(6.00\times10^{-3}\,\text{s})} = 6.85\times10^{-2}\,\text{A/s}.$$

(c) The dissipation rate is given by $P(t) = i^2(t)R_2$. Thus, the rate of change of $P(t)$ at t = 4.00 ms is

$$\frac{dP(t)}{dt} = \frac{d}{dt}\left[i^2(t)R_2\right] = 2R_2 i(t)\frac{di(t)}{dt}$$
$$= 2(15.0\times10^3\,\Omega)(4.11\times10^{-6}\ \text{A})(6.85\times10^{-2}\,\text{A/s}) = 0.844\ \text{W/s}.$$

P14-27) The resistances in Figs. 27-45*a* and *b* are all 4.0 Ω, and the batteries are ideal 12 V batteries. (a) When switch S in Fig. 27-45*a* is closed, what is the change in the electric potential *V*1 across resistor 1, or does *V*1 remain the same? (b) When switch S in Fig. 27-45*b* is closed, what is the change in *V*1 across resistor 1, or does *V*1 remain the same

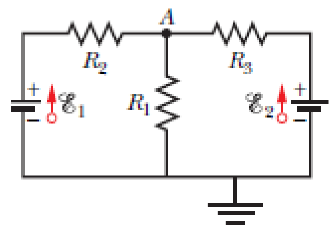


(*a*) (*b*)

14. (a) By the loop rule, it remains the same. This question is aimed at student conceptualization of voltage; many students apparently confuse the concepts of voltage and current and speak of "voltage going through" a resistor – which would be difficult to rectify with the conclusion of this problem.

(b) The loop rule still applies, of course, but (by the junction rule and Ohm's law) the voltages across R_1 and R_3 (which were the same when the switch was open) are no longer equal. More current is now being supplied by the battery, which means more current is in R_3, implying its voltage drop has increased (in magnitude). Thus, by the loop rule (since the battery voltage has not changed) the voltage across R_1 has decreased a corresponding amount. When the switch was open, the voltage across R_1 was 6.0 V (easily seen from symmetry considerations). With the switch closed, R_1 and R_2 are equivalent (by Eq. 27-24) to 2.0 Ω, which means the total load on the battery is 6.0 Ω. The current therefore is 2.00 A, which implies that the voltage drop across R_3 is 8.0 V. The loop rule then tells us that the voltage drop across R_1 is 12 V – 8.0 V = 4.0 V. This is a decrease of 2.0 volts from the value it had when the switch was open.

P20-27) In Fig, ε_1 = =.00 V, ε_2 =12.0 V, $R1$ = 100 Ω, $R2$ =200 Ω, and $R3$= 300 Ω. One point of the circuit is grounded (V = 0). What are the (a) size and (b) direction (up or down) of the current through resistance 1, the (c) size and (d) direction (left or right) of the current through resistance 2, and the (e) size and (f) direction of the current through resistance 3? (g) What is the electric potential at point A?



20. (a) Using the junction rule ($i_1 = i_2 + i_3$) we write two loop rule equations:

$$\varepsilon_1 - i_2R_2 - (i_2 + i_3)R_1 = 0$$
$$\varepsilon_2 - i_3R_3 - (i_2 + i_3)R_1 = 0.$$

Solving, we find i_2 = 0.0182 A (rightward, as was assumed in writing the equations as we did), i_3 = 0.02545 A (leftward), and $i_1 = i_2 + i_3$ = 0.04365 A (downward).

(b) The direction is downward. See the results in part (a).

(c) I_2 = 0.0182 A. See the results in part (a).

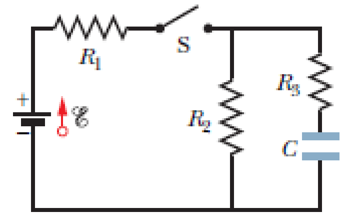
(d) The direction is rightward. See the results in part (a).

(e) i_3 = 0.0254 A. See the results in part (a).

(f) The direction is leftward. See the results in part (a).

(g) The voltage across R_1 equals V_A: (0.0382 A)(100 Ω) = +4.37 V.

P47-27) In the circuit, ε_1 = 1.2 kV, C = 6.5 μF, $R1 = R2 = R3$ = 0.73 M Ω. With C completely uncharged, switch S is suddenly closed (at t = 0). At t = 0, what are (a) current $i1$ in resistor 1, (b) current $i2$ in resistor 2, and (c) current $i3$ in resistor 3? At t = infinity (that is, after many time constants), what are (d) $i1$, (e) $i2$, and (f) $i3$? What is the potential difference $V2$ across resistor 2 at (g) t = 0 and (h) t = infinity? (i) Sketch $V2$ versus t between these two extreme times.



47. **THINK** We have a multi-loop circuit with a capacitor that's being charged. Since at $t = 0$ the capacitor is completely uncharged, the current in the capacitor branch is as it would be if the capacitor were replaced by a wire.

EXPRESS Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is downward. Let i_3 be the current in R_3 and take it to be positive if it is downward. The junction rule produces $i_1 = i_2 + i_3$, the loop rule applied to the left-hand loop produces

$$\varepsilon - i_1R_1 - i_2R_2 = 0,$$

and the loop rule applied to the right-hand loop produces

$$i_2R_2 - i_3R_3 = 0.$$

Since the resistances are all the same we can simplify the mathematics by replacing R_1, R_2, and R_3 with R.

ANALYZE (a) Solving the three simultaneous equations, we find

$$i_1 = \frac{2\varepsilon}{3R} = \frac{2(1.2\times10^3\ \text{V})}{3(0.73\times10^6\ \Omega)} = 1.1\times10^{-3}\ \text{A},$$

(b) $i_2 = \dfrac{\varepsilon}{3R} = \dfrac{1.2\times10^3\ \text{V}}{3(0.73\times10^6\ \Omega)} = 5.5\times10^{-4}\ \text{A},$

(c) and $i_3 = i_2 = 5.5\times10^{-4}\ \text{A}.$

At $t = \infty$ the capacitor is fully charged and the current in the capacitor branch is 0. Thus, $i_1 = i_2$, and the loop rule yields $\varepsilon - i_1R_1 - i_1R_2 = 0$.

(d) The solution is $i_1 = \dfrac{\varepsilon}{2R} = \dfrac{1.2\times10^3\ \text{V}}{2(0.73\times10^6\ \Omega)} = 8.2\times10^{-4}\ \text{A}$

(e) and $i_2 = i_1 = 8.2\times10^{-4}\ \text{A}.$

(f) As stated before, the current in the capacitor branch is $i_3 = 0$.

We take the upper plate of the capacitor to be positive. This is consistent with current flowing into that plate. The junction equation is $i_1 = i_2 + i_3$, and the loop equations are

$$\varepsilon - i_1 R - i_2 R = 0$$

$$-\frac{q}{C} - i_3 R + i_2 R = 0.$$

We use the first equation to substitute for i_1 in the second and obtain

$$\varepsilon - 2i_2 R - i_3 R = 0.$$

Thus $i_2 = (\varepsilon - i_3 R)/2R$. We substitute this expression into the third equation above to obtain

$$-(q/C) - (i_3 R) + (\varepsilon/2) - (i_3 R/2) = 0.$$

Now we replace i_3 with dq/dt to obtain

$$\frac{3R}{2}\frac{dq}{dt} + \frac{q}{C} = \frac{\varepsilon}{2}.$$

This is just like the equation for an RC series circuit, except that the time constant is $\tau = 3RC/2$ and the impressed potential difference is $\varepsilon/2$. The solution is

$$q = \frac{C\varepsilon}{2}\left(1 - e^{-2t/3RC}\right).$$

The current in the capacitor branch is

$$i_3(t) = \frac{dq}{dt} = \frac{\varepsilon}{3R} e^{-2t/3RC}.$$

The current in the center branch is

$$i_2(t) = \frac{\varepsilon}{2R} - \frac{i_3}{2} = \frac{\varepsilon}{2R} - \frac{\varepsilon}{6R} e^{-2t/3RC} = \frac{\varepsilon}{6R}\left(3 - e^{-2t/3RC}\right)$$

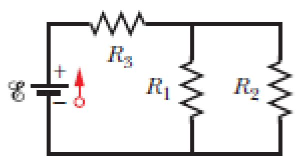
and the potential difference across R_2 is $V_2(t) = i_2 R = \frac{\varepsilon}{6}\left(3 - e^{-2t/3RC}\right)$.

(g) For $t = 0$, $e^{-2t/3RC} = 1$ and $V_2 = \varepsilon/3 = \left(1.2\times10^3\ \mathrm{V}\right)/3 = 4.0\times10^2\ \mathrm{V}$.

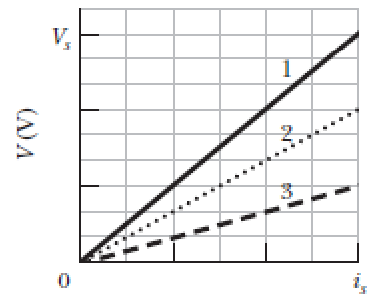
(h) For $t = \infty$, $e^{-2t/3RC} \to 0$ and $V_2 = \varepsilon/2 = \left(1.2\times 20^3\ \mathrm{V}\right)/2 = 6.0\times10^2\ \mathrm{V}$.

(i) A plot of V_2 as a function of time is shown in the following graph.

P56-27) The ideal battery in Fig. 27-39*a* has emf ε_1 = 10.0 V. Plot 1 in Fig. 27-39*b* gives the electric potential difference *V* that can appear across resistor 1 of the circuit versus the current *i* in that resistor. The scale of the *V* axis is set by *Vs* = 18.0 V, and the scale of the *i* axis is set by i_s = 3.00 mA. Plots 2 and 3 are similar plots for resistors 2 and 3, respectively.What is the current in resistor 2?



(*a*)



(*b*) *i* (mA)

56. Line 1 has slope R_1 = 6.0 kΩ. Line 2 has slope R_2 = 4.0 kΩ. Line 3 has slope R_3 = 2.0 kΩ. The parallel pair equivalence is $R_{12} = R_1R_2/(R_1+R_2)$ = 2.4 kΩ. That in series with R_3 gives an equivalence of

$$R_{123} = R_{12} + R_3 = 2.4\ \text{k}\Omega + 2.0\ \text{k}\Omega = 4.4\ \text{k}\Omega.$$

The current through the battery is therefore $i = \varepsilon / R_{123}$ = (10 V)/(4.4 kΩ) = 2.27 mA and the voltage drop across R_3 is $V_3 = iR_3 = (2.27 \times 10^{-3}\text{A})(2.0\ \text{k}\Omega) = 4.55$ V. Subtracting this (because of the loop rule) from the battery voltage leaves us with the voltage across R_2:

$$V_2 = \varepsilon - V_3 = 10.0\ \text{V} - 4.55\ \text{V} = 5.45\ \text{V}.$$

Then Ohm's law gives the current through R_2:

$$i_2 = \frac{V_2}{R_2} = \frac{5.45\ \text{V}}{4.0\ \text{k}\Omega} = 1.4\ \text{mA}.$$