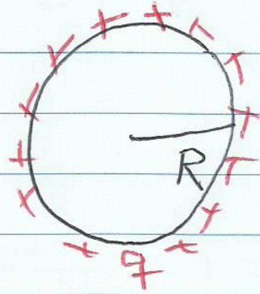
25- Capacitance

Electric Capacitance: is the amount (quantity) of charge needed to change the Potential of a Conductor by 1V

$$C = \frac{q}{V} \quad C/V = \text{Farad}$$

example:- A conducting sphere of radius = R = 90cm
Find its Electric Capacitance ?

$C = \frac{q}{V}$, Assume the conducting sphere has a charge q.



$$V_{sphere} = \frac{kq}{R} = \frac{q}{4\pi \epsilon_0 R}$$

$$C = \frac{q}{V} = q \div \left(\frac{q}{4\pi \epsilon_0 R}\right)$$

$$C_{sphere} = 4\pi \epsilon_0 R$$

depends on:
1) Geometry
2) Medium air (ϵ_0)

$R = 0.9m \Longrightarrow C_{sphere} = \frac{0.9}{9\times10^{9}} = 1\times10^{-10} F$

$= 0.1\times10^{-9} F = 0.1\,nF$

$= 100\times10^{-12} F$

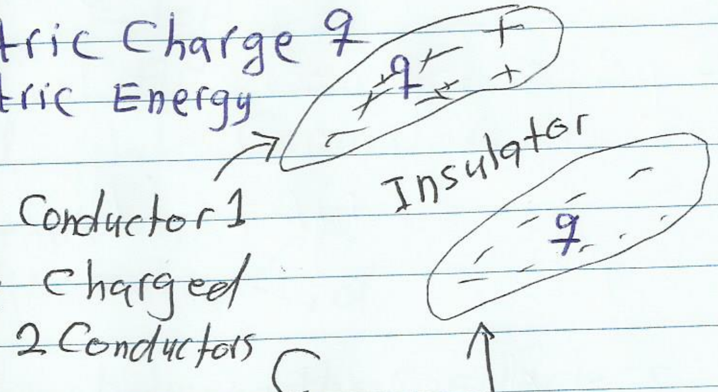
$= 100\,\mu\mu F$

$= 100\,pF$

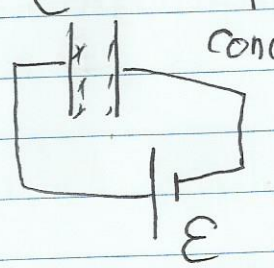
A Capacitor:-

consist. from 2 conductors with insulating material between them.

to store: 1) Electric Charge q
2) Electric Energy U



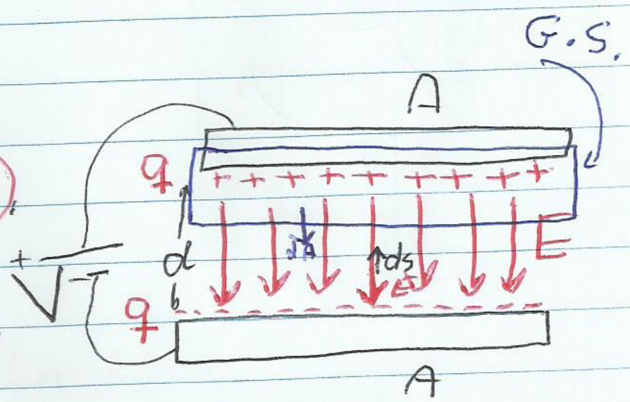
The Capacitor could be charged by connecting its 2 Conductors to a Battery



1) Parallel-plate Capacitor.

2) Cylindrical Capacitor.

3) Spherical Capacitor



1) A parallel-plate Capacitor.

each plate has Area = A
conducting

distance between the plates = d

(air) is the insulating material between the plates

To find C:

1) Find E between the plates by using Gauss' Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \quad \text{Gauss' Law}$$

$$EA\cos 0 = \frac{q}{\epsilon_0} \Rightarrow E = q/\epsilon_0 A$$

$$q = \epsilon_0 E A$$

2) Find ΔV between the plates:

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} \Rightarrow$$

$$V_{f(+)} - V_{(-)i} = -\int_{(-)}^{(+)} \vec{E}\cdot d\vec{s} = -\int_{(-)}^{(+)} E\cos 180\, ds = \int_{-}^{+} E\,ds = E\int_{(-)}^{(+)} ds$$

$$V = Ed$$

3) Find C:
$$C = \frac{q}{V} = \frac{\epsilon_0 E A}{E d}$$

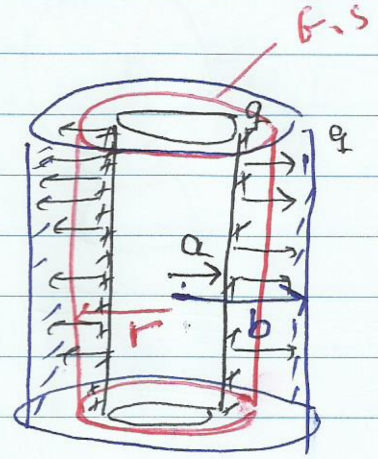
$$C = \frac{\epsilon_0 A}{d}$$ Parallel-plate Capacitor

1) depends on: geometry A, d

2) medium between the conductors ϵ_0

(2) A Cylindrical Capacitor:

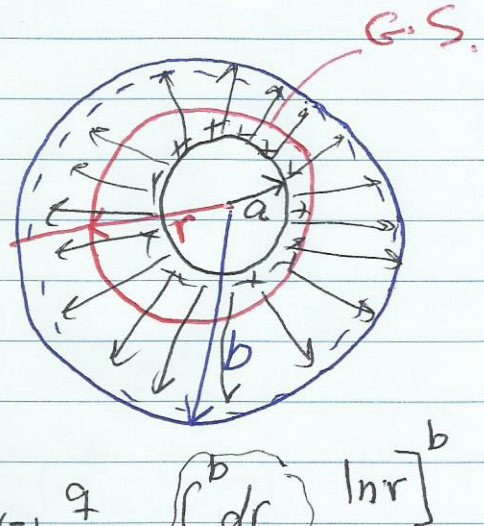
2 Concentric Conducting cylinders of radii, (a, b)
Length L > b



To find C:

1) Find E between the cylinders

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$ Gauss' Law



$$E(2\pi r L) = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{2\pi\epsilon_0 L r}, \quad \lambda = \frac{q}{L}$$

2) Find $\Delta V = V_- - V_+$

$$V_- - V_+ = -\int_{+}^{-} \vec{E}\cdot d\vec{s} = (-)\int_a^b \frac{q\cos 0}{2\pi\epsilon_0 L r}\,dr = (-)\frac{q}{2\pi\epsilon_0 L}\int_a^b \frac{dr}{r} \quad \ln r\Big]_a^b$$

$$V_- - V_+ = (-)\frac{q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right) \Rightarrow V_+ - V_- = \frac{q}{2\pi\epsilon_0 L}\ln(b/a)$$

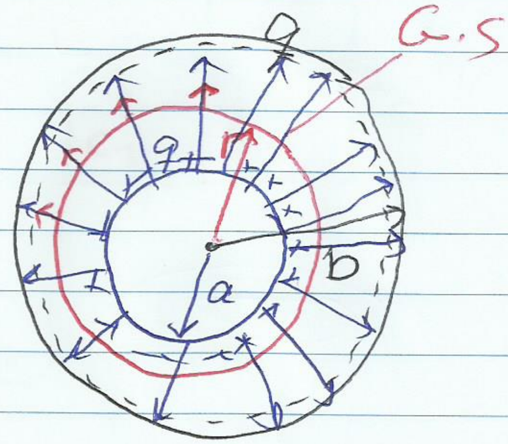
3) Find C:

$$C = \frac{q}{V} \quad , V = \frac{q}{2\pi\epsilon_0 L} \ln\left(\frac{b}{a}\right)$$

$$C = 2\pi\epsilon_0 L / \ln\left(\frac{b}{a}\right)$$ depends on Geometry and the material between the 2 cylinders Conducting

3) A Spherical Capacitor

2 Concentric Conducting spherical shells, of radii a, b



To find C:

1) Find E between the spherical shells

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{en}}{\epsilon_0} \quad \text{Gauss' Law}$$

$$E \, 4\pi r^2 \cos 0 = \frac{q}{\epsilon_0} \qquad E = \frac{q}{4\pi\epsilon_0 r^2}$$

2) Find ΔV

$$V_- - V_+ = -\int_+^- \vec{E} \cdot d\vec{s} = -\int_a^b \frac{q}{4\pi\epsilon_0 r^2} dr \cos 0$$

$$= (-)\frac{q}{4\pi\epsilon_0} \int_a^b \frac{dr}{r^2} = (-)\frac{q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_a^b$$

$$V_- - V_+ = (-)\frac{q}{4\pi\epsilon_0}\left[-\frac{1}{b} + \frac{1}{a}\right] = (-)\frac{q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)$$

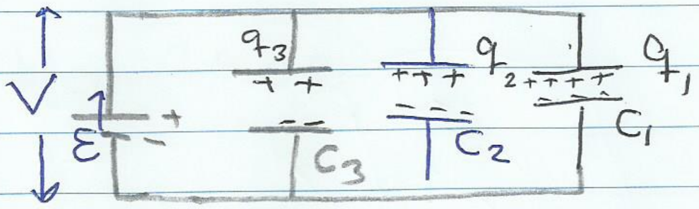
$$V_+ - V_- = \frac{q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)$$

3) Find C :

$$C = \frac{q}{V} \quad , V = \frac{q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)$$

$$C = 4\pi\epsilon_0\left(\frac{ab}{b-a}\right)$$ → depends on Geometry $\lessgtr^{a}_{b}$ and the medium (ϵ_0) air between them.

Capacitors in Parallel:



$$V = V_1 = V_2 = V_3$$

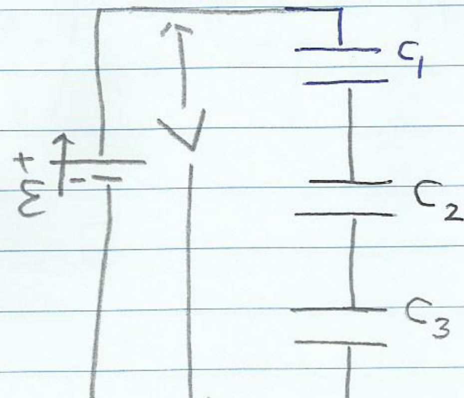
$$V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_3}{C_3} \quad , \quad V = \frac{q_{tot}}{C_{eq}}$$

$$\frac{q_{tot}}{C_{eq}} = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_3}{C_3}$$

$$q_{tot} = q_1 + q_2 + q_3$$
$$= C_1V + C_2V + C_3V$$
$$\frac{q_{tot}}{V} = C_1 + C_2 + C_3$$

$$C_{eq} = C_1 + C_2 + C_3$$

Capacitor in Series:



$$q = q_1 = q_2 = q_3$$
$$q = C_1V_1 = C_2V_2 = C_3V_3$$
$$C_{eq}V = C_1V_1 = C_2V_2 = C_3V_3$$
$$\cancel{q} = \cancel{q}$$

$$V = V_1 + V_2 + V_3$$

$$V = \frac{q_1}{C_1} + \frac{q_2}{C_2} + \frac{q_3}{C_3} \quad ; q_1 = q_2 = q_3 = q$$

$$V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\frac{V}{q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad ; 1/C_{eq} = \frac{V}{q}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Solve Sample Problem 25.02.

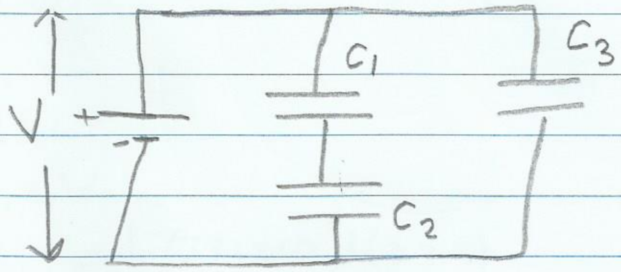
Problem (25.06)

$V = 100V$

$C_1 = 10 \mu F$

$C_2 = 5 \mu F$

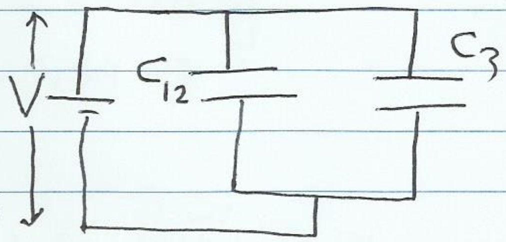
$C_3 = 2 \mu F$



For each Capacitor Find q, V, U?

Find C_{eq}:

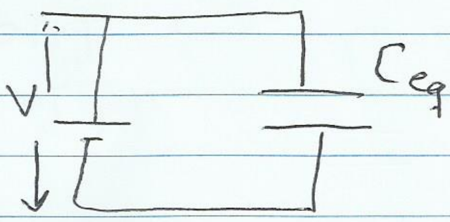
C_1 & C_2 in Series



$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10} + \frac{1}{5} = \frac{3}{10}$$

$$C_{12} = \frac{10}{3} = 3.33 \mu F$$

C_{12} and C_3 are in Parallel



$$C_{eq} = C_{12} + C_3 = 3.33 + 2 = 5.33 \mu F$$

To find the charges:

$$q_{tot} = C_{eq} V = 5.33(100) = 533 \mu C$$

$$V_3 = V_{12} = V = 100V$$

$$q_3 = C_3 V_3 = 2(100) = 200 \mu C$$

$C_3 = 2\mu F \rightarrow V_3 = 100V$

$\rightarrow q_3 = 200\mu C$

$\rightarrow U_3 = \frac{1}{2} q_3 V_3 = \frac{1}{2}(200\times10^{-6})(100)$

$= 0.01 J$

$C_1 + C_2$ in series ; $= 10 mJ$

$q_{12} = q_1 = q_2$

$V_{12} = 100V$

$q_{12} = C_{12} V_{12} = (3.33)(100) = 333 \mu C$

$q_1 = q_2 = 333 \mu C$

$V_1 = \frac{q_1}{C_1} = \frac{333\mu C}{10\mu F} = 33.3V$

$V_2 = \frac{q_2}{C_2} = \frac{333\mu C}{5\mu F} = 66.7V$

$C_1 = 10\mu F \rightarrow q_1 = 333\mu C$

$\rightarrow V_1 = 33.3V$

$\rightarrow U_1 = \frac{1}{2} q_1 V_1 = \frac{1}{2}(333\times10^{-6})(33.3)$

$= 5.54\times10^{-3} J$

$= 5.54 mJ$

$C_2 = 5\mu F \rightarrow q_2 = 333\mu C$

$\rightarrow V_2 = 66.7V$

$\rightarrow U_2 = \frac{1}{2} q_2 V_2 = \frac{1}{2}(333\times10^{-6})(66.7)$

$= 0.011 J$

$= 11 mJ$

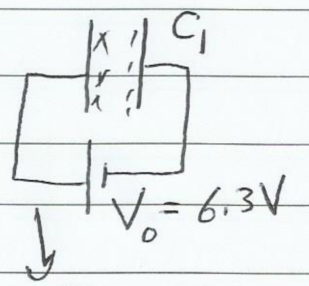
(Sample Problem 25.03)

$C_1 = 3.55\,\mu F$, $V_0 = 6.3V$

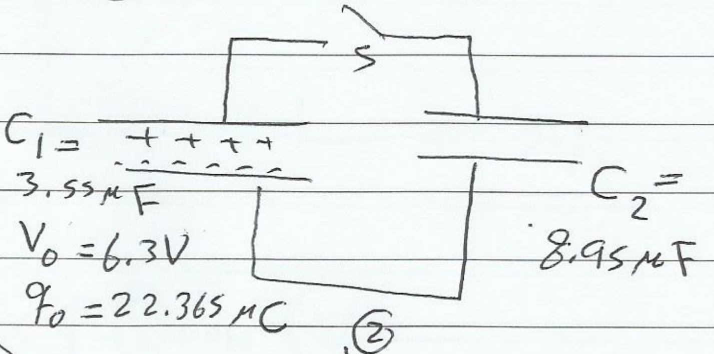
$\mathcal{E} = 6.3V$, $q_0 = C_1 V_0$

$= (3.55)(6.3)$

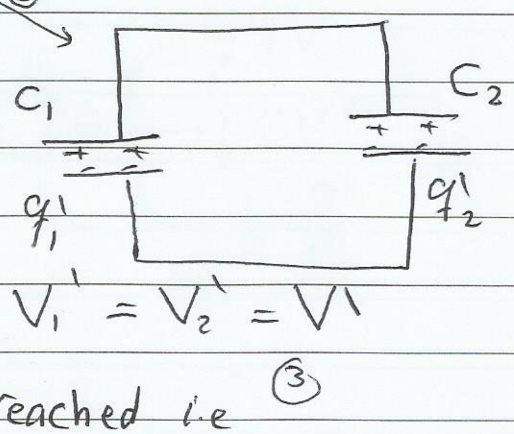
$= 22.365\,\mu C$



C_1 is removed from the battery and connected as shown in the ~~last~~ figure with uncharged capacitor $C_2 = 8.95\,\mu F$



Find the charge on each capacitor after connecting them?



Find $q_1' + q_2'$?

After connecting $C_1 + C_2$ the charge will move from C_1 to C_2 until the equilibrium is reached i.e

The charge will move from $C_1 \longrightarrow C_2$

Until $\longrightarrow$ $V_1' = V_2' = V$

$$\frac{q_1'}{C_1} = \frac{q_2'}{C_2} = \frac{q}{C_{12}} \quad , \quad C_{12} = C_1 + C_2 \text{ , They are in Parallel}$$

$$= 3.55 + 8.95$$

$$= 12.5$$

$$\frac{q_1}{3.55} = \frac{q_2'}{8.95} = \frac{q}{(12.5)} \quad ①$$

$$\frac{q_1'}{3.55} = \frac{q_2'}{8.95} = \frac{22.365}{12.5} \quad ①$$

$$\frac{q_1'}{3.55} = \frac{q_2'}{8.95} = 1.789 \quad ①'$$

From conservation of charge

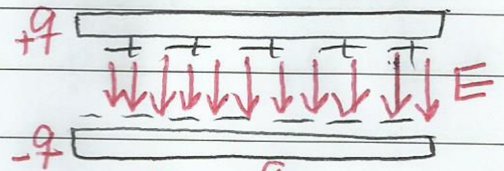
$$q_{\substack{\text{before}\\ \text{connection}}} = q_{\substack{\text{after}\\ \text{connection}}}$$

$$22.365 = q_1' + q_2' \quad ②$$

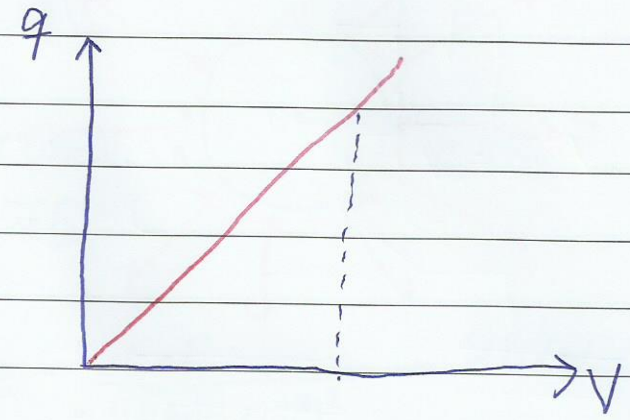
$q_1' = 3.55(1.789) = 6.35\,\mu C$

$q_2' = 8.95(1.789) = 16\,\mu C$

Energy Stored in an Electric Field:-



$C = \frac{q}{V}$



$\text{Slope} = \frac{\Delta q}{\Delta V} = C$

U = Area Under the curve of q Versus V

$= \frac{1}{2} V q$ Joul

$U = \frac{1}{2} q V$

$= \frac{1}{2}(CV)V = \frac{1}{2} C V^2$ Joul

$U = \frac{1}{2} q \left(\frac{q}{C}\right) = \frac{1}{2}\frac{q^2}{C}$

$$U = \begin{cases} \frac{1}{2} q V \\ \frac{1}{2} C V^2 \\ \frac{1}{2}\frac{q^2}{C} \end{cases}$$

$\text{Energy Density} = \frac{\text{Energy}}{\text{Volume}} = \frac{U}{\text{Volume}}$

$u = \frac{U}{\text{Volume}}$

$= \frac{\frac{1}{2} C V^2}{Ad}$

$= \frac{\epsilon_0 A/d}{2Ad} V^2$

for parallel plate Capacitor

$C = \epsilon_0 A/d$

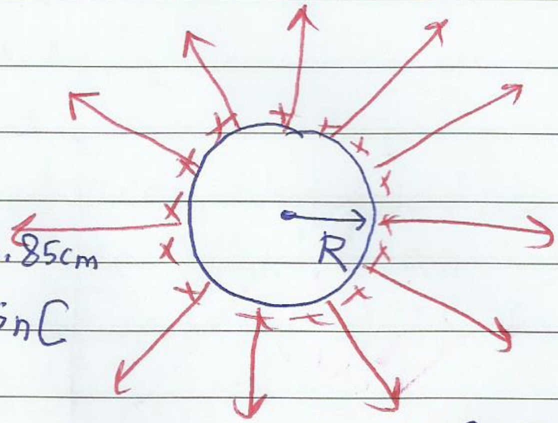
$\text{Volume} = Ad$

$U = \frac{1}{2} C V^2$

$= \frac{1}{2}\epsilon_0 \frac{V^2}{d^2} = \frac{1}{2}\epsilon_0 \left(\frac{V}{d}\right)^2 = \frac{1}{2}\epsilon_0 E^2 \ J/m^3$

$u = \frac{1}{2}\epsilon_0 E^2 \ J/m^3$

Sample Problem 25.04:



An isolated Conducting sphere

→ radius $= R = 6.85\,cm$

→ Charge $= 1.25\,nC$

1) → $\rho = 0$

2) → $\sigma = \dfrac{q}{4\pi R^2} = \dfrac{1.25\,nC}{4\pi(6.85\times10^{-2})^2} = 21.2\,nC/m^2$

3) → $E = 0$, inside the Conducting sphere

4) $E = 0$, $r < R$

5) → $E_s = \dfrac{q}{4\pi\epsilon_0 R^2} = \dfrac{9\times10^{9}\times1.25\times10^{-9}}{(6.85\times10^{-2})^2}$

$= 2.4\times10^{3}\ N/C$

6) → $E = \dfrac{q}{4\pi\epsilon_0 r^2}$, $r \geq R$

→ $V_s = \dfrac{q}{4\pi\epsilon_0 R} = \dfrac{9\times10^{9}\times1.25\times10^{-9}}{6.85\times10^{-2}}$

$= 164\ V$

→ $V_{center} = 164\,V$

مراجعة

→ $C = 4\pi\epsilon_0 R = \dfrac{6.85\times10^{-2}}{9\times10^{9}}$

$= 7.61\times10^{-12}\ F$

$= 7.61\ pF$

هنا مطلوب → $U = \dfrac{q^2}{2C} = \dfrac{(1.25\times10^{-9})^2}{2(7.61\times10^{-12})}$

$= 1.026\times10^{-7}\ J$

$= 102.6\ nJ = 103\ nJ$

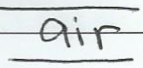
هنا مطلوب → $u_s = \frac{1}{2}\epsilon_0 E_s^2$

$= \frac{1}{2}\epsilon_0\left(\dfrac{q}{4\pi\epsilon_0 R^2}\right)^2 = 2.54\times10^{-5}\ J/m^3$

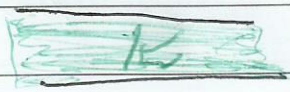
Capacitor With a Dielectric:

If you fill the space between the plates of a Capacitor with a dielectric, Which is an insulating material, the Capacitance Will increase by a factor of (K) the Dielectric Constant

$$C_0 = \frac{\epsilon_0 A}{d}$$



$$C = K C_0 \quad , \quad K > 1$$



For each Dielectric (insulating material)

- Dielectric Constant $K = \frac{\epsilon}{\epsilon_0}$
- Dielectric strength is the maximum Electric field (E_{max}) the material Can withstand

In Dielectric material replace in all equation $\epsilon_0 \rightarrow K\epsilon_0$

$$E = \frac{q}{4\pi\epsilon_0 r^2} \longrightarrow E = \frac{q}{4\pi K\epsilon_0 r^2}$$

$$V = \frac{q}{4\pi\epsilon_0 r} \longrightarrow V = \frac{q}{4\pi(K\epsilon_0) r}$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0} \longrightarrow \oint \vec{E}\cdot d\vec{A} = \frac{q}{K\epsilon_0} \Rightarrow \oint K\vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0}$$

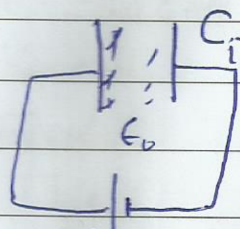
$$E = \frac{\sigma}{2\epsilon_0} \longrightarrow E = \frac{\sigma}{2(K\epsilon_0)}$$

$$u = \frac{1}{2}\epsilon_0 E^2 \longrightarrow u = \frac{1}{2}(K\epsilon_0)E^2$$

Sample Problem 25.05

Work and Energy when a dielectric is inserted into a Capa

$C_i = 13.5\,pF$

① 

$V_i = 12.5\,V$

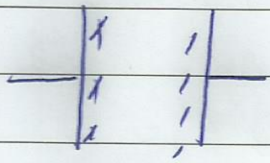
a) $q_i = C_i V_i = (13.5\,pF)(12.5\,V)$

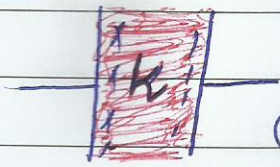
$= 168.75\,pC$

$U_i = \frac{1}{2} C_i V_i^2 = \frac{1}{2}(13.5\times10^{-12})(12.5)^2$

$= 1.055\times10^{-9}\,J$

$= 1.055\,nJ$

②  U_i, q_i, V_i, C_i

③ 

b) C_f, q_f, V_f, U_f ? $\kappa = 6.5$

$C_f = \kappa C_i = (6.5)(13.5\,pF)$

$= 87.75\,pF$

$q_f = q_i = 168.75\,pC$ (q does not change because No Source (Battery) for the Capacitor to gain Charge)

$V_f = \frac{q_f}{C_f} = \frac{168.75\,pC}{87.75\,pF}$

$V_f = 1.92\,V$

$U_f = \frac{1}{2}\frac{q_f^2}{C_f} = \frac{1}{2}\left[\frac{[168.75\times10^{-12}]^2}{87.7\times10^{-12}}\right.$

$U_f = 1.62\times10^{-10}\,J = 0.162\,nJ$

Find the Work done by E during inserting the slab?

$W_E = -\Delta U = -[U_f - U_i] = -[0.162 - 1.055] = -[-0.893]$

$= 0.893\,nJ$

$= 893\,pJ$, the slab will be sucked in Not pushed

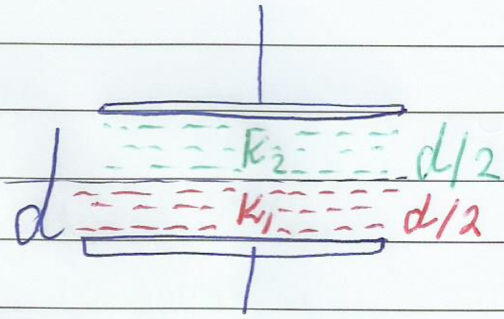
Problem (25-31)

$A = 7.89\,cm^2$

$d = 4.62\,mm$

$K_1 = 11$,, $K_2 = 4$

Find C?



This Capacitor Will be considered to be 2 Capacitors in series

$$C_1 = \frac{K_1 \epsilon_0 A}{d/2} = 2\frac{K_1 \epsilon_0 A}{d} = 2K_1\left(\frac{\epsilon_0 A}{d}\right) = 2(11)[1.51\times10^{-12}]$$
$$= 33.25\,pF$$

$$C_2 = \frac{K_2 \epsilon_0 A}{d/2} = 2\frac{K_2 \epsilon_0 A}{d} = 2(4)\left(\frac{\epsilon_0 A}{d}\right) = 2(4)(1.51\times10^{-12})$$
$$= 12.08\,pF$$

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K_1\epsilon_0 A} + \frac{d}{2K_2\epsilon_0 A} = \frac{d}{2\epsilon_0 A}\left[\frac{1}{K_1} + \frac{1}{K_2}\right]$$

$$\frac{1}{C_{12}} = \frac{d}{2\epsilon_0 A}\left[\frac{K_2 + K_1}{K_1 K_2}\right]$$

$$C_{12} = \frac{2\epsilon_0 A}{d}\left[\frac{K_1 K_2}{K_1 + K_2}\right] = \frac{\epsilon_0 A}{d}\left[\frac{2K_1 K_2}{K_1 + K_2}\right]$$

$$C_{12} = (1.51\,pF)\left[\frac{2(4)(11)}{11+4}\right] = 1.51\,pF\left[\frac{88}{15}\right] = 1.51\,pF(5.8667)$$

$$C_{12} = 8.86\,pF$$

Problem (25 – 37)

Dielectric material → $K = 5.6$

Dielectric material → Dielectric strength $= 18\,MV/m$

$E_{max} = 18\,MV/m$

$C = 3.9 \times 10^{-2}\,\mu F$

A_{min} ? For C to withstand a potential difference of 4 kV.

C →
- $3.9 \times 10^{-2}\,\mu F$
- $K = 5.6$
- $E_{max} = 18\,MV/m$
- $V_{max} = 4\,kV$
- A_{min} ?
- d ?

$$C = \frac{K \epsilon_0 A}{d} \quad ,, \quad \vec{E} \cdot \vec{d} = \Delta V \quad ,, \quad d = \frac{V}{E}$$

$$A = \frac{C \cdot d}{K \epsilon_0} = \frac{C}{K \epsilon_0} \left(\frac{V_{max}}{E_{max}} \right)$$

(A is min. When E is max)

$$A_{min} = \frac{3.9 \times 10^{-2} \times 10^{-6}}{(5.6)(8.85 \times 10^{-12})} \left(\frac{4 \times 10^{3}}{18 \times 10^{6}} \right)$$

$$= \frac{3.9 \times 10^{-8}}{49.56 \times 10^{-12}} \left(2.222 \times 10^{-4} \right)$$

$$A_{min} = 0.175\ m^2$$