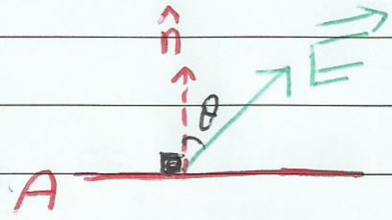
Chapter 23 Gauss' Law

Electric Flux: is directly Proportional to the number of Electric Field lines piercing تخترق the surface Perpendicularly "عموديا"

$$\Phi_E = \vec{E} \cdot \vec{A} \quad N \cdot m^2/C$$
$$= EA\cos\theta \quad N \cdot m^2/C$$



$\vec{A} = A\hat{n}$

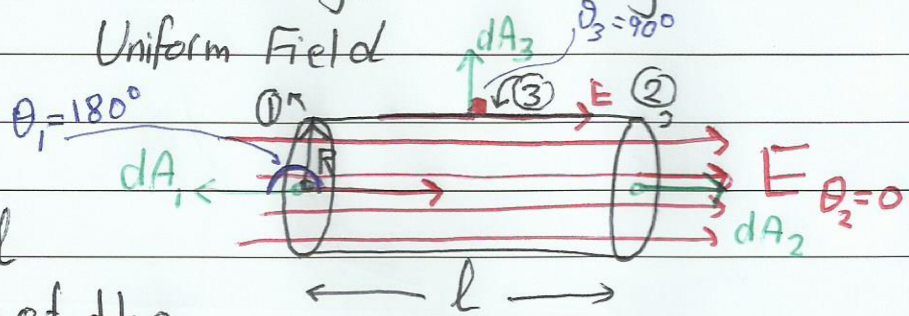
$\Phi_E = \vec{E} \cdot \vec{A}$, for Uniform Field

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

An inward piercing field is negative flux $(180° > \theta > 90)$
An outward piercing field is positive flux $(90° > \theta > 0)$
The lectric Flux = 0 for $\theta = 90°$
skimming field has zero flux
(الحقل الموازي للسطح)

Do Checkpoint 1

Sample Problem 23.01: Flux through a closed Cylinder
Uniform Field



closed cylinder
radius = R Length = l
E // to the length of the cylinder, and Uniform
Find Φ_E from left face? ① right face? ② curved face ③

$(\Phi_E)_1 = \vec{E}\cdot\vec{A} = E\pi R \cos 180 = -E\pi R^2$

$(\Phi_E)_2 = \vec{E}\cdot\vec{A} = E\pi R^2 \cos 0 = E\pi R^2$

$(\Phi_E)_3 = \vec{E}\cdot\vec{A} = E(2\pi R \ell)\cos 90 = 0$

$(\Phi_E)_{cylinder} = \Phi_E 1 + \Phi_E 2 + \Phi_E 3$

$\Phi_{cylinder} = 0$, Cylinder is closed surface

Sample Problem 23.02 : Electric Flux through a Closed Cube nonuniform field

Nonuniform E is given by

$\vec{E} = (3x)\hat{i} + 4\hat{j}$ N/C

$E_x = 3x$ Nonuniform Component

$E_y = 4$ Uniform component

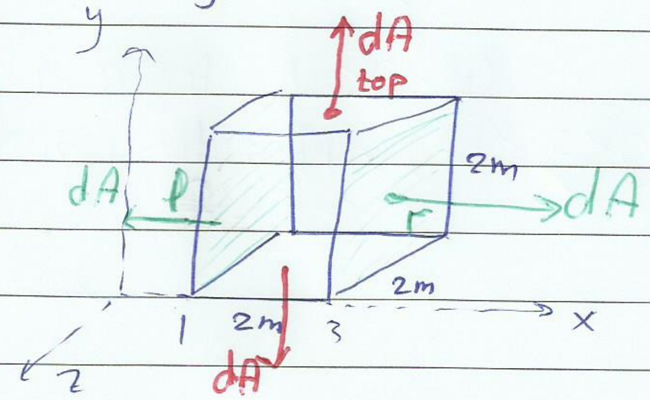
pierces a closed cube of edge length = 2m

What is the elelectric Flux

1) Through the right face

2) Through the left face

3) Through the top face.



"Depending on the diagramms in the book"

1) right face $\vec{dA} = \hat{i}\,dA$

$\Phi_r = \int \vec{E}\cdot\vec{dA} = \int (3x\hat{i} + 4\hat{j})\cdot\hat{i}\,dA$

rememeber
$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$
$\hat{i}\cdot\hat{j} = 0$

$= \int (3x\,dA)(\hat{i}\cdot\hat{i}) + \int 4\,dA\,\hat{j}\cdot\hat{i}$

$= \int 3x\,dA$, All parts of right face at $x = 3$

$= \int 3(3)\,dA = 9\int dA = 9A = 9(\ell)^2 = 9(2)^2$

$\Phi_r = 36\ Nm^2/C$

2) left face $d\vec{A} = -\hat{i}\,dA \implies \Phi_\ell = \int \vec{E} \cdot d\vec{A}$

$\Phi_\ell = \int (3x^2\hat{i} + 4\hat{j}) \cdot -\hat{i}\,dA$

$= \int -3x\,dA = -3\int x\,dA$, left face is at $x = 1$

$= -3\int dA = -3A = -3(2)^2$

$\Phi_\ell = -12\ N m^2/C$

3)

topface $\Phi_t = \int \vec{E} \cdot d\vec{A}$, $d\vec{A} = \hat{j}\,dA$

$\Phi_t = \int (3x\hat{i} + 4\hat{j}) \cdot \hat{j}\,dA = \int 4\,dA$

$\Phi_t = 4\int dA = 4A = 4(2)^2 = 16\ N m^2/C$

$\Phi_{top} = 16\ N m^2/C$

Addition: find

(4) Φ through bottom face $d\vec{A} = -\hat{j}\,dA$

$\Phi_b = \int \vec{E} \cdot d\vec{A} = \int (3x\hat{i} + 4\hat{j}) \cdot -\hat{j}\,dA$

$= -4\int dA = -4(2)^2 = -16\ N m^2/C$

$\Phi_b = -16\ N m^2/C$

5) find Φ through the front face

$\Phi_f = \int \vec{E} \cdot d\vec{A}$, $d\vec{A} = \hat{k}\,dA \Rightarrow$ for front face

$\Phi_f = \int (3x\hat{i} + 4\hat{j}) \cdot \hat{k}\,dA = 0$ $\quad \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = 0$

6) find Φ throug back face? $d\vec{A} = -\hat{k}\,dA$ for back face

$\Phi_{ba} = \int \vec{E} \cdot d\vec{A} = 0$

$\Phi_{cub} = \Phi_r + \Phi_\ell + \Phi_t + \Phi_b + \Phi_f + \Phi_{ba}$

$= 36 + {-12} + 16 + {-16} + 0 + 0$

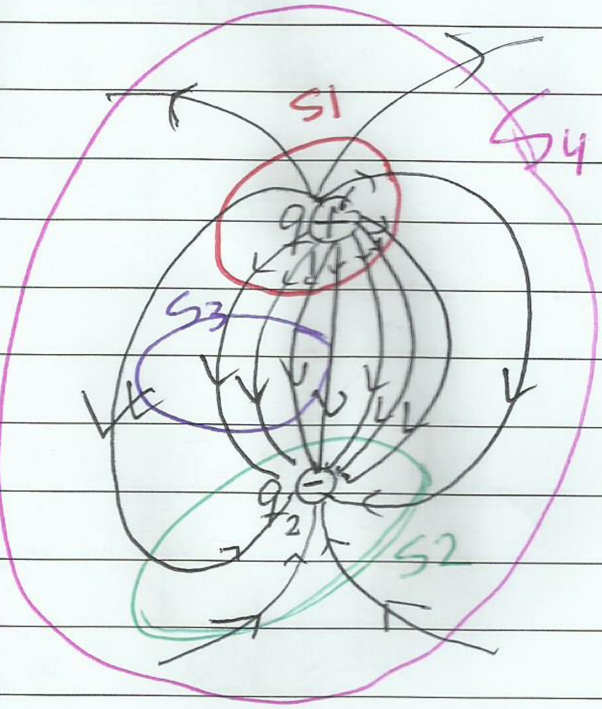
$\Phi_{cube} = 24\ N m^2/C$

Gauss' Law:

The electric flux through a closed surface equal to the total charge inside the surface divided by ϵ_0

$$\underset{\substack{\text{closed}\\\text{surface}}}{\Phi_E} = \frac{q_{en}}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$



example: $q_1 = +17.7\,\mu C$

$q_2 = -17.7\,\mu C$

find: ① $\Phi_{S1} = \frac{q_{enc}}{\epsilon_0} = \frac{q_1}{\epsilon_0}$

$= + \frac{17.7 \times 10^{-6}}{8.85 \times 10^{-12}}$

$\Phi_{S1} = 2 \times 10^6\ Nm^2/C$

$\Phi_{S2} = \frac{q_{enc}}{\epsilon_0} = \frac{q_2}{\epsilon_0}$

$\Phi_{S2} = \frac{-17.7 \times 10^{-6}}{8.85 \times 10^{-12}} = -2 \times 10^6\ Nm^2/C$

$\Phi_{S3} = \frac{q_{enc}}{\epsilon_0}$

$\Phi_{S3} = 0$ No charge inside S_3

$\Phi_{S} = \frac{q_{enc}}{\epsilon_0} = \frac{q_1 + q_2}{\epsilon_0} = 0$

Gauss' Law is usefull in Calculating E due to a Continous Charg distribution

Having High Symmetry

We have three Symmetries

I 1) Volume Symmetry

II 2) Linear Symmetry (Cylindical Symmetry)

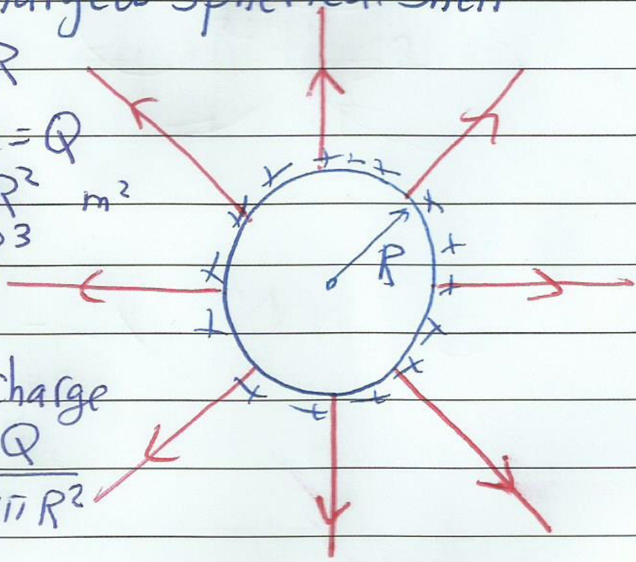
III 3) Surface Symmetry (planar Symmetry)

Applications on Gauss' Law:

$\boxed{I}$ Volume Symmetry

1] E due to a Uniformly charged Spherical Shell

A spherical shell
- radius $= R$
- charge $= Q$
- $A = 4\pi R^2$ m^2
- $V = \frac{4}{3}\pi R^3$
- σ surface charge density $= \frac{Q}{4\pi R^2}$



The charge distributed on the surface of the sphere

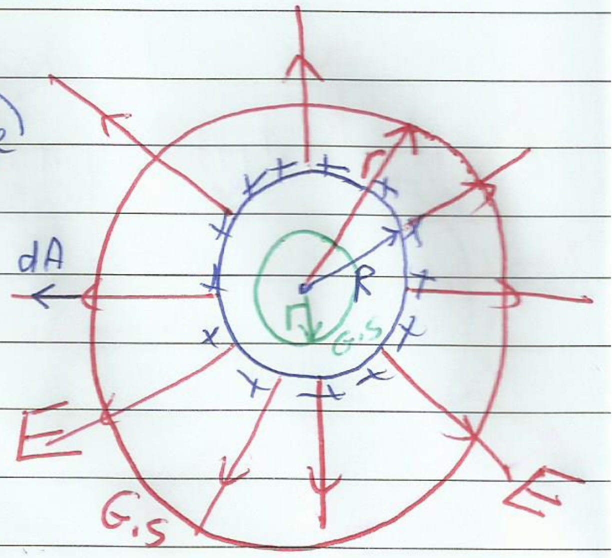
find E at ① $r > R$; outside the spherical shell

② $r < R$; inside the spherical shell

③ $r = R$.

1) E at r>R

Draw a Gaussian surface to be a sphere of radius = r



$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$\oint E\cos 0\, dA = \frac{Q}{\epsilon_0}$$

$$E\oint dA = \frac{Q}{\epsilon_0}$$

$$E\,4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \quad r \geq R$$

2) E at r<R

Draw a Gaussian surface to be a sphere of radius r

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$E\oint dA = 0$ (No charge inside the sphere

$E = 0$ at $r<R$

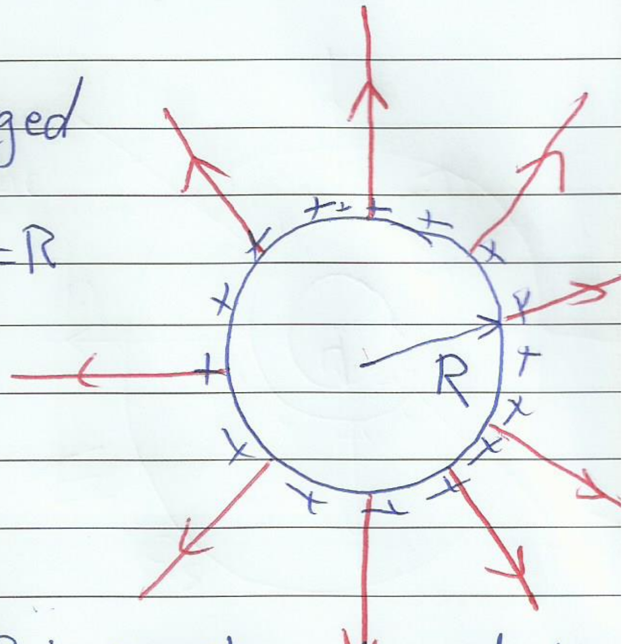
2] E due to a Uniformly charged Solid Conducting sphere

A conducting sphere of radius $= R$ is Charge by a charge $+Q$

(1) find E at $r > R$

(2) find E at $r = R$

(3) find E at $r < R$



In Electrostatic: All charge Q in any charged conductor will sit on the outer Surface only, No charge inside the Conductor

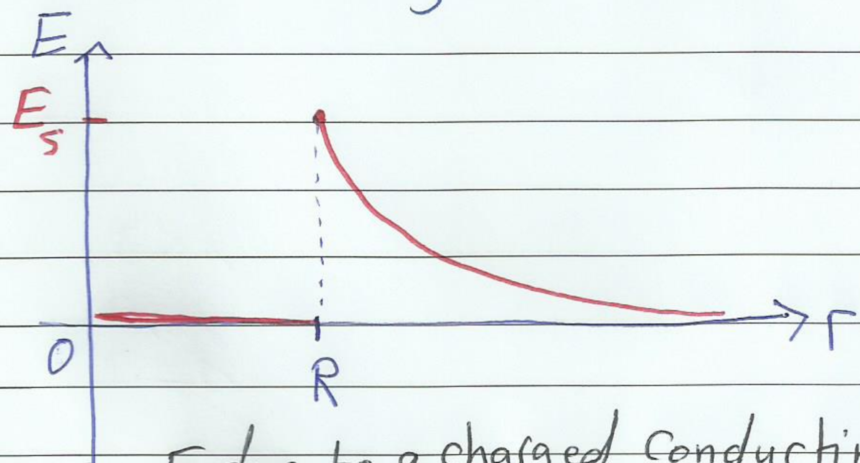
$\Longrightarrow$ Charged Conducting sphere will behave as a spherical shell (the same Solution)

$$\sigma = \frac{Q}{4\pi R^2} \text{ (Charged Conducting sphere)}$$

$$E = \frac{Q}{4\pi \epsilon_0 r^2} , \quad r > R$$

$$E_s = \frac{Q}{4\pi \epsilon_0 R^2} , \quad r = R \text{ at the surface}$$

$E = 0$ inside the Charged Conductor



E due to a charged Conducting sphere.

3] E due to a Uniformly Charged Nonconducting sphere

A nonconducting sphere of radius $= R$

Charge $= Q$

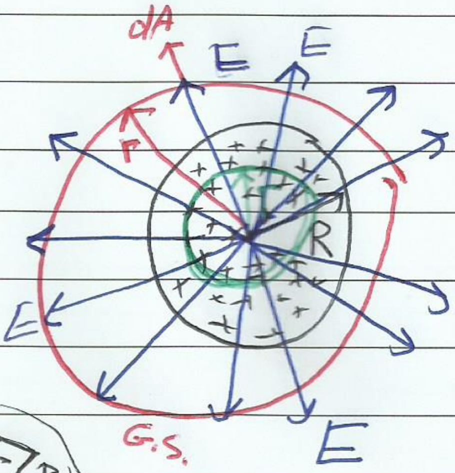
distributed Uniformoly through its volume from the center to the outer surface,

Find 1) E at $r > R$ 2) E at $r < R$ 3) E at $r = R$

The volume charge density $\rho = \frac{Q}{\text{Volume}} = \frac{Q}{\frac{4}{3}\pi R^3}$

① To find E at $r > R$

Draw a Gaussian surface to be a sphere of radius $r > R$



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$\oint E \cos 0 \, dA = \frac{Q}{\epsilon_0}$$

$$E \oint dA = \frac{Q}{\epsilon_0}$$

$$E 4\pi r^2 = \frac{Q}{\epsilon_0} \Rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2}, \; r > R \quad ①$$

2) To find E at $r < R$ Draw a sphere of radius $= r$

Gaussian surface is closed sphere of radius $= r$

$$\oint \vec{E} \cdot dA = \frac{q_{enc}}{\epsilon_0}$$

$$E 4\pi r^2 = \frac{1}{\epsilon_0}\left[\rho \, \text{Volume}\right] = \frac{1}{\epsilon_0}\left[\rho \cdot \frac{4}{3}\pi r^3\right]$$

$$E = \frac{\rho r}{3\epsilon_0} \quad ② \quad r < R \qquad \rho = \frac{Q}{\frac{4\pi}{3} R^3}$$

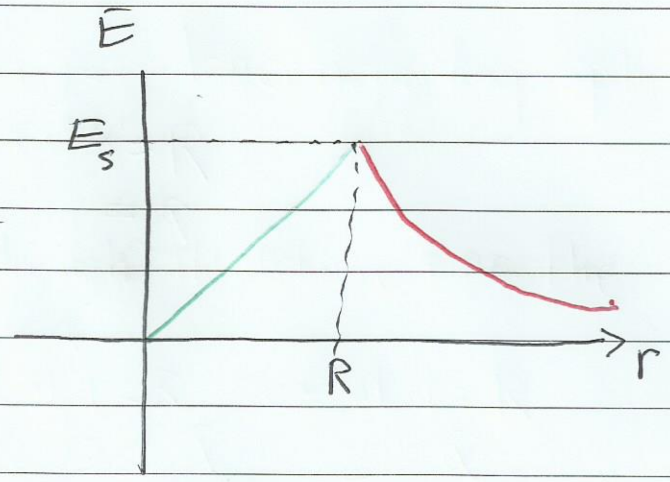
$$E = \left[\frac{Q}{\frac{4}{3}\pi R^3}\right] \cdot \frac{r}{3\epsilon_0} = \frac{Q}{4\pi\epsilon_0 R^3} r \quad r < R \quad ②'$$

3) E at $r = R$ from ① or from ②'

$$E_{surface} = \frac{Q}{4\pi\epsilon_0 R^2} \quad ③$$

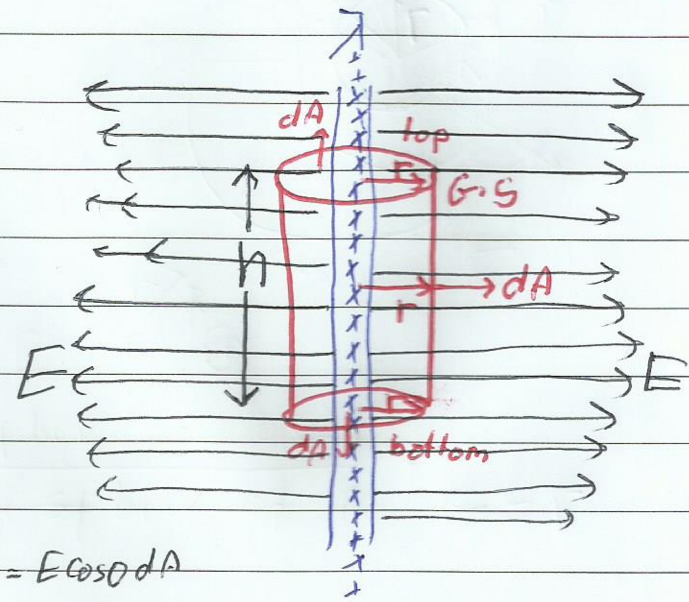
E due to a charged

Nonconducting sphere



II] Linear Symmetry:-

E due to an infinitely long plastic rod with a Uniform linear charge Density λ
Find E near the rod a distance r from the rod



Draw a Gaussian surface to be a Cylinder of radius = r, and length = h

$E_{top} = 0 \quad \vec{E} \perp \vec{A}$ $\quad \cos 90 = 0$

$E_{bottom} = 0 \quad \vec{E} \perp \vec{A}$

from curved surface

$A = 2\pi r h$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{en}}{\epsilon_0} \qquad \vec{E} \cdot d\vec{A} = E \cos 0 \, dA = E \, dA$$

$$E \oint dA = \frac{\lambda h}{\epsilon_0} \quad , \cos 0 = 1$$

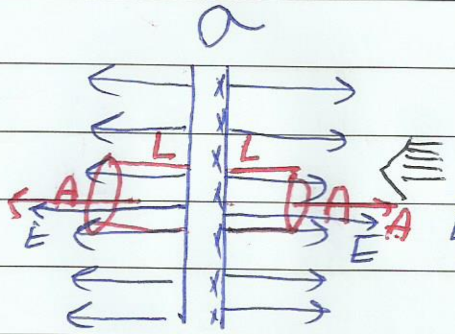
$$E(2\pi r h) = \frac{\lambda h}{\epsilon_0}$$

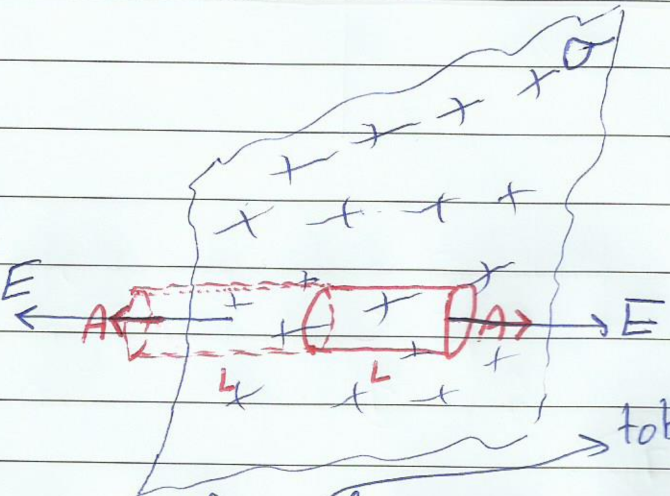
$$E = \frac{\lambda}{2\pi \epsilon_0 r}$$ very long charged rod

III Planar Symmetry:

E due to a Uniformly Charged, thin, infinite Nonconducting sheet (Surface)

surface charge density $= \sigma$ C/m^2 in one surface





tobe a cylinder

Draw the Gaussian surface of Length = 2L crossing the sheet, of Length = L from each side

$$\oint \vec{E} \cdot dA = \frac{q_{enc}}{\epsilon_0}$$

$$E\cos 0 A + E\cos 0 A = \frac{\sigma A}{\epsilon_0}$$

$$EA + EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$ constant.

Solve Sample Problems (23.03)

(23.04)

(23.05)

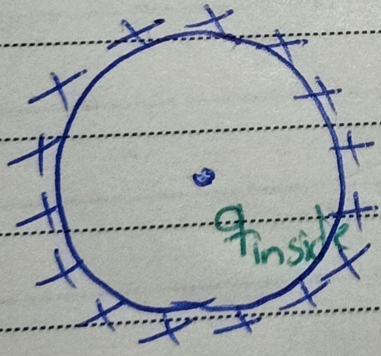
The two shell theories for electrostatics

[1] Shell Theorem 1:

A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center

[2] Sell Theorem 2:

A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.

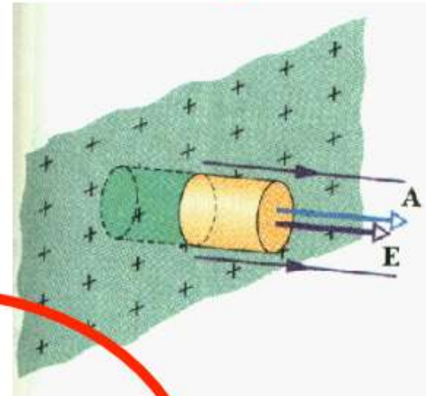


q_{out} $\Rightarrow$ By Gauss' Law $\vec{E}_{out}$ the shell

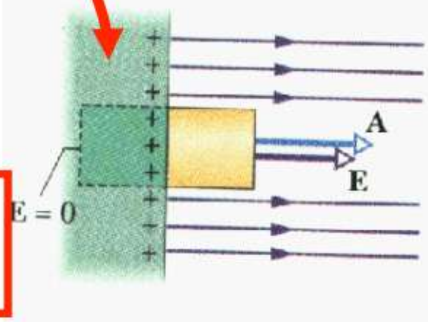
$E_{inside} = \text{zero}$

Gauss' Law: Conducting Plane

- Infinite CONDUCTING plane with uniform areal charge density s
- E is NORMAL to plane
- Construct Gaussian box as shown.
- Note that E = 0 inside conductor



Applying Gauss' law, we have, $\dfrac{A\sigma}{\varepsilon_0} = AE$

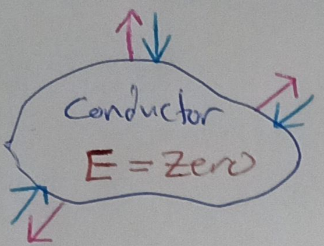


Solving for the electric field, we get $\boxed{E = \dfrac{\sigma}{\varepsilon_0}}$

- Gauss' Law $\Rightarrow$ $\epsilon_0 \Phi = q_{enc}$; $\Phi = \oint \vec{E} \cdot d\vec{A}$

[1] Isolated charged conductor

Electric field near the surface is perpendicular to the surface



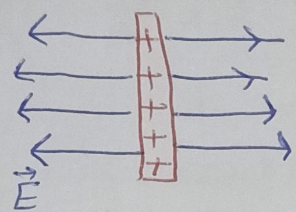
$$E = \frac{\sigma}{\epsilon_0}$$

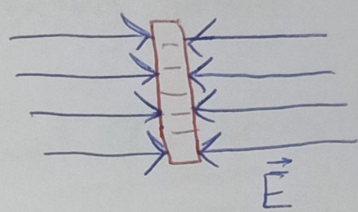
q is positive $\Rightarrow$ outward

q is negative $\Rightarrow$ Inward

[2] Infinite non-conducting sheet with uniform surface charge density σ is perpendicular to the plane of the sheet

$$E = \frac{\sigma}{2\epsilon_0}$$





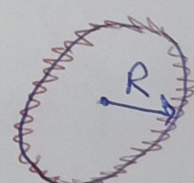
[3] Infinite line of charge (uniform linear charge density λ)

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

" r is the perpendicular distance from the line of charge to the point "

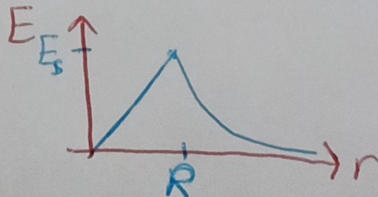
[4] Spherical shell of charge with radius R and charge q

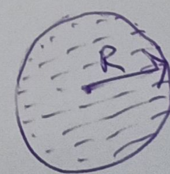
$$E = \begin{cases} \frac{kq}{r^2} & ; \; r \geq R \\ \text{zero} & ; \; r < R \end{cases}$$



[5] charged sphere (non-conducting; uniform volume charge distribution)

$$E = \begin{cases} \frac{kq}{r^2} & ; \; r \geq R \\ \frac{kq}{R^3} r & ; \; r \leq R \end{cases}$$

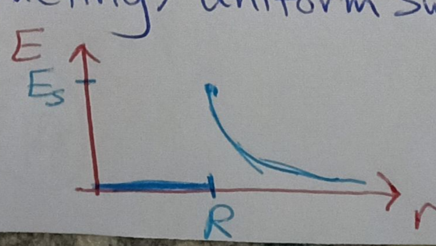




ρ is Const

[6] charged sphere (conducting; uniform surface charge density

$$E = \begin{cases} \frac{kq}{r^2} & ; \; r \geq R \\ \text{zero} & ; \; r < R \end{cases}$$



$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{A\epsilon_0} = \frac{q}{4\pi r^2 \epsilon_0}$$