Chapter 28 – Magnetic Fields $\vec{B}$

Introduction:-

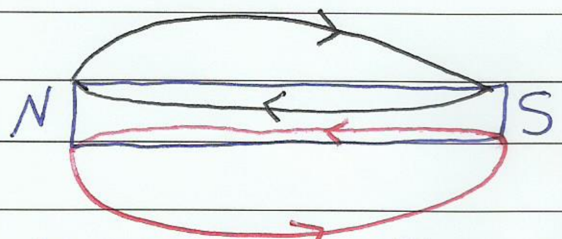
1) All Magnets → Natural Magnets
   → Industrial Magnets

have 2 poles → South Pole
   → North pole

2) There is A repulsive Magnetic Force between like poles N—N
   S—S

3) There is An Attractive magnetic Force between opposite poles
   N—S

4) Magnetic Field lines extends From North poles → to South pole outside the magnet
and From South pole → North pole
inside the magnet



Magnetic Field lines Form a closed loop

5) No magnetic Monopole

6) Magnetic Flux through a closed surface = zero

$$\oint \vec{B} \cdot d\vec{A} = 0$$

7) Magnetic Field lines do not cross each other

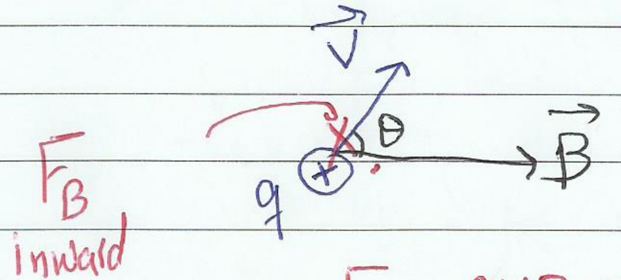
Magnetic Force on a Charged Particles:

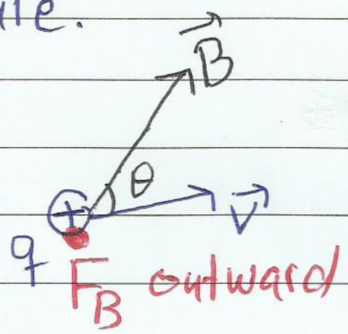
$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

N → $\vec{F}_B$, C → q, m/s → $\vec{v}$, Tesla → $\vec{B}$

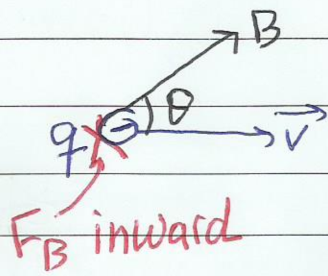
Magnetic Force F_B has the following Properties:

1) $\vec{F}_B \perp \vec{v}$ & $\vec{F}_B \perp \vec{B}$
to find the direction of $\vec{F}_B$ you have to use the right hand rule.

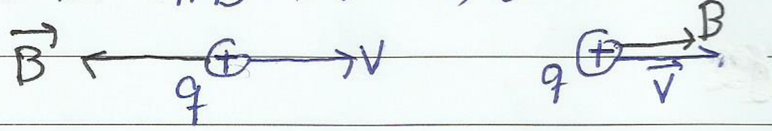




$$F_B = qvB\sin\theta$$



2) $\vec{F}_B = 0$ → for $v=0$ / at rest (q)
→ for $v \parallel B$, $\theta = 0$, $\theta = 180^\circ$



3) F_B: do not change the magnitude of v.
because $\vec{F}_B \perp \vec{v}$

F_B: change the direction o $\vec{v}$ only

4) F_B: do not chang kinetic energy

5) $\vec{F}_B$: Change the linear Momentum of q, by changing the direction.

6) The charge will move in a Uniform Circular Motion if $\vec{B} \perp \vec{V}$ $\theta = 90°$ between $\vec{V}$ + $\vec{B}$

7) The charge will move in a helical motion (Spiral Motion) if $\theta \neq 90°$ between $\vec{V}$ + $\vec{B}$

Sample Problem 28.01:

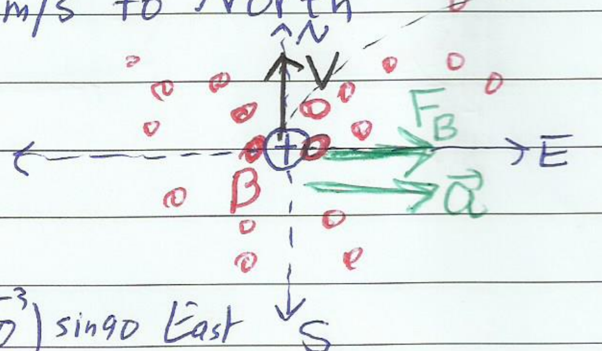
$B = 1.2\,mT$ upward
$= 1.2 \times 10^{-3}\,T$ upward

$q_p = +1.6 \times 10^{-19}\,C$ } with kinetic energy moves from S → N
$m_p = 1.67 \times 10^{-27}\,kg$ } $K = 5.3\,MeV$
$= 5.3 \times 10^{6}\,eV$
$= (5.3 \times 10^{6})(1.6 \times 10^{-19})\,J$

Find 1) $\vec{F}_B$? Magnetic Force ?
2) $\vec{a}$? acceleration

1) $K = \frac{1}{2} m v^2$ $\quad V = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(5.3 \times 10^{6})(1.6 \times 10^{-19})}{1.67 \times 10^{-27}}}$

$= 3.2 \times 10^{7}\,m/s$ to North



$\vec{Force} = q\vec{V} \times \vec{B}$
$= qVB \sin 90$ East

$= (1.6 \times 10^{-19})(3.2 \times 10^{7})(1.2 \times 10^{-3}) \sin 90$ East
$= 6.12 \times 10^{-15}\,N$ East

$\vec{a} = \frac{\vec{F}_B}{m} = \frac{6.12 \times 10^{-15}}{1.67 \times 10^{-27}} = 3.66 \times 10^{12}\,m/s^2$ East

Repeat Solving the Problem for electron?
Find $\vec{F}_B$? $\vec{a}$?

$$q_e = -1.6\times10^{-19}\,C \quad ,, \quad m_e = 9.11\times10^{-31}\,kg.$$

(Problem 28-56)

A proton moves at t_1 with velocity $\vec{V} = V_x\hat{i} + V_y\hat{j} + 2\times10^{3}\hat{k}$ m/s through a magnetic Field $\vec{B} = 10\hat{i} - 20\hat{j} + 25\hat{k}$ mT

the magnetic Force on the proton $\vec{F}_B = (4\hat{i} + 2\hat{j})\times10^{-17}\,N$
Find V_x ? V_y ?

$$\vec{F}_B = q\vec{V}\times\vec{B} = 1.6\times10^{-19}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ V_x & V_y & 2\times10^{3} \\ 10\times10^{-3} & -20\times10^{-3} & +25\times10^{-3} \end{vmatrix}$$

$$= 1.6\times10^{-19}\Big[\hat{i}\,(25\times10^{-3}V_y - (2\times10^{3})(-20\times10^{-3})) - \hat{j}\,(25\times10^{-3}V_x - 2\times10^{3}(10\times10^{-3})) + \hat{k}\,(-20\times10^{-3}V_x - 10\times10^{-3}V_y)\Big]$$

$$= 1.6\times10^{-19}\Big[\hat{i}(0.025V_y + 40) + \hat{j}(20 - 0.025V_x) + \hat{k}(-0.02V_x - 0.01V_y)\Big]$$

$$(4\hat{i} + 2\hat{j} + 0\hat{k})10^{-17} =$$

نقسم طرفي المعادلة على $10^{-17} \Leftarrow$

$$4\hat{i} + 2\hat{j} + 0\hat{k} = 1.6\times10^{-2}\Big[\hat{i}(25\times10^{-3}V_y + 40) + \hat{j}(20 - 25\times10^{-3}V_x) + \hat{k}(-20\times10^{-3}V_x - 10\times10^{-3}V_y)\Big]$$

$$4 = 1.6\times10^{-2}(25\times10^{-3}V_y + 40) \Rightarrow 250 = 25\times10^{-3}V_y + 40$$

$$V_y = \frac{250 - 40}{25\times10^{-3}} = 8.4\times10^{3}\ m/s$$

$$2 = 1.6\times10^{-2}(20 - 25\times10^{-3}V_x) \Rightarrow 125 = 20 - 25\times10^{-3}V_x$$

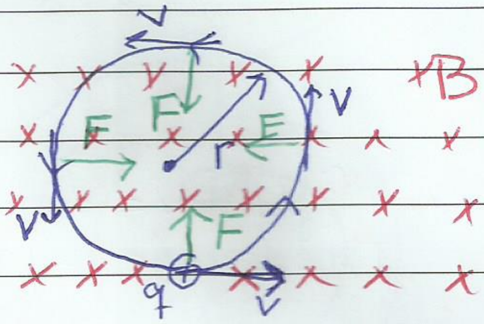
$$V_x = \frac{125 - 20}{-25\times10^{-3}} = -4.2\times10^{3}\ m/s$$

$$\vec{V} = -4.2\hat{i} + 8.4\hat{j} + 2\hat{k}\ km/s$$

Applications on $q\vec{v}\times\vec{B}$:

1) The charge moves in A Uniform circular Motion:-

Apply $\vec{B}$ perpendicular on $\vec{v}$
the charge will moves
in A Uniform Circular Motion



$\vec{B} \perp \vec{v} \Rightarrow$ Uniform Circular Motion

$$\vec{F_B} = q\vec{v}\times\vec{B} = qvB\sin 90$$

Apply $\Sigma \vec{F} = m\vec{a}$ Newton's 2nd Law

$ma = qvB \qquad a = \frac{v^2}{r}$ Centrepital acceleration

$\frac{mv^2}{r} = qvB \Rightarrow \qquad r = \frac{mv}{qB}$ radius

$T = \frac{2\pi r}{v}$ (Periodic Time) sec.

$$T = \frac{2\pi}{v}\left(\frac{mv}{qB}\right) = \frac{2\pi m}{qB}$$

$f = \frac{1}{T} = \frac{qB}{2\pi m}$ frequency (Hz)

$W = 2\pi f = \frac{qB}{m}$ rad/s

Note that: f depend on, the charge Properties (q & m) and B.

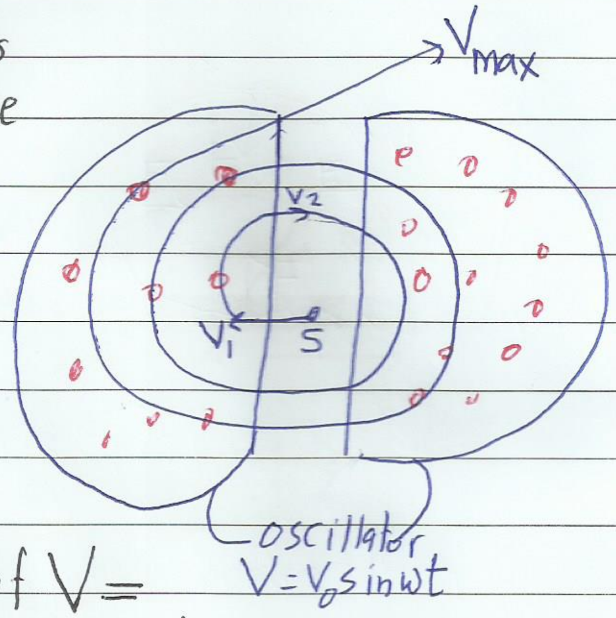
f: does not depend on
- r (radius)
- v (speed).

2) The cyclotron:- To Accelerate Charged Particles

It Consists from:

The 2 hollow D-shaped objects each open on its straight edge with radius $= R$

There is An Oscillating electric Potential Defference between the gap- between the 2 Dees



the Oscillating frequency of $V =$ the frequency of the circulating charge

$$f_{cyclotron} = f_{circulating\ q} = \frac{qB}{2\pi m}$$

the magnetic field $\vec{B} \perp \vec{v} \Rightarrow$ the charged particle will move in a circle.

V: Potential difference between the 2 Dees to accelerate the charge.

In each cycle the charge gains kinetic energy $= 2(qV)$

$K_{max} = N(2qV)$ number of circles the charge moves.

$= \frac{1}{2} m v^2_{max}$

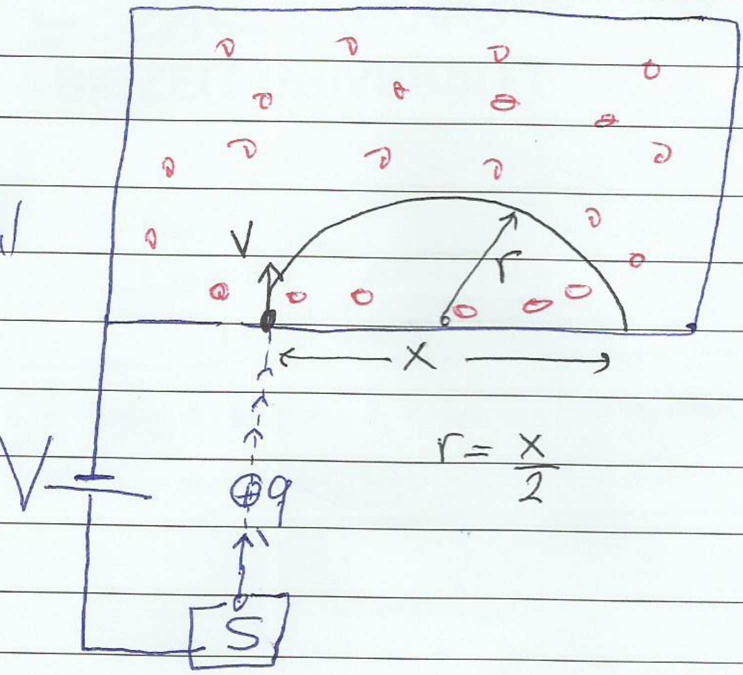
V_{max} could be found from $\frac{mv^2}{r} = qvB$

$V = \frac{qBr}{m} \Rightarrow V_{max} = \left(\frac{qB}{m}\right)R$ radius of the cyclotron.

3) Spectrometer :- Is a device to measure the
Mass - spectrometer: mass of an ion

To measure the mass of the ion (q, m)

1) Accelerate the ion by An Electric Potential Difference V

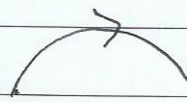


$$K = qV$$

$$\frac{1}{2}mv^2 = qV$$

$$v = \sqrt{\frac{2qV}{m}} \quad (1)$$

2) Apply $\vec{B} \perp \vec{v} \Rightarrow$ the ion will move in a circle of radius $= r = \frac{x}{2}$



$$\frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}}$$

$$r = \sqrt{\frac{2mV}{q}} \Big/ B$$

$$r^2 = \frac{1}{B^2} \cdot \frac{2mV}{q} \Rightarrow m = \frac{qB^2r^2}{2V}, \quad r = \frac{x}{2}$$

$$m = \frac{qB^2x^2}{8V}$$

B, V, x are measured
q is known

Solve Sample Problem (28.04)

Sample Problem (28 ~~-0~~.5)

Cyclotron
- operates at $f = 12 MHz = 12 \times 10^6 Hz$
- Radius = $R = 53 cm$ of each D
- B? to accelerate deuteron
  - $q = +1.6 \times 10^{-19} C$
  - $m = 3.34 \times 10^{-27} kg$

1) Find B ?

$$f_{cyc} = \frac{Bq}{2\pi m}$$

$$B = \frac{2\pi m f}{q} = \frac{(2\pi)(3.34 \times 10^{-27})(12 \times 10^6)}{1.6 \times 10^{-19}}$$

$$= 1.57 T = 1.6 T$$

2) What is the resulting Kinetic Energy of the deuterons?

$$\frac{mv^2}{R} = qvB$$

$$V = \frac{RqB}{m} , \quad V^2 = \frac{R^2 q^2 B^2}{m^2}$$

$$V = \frac{53 \times 10^{-2} \times 1.6 \times 10^{-19} \times 1.57}{3.34 \times 10^{-27}}$$

$$V = 3.99 \times 10^7 m/s$$

$$K.E = \frac{1}{2} mv^2 = \frac{1}{2} \frac{R^2 q^2 B^2}{m}$$

$$K.E = \frac{1}{2} (3.34 \times 10^{-27})(3.99 \times 10^7)^2$$

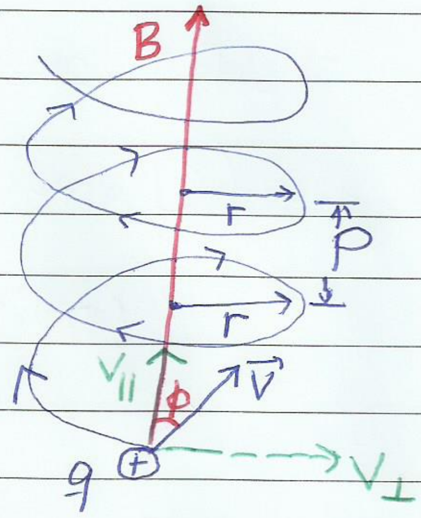
$$= 2.7 \times 10^{-12} J$$

$$= \frac{2.7 \times 10^{-12}}{1.6 \times 10^{-19}} eV = 1.69 \times 10^7 eV = 1.7 \times 10^7 eV$$

$$= 17 \times 10^6 eV$$

$$= 17 MeV$$

4) Helical Motion (Spiral Motion)



If the angle (ϕ) between $\vec{B}$ + $\vec{V}$ is not $90°$

$\phi \neq 90° \Rightarrow$ the charged Particle will move in a helix

Find r (radius)? f (frequency)? Pitch (P)?

To study this motion:

1) $\vec{V}$: has 2 components $\to V_{\perp} = V\sin\phi$, $V_{\parallel} = V\cos\phi$,

$V_{\perp} = V\sin\phi$, Cause circular Motion around B

$V_{\parallel} = V\cos\phi$, Cause linear Motion along B

$V_{\perp} = V\sin\phi$ Causes Circular Motion with radius = r and frequency = f

$$\frac{mV_{\perp}^2}{r} = qV_{\perp}B$$

$$r = \frac{mV_{\perp}}{qB} = \frac{mv\sin\phi}{qB}$$

$$T = \frac{2\pi r}{V_{\perp}}$$

$$f = \frac{V_{\perp}}{2\pi r} = \frac{V_{\perp}}{2\pi(mV_{\perp})/qB}$$

$$f = \frac{qB}{2\pi m}$$ does not depend on $\to$ r + v

$V_{\parallel} = V\cos\phi$ causes linear Motion

the pitch $= V_{\parallel}T = (V\cos\phi)\left(\frac{2\pi m}{qB}\right)$

Solve Sample Problem 28.03

6) Lorentz' Force:

When a charged particle moves in electric field $\vec{E}$ and magnetic field $\vec{B}$,
the net force on the charged particle is given by

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \quad \text{(Lorentz' Force)}.$$

Problem (28-50)

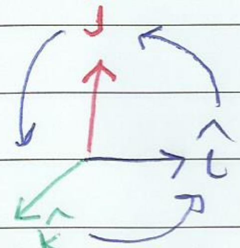
A proton travels through a uniform electric + magnetic fields

$\vec{B} = -3.25\hat{i}$ mT

$\vec{v} = 2000\hat{j}$ m/s

What is the net force acting on the proton

If: a) $\vec{E} = 4\hat{k}$ V/m



$\hat{j} \times \hat{i} = -\hat{k}$

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

$$= {}^{+}1.6\times10^{-19}\left[4\hat{k} + 2000\hat{j} \times -3.25\times10^{-3}\hat{i}\right]$$

$$= +1.6\times10^{-19}\left[4\hat{k} \,(-)\, 6.5(-\hat{k})\right] = {}^{+}1.6\times10^{-19}\left[4\hat{k} + 6.5\hat{k}\right]$$

$$\vec{F}_{net} = 1.6\times10^{-19}\left[10.5\hat{k}\right] = +1.68\times10^{-18}\hat{k} \text{ N}$$

do not changed

b) $\vec{E} = -4\hat{k}$ V/m

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

$$= 1.6\times10^{-19}\left[-4\hat{k} + \vec{v} \times \vec{B}\right] = 1.6\times10^{-19}\left[-4\hat{k} + 6.5\hat{k}\right]$$

$$= {}^{+}1.6\times10^{-19}\left[2.5\hat{k}\right] = 4\times10^{-19}\hat{k} \text{ N}$$

c) $\vec{E} = 4\hat{i}$ V/m

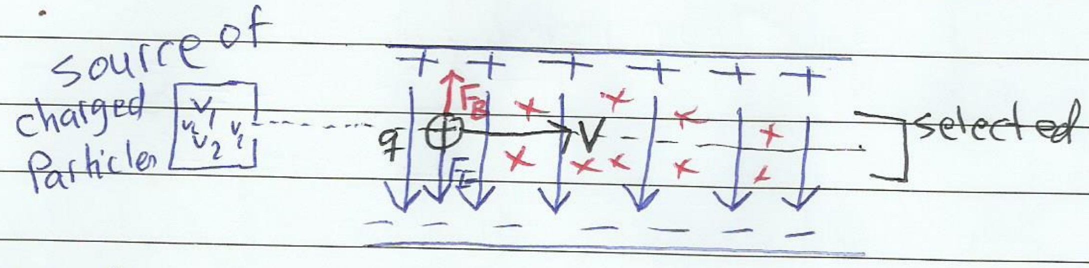
$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} = {}^{+}1.6\times10^{-19}\left[4\hat{i} + 6.5\hat{k}\right]$$

$$\vec{F} = 6.4\times10^{-19}\hat{i} + 10.4\times10^{-19}\hat{k} \text{ N}$$

7) Crossed Fields (Velocity Selector):-

Velocity Selector is a device to select charged Particles with certain velocity

By using 2 fields. $\vec{E} \perp \vec{B}$ acting on moving charge to produce No net Force as follows:-

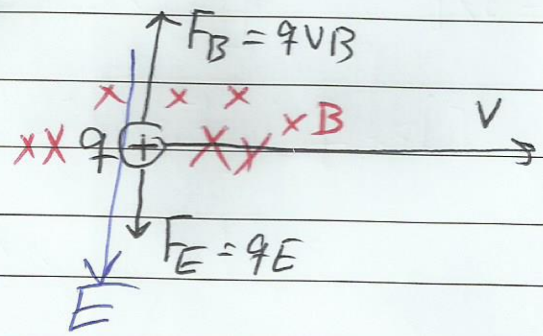


1) let $\vec{E}$ as shown in (-y)

$\vec{F}_E = q\vec{E}$ in (-y)

2) Apply $\vec{B} \perp \vec{E}$, $\vec{B}$ inward to produce F_B opposes F_E

$\vec{F}_B = q\vec{v} \times \vec{B} = qvB$ in (+y)



For certain $\vec{v}$

$\vec{F}_{net} = q\vec{E} + q\vec{v} \times \vec{B} = 0$

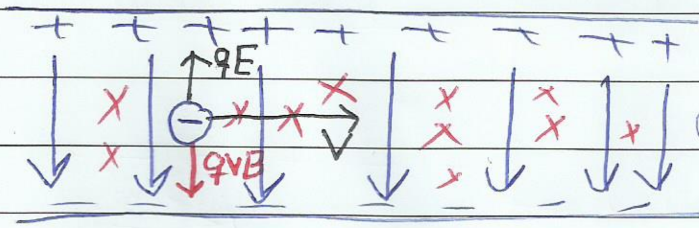
$-qE + qvB = 0 \Rightarrow V = \frac{E}{B}$ selected $\vec{v}$

$V = \frac{E}{B}$ selected Particles with this speed.

Problem (28-60)

$E = 1.50\,kV/m \perp B = 0.350\,T$

Both fields are acting on electron to produce
No net force on the electron?
find v?



suppose: $\vec{V}$ in $+x$, E in $-y$

$\Downarrow$

B must be inward $\Downarrow$ to produce $\vec{F}_{net} = 0$

$\vec{F}_{net} = 0$

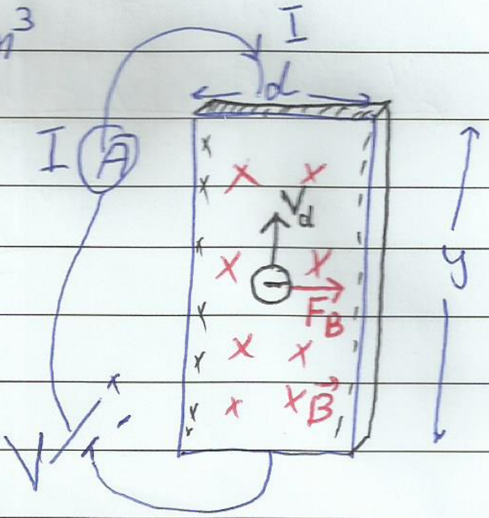
$+qE + -qvB = 0 \Rightarrow qE = qvB$

$V = \frac{E}{B} = \frac{1.5 \times 10^3}{0.350} = 4.3 \times 10^3\ m/s = 4.3\ km/s$

8) Hall Effect (crossed Fields $\vec{B} \perp \vec{E}$)

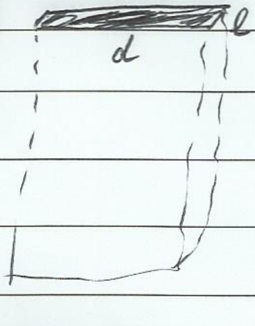
The aim of Hall Effect:

1) To find $\boxed{n}$ number of free electrons/m^3 in the Conductor
$I = neAV_d$
2) To find drift speed V_d?
3) To know the type of charge carriers.



the metalic strip: length $= y$, width $= d$, thickness $= l$

1) Apply V between the length of the strip $\Rightarrow$ I will flow as show
$\Rightarrow$ free electrons will move with V_d as shown
$I = neAV_d$, $A = ld$



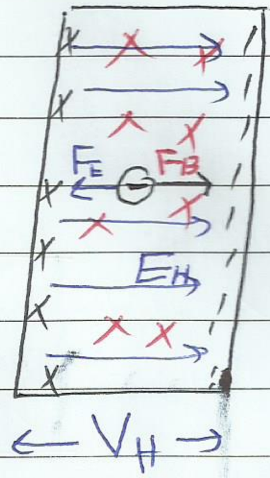
2) Apply $\vec{B} \perp yd$ plane inward

$\vec{F} = q\vec{v} \times \vec{B}$

$\vec{F}_B = eV_dB\sin 90 = eV_dB$ in $\boxed{+x}$

This magnetic Force on each moving electron toward $+x$ $ev_dB \xrightarrow{+x}$

~~will creat Potential difference between the left face (High V) + the right face (low V)~~



This Potential difference is Called

V_H Hall Potential difference

$$V_H = E_H d$$

$\vec{F}_E = q\vec{E}$, $F_E = eE_H$ to the $-x$

At equilibrium $q\vec{E} + q\vec{v}\times\vec{B} = 0 \Rightarrow$

$$eE_H = ev_dB$$

$$v_d = \frac{E_H}{B} = \frac{V_H/d}{B} = \frac{V_H}{Bd}$$

$$v_d = \frac{V_H}{Bd}$$ could be found

To find n

$$I = neAv_d$$

$$I = ne(ld)\left(\frac{V_H}{Bd}\right)$$

$$I = \frac{nelV_H}{B}$$

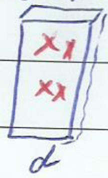
$$n = \frac{BI}{eV_H l}$$

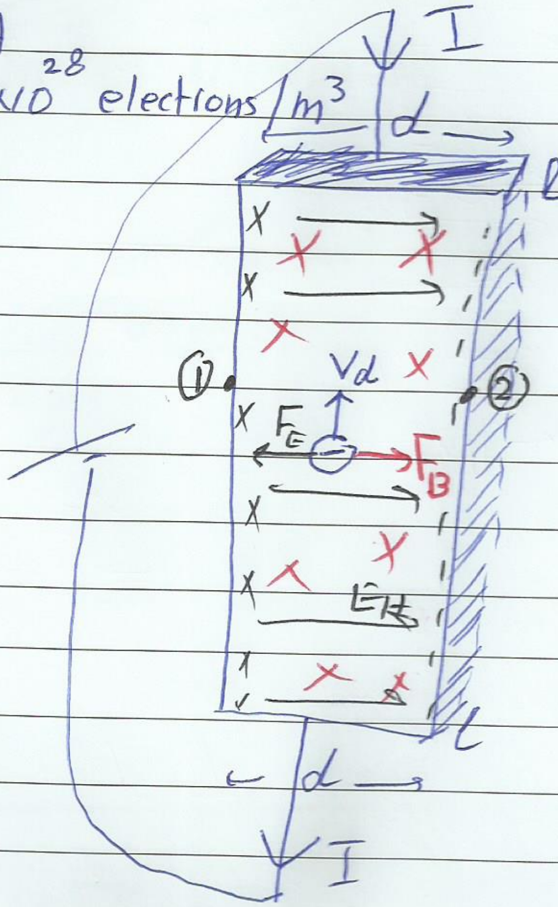
Solve Sample Problem 28.02

(Problem 28 − 47)

A strip of Cu
- → thickness = l = 75 µm
- → width = d = 4.5 mm
- → i = 57 A
- → $n = 8.47\times10^{28}$ electrons / m^3

Apply $B = 0.65$ T





Find V_H ?

$$V_H = V_{12} = E_H d$$

At equilibrium $eE_H = ev_d B$

$$v_d = \frac{E_H}{B} = \frac{V_H}{Bd} \quad ①$$

$$I = neAv_d$$

$$v_d = \frac{I}{neA} \quad ②$$

$$v_d = \frac{I}{neld}$$ ② from this equation you can find v_d.

From ① + ②

$$\frac{V_H}{Bd} = \frac{I}{neld} \Rightarrow \frac{V_H}{B} = \frac{I}{nel}$$

$$V_H = \frac{IB}{nel} = \frac{(57)(0.65)}{8.47\times10^{28}\times1.6\times10^{-19}\times75\times10^{-6}}$$

$$= 3.65\times10^{-5}\ \text{V} = 36.5\ \mu\text{V}$$

Find v_d?

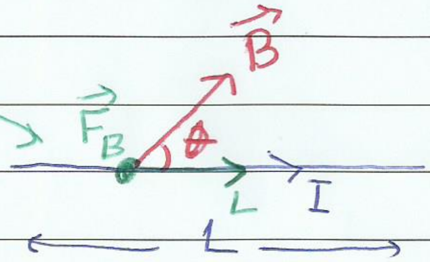
$$v_d = \frac{V_H}{Bd} = \frac{3.65\times10^{-5}}{(0.65)(4.5\times10^{-3})} = 12.5\times10^{-3}\ \text{m} = 12.5\ \text{mm}$$

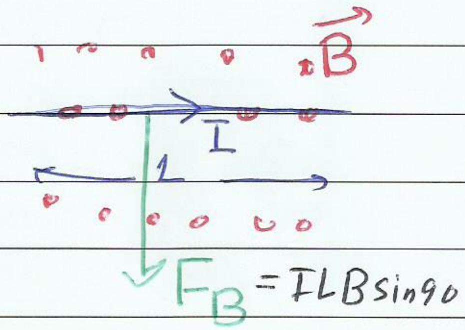
Magnetic Force on a Current-Carrying Wire:-

$$\vec{F}_B = I\vec{L} \times \vec{B}$$

the direction of the Length vector ($\vec{L}$) is that of the current (i)

$\vec{F}_B = ILB\sin\theta$ outward



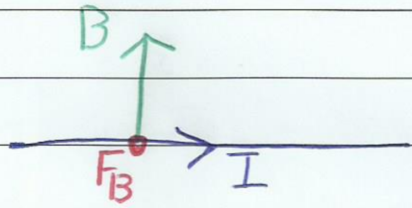


Sample Problem 28.06

A straight horezontal wire

$I = 28A$, $\frac{m}{L} = 46.6 g/m$

Find $\vec{B}$? needed to suspend the wire in air?



Apply $\vec{B}$ in the direction of $+y$, wire (Length) $+x$

B $+y$

$\vec{F}_B = I\vec{\ell} \times \vec{B} = I\ell B \sin 90$ upward

mg downward

$\Rightarrow \vec{F}_{net} = 0 \Rightarrow I\ell B = mg$

$$B = \left(\frac{m}{\ell}\right)\frac{g}{I} = \frac{(46.6\times10^{-3})(9.8)}{28} = 1.6\times10^{-2}\,T$$ horizontally $+y$

$= 16\,mT$

Problem (28-10)

$l_{ab} = l_{bc} = 2\,m$

$\theta = 60^\circ$

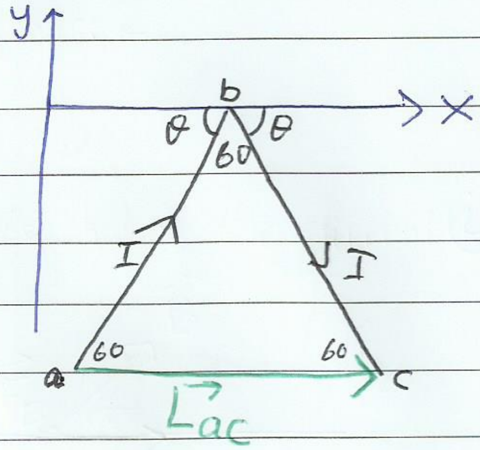
$I = 3.5\,A$

Find $\vec{F}_{net}$ on the wire

a) $\vec{B} = 4\hat{k}\ T$

b) $\vec{B} = 4\hat{i}\ T$

c) $\vec{B} = 4\hat{j}\ T$



$L_{ab} = L_{bc} = L_{ac} = 2\,m$

$\vec{L}_{ac} = 2\hat{i}\ m$

Note:

$\vec{L}_{ab} = L_{ab}\cos 60\,\hat{i} + L_{ab}\sin 60\,\hat{j}$

$= 2\cos 60\,\hat{i} + 2\sin 60\,\hat{j}$

$\vec{L}_{ab} = 1\hat{i} + 1.73\hat{j}$

$\vec{L}_{bc} = 2\cos 60\,\hat{i} - 2\sin 60\,\hat{j}$

$\vec{L}_{bc} = 1\hat{i} - 1.73\hat{j}$

هذا الشي فقط للعلم

a) $\vec{B} = 4\hat{k}\ T$

$\vec{F} = I\vec{L} \times \vec{B}$

$\vec{F}_{ab} = I\vec{L}_{ab} \times \vec{B}$

$\vec{F}_{bc} = I\vec{L}_{bc} \times \vec{B}$

$\vec{F}_{ab} + \vec{F}_{bc} = I\vec{L}_{ab} \times \vec{B} + I\vec{L}_{bc} \times \vec{B}$

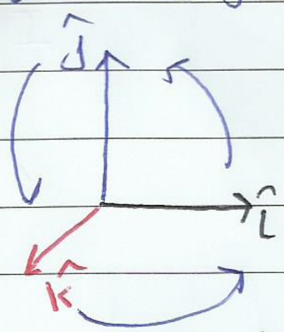
$\vec{F}_{net} = I\left[\vec{L}_{ab} + \vec{L}_{bc}\right] \times \vec{B}$

$= I\ \vec{L}_{ac} \times \vec{B}$

$\vec{F}_{net} = (3.5)(2\hat{i}) \times 4\hat{k}$

$= 28(\hat{i} \times \hat{k})$

$\vec{F}_{net} = -28\hat{j}\ N$



$\hat{i} \times \hat{k} = -\hat{j}$

$\hat{i} \times \hat{i} = 0$

$\hat{i} \times \hat{j} = \hat{k}$

b) $\vec{B} = 4\hat{i}\ T$

$\vec{F}_{net} = I\vec{L}_{ac} \times \vec{B}$

$= (3.5)(2\hat{i}) \times (4\hat{i})$

$= 0$

c) $\vec{B} = 4\hat{j}\ T$

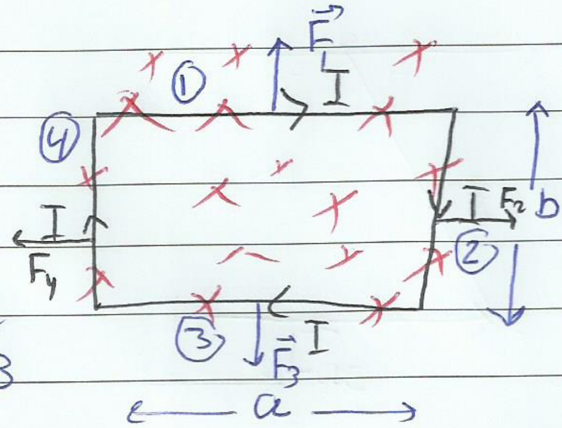
$\vec{F}_{net} = I\vec{L}_{ac} \times \vec{B} = (3.5)(2\hat{i}) \times 4\hat{j}$

$= 28(\hat{i} \times \hat{j})$

$= 28\hat{k}\ N$

Torque on a Current loop:

Case 1:- loop has 4 sides
I flows as show
$\vec{B}$ inward, $\vec{B} \perp (a \times b)$ face
$\vec{B} \perp (a \times b)$ plane



Find $\vec{F_B}$ on each side)? $\vec{F_B} = I\vec{L} \times B$

$\vec{F_1} = IaB\sin 90\,\hat{j} = IaB\,\hat{j}$

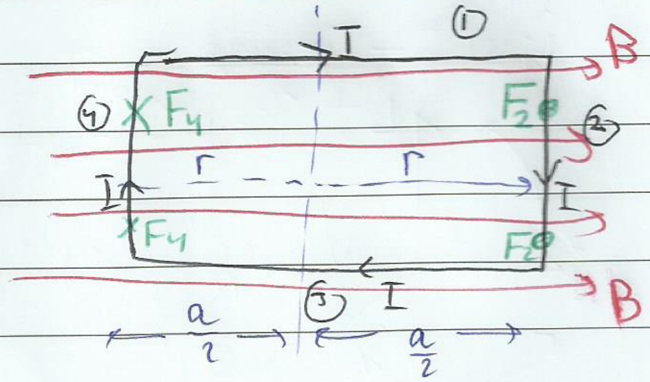
$\vec{F_3} = IaB(-\hat{j})$

$\vec{F_2} = IbB(+\hat{i})$

$\vec{F_4} = IbB(-\hat{i})$

$\vec{F}_{net} = 0$, $\vec{\tau}_{net} = 0$, $\vec{F_1}$ + $\vec{F_3}$ along the same line
$\vec{F_2}$ + $\vec{F_4}$ along the same line

Cas 2: Apply $\vec{B} \parallel (a \times b)$ plane
$\vec{B} \parallel$ side ① + ②
$\vec{B} \perp$ side 2 + sid ④



Find $\vec{F_1}, F_2, F_3, \vec{F_4}$? $\vec{F}_{net}$? $\vec{\tau}_{net}$

$F_1 = IaB\sin 0 = 0$

$F_3 = IaB\sin 180 = 0$

$\vec{F_2} = IbB\sin 90\,\hat{k} = IbB\,\hat{k}$ outward

$\vec{F_4} = IbB(-\hat{k})$ inward

$\vec{F}_{net} = 0$ $\vec{\tau}_{net} \neq 0$ $F_2 + F_4$ are equal + in opposite direction + not along the same line $\Longrightarrow$ They will Produce torque

$\vec{F_2}$; will cause side② to move upward
$\vec{F_4}$; will cause side(4) to move inward
the loop will rotate

$\vec{\tau} = \vec{r} \times \vec{F}$

$\tau_2 = F_2 r \sin 90 = \frac{IbBa}{2}$ N.m counter clockwise

$\tau_4 = \frac{IbBa}{2}$ N.m Counter Clockwise

$\vec{\tau}_{net} = \vec{\tau_2} + \vec{\tau_4}$

$\tau_{net} = IabB$ counter clockewise

Case 3: general Case

There is an angle θ between $\vec{B}$ & the normal of the plane

هنا زاوية θ بين B والعمودي على المستوى

$$\tau = I a b B \sin\theta$$

$$\tau = N I a b B \sin\theta \qquad = (N I A) B \sin\theta$$

Magnetic Dipole Moment:

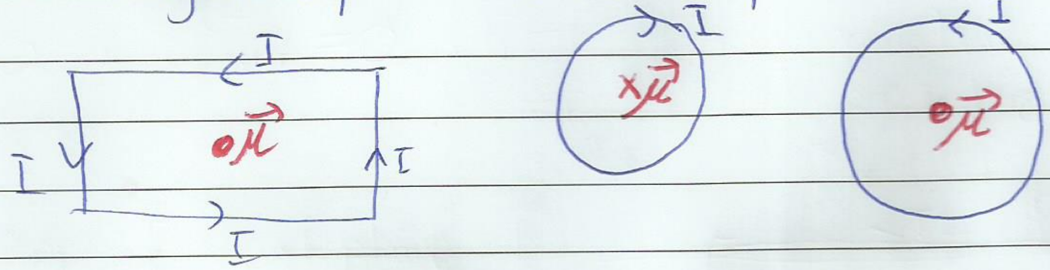
let $\vec{\mu} = N I A$ Ampere $\cdot$ m^2

N — number of turns

I — current

A — $(a \times b)$ Area

$\vec{\mu}$ — magnetic Dipole Moment (Vector quantity)



$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad N \cdot m$$

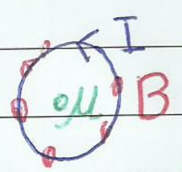
At the same time Potential energy will stored as magnetic energy

$$U_B = -\vec{\mu} \cdot \vec{B} \qquad \text{Joul}$$

$$W_B = -\Delta U = -[U_f - U_i] \qquad J$$

$$W_{F_{ext.}} = \Delta U = U_f - U_i \qquad J$$

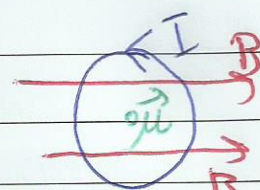
Notes:



$U_B = (-)\mu B \cos 0$

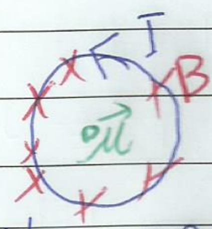
$= -\mu B$

minimum energy



$U_B = (-)\mu B \cos 90$

$= 0$



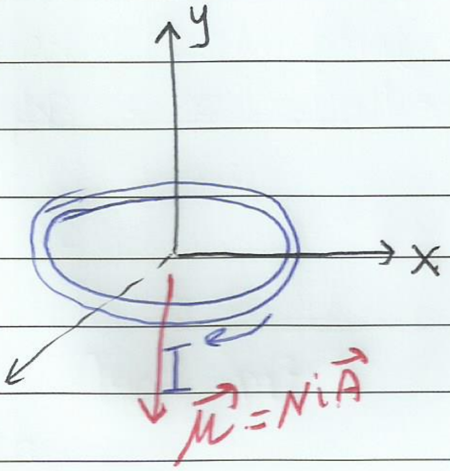
$U_B = (-)\mu B \cos 180$

$= +\mu B$

Maximum energ

(Problem 28-49)

The coil → $i = 4.6\,A$
→ in xz-plane
→ $N = 300$ turns
→ $A = 4\times10^{-3}\ m^2$



$\vec{B} = (3\hat{i} - 3\hat{j} - 4\hat{k})\ mT$

Find U? $\vec{\tau}$?

1) $\vec{\mu} = Ni\vec{A} = (300)(4.6)(4\times10^{-3})(-\hat{j})$ Amp.m²

$= (-)\,5.52\hat{j}$ Amp.m²

$U = -\vec{\mu}\cdot\vec{B}$ $\qquad \hat{j}\cdot\hat{i} = \hat{j}\cdot\hat{k} = 0$

$= (-)\left[(-5.52\hat{j})\cdot(3\hat{i} - 3\hat{j} - 4\hat{k})10^{-3}\right]$ $\qquad \hat{j}\cdot\hat{j} = 1$

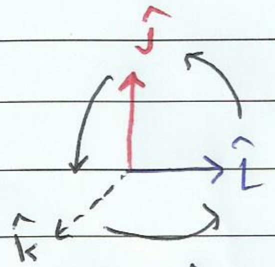
$= +10^{-3}\left[(5.52\hat{j})\cdot(3\hat{i} - 3\hat{j} - 4\hat{k})\right]$

$= +10^{-3}(5.52)(-3)(\hat{j}\cdot\hat{j})$

$= -16.56\times10^{-3}\ J = -16.56\ mJ$

2) $\vec{\tau} = \vec{\mu}\times\vec{B}$

$= [(-5.52)\hat{j}]\times[3\hat{i} - 3\hat{j} - 4\hat{k}]\times10^{-3}$



$= [-5.52\times10^{-3}]\left[\hat{j}\times3\hat{i}\ (+)\ \hat{j}\times-3\hat{j} + \hat{j}\times-4\hat{k}\right]$ $\qquad \hat{j}\times\hat{i} = -\hat{k}$

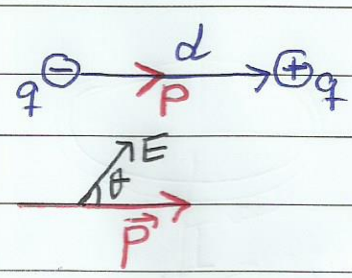
$\hat{j}\times\hat{j} = 0$

$= [-5.52\times10^{-3}]\left[3(-\hat{k}) + 0\ \ -4(\hat{i})\right]$ $\qquad \hat{j}\times\hat{k} = \hat{i}$

$\vec{\tau} = 22.1\times10^{-3}\hat{i} + 16.56\times10^{-3}\hat{k}$ N.m

Remember:-

Electric Dipole moment $\vec{p} = q\vec{d}$



Apply $\vec{E}$ on $\vec{p}$

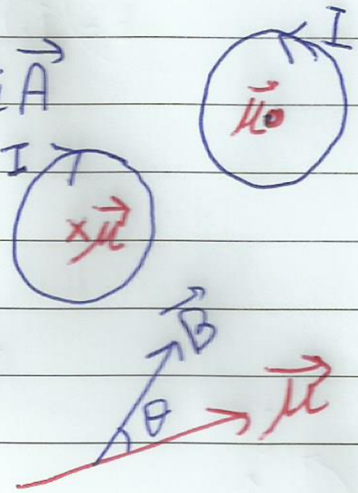
$$\vec{\tau} = \vec{p} \times \vec{E} \quad N.m$$

$$U_E = -\vec{p} \cdot \vec{E} \quad \text{Joul}$$

$$W_{E_{i \to f}} = -\Delta U = -[U_f - U_i]$$

$$W_{F_{ext\, i \to f}} = +\Delta U = [U_f - U_i]$$

Magnetic Dipole moment $\vec{\mu} = Ni\vec{A}$



Apply $\vec{B}$ on $\vec{\mu}$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$U_B = -\vec{\mu} \cdot \vec{B}$$

$$W_{B_{i \to f}} = -\Delta U = -[U_f - U_i]$$

$$W_{F_{ext}\, i \to f} = \Delta U = U_f - U_i$$