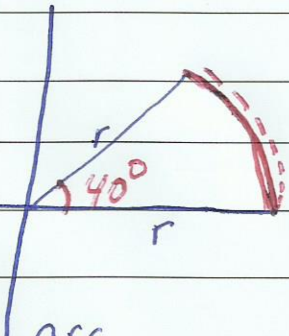
Discussion

(22-4) Charge Density:

a) $q = -300e = -300 \times 1.6 \times 10^{-19}$
$= -4.8 \times 10^{-17} C$
is Uniformly distributed along a circular arc → radius = 4 cm
→ subtends an angle = 40°



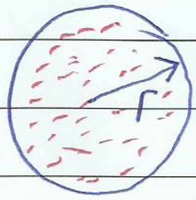
What is the linear charge density along the arc

$\ell = r\theta$, $\theta = \left(\frac{40°}{180°}\right)(\pi \text{ rad}) = \left(\frac{40}{180}\right)(3.14) = 0.7 \text{ rad}$

$\ell = (4 \times 10^{-2})(0.7)$
$= 2.8 \times 10^{-2}$

$\lambda = \frac{q}{\ell} = \frac{-4.8 \times 10^{-17}}{2.8 \times 10^{-2}} = -1.71 \times 10^{-15} C/m$

b) $q = -300e = -4.8 \times 10^{-15} C$ is Uniformly distributed over one face of a circular disk of radius = 2 cm



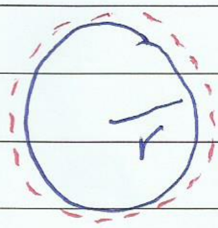
What is the surface charge density

$\sigma = \frac{q}{Area} = \frac{q}{\pi r^2}$, $A = \pi r^2 = \pi (0.02)^2$
$= 1.257 \times 10^{-3} m^2$

$\sigma = \frac{-4.8 \times 10^{-17}}{1.257 \times 10^{-3}} = 3.82 \times 10^{-14} C/m^2$

c) $q = -300e = -4.8 \times 10^{-17} C$ distributed uniformly over the surface of a sphere of radius = 4 cm
What is the surface charge density σ?

$\sigma = \frac{q}{Area}$, $A = 4\pi r^2 = 4\pi (0.04)^2$
$= 2 \times 10^{-2} m^2$



$= \frac{-4.8 \times 10^{-17}}{2 \times 10^{-2}} = (-)\, 2.4 \times 10^{-15} C/m^2$

d) $q = -300e = -4.8 \times 10^{-17} C$ Uniformly Spread through the volume of a sphere of radius = 2 cm
What is the volume charge density

$$\rho = \frac{q}{\text{Volume}} = \frac{q}{\frac{4}{3}\pi r^3}, \quad \text{Volume} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.02)^3$$

$$= 3.35 \times 10^{-5}\ m^3$$

$$= -\frac{4.8\times10^{-17}}{3.35\times10^{-5}} = -1.43\times10^{-12}\ C/m^3$$

(21-13)

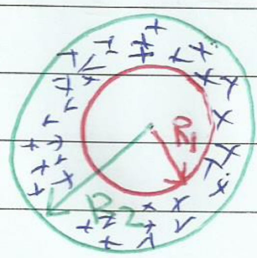
A nonconducting spherical shell has a charge spread nonuniformly through its volume between R_1 + R_2

$R_1 = 4cm$

$R_2 = 5cm$

the volume charge density $\rho = \frac{b}{r}$

$b = 3\mu C/m^2$



What is the net charge in the shell?

$$q = \int \rho\, dV, \quad dV = 4\pi r^2 dr$$

dr

spherical shell of radius $= r$, thickness $= dr$

$$q = \int_{R_1}^{R_2} \frac{b}{r}(4\pi r^2 dr) = 4\pi b \int_{R_1}^{R_2} r\,dr = 4\pi b\left[\frac{r^2}{2}\right]_{R_1}^{R_2}$$

$$q = 2\pi b\left[R_2^2 - R_1^2\right]$$

$$= 2\pi(3\times10^{-6})\left[(0.05)^2 - (0.04)^2\right] = 6\pi\times10^{-6}(9\times10^{-4})$$

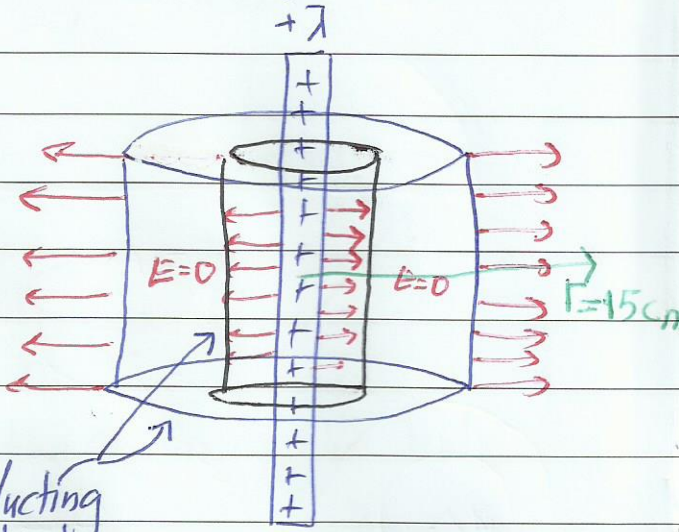
$$= {}^{+}1.7\times10^{-8}\ C = 17\ nC$$

$$= 1.7\times10^{-2}\ \mu C$$

$=$

(23-4) $\lambda = +1.5\,nC/m$ long noncconducting rod

a) Find E at $r = 15\,cm$ from the rod



$$E = \frac{\lambda}{2\pi\epsilon_0 r}\ N/C$$

$$E = \frac{2\lambda}{4\pi\epsilon_0 r}, \quad \frac{1}{4\pi\epsilon_0} = 9\times10^{9}$$

$$= \frac{2\times1.5\times10^{-9}\times9\times10^{9}}{15\times10^{-2}}$$

$$= 1.8\times10^{2}\ N/C$$

Conducting cylindrical spherical shell $\rightarrow R_1 = 5\,cm$, $R_2 = 10\,cm$

b) No Electric Field inside the Conductor

$$\Rightarrow q_{\text{inside surface of the conducting shell}} = -q = -\lambda l \qquad l = \text{length of the cylinder}$$

$$q_{\text{outside surface of the Conducting shell}} = +q = +\lambda l$$

$$\sigma_{\text{inner surface}} = \frac{q_{inner}}{Area} = \frac{-\lambda l}{2\pi R_1 l} = \frac{-\lambda}{2\pi R_1}$$

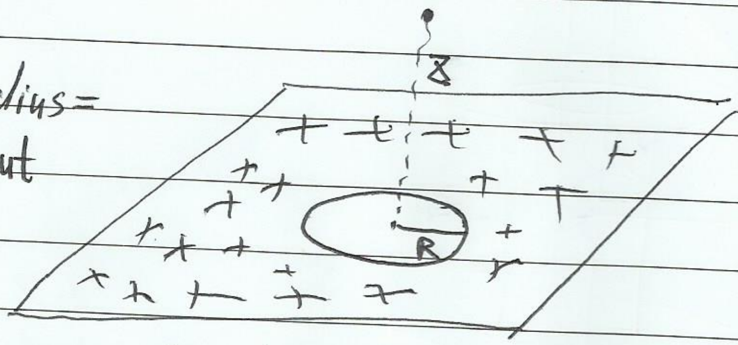
$$= \frac{-1.5\times10^{-9}}{2(3.14)(0.05)} = -4.8\,nC/m^2 = -4.8\times10^{-9}\,C/m^2$$

$$\sigma_{\text{outer surface}} = \frac{q_{outer}}{Area} = \frac{+\lambda l}{2\pi R_2 l} = \frac{+\lambda}{2\pi R_2} = \frac{+1.5\times10^{-9}}{2(3.14)(0.1)}$$

$$= 2.4\times10^{-9}\,C/m^2$$

(23 - 8) An infinite flat nonconducting sheet of surface charge density

$\sigma = +4.5\,pC/m^2$

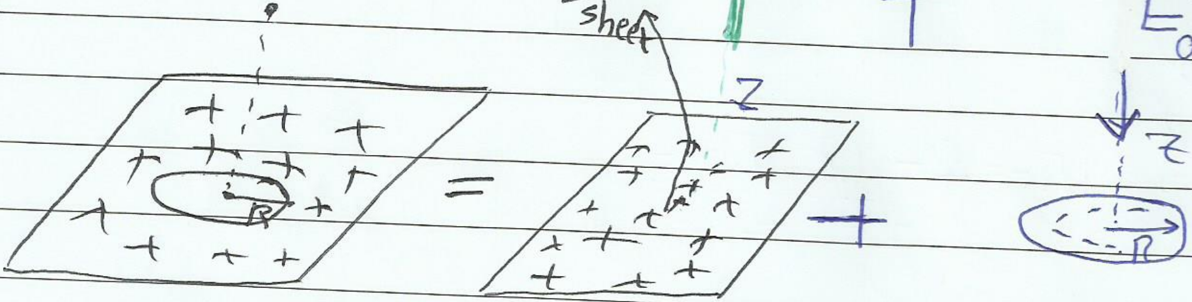


A circular hole of radius = $R = 1.3\,cm$ has been cut from the sheet

Find $\vec{E}$ at point

Ⓟ a distance $z = 2.56\,cm$ above the center of the hole.

$$\vec{E} = \vec{E}_{sheet} + \vec{E}_{disk}$$



$$\vec{E}_{net} = \frac{\sigma}{2\epsilon_0}\hat{k} + \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right](-\hat{k})$$

$$= \frac{\sigma z}{2\epsilon_0\sqrt{(R^2+z^2)}}\hat{k}$$

$$= \frac{(4.5\times10^{-12})(2.56\times10^{-2})}{2(8.85\times10^{-12})\left[(1.3\times10^{-2})^2+(2.56\times10^{-2})^2\right]^{1/2}}$$

$$= 0.227\,N/C\,\hat{k}$$

Note $\vec{E}_{sheet}$ without hole $= \frac{\sigma}{2\epsilon_0}\hat{k}$

$$= \frac{4.5\times10^{-12}}{2(8.85\times10^{-12})}$$

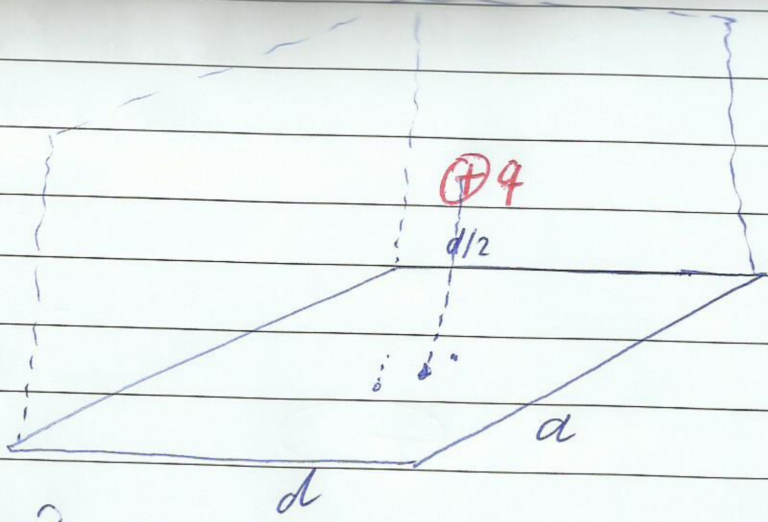
$$= 0.254\,N/C\,\hat{k}$$

(23-17)

$q = +e$

sheet $(d \times d)$

$q = +e$ above the center of the square sheet at $\frac{d}{2}$.



Find Φ from the square?

Draw a Gaussian surface around q to be cube of edge $= d$

$$\Phi_{cube} = \frac{+q}{\epsilon_0} = \frac{+e}{\epsilon_0} = +1.81 \times 10^{-8}\ Nm^2/C$$

$$\Phi_{oneface} = \frac{\Phi_{cube}}{6} = \frac{+e}{6\epsilon_0}$$

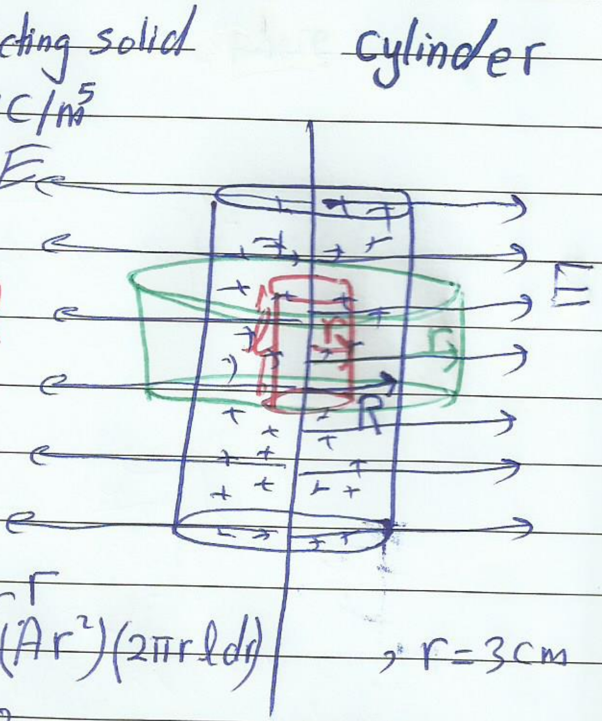
$$\Phi_{square} = \frac{+1.6 \times 10^{-19}}{6(8.85 \times 10^{-12})} = 3.0 \times 10^{-9}\ Nm^2/C$$

(23-18) $R = 4\,cm$ (Cylinder) Nonconducting solid cylinder

$\rho = Ar^2\ C/m^3$, $A = 6.3\,\mu C/m^5$

What is a) E at $r = 3\,cm$

Draw a Gaussian surface to be a cylinder of length $= \ell$ radius $r = 3\,cm$



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$E(2\pi r \ell) = \frac{1}{\epsilon_0}\int \rho\, dV = \frac{1}{\epsilon_0}\int_0^r (Ar^2)(2\pi r \ell\, dr) \quad , r = 3\,cm$$

$$E(2\pi r \ell) = \frac{2\pi \ell A}{\epsilon_0}\int_0^r r^3\, dr = \frac{2\pi \ell A}{\epsilon_0}\left[\frac{r^4}{4}\right]_0^{r=3cm}$$

$$E(2\pi r l) = \frac{2\pi l A}{\epsilon_0}\frac{r^4}{4}$$

$$E = \frac{A}{4\epsilon_0} r^3 \text{ ①}, \quad r \leq R$$

$$E = \frac{6.3\times10^{-6}(3\times10^{-2})^3}{4(8.85\times10^{-12})}$$

$$= 4.81 \text{ N/C}$$

b) Find E at $r = 5cm$, $r > R$

Draw a Gaussian surface to be a cylinder of
- Length $= l$
- radius $r = 5cm$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{en}}{\epsilon_0} = \frac{1}{\epsilon_0}\int \rho\, dV = \frac{1}{\epsilon_0}\int_0^R (Ar^2)(2\pi r l\, dr)$$

$$E(2\pi r l) = \frac{2\pi l A}{\epsilon_0}\int_0^R r^3 dr = \frac{2\pi l A}{\epsilon_0}\left[\frac{r^4}{4}\right]_0^R$$

$$E(2\pi r l) = \frac{2\pi l A}{4\epsilon_0} R^4$$

$$E = \frac{AR^4}{4\epsilon_0}\left(\frac{1}{r}\right) \text{ ②}, \quad r \geq R$$

For $r = 5cm$

$$E = \frac{6.3\times10^{-6}(0.04)^4}{4(8.85\times10^{-12})}\left(\frac{1}{0.05}\right) = 0.4556\left(\frac{1}{0.05}\right)$$

$$= 9.11 \text{ N/C}$$

c) At $r = R$ in ① $E = \frac{A}{4\epsilon_0} R^3$

At $r = R$ in ② $E = \frac{A R^3}{4\epsilon_0}$

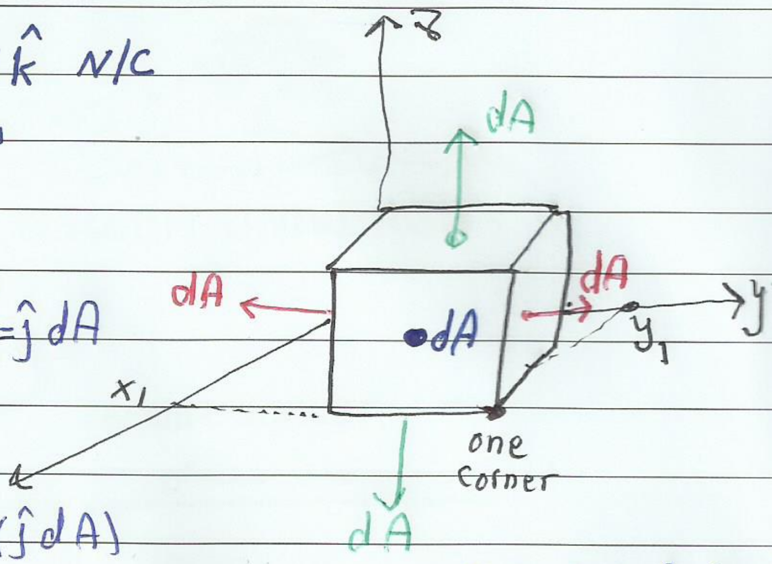
$$E_{surface} = \frac{6.3\times10^{-6}\times(0.04)^3}{4(8.85\times10^{-12})} = 11.4 \text{ N/C}$$

(23-23)

(23-23) $\vec{E} = +23\hat{i} - 2y^2\hat{j} - 16\hat{k}$ N/C

th cube edge length = 2m

$x_1 = 5m, \; y_1 = 4m$



$$\Phi_{\text{right face}} = \int \vec{E} \cdot d\vec{A} \quad , \; d\vec{A} = \hat{j}\, dA$$

$$= \int (23\hat{i} - 2y^2\hat{j} - 16\hat{k}) \cdot (\hat{j}\, dA)$$

$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$$= \int -2y^2 dA \quad , \text{ All right face Points at } y = 4m$$

$$= \int -2(4)^2 dA = -32\int dA = -32(A) = -32(2)^2$$

$$\Phi_{right} = -128 \text{ Nm}^2/C$$

$$\Phi_{Left} = \int \vec{E}\cdot d\vec{A} = \int (23\hat{i} - 2y^2\hat{j} - 16\hat{k}) \cdot (-\hat{j}\, dA) \quad , \; d\vec{A} = -\hat{j}\, dA$$

$$= \int 2y^2 dA \quad , \text{ All points of left face at } y = 2m$$

$$= \int 2(2)^2 dA = 8\int dA = 8A = 8(2)^2$$

$$\Phi_{left} = 32 \text{ Nm}^2/C$$

$$\Phi_{front} = \int (23\hat{i} - 2y^2\hat{j} - 16\hat{k}) \cdot (\hat{i}\, dA) = \int 23\, dA = 23A = 92$$

$$\Phi_{back} = \int (23\hat{i} - 2y^2\hat{j} - 16\hat{k})(-\hat{i}\, dA) = -23\int dA = -23A = -92$$

$$\Phi_{top} = \int (23\hat{i} - 2y^2\hat{j} - 16\hat{k}) \cdot (\hat{k}\, dA) = -16\int dA = -16A = -64$$

$$\Phi_{bottom} = \int (23\hat{i} - 2y^2\hat{j} - 16\hat{k}) \cdot (-\hat{k}\, dA) = 16\int dA = 16A = +64$$

$$\Phi_{cube} = \Phi_r + \Phi_\ell + \Phi_{top} + \Phi_{bottom} + \Phi_{front} + \Phi_{back}$$

$$= -128 + 32 + \cancel{-64} + \cancel{64} + \cancel{92} + \cancel{-92}$$

$$= -96$$

7

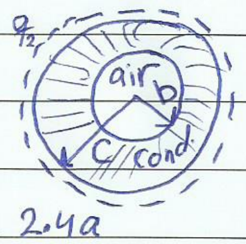
$$\Phi_{cube} = \frac{q_{enc}}{\epsilon_0} \Longrightarrow q_{enc} = \epsilon_0 \Phi_{cub} = -96\epsilon_0 = -8.5\times10^{-10}\, C$$

(23-29)

Solid sphere of radius $= a = 2cm$

$q_1 = +2fC = 2\times10^{-15}C$

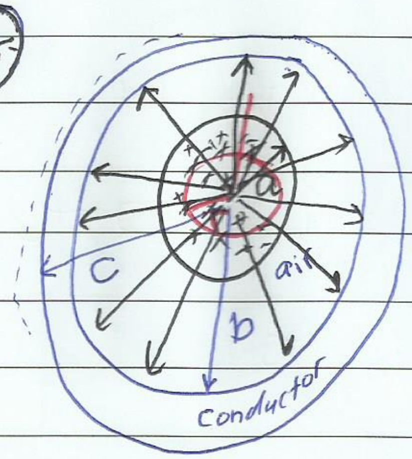
is Concentric with with a spherical Conducting shell of



inner radius $b = 2a$

outer radius $= c = 2.4a$

$q_2 = -q_1 = -2\times10^{-15}C$



What is the electric field at radial distances.

a) $r = 0 \Rightarrow E = 0$

b) E ? at $r = \frac{a}{2}$

Draw a Gaussian surface to be a sphere of radius, $r = \frac{a}{2}$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$\oint E\cos 0\, dA = \frac{1}{\epsilon_0}\int \rho\, dV \quad , \rho = \frac{q_1}{\frac{4}{3}\pi a^3} \text{ Constant}$$

$$E 4\pi r^2 = \frac{1}{\epsilon_0}\rho\int_0^{r=a/2} 4\pi r^2 dr = \frac{1}{\epsilon_0}\rho\left[\frac{4}{3}\pi r^3\right]$$

$$E = \frac{\rho}{3\epsilon_0} r \rightarrow r \le a \quad (1)$$

at $r = \frac{a}{2}$

$$E = \frac{q_1}{3\epsilon_0\left[\frac{4}{3}\pi a^3\right]}\left(\frac{a}{2}\right) = \frac{q_1}{2(4\pi\epsilon_0)a^2} = \frac{9\times10^9\times2\times10^{-15}}{2\left[2\times10^{-2}\right]^2}$$

$E = 2.25\times10^{-2}\ N/C$ outward.

c) E ? at $r = a$

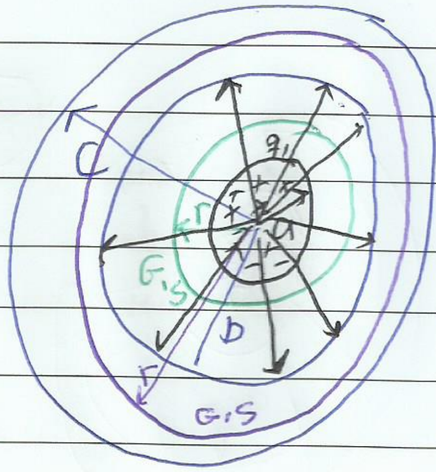
$$E = \frac{\rho}{3\epsilon_0} r \ , r \le a$$

at $r = a$

$$E = \frac{q_1}{\left[\frac{4}{3}\pi a^3\right]3\epsilon_0}\cdot a = \frac{q_1}{4\pi\epsilon_0 a^2} = \frac{(9\times10^9)(2\times10^{-15})}{(2\times10^{-2})^2}$$

$= 4.5\times10^{-2}\ N/C$ outward.

d) E? at $r = 1.5a$



Draw a G.S. to be a sphere of radius = $r = 1.5a$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$E\cos 0(4\pi r^2) = \frac{q_1}{\epsilon_0}, \quad r = 1.5a$$

$$E = \frac{q_1}{4\pi\epsilon_0 r^2} \quad a \leq r \leq b$$

at $r = 1.5a = 3\times10^{-2}$ m

$$E = \frac{9\times10^{9}(2\times10^{-15})}{(3\times10^{-2})^2} = 2\times10^{-2} \text{ N/C outward}$$

e) E at $r = 2.3a$

this point inside the conductor $E = 0$

f) E? at $r = 3.5a = 7\times10^{-2}$ m

Draw a G.S. to be a sphere of radius $r = 3.5a$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{q_1 + q_2}{\epsilon_0} = \frac{q_1 + -q_1}{\epsilon_0} = 0$$

$$E = 0, \quad r \geq 2.4a$$

g) Find $E_{inner\ surface}$? h) Find $E_{outer\ surface}$

at $r = 2.3a$ (inside the conductor)

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

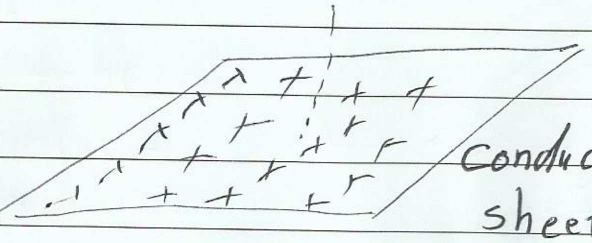
$$0 = \frac{q_1 + q_{inner}}{\epsilon_0} \Rightarrow q_{inner} = -q_1 = -2fC$$

$$q_{inner} + q_{outer} = q_2$$

$$-2fC + q_{outer} = -2fC$$

$$q_{outer} = 0$$

(23 – 44)



$E = \frac{\sigma}{\epsilon_0}$ → near the charged conducting sheet

$\sigma = \epsilon_0 E = (8.85 \times 10^{-12})(1.9 \times 10^{5})$

$= 16.82 \times 10^{-7} C/m^2$

$= 1.68 \mu C/m^2$

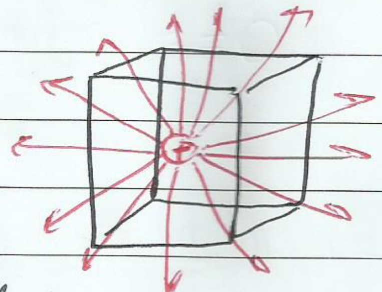
Remember → $E = \frac{\sigma}{2\epsilon_0}$ near the charged Nonconducting sheet

(23 – 43) $q = 6.3 \mu C$ at the Center of a Gaussian Cube

a) $\Phi_{cube} = \frac{q_{enc}}{\epsilon_0}$ $\ell = 92 cm$

$= \frac{+6.3 \times 10^{-6}}{8.85 \times 10^{-12}}$

$= 0.71 \times 10^{6} Nm^2/C$



b) the edge length is doubled $\ell_2 = 2\ell$

find Φ_{cube}

$\Phi_{cube} = \frac{q_{en}}{\epsilon_0} = \frac{+6.3 \times 10^{-6}}{8.85 \times 10^{-12}} = 0.71 \times 10^{6} Nm^2/C$

c) (extra) find Φ from each face?

q at the center of the cube ⟹

$\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4 = \Phi_5 = \Phi_6 = \frac{q_{enc}}{6\epsilon_0}$

$\Phi_{oneface} = \frac{1}{6}\left(\frac{q}{\epsilon_0}\right) = \frac{0.71 \times 10^{6}}{6}$

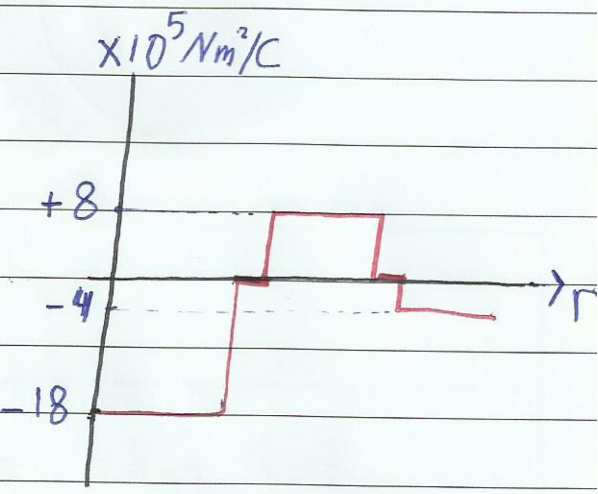
$= 0.12 \times 10^{6} Nm^2/C$

(23-50) $\Phi_{\text{closed surface}} = \frac{q_{enc}}{\epsilon_0}$

$q_{enc} = \epsilon_0 \Phi_{\text{closed surface}}$

$, \Phi_s = 10 \times 10^5\ Nm^2/C$

a) $q_{center} = \epsilon_0[-1.8\Phi_s] = \epsilon_0[-1.8(10\times10^5]$



$= (-\epsilon_0)(18\times10^5)$

$= -8.85\times10^{-12}\times 18\times10^5$

$= -1.6\times10^{-5}C = -16\mu C$

$q_{center} = -16\ \mu C$

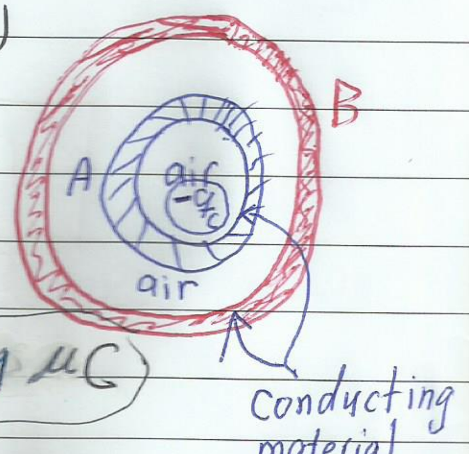
b) E_{inside} the conducting shell $A = 0$

flux between conducting shells $A + B = {}^{+0.8}\Phi_s = +8\times10^5$

$q_{enc} = \epsilon_0 0.8\Phi_s = (8.85\times10^{-12})(+8\times10^5)$

$= {}^+ 7.1\ 10^{-6}\ C$

$q_{center} + q_A = +7.1\ \mu C$



$-16\ q_A = +7.1$

$q_A = +16 + 7.1 = {}^+23.1\ \mu C$

c) E_{inside} the conducting shell $B = 0$

flux outside shell $B = -4\times10^5$

$q_{enc} = \epsilon_0 \Phi = (8.85\times10^{-12})(-4\times10^5)$

$= -3.54\times10^{-6} = -3.54\mu C$

$q_c + q_A + q_B = \epsilon_0(-4\times10^5) = (-)3.54\times10^{-6} = -3.54\ \mu C$

$-16 + {}^+23.1 + q_b = -3.54$

$-16 + {}^+23.1 + q_b = -3.54$

$q_B = 16 - 23.1 - 3.54$

$q_B = -10.64\ \mu C$