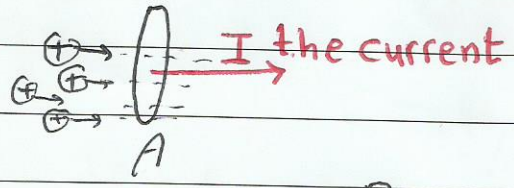
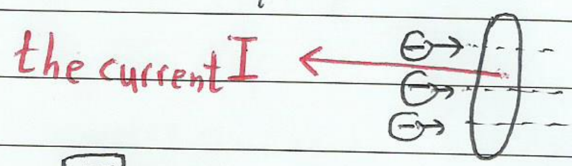
Chapter 26. Current and Resistance

Electric current in a Conductor

$i = \frac{dq}{dt}$ C/s = Ampere [The time rate of +ve Charges crossing the surface]





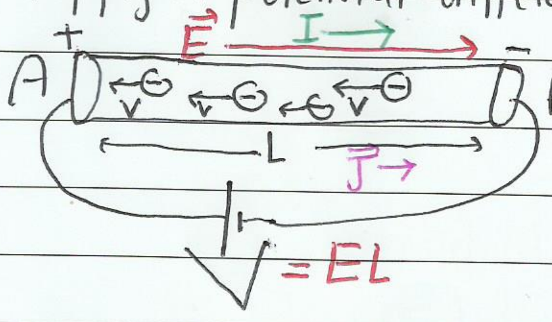
[I] is the Scalar Quantity

Conductor
- → of Length = L
- → Cross sectional Area = A
- → n ≐ the density of free electrons free e/m^3 (free electrons/m^3)

$ne = C/m^3$ ≐ Carrier Charge Density

(n) depends on the metal, Cu, Fe, Al, ...

Apply A potential difference V



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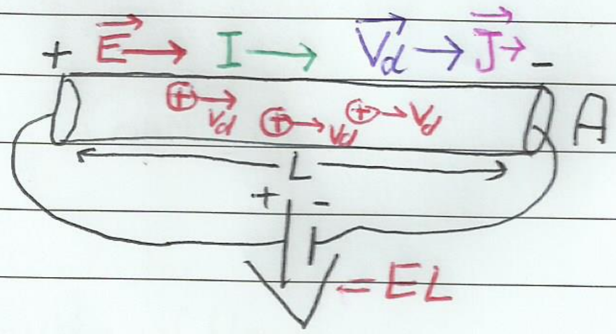


The number of charge Carriers in the wire (Conductor) = nAL

The total charge of the Carriers in the Wire $= (nAL)e = nALe$ (q)

The electric current $\frac{q}{t} = \frac{nALe}{t}$

$I = nA\left(\frac{L}{t}\right)e$

$\boxed{I = nAe\,V_d}$, $V_d = \frac{L}{t}$ = the drift speed of each charge carrier

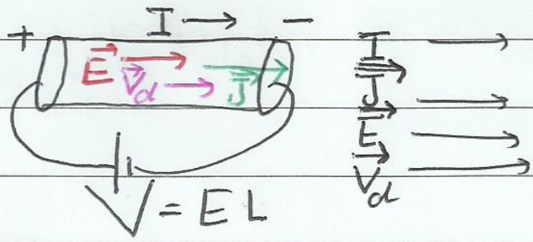
The Current Density $= \frac{I}{A}$ Ampere/m^2 is a vector quantity

$$J = \frac{I}{A} \Rightarrow I = \vec{J} \cdot \vec{A} \text{ , for constant } J$$

$$I = \int \vec{J} \cdot d\vec{A} \text{ , for variable } J$$

$\vec{J} = ne\vec{V_d}$

$I = neAV_d$



$V = EL$

The Current In the Conductor depends on:

1) Geometry of the Conductor $< A, L$
2) The type of the Conductor $\longrightarrow n$
3) The Applied Voltage (V) (or) $\longrightarrow V = EL$
   The Applied Electric Field $E = V/L$

OHM'S Law :

The Current Density in the Conductor is Directly proportional to Applied Electric Field

$$J \propto E \Rightarrow \vec{E} = \rho \vec{J} \quad \text{OHM'S Law}$$

ρ is the resistivity of the Conductor

$\sigma = \frac{1}{\rho}$, σ is the conductivity of the Conductor

examples

$$\rho_{Cu} = 1.69 \times 10^{-8} \ \Omega \cdot m$$

$$\rho_{Al} = 2.75 \times 10^{-8} \ \Omega \cdot m$$

⋮

$E = \rho J$

$\rightarrow J = \frac{I}{A}$

$\rightarrow EL = V$

$$\frac{V}{L} = \rho \frac{I}{A}$$

$$V = \left(\frac{\rho L}{A}\right) I$$

$$V = RI$$ OHM's Law

(V → Volt, I → Ampere)

Where $R = \rho \frac{L}{A}$ → depends on:

① geometry $< \begin{matrix} L \\ A \end{matrix}$

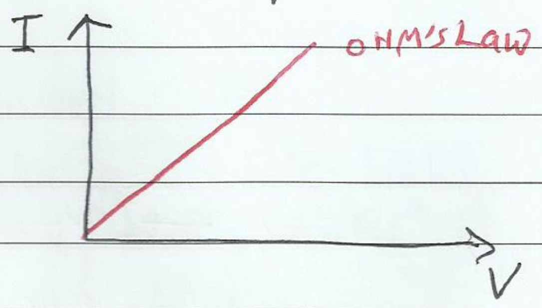
② The type of the Conductor ρ.

R → Ω

$\rho = \Omega \cdot m$

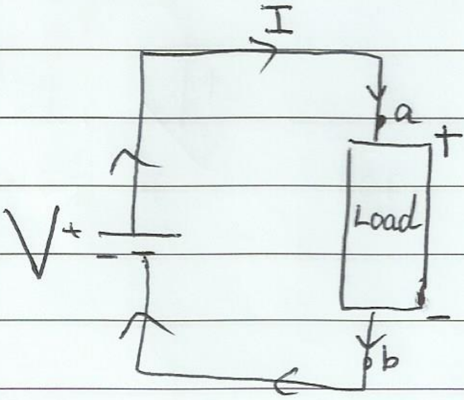
$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$, ρ & R depends on Tempreture

$R - R_0 = R_0 \alpha (T - T_0)$



Power in Electric Circuit:

When a charge dq moves through the Load from ⓐ → ⓑ its Potential energy $dU = V dq$ will decrease because it moves from $V_a \rightarrow V_b$ & $V_a > V_b$



Change of Energy $= V dq$

$$\frac{dU}{dt} = V \frac{dq}{dt}$$

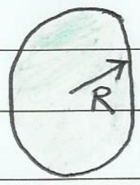
$$\text{Power} = VI \begin{cases} I^2 R \\ \frac{V^2}{R} \end{cases} \text{Watt.}$$

Solve Sample Problems 26.02 + 26.03 + 26.04 + 26.06

(Problem 26-54)

$J = (3\times10^{8})r^{2}$

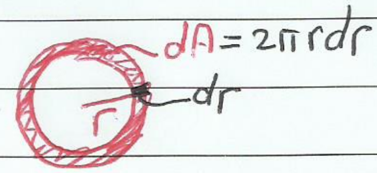
$J = \alpha r^{2}, \ \alpha = 3\times10^{8}$



$R = 2.5\ mm$

a) What is the current through the outer section from $r_1 = 0.9R \longrightarrow r_2 = R$

$I = \int \vec{J}\cdot d\vec{A}$,



$= \int_{r_1}^{r_2} (\alpha r^{2})(2\pi r\, dr)$

$= 2\pi\alpha \int_{0.9R}^{R} r^{3} dr = 2\pi\alpha \left[\frac{r^{4}}{4}\right]_{0.9R}^{R} = \frac{2\pi\alpha}{4}\left[R^{4} - (0.9R)^{4}\right]$

$I_{(0.9-1)R} = \frac{2\pi\alpha R^{4}}{4}\left[1 - (0.9)^{4}\right] = \frac{2\pi\alpha R^{4}}{4}\left[1 - 0.656\right]$

$= 0.3441\left(\frac{2\pi}{4}\right)\alpha R^{4} = 0.54\,\alpha R^{4}$

$= 0.54\alpha R^{4} = 6.33\times10^{-3} A$

$= 6.33\ mA$

b) Find the total current in the Wire?

$I = \int \vec{J}\cdot d\vec{A} = \int_{0}^{R} \alpha r^{2}(2\pi r\, dr) = 2\pi\alpha \int_{0}^{R} r^{3} dr = 2\pi\alpha\left[\frac{r^{4}}{4}\right]_{0}^{R}$

$I_{tot.} = \frac{2\pi\alpha}{4} R^{4} = \frac{\pi\alpha R^{4}}{2} = 1.57\,\alpha R^{4}$

$I_{tot} = \frac{\pi\alpha}{2} R^{4} = 184\times10^{-2} A = 18.4\ mA$

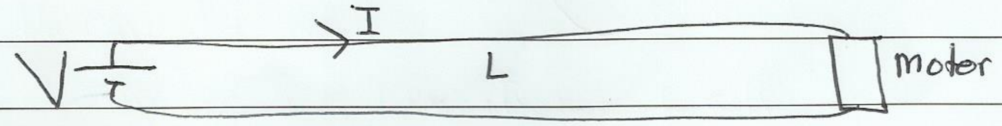
(26-43)

$I = 285A$

$V_d = ?$ $A = 0.17 cm^2$

time? $L = 0.43 m$

$n = 8.49 \times 10^{28}$ free electrons/m^3



$I = neAV_d$, $V_d = \frac{I}{neA}$

$$V_d = \frac{285}{(8.49\times10^{28})(1.6\times10^{-19})(0.17\times10^{-4})} = 1.23\times10^{-3} \text{ m/s}$$

$$t = \frac{L}{V_d} = \frac{0.43}{1.23\times10^{-3}} = 348 \text{ sec} = 5.8 \text{ min.}$$

(26-37) $V = 120V$

$P = 1500W$

a) R? $P = \frac{V^2}{R}$, $R = \frac{V^2}{P} = \frac{(120)^2}{1500} = 9.6\,\Omega$

b) $P = IV$

$$I = \frac{P}{V} = \frac{1500}{120} = 12.5 A = 12.5 C/s$$

$$\frac{12.5}{1.6\times10^{-19}} = 7.8\times10^{19} \text{ electrons/s}$$