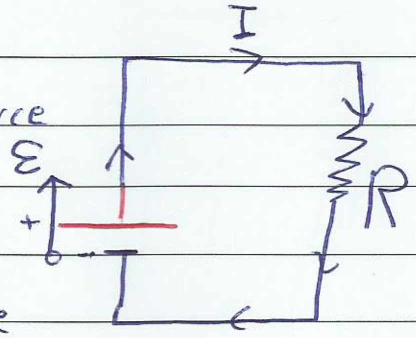
Chapter 27- Circuits

Single-loop circuit:



Electromotive force of the power source (Battery : is the work done by the Battery to move +1C from negative terminal ⟶ to positive terminal inside the source

$$\mathcal{E} = \frac{dW}{dq} \quad J/C = \text{Volt}$$

$dW = \mathcal{E}\, dq$ is the work done by the Battery to move $+dq$ from $((-) \longrightarrow (+))$ inside the Battery

$$\frac{dW}{dt} = \mathcal{E} \frac{dq}{dt}$$

Power of the Battery $= \mathcal{E} I$ Watt.

$P_{\mathcal{E}} = \mathcal{E} I$ The amount of power supplied by $\mathcal{E}$ to the circuit

$P_R = I^2 R$ The thermal Power in R or ↳ The Power Consumed by R

from Conservation of Energy:

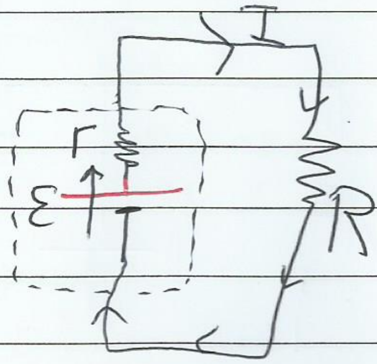
$$P_{\mathcal{E}}(\text{supplied}) = P_R(\text{Consumed})$$

$$\mathcal{E} I = I^2 R$$

$\boxed{I = \frac{\mathcal{E}}{R}}$ ⟶ Single-loop circuit equation for ideal Power Source

Real Power Sourges (Real Battery)
has internal resistan = r

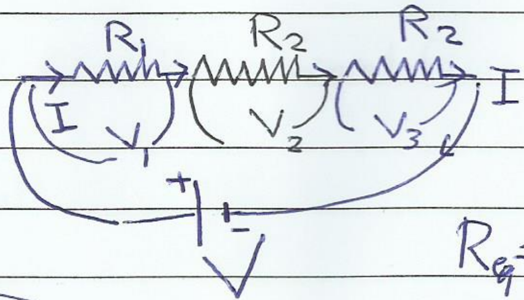
$$I = \frac{\mathcal{E}}{R+r}$$ single-loop circuit equation



Resistance in Series

① The same current is Passing through each resistor

$I = I_1 = I_2 = I_3$



$R_{eq} = \sum_{j=1}^{n} R_j$

$V = RI$ OHM's Law

② $V = V_1 + V_2 + V_2$

$V = IR_1 + IR_2 + IR_3$

$V = I(R_1 + R_2 + R_3)$

$\frac{V}{I} = R_1 + R_2 + R_3$, $R_{eq} = \frac{V}{I}$

$R_{eq} = R_1 + R_2 + R_3 + \dots$

Resistance in Parallel:

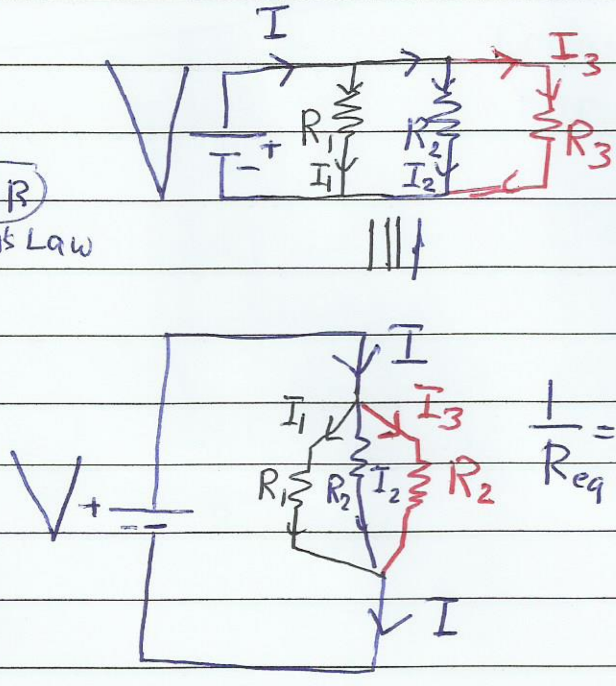
. $V = V_1 = V_2 = V_3$

$V = IR$ OHM's Law

: $I = I_1 + I_2 + I_3$

$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

$I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$



$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$

$\frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

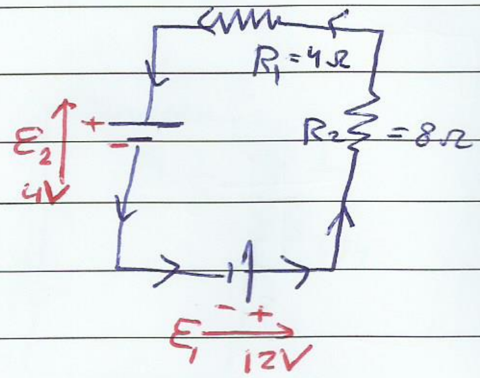
$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Problem (27 - 61)

$\mathcal{E}_1 = 12V, \ \mathcal{E}_2 = 4V$

$I = \dfrac{\Sigma \mathcal{E}}{R_{eq}}$

$= \dfrac{12 + -4}{4+8} = \dfrac{8}{12} = 0.67A$



$I = I_1 = I_2 = 0.67A$ in each resistor

$P_{R_1} = I^2 R_1 = (0.67)^2(4) = 1.78$ Watt thermal Power

$P_{R_2} = I^2 R_2 = (0.67)^2(8) = 3.56$ Watt, thermal Power

$P_{\mathcal{E}1} = I\mathcal{E}_1 = (0.67)(12) = 8$ Watt

$P_{\mathcal{E}2} = I\mathcal{E}_2 = (0.67)(4) = 2.67$ Watt

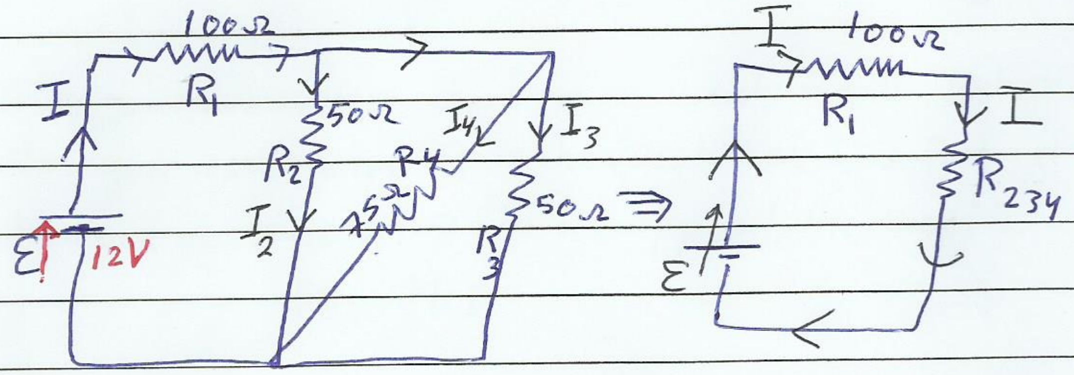
8 J/s Chemical energy lost by $\mathcal{E}_1$ →

- Thermal energy Produced by R_1 — 1.78 J/s
- Thermal energy Produced by R_2 — 3.56 J/s
- Chemical Energy Stored in $\mathcal{E}_2$ — 2.67 J/s

+ ______

8 J/s supplied to the circuit from $\mathcal{E}_1$ → 8 J/s Consumed in the Circuit

Problem (27-8)



• R_2, R_3, R_4 all in Paratlelel

$$\frac{1}{R_{234}} = \frac{1}{50} + \frac{1}{50} + \frac{1}{75} = \frac{3+3+2}{150} = \frac{8}{150}$$

$$R_{234} = \frac{150}{8} = \frac{75}{4} = 18.75\,\Omega$$

$$R_{eq} = R_1 + R_{234} = 118.75\,\Omega$$

$$I = \frac{\varepsilon}{R_{eq}} = \frac{12}{118.75} = 0.1\text{Amp in } R_1$$

to find I_2, I_3, I_4

① $I = I_2 + I_3 + I_4$

$$V_{234} = V_2 = V_3 = V_4$$

$$I R_{234} = I_2 R_2 = I_3 R_3 = I_4 R_4$$

$$(0.01)(18.75) = 50 I_2 = 50 I_3 = 75 I_4$$

$$I_2 = \frac{(0.1)(18.75)}{50} = 0.0375\,A = 3.75\times10^{-3}\,A$$

$$I_3 = \frac{(0.1)(18.75)}{50} = 0.0375\,A = 3.75\times10^{-3}\,A$$

$$I_4 = \frac{(0.1)(18.75)}{75} = 0.025\,A = 25\times10^{-3}\,A$$

+

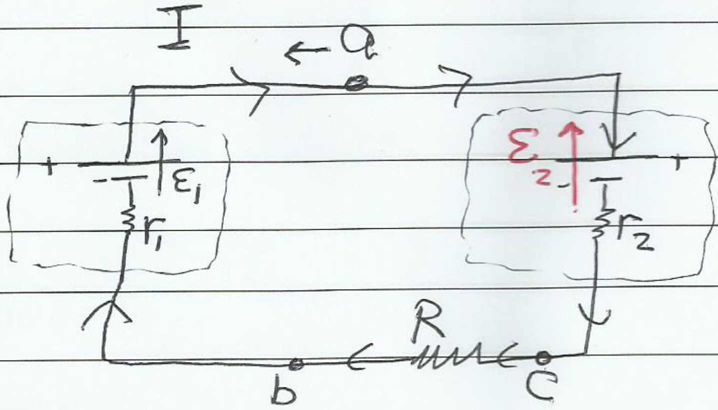
0.1A

Solve Sample Problem 27.02

Potential difference between 2 points ($V_{ab} = V_a - V_b$)

- When you move across $\mathcal{E}$ from (−) → (+) terminals put $+\mathcal{E}$
- When you move across R in the direction of I, put $-RI$
- When you move across R in the opposite direction of I
  Put $+RI$

Sample Problem 27.01



$\mathcal{E}_1 = 4.4\,V$
$\mathcal{E}_2 = 2.1\,V$
$r_1 = 2.3\,\Omega$
$r_2 = 1.8\,\Omega$
$R = 5.5\,\Omega$

Find I? $I = \dfrac{\sum \mathcal{E}}{r_1 + r_2 + R} = \dfrac{^{+}4.4 + ^{-}2.1}{2.3 + 1.8 + 5.5} = \dfrac{2.3}{9.6}$

$I = 0.24\,A$

find V_{ab}? find $V_a - V_b$?

$V_a + ^{-}\mathcal{E}_1 + ^{+}Ir_1 = V_b$

$V_a - 4.4 + (0.24)(2.3) = V_b \Rightarrow V_a - 4.4 + 0.55 = V_b$

$V_a - 3.85 = V_b$

$V_a - V_b = +3.85\,V$ → Note that $V_b - V_a = -3.85\,V = V_{ba}$

find V_{ac}? find $V_a - V_c$?

$V_a + ^{-}\mathcal{E}_2 + ^{-}Ir_2 = V_c$

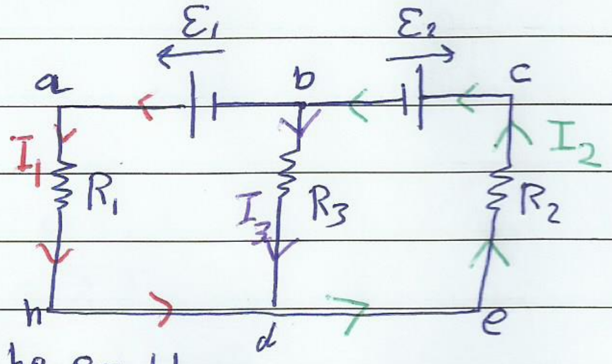
$V_a - 2.1 - (0.24 \times 1.8) = V_c \Rightarrow V_a - 2.1 - 0.43 = V_c$

$V_a - 2.53 = V_c \Rightarrow V_a - V_c = 2.53\,V$

Note that $V_{ab} = -V_{ba}$ Note that $V_c - V_a = -2.53\,V$

Multiloop Circuits

Kirchhoff's rules:-



[1] Junction Rule:
From Conservation of charge
"The sum of the currents entering any junction must be equal to the sum of currents leaving that junction"

At a point (b) $\sum I_{entering} = \sum I_{Leaving}$

$$I_2 = I_1 + I_3 \qquad (1)$$

From Conservation of energy

[2] Loop Rule:- "The algebraic sum of the changes in Potential encountered in a complete traversal of any loop of a circuit must be zero"

$\sum V = 0$ around any closed loop

$$\sum V_{abdha} = 0 \Rightarrow -\mathcal{E}_1 + {}^{-}I_3R_3 + I_1R_1 = 0 \qquad (2)$$

$$\sum V_{bcedb} = 0 \Rightarrow {}^{+}\mathcal{E}_2 + {}^{+}I_2R_2 + {}^{+}I_3R_3 = 0 \qquad (3)$$

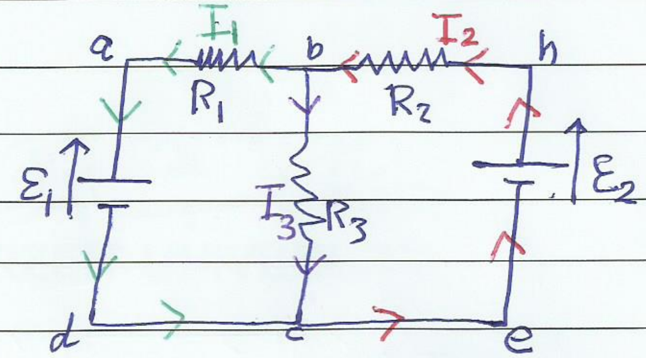
also you can consider the third loop aceha

$$\sum V_{aceha} = 0 \Rightarrow -\mathcal{E}_1 + {}^{+}\mathcal{E}_2 + I_2R_2 + I_1R_1 = 0 \qquad (4)$$

equation (4) is the sum of (2) + (3), you will not need it if you write (3) + (2)

Solve Sample Problem 27.04

Problem(27-3)



$\mathcal{E}_1 = 1V$, $\mathcal{E}_2 = 3V$

$R_1 = 4\Omega$, $R_2 = 2\Omega$, $R_3 = 5\Omega$

What is the rate at which energy dissipated in:

a) R_1? b) R_2? c) R_3?

What is the power of d) $\mathcal{E}_1$? e) $\mathcal{E}_2$?

At point $\boxed{b}$ $\sum I_{entering} = \sum I_{Leaving}$

$$I_2 = I_1 + I_3 \quad ①$$

$$\sum V_{abcda} = 0 \Rightarrow +I_1R_1 + {}^-I_3R_3 + \mathcal{E}_1 = 0$$

$$4I_1 - 5I_3 + 1 = 0$$

$$1 = 5I_3 - 4I_1 \quad ②$$

$$\sum V_{bhecb} = 0 \Rightarrow +I_2R_2 + {}^-\mathcal{E}_2 + I_3R_3 = 0$$

$$2I_2 - 3 + 5I_3 = 0$$

$$3 = 2I_2 + 5I_3 \quad ③$$

Note: that

$P_{\mathcal{E}2} \to I_1^2R_1$, $I_2^2R_2$, $I_3^2R_2$, $P_{\mathcal{E}1}$

i.e

$$P_{\mathcal{E}2} = P_{\mathcal{E}1} + I_1^2R_1 + I_2^2R_2 + I_3^2R_3^2$$

From ① in ③ $\Rightarrow$ $3 = 2(I_1 + I_3) + 5I_3$

$$3 = 2I_1 + 7I_3 \quad 3'$$

you have to slove ② with ③' $\Rightarrow$ multiply ③' by 2 $\Rightarrow$

$$6 = 4I_1 + 14I_3 \quad ③'$$

$$1 = -4I_1 + 5I_3 \quad ② \text{ Add}$$

$$7 = 19I_3$$

$$I_3 = \frac{7}{19} = 0.368\text{ A}$$

$I_3 = 0.368A$ in ② $\Rightarrow$

$$4I_1 = 5I_3 - 1$$

$$I_1 = \frac{5(0.368) - 1}{4} = 0.211A \qquad I_1 = 0.211A$$

from ① $I_2 = I_1 + I_3$

$$I_2 = 0.579A$$

a) $P_{R_1} = I_1^2R_1 = (0.211)(4) = 0.178$ Watt

b) $P_{R_2} = I_2^2R_2 = (0.579)^2(2) = 0.670$ Watt

c) $P_{R_3} = I_3^2R_3 = (0.368)^2(5) = 0.677$ Watt

d) $P_{\mathcal{E}1} = I_1\mathcal{E}_1$

$= (-)(0.211)(1)$

$= -0.211$ Watt

absorbed

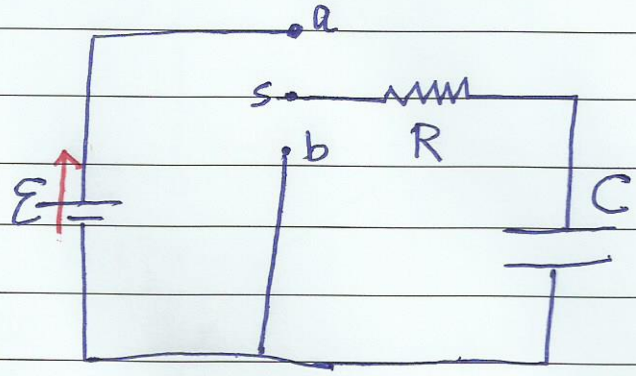
e) $P_{\mathcal{E}2} = I_2\mathcal{E}_2 = 0.579(3)$

$= +1.737$ Watt

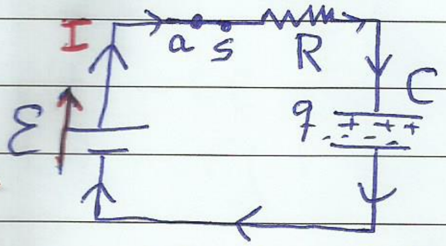
Provided

RC – Circuits:-

Charging a Capacitor:



Connecting (s + a)



remember $\rightarrow V_c = \frac{q}{C}$

$\rightarrow V_R = RI$

$q_o = 0$ intial chargeon the Capacitor,

the charge will increase gradually on the Capacitor to reac q_{max} after a long time

$q_{max} = C\mathcal{E}$

To find the charge on the Capacitor at any time .....

$\sum V = 0$ around the closed loop

$$\mathcal{E} + {-RI} + \frac{-q}{C} = 0 \quad , I = \frac{dq}{dt}$$

$$\mathcal{E} = \frac{q}{C} + R\frac{dq}{dt}$$

$$\mathcal{E} - \frac{q}{C} = R\frac{dq}{dt} \Rightarrow \frac{\mathcal{E}C - q}{C} = R\frac{dq}{dt}$$

$$\frac{\mathcal{E}C - q}{RC} = \frac{dq}{dt} \Rightarrow \frac{q - C\mathcal{E}}{RC} = (-)\frac{dq}{dt}$$

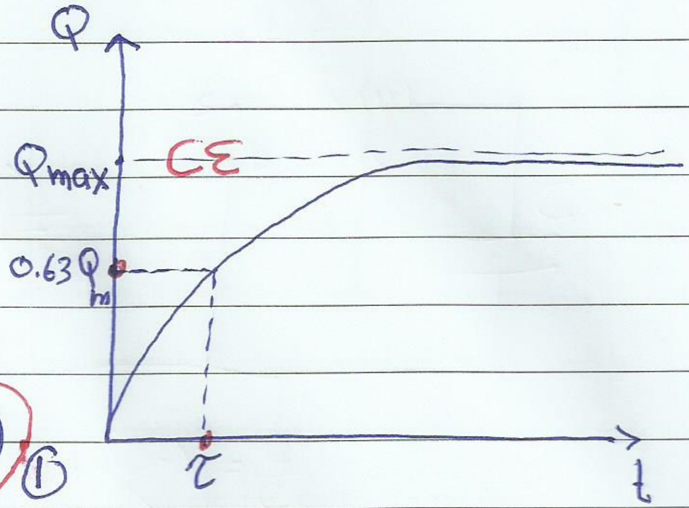
$$\frac{dq}{q - C\mathcal{E}} = (-)\frac{dt}{RC}$$ integrate from $t = 0 \rightarrow t = t$

$$\int_0^q \frac{dq}{q - C\mathcal{E}} = (-)\frac{1}{RC}\int_0^t dt$$

$$\ln\left[q - C\mathcal{E}\right]_0^q = \frac{-t}{RC}$$

$$\ln(q - C\mathcal{E}) - \ln(-C\mathcal{E}) = \frac{-t}{RC}$$

$$\ln\left(\frac{Q - C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$



$$\frac{Q - C\mathcal{E}}{-C\mathcal{E}} = e^{-t/RC}$$

$$Q - C\mathcal{E} = -C\mathcal{E}\, e^{-t/RC}$$

$$Q(t) = C\mathcal{E} - C\mathcal{E}\, e^{-t/RC}$$

$$Q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right) \quad (1)$$

(charging a capacitor)

The time constant of the circuit $= RC$

$\Omega \quad F \quad = \text{sec.}$

$$\tau = RC$$

$$Q(t) = C\mathcal{E}\left(1 - e^{-t/\tau}\right) \quad (1)$$

1) to find Q_{max}, put $t \to \infty$ $\quad Q_{max} = C\mathcal{E}$

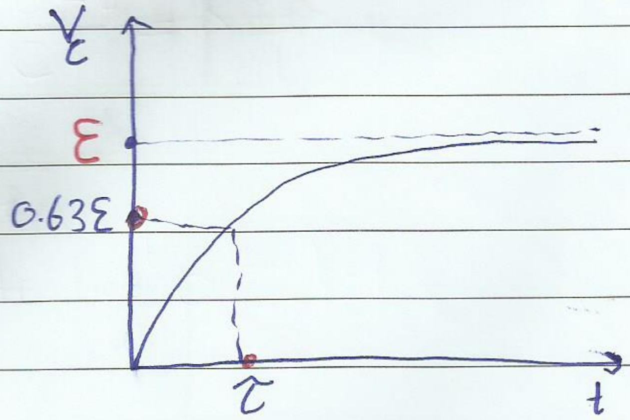
2) Find Q at $t = \tau \Rightarrow Q(\tau) = C\mathcal{E}(1 - e^{-1})$

$$= C\mathcal{E}(1 - 0.37)$$

$$Q(\tau) = 0.63\, C\mathcal{E} = 0.63\, Q_{max}$$

$\Rightarrow$ At $t = \tau \Rightarrow$ the charge on the capacitor will reach $0.63\, Q_{max}$

$$V_c(t) = \frac{Q(t)}{C} = \mathcal{E}\left(1 - e^{-t/\tau}\right)$$



Find the current at any time?

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left[C\mathcal{E}\left(1 - e^{-t/RC}\right)\right]$$

$$= 0 - C\mathcal{E}\left(\frac{-1}{RC}\right) e^{-t/RC}$$

$$I(t) = \frac{\mathcal{E}}{R}\, e^{-t/RC} \quad , \quad \frac{\mathcal{E}}{R} = I_0 \text{ at } t = 0$$

charging C

$$V_R = RI$$

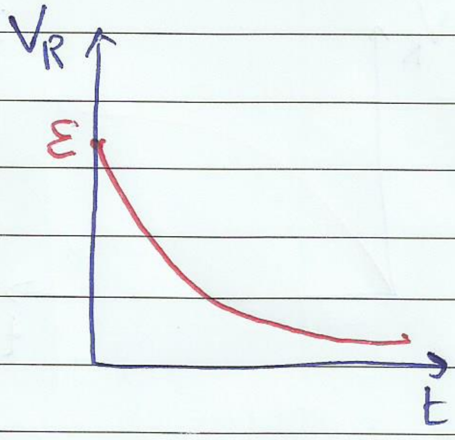
$$V_R(t) = \mathcal{E}\, e^{-t/RC}$$

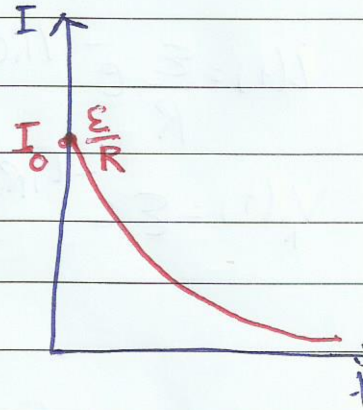
$$I(t) = \frac{\varepsilon}{R}\left(e^{-t/\tau}\right)$$

$$V_R(t) = \varepsilon e^{-t/\tau}$$

as $t \to \infty$ $\quad I = 0$

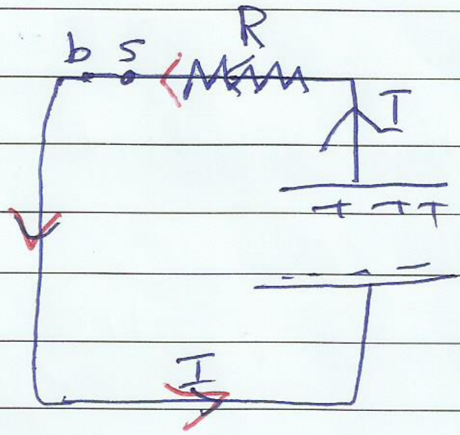
$\qquad V_R = 0$





Discharging a Capacitor

Connect b & S



The charge will move from (+) plate → (−) plate through R

$q_0 = C\varepsilon = q_m$

$q_{finally} = 0$ as $t \to \infty$

find q at any time

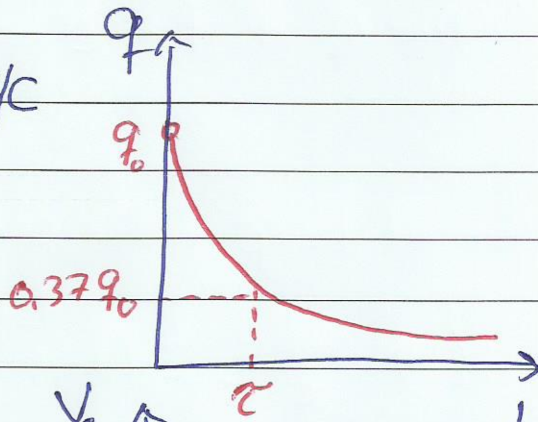
$$RI + \frac{q}{C} = 0 \Rightarrow R\frac{dq}{dt} = -q/C$$

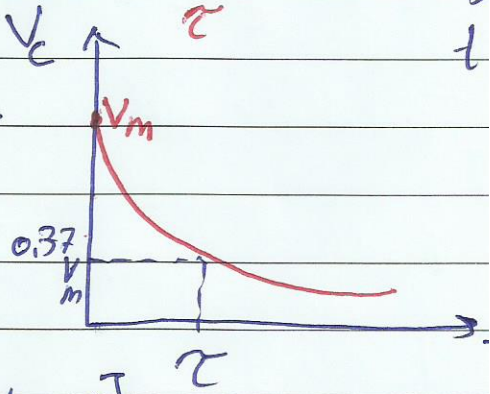
$$\frac{dq}{q} = -\frac{dt}{RC} \qquad \text{integrate}$$

$$\int_{q_0}^{q} \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q}{q_0}\right) = -\frac{t}{RC} \Rightarrow \frac{q}{q_0} = e^{-t/RC}$$

$$q(t) = q_0 e^{-t/RC} \Rightarrow q(t) = q_0 e^{-t/\tau}$$

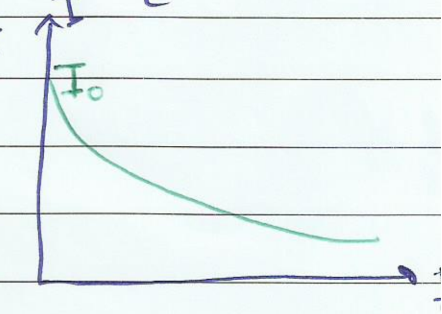




$$i(t) = \frac{dq}{dt} = q_0\left(\frac{-1}{RC}\right)e^{-t/RC} = \left(\frac{-q_0}{RC}\right)e^{-t/\tau}$$

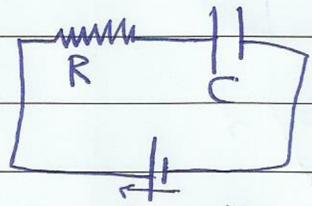
$$I_0 = \frac{q_0}{RC}$$

$$I_{finally} = 0$$



Problems (27-40)

$R = 1.4\,M\Omega$, $C = 2.7\,\mu F$



$\mathcal{E} = 12V$

Find:

a) τ?

$$\tau = RC = (1.4\times10^{6})(2.7\times10^{-6}) = 3.78 \text{ sec.}$$

b) $Q(t) = C\mathcal{E}(1 - e^{-t/\tau})$

Q_{max} at $t \to \infty \Rightarrow Q_{max} = C\mathcal{E} = (2.7\times10^{-6})(12)$

$= 32.4\times10^{-6}\,C$

c) Find t? when $Q = 16\,\mu C$

$$Q(t) = C\mathcal{E}(1 - e^{-t/\tau})$$

$$16\times10^{-6} = 32.4\times10^{-6}(1 - e^{-t/\tau})$$

$$0.494 = 1 - e^{-t/\tau} \Rightarrow e^{-t/\tau} = 1 - 0.494 = 0.506$$

$$\frac{-t}{\tau} = \ln(0.506) \Rightarrow t = -\tau \ln(0.506)$$

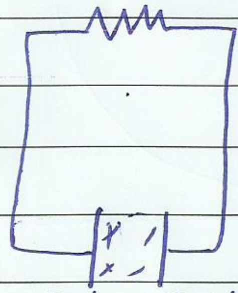
$$= -\tau(-0.68)$$

$$t = 0.68(3.78)$$

$$t = 2.575\,s$$

Problem (27-18)

R



$V_{c0} = 80V$

at $t = 0$, $V_{c0} = 80V$

at $t = 10s$, $V_c = 1V$

a) Find τ?

$$q(t) = q_0 e^{-t/\tau}$$

$$V_c(t) = V_{c0} e^{-t/\tau} , \quad V_c(t) = 80 e^{-t/\tau}$$

$$1 = 80 e^{-10/\tau} \Rightarrow \frac{1}{80} = e^{-10/\tau} \Rightarrow \frac{-10}{\tau} = \ln\left(\frac{1}{80}\right)$$

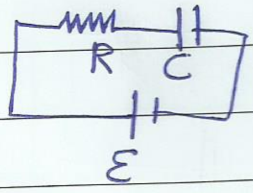
$$\tau = \frac{-10}{\ln(1/80)} = \frac{-10}{-4.382} = 2.28s \qquad \tau = 2.28s$$

b) Find V_c? at $t = 17s$. $V_c = 80 e^{-17/2.28} = 80 e^{-7.45} = 4.65\times10^{-2}\,V$

$$V_c = 46.5\,mV$$

Problem (27-15)

Find t interms of τ for C to reach 89% Q_f



$Q(t) = C\varepsilon(1 - e^{-t/\tau})$

$\frac{89}{100}(C\varepsilon) = C\varepsilon(1 - e^{-t/\tau})$

$0.89 = 1 - e^{-t/\tau} \longrightarrow e^{-t/\tau} = 1 - 0.89 = 0.11$

$-\frac{t}{\tau} = \ln 0.11 \Longrightarrow t = -\tau \ln 0.11$

$t = 2.21\tau$ for $Q = 0.89\, Q_{max}$

b)*

find t? for $Q = 99\%\, Q_0$

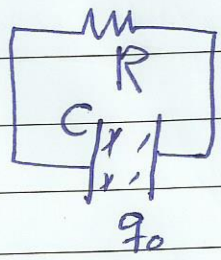
$0.99C\varepsilon = C\varepsilon(1 - e^{-t/\tau}) \Rightarrow$

$t = -\tau \ln(0.01)$

$t = 4.6\tau$ for $Q = 0.99\, Q_{max}$

Problem (27-30)

a) Find t? for the Capacitor to lose the first 25% of its q_0



$q(t) = q_0 e^{-t/\tau}$

$0.75 q_0 = q_0 e^{-t/\tau} \Longrightarrow 0.75 = e^{-t/\tau}$

$-\frac{t}{\tau} = \ln 0.75 \Longrightarrow t = -\tau \ln 0.75$

$t = 0.29\tau$ to lose 25%

b) Find t? to lose 50% q_0

$0.5 q_0 = q_0 e^{-t/\tau} \Longrightarrow t = -\tau \ln 0.5 = 0.69\tau$ to lose 50%

c) Find t? to lose one third of its q_0.

$\frac{2}{3} q_0 = q_0 e^{-t/\tau} \longrightarrow t = -\tau \ln\left(\frac{2}{2}\right) = 0.41\tau$ to lose $\frac{1}{3} q_0$