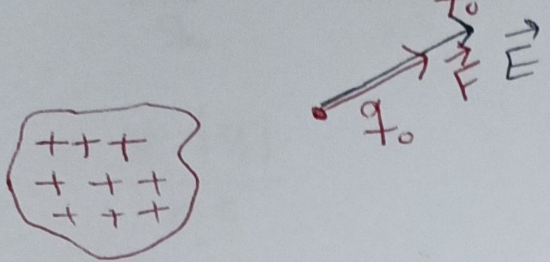
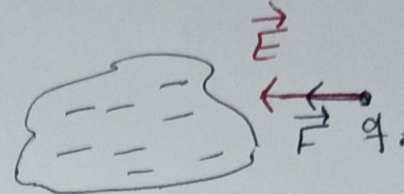
Chapter 22: Electric Fields

- The electric field $\vec{E}$ at any point is defined in terms of the electrostatic force $\vec{F}$ that would be exerted on a positive test charge q_0 placed there. $\vec{E} = \frac{\vec{F}}{q_0}$ $\quad [\vec{E}] = N/C$

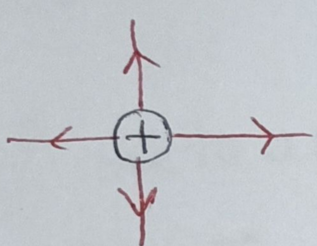


positive charge distribution

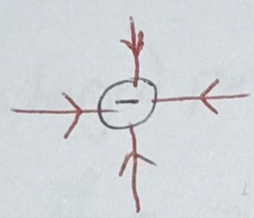


negative charge distribution

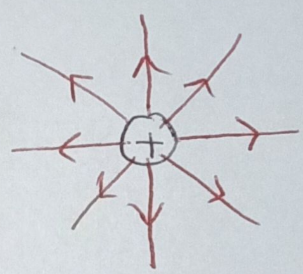
Electric field lines:



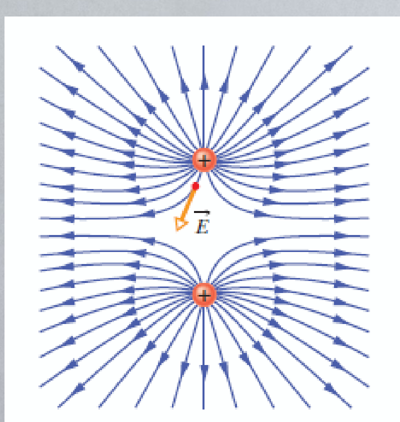
$+q$

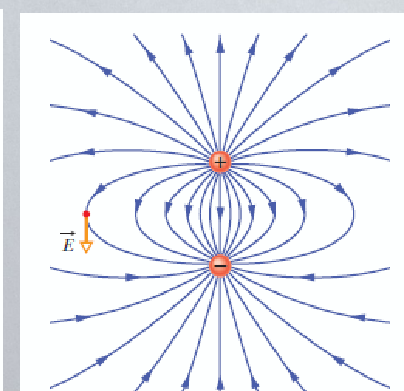


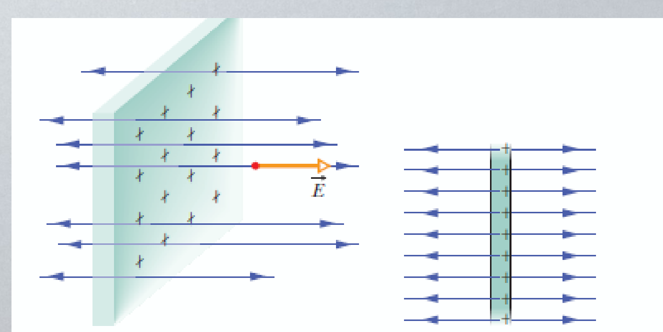
$-q$



$+2q$







Very large, non-conducting sheet with uniform positive charge on one side

$\boxed{1}$ Electric field due to a charged particle

$$\vec{F_e} = \frac{k\, q\, q_0}{r^2}\,\hat{r}$$

$$\vec{E} = \frac{\vec{F_e}}{q_0} = \frac{k\, q}{r^2}\,\hat{r}$$

$$E = \frac{k\,|q|}{r^2}$$

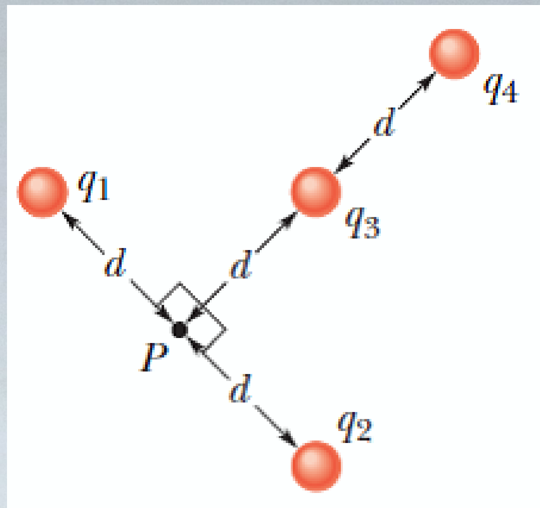
$\Rightarrow$ The electric field vectors set up by a positively charged particle all point directly away from the particles. Those set up by a negatively charged particles all point directly toward the particle.

$\boxed{2}$ $\vec{E}$ due to a set of charged particles

$$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \cdots$$

22-32 The four particles are fixed in place and have charges $q_1 = q_2 = +5e$, $q_3 = +3e$, and $q_4 = -12e$. Distance d $= 8.0\,\mu m$

What is the magnitude of the net electric field at point P due to the particles?

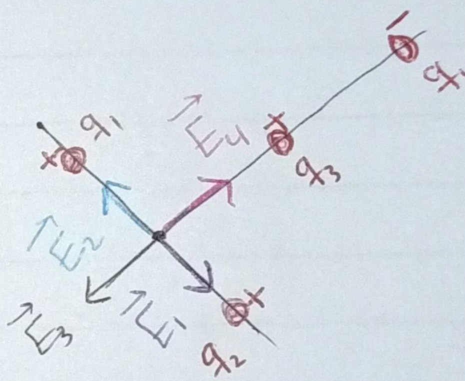


- $E_1 = E_2 \Rightarrow \vec{E}_1 + \vec{E}_2 = \text{Zero}$

- $E_3 = \dfrac{k q_3}{d^2} = \dfrac{9\times10^9 \times 3(1.6\times10^{-19})}{(8\times10^{-6})^2}$

$E_3 = 67.5\ N/C$

- $E_4 = \dfrac{k q_4}{(2d)^2} = \dfrac{9\times10^9 (12)(1.6\times10^{-19})}{(16\times10^{-6})^2}$



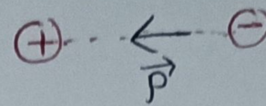
$E_4 = 67.5\ N/C$

$\Rightarrow \vec{E}_3 + \vec{E}_4 = \text{Zero}$

$$\boxed{\vec{E}_p = \text{Zero}}$$

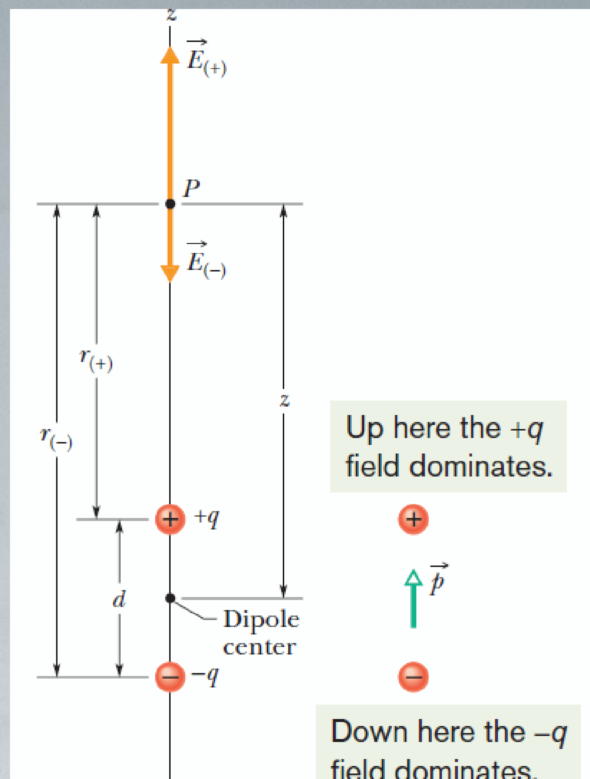
• The electric field due to a dipole :

⇒ An electric dipole consists of two particles with charges of equal magnitude q but opposite signs, seperated by a small distance d.



• $\vec{p}$ = Electric dipole moment

$p = qd$ "from negative to positive"



⇒ $\vec{E}$ at point on a dipole axis

$$E_+ = \frac{kq}{r_+^2} \quad \text{"upward"}$$

$$E_- = \frac{kq}{r_-^2} \quad \text{"downward"}$$

$$r_+ = z - \frac{d}{2} = z - a \; ; \; a = \frac{d}{2}$$

$$r_- = z + \frac{d}{2} = z + a$$

$$E = E_+ - E_- = \frac{kq}{(z-a)^2} - \frac{kq}{(z+a)^2}$$

$$= kq\left[\frac{1}{(z-a)^2} - \frac{1}{(z+a)^2}\right] = kq\left[\frac{(z+a)^2 - (z-a)^2}{(z-a)^2(z+a)^2}\right]$$

$$E = kq\left[\frac{z^2 + 2az + a^2 - z^2 + 2az - a^2}{(z^2 - a^2)^2}\right] = kq\left[\frac{4az}{(z^2-a^2)^2}\right]$$

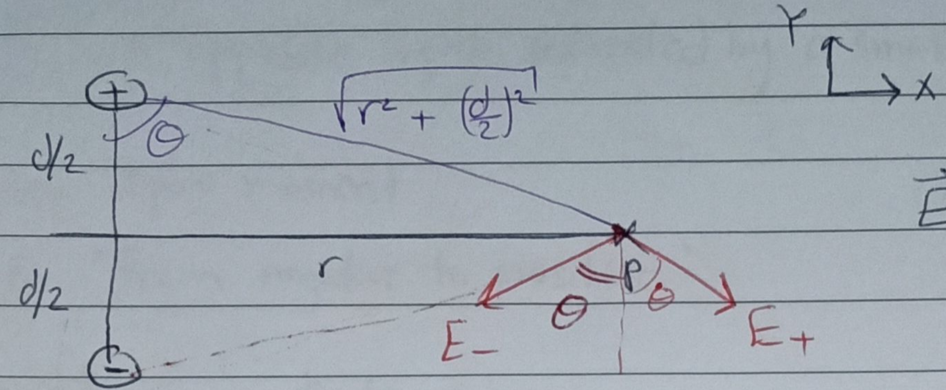
For $z \gg d$; $z \gg a$; $E = \frac{kq\,4az}{z^4} = \frac{4kqa}{z^3}$

we $a = \frac{d}{2}$; $E = \frac{4kqd}{2z^3} = \frac{2kp}{z^3}$

$$E = \frac{1}{2\pi\epsilon_0}\frac{p}{z^3} \quad \text{"upward"}$$

Z is the distance between the dipole center and the point that we calculated the electric field at

22-55 Electric dipole



$\vec{E}(P) = ??$
$r \gg d$

By symmetric E_x cancelled

$$E_{y+} = \frac{kq}{\left[r^2 + (\frac{d}{2})^2\right]} \cos\theta \quad , \cos\theta = \frac{d/2}{\left[r^2 + (\frac{d}{2})^2\right]^{1/2}}$$

$$= \frac{kqd}{2\left[r^2 + (\frac{d}{2})^2\right]^{3/2}}$$

$$E_{y+} = E_{y-}$$

$$\vec{E}(P) = -\frac{kqd}{\left[r^2 + (\frac{d}{2})^2\right]^{3/2}} \hat{j}$$

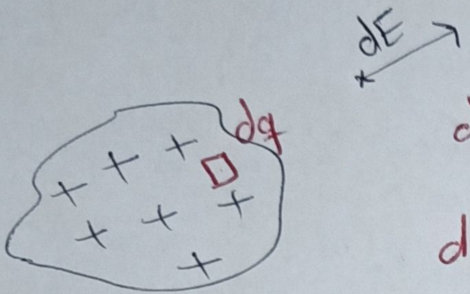
$$r \gg d \Rightarrow \vec{E} = -\frac{kqd}{r^3} \hat{j}$$

$\boxed{3}$ $\vec{E}$ due to a continuous charge distribution $\Rightarrow$ Integration

λ = Linear charge density C/m

σ = Surface charge density C/m^2

ρ = Volume charge density C/m^3

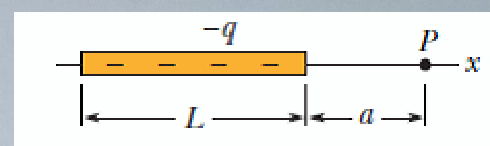


dq = charge element

$$dE = \frac{k\,dq}{r^2} \Rightarrow E = \int dE$$

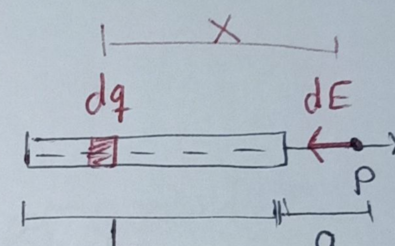
$d\vec{E}$ → magnitude, direction } symmetry allows us to cancel out any of the components of the fields, to simplify the integration.

22-13 A non conducting rod of length $L = 8.15$ cm has a charge $-q = -4.23\,fC$ uniformly distributed along its length. (a) What is the linear charge density of the rod? What are the (b) magnitude and (c) direction (relative to the positive direction of the x axis) of the electric field produced at point P at distance $a = 6.00$ cm from the rod? What is the electric field magnitude produced at distance $a = 50$m by (d) the rod and (e) a particle of charge $-q = -4.23\,fC$ that we use to replace the rod? (At that distance, the rod "looks" like a particle?



(a) $\lambda = \frac{q}{L} = \frac{-4.32\times10^{-15}}{8.15\times10^{-2}} = -5.19\times10^{-14}\,\frac{C}{m}$

(b) $dE = k\frac{dq}{x^2}$; $dq = \lambda\, dx$



$$dE = \frac{k\lambda\, dx}{x^2}$$

"The charge extends from $x_1 = a$ to $x_2 = L + a$"

$$E = \int_a^{L+a} k\lambda \frac{dx}{x^2} = k\lambda \int_a^{L+a} x^{-2}dx = k\lambda\left[-\frac{1}{x}\right]_a^{L+a}$$

$$E = k\lambda\left[-\frac{1}{L+a} + \frac{1}{a}\right] = k\lambda\left[\frac{-a+L+a}{a(L+a)}\right]$$

$$E = \frac{k\lambda L}{a(L+a)} = \frac{kq}{a(L+a)}$$

$$\boxed{\vec{E}_p = \frac{-kq}{a(L+a)}\,\hat{\imath}}$$

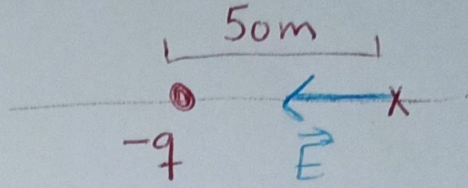
use $q = 4.23\times10^{-15}$C
$a = 6.00\times10^{-2}$m
$L = 8.15\times10^{-2}$m

$\vec{E}_p = 4.48\times10^{-3}\,\frac{N}{C}\,\hat{\imath}$

d) $a = 50m$

$$\vec{E}_p = \frac{-kq}{a(L+a)}\hat{\imath} = -1.52 \times 10^{-8}\ N/C\ \hat{\imath}$$

(e) charged particle $-q = -4.23fC$



$$E = k\frac{q}{r^2} = \frac{9\times10^{9} \times 4.23\times10^{-15}}{(50)^2}$$

$$\vec{E} = -1.52 \times 10^{-8}\ N/C$$

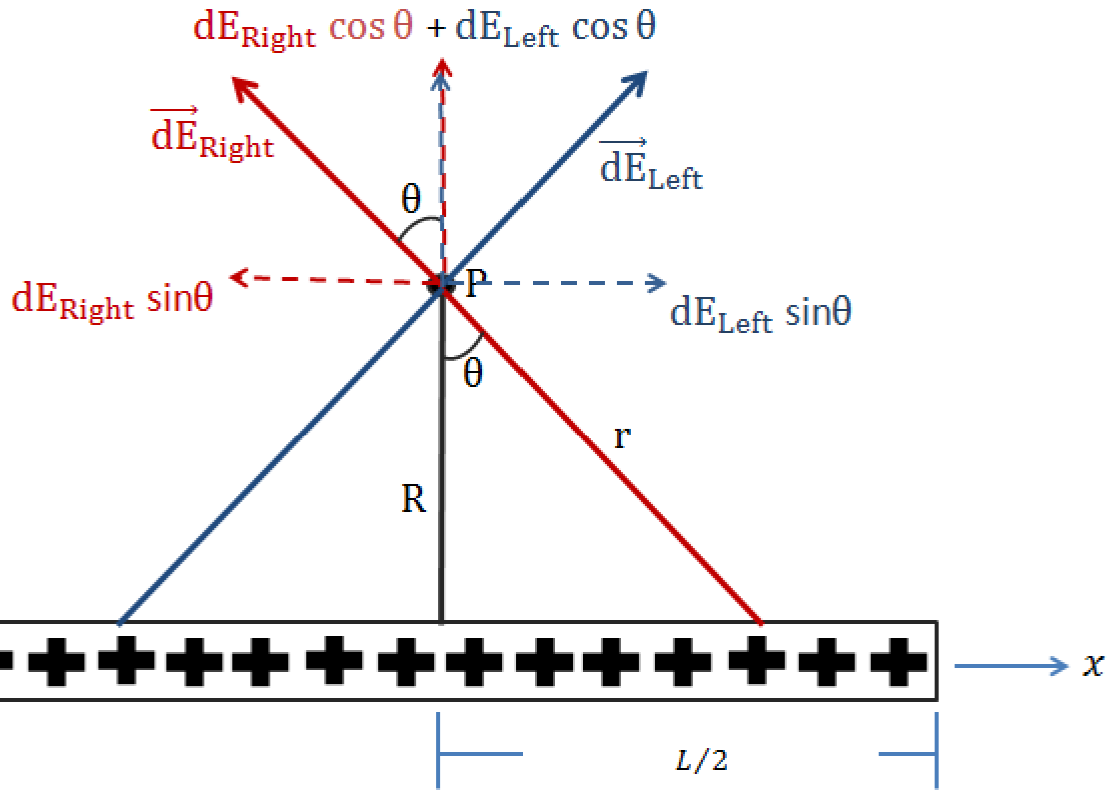
Note $a \gg L$

$a(L+a) = a^2 \Rightarrow$ The rod treats as a particle.

$$E = \frac{kq}{a^2}$$

Problem 22-10: Positive charge (q = 9.25 pC = 9.25×10^{-12} C) is spread uniformly along a thin non-conducting rod of length (L = 16.0 cm = 0.16 m). What is the electric field produced at point P, at distance (R = 6.00 cm = 0.06 m) from the rod along its perpendicular bisector?

$$\overrightarrow{dE} = \frac{k\,dq}{r^2}\hat{r}$$



$dE_{Left} = dE_{Right}$ (both have same magnitude)

$dE_x = -dE_{Right}\, sin\theta\,\hat{\imath} + dE_{Left}\, sin\theta\,\hat{\imath} = zero.$ (by symmetric)

$$\vec{E} = \int dE_y = \int dE\cos\theta\hat{\jmath} = \int \frac{k\,dq}{r^2}\cos\theta\hat{\jmath}$$

Using q =λ x; linear charge density. dq = λ dx

$r^2 = (R^2 + x^2)$ and $\cos\theta = \frac{R}{r} = \frac{R}{(R^2+x^2)^{1/2}}$

$$\vec{E} = \int \frac{k\,dq}{r^2}\ cos\,\theta\hat{\jmath} = \int_{-L/2}^{L/2} \frac{k\,\lambda\,\mathrm{dx}}{(R^2+x^2)}\ \frac{R}{(R^2+x^2)^{1/2}}\ \hat{\jmath}$$

$$= k\,\lambda\,\mathrm{R}\,\hat{\jmath}\int_{-L/2}^{L/2} \frac{dx}{(R^2+x^2)^{3/2}}\ = 2\,k\,\lambda\,\mathrm{R}\,\hat{\jmath}\int_{0}^{L/2} \frac{dx}{(R^2+x^2)^{3/2}}$$

$$\left\{\begin{matrix} take\tan\theta = \dfrac{x}{R}, x = R\tan\theta \ \to\to \mathrm{dx} = \mathrm{R}\,sec^2\theta\,d\theta \\ (R^2+x^2)^{3/2} = (R^2+R^2tan^2\theta)^{3/2} = \ (R^2sec^2\theta)^{3/2} = \ R^3sec^3\theta \end{matrix}\right\}$$

$$\vec{E} = \ 2\,k\,\lambda\,\mathrm{R}\,\hat{\jmath}\int \frac{\mathrm{R}\,sec^2\theta\,d\theta}{R^3sec^3\theta} = \frac{2\,k\,\lambda}{\mathrm{R}}\hat{\jmath}\int \frac{d\theta}{\sec\theta} = \frac{2\,k\,\lambda}{\mathrm{R}}\hat{\jmath}\int \cos\theta\,d\theta$$

$$= \ \frac{2\,k\,\lambda}{\mathrm{R}}\hat{\jmath}\sin\theta$$

Use $\sin\theta = \ \frac{x}{(R^2+x^2)^{1/2}}$

$$\Rightarrow\ \vec{E} = \ \left[\frac{2\,k\,\lambda}{\mathrm{R}}\frac{x}{(R^2+x^2)^{1/2}}\ \ \hat{\jmath}\right]_0^{L/2} = \ \frac{k}{\mathrm{R}}\frac{\lambda\,L}{(R^2+(L/2)^2)^{1/2}}\ \ \hat{\jmath}$$

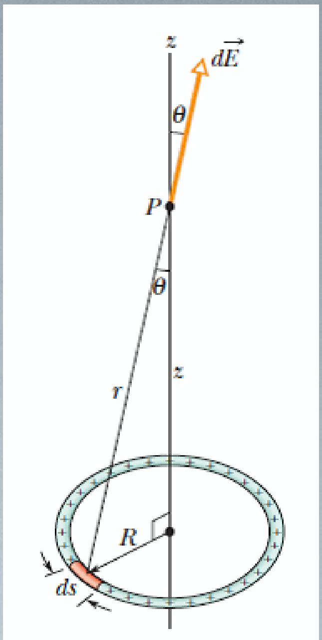
Substituted q = 9.25 pC = 9.25×10^{-12} C = λ L, L = 16.0 cm = 0.16 m and R = 6.00 cm = 0.06 m.

$$\boxed{\vec{E} = 13.86\,N/C\ \hat{\jmath}}$$

• $\vec{E}$ due to a ring of uniform positive charge

$\Rightarrow \vec{E}$ at point on a perpendicular axis to the ring plane.

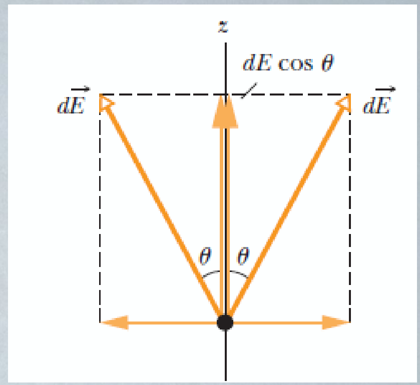
$$dE = k\frac{dq}{r^2} = \frac{k\lambda\, ds}{r^2} = \frac{k\lambda\, ds}{R^2+Z^2}$$



By the symmetry the components perpendicular to the Z-axis cancel.

$$E_{Ring} = \int dE\cos\theta = \frac{k\lambda\, ds}{R^2+Z^2}\,\frac{Z}{(Z^2+R^2)^{1/2}}$$

$$= \frac{kZ\lambda}{(Z^2+R^2)^{3/2}}\int_0^{2\pi R} ds$$



$$E = \frac{k\lambda Z\, 2\pi R}{(Z^2+R^2)^{3/2}}$$

total charge of the ring $q = \lambda(2\pi R)$

$$E = \frac{kqZ}{(Z^2+R^2)^{3/2}}$$

$$ds = R\, d\theta$$
$$\int ds = R\int_0^{2\pi} d\theta = 2\pi R$$

$$\vec{E} = +\frac{kqZ}{(Z^2+R^2)^{3/2}}\,\hat{k}$$

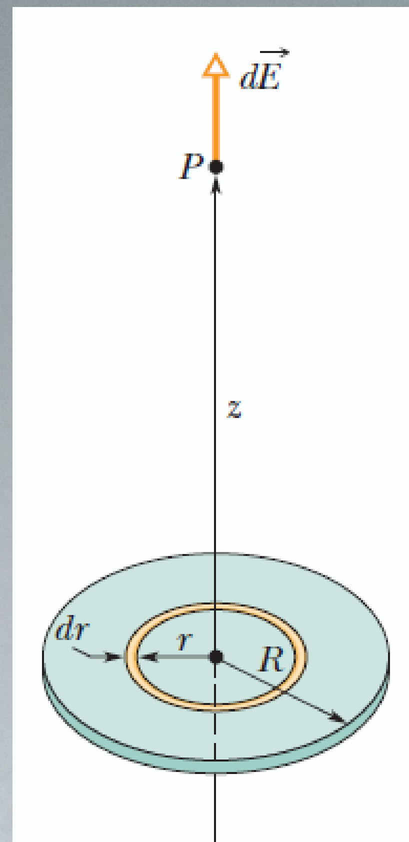
$Z \gg R \Rightarrow E = \frac{kq}{Z^2}$ "like charged point particle"

$E_{\text{at the ring center}}$ = Zero

- Electric field due to a charged disk

⇒ A disk of radius R and uniform positive charge

σ = surface charge density



- Take a ring on the disk has radius r ; $r \leq R$, then integrate from the center of the disk to its rim ⇒ $(0 \rightarrow R)$

$$dE = \frac{dq\, z}{4\pi\epsilon_0 (z^2 + r^2)^{3/2}}$$ (in the positive direction of the z-axis)

use $dq = \sigma\, dA = \sigma(2\pi r\, dr)$

$$E = \int dE = \frac{\sigma z}{4\epsilon_0} \int_0^R (z^2 + r^2)^{-3/2}\, 2r\, dr$$

use $\int X^m dX = \frac{X^{m+1}}{m+1}$ ⇒ $\begin{cases} X = z^2 + r^2 \\ m = -3/2 \\ dX = 2r\,dr \end{cases}$

$$E = \frac{\sigma z}{4\epsilon_0} \left[\frac{(z^2 + r^2)^{-\frac{1}{2}}}{-\frac{1}{2}} \right]_0^R$$

$$= \frac{\sigma z}{4\epsilon_0} \left[\frac{(z^2 + R^2)^{-\frac{1}{2}} - z^{-1}}{-\frac{1}{2}} \right]$$

$$E = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{z^2 + R^2}} \right]$$

* The electric field due to a charged disk at point on the central axis

$$E = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{z^2 + R^2}} \right]$$

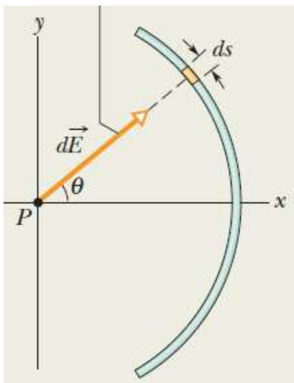
- Positive charged disk ⇒ Electric field lines tends away from the disk
- negative charged disk ⇒ Electric field lines tends toward the disk

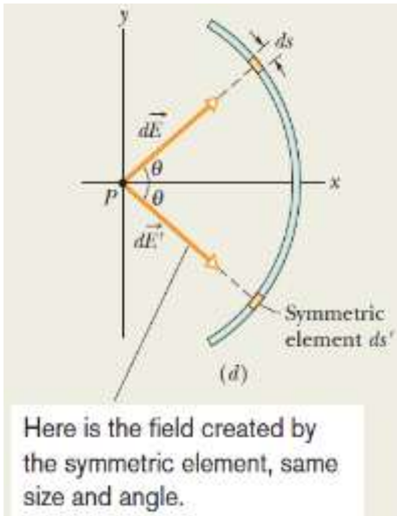
- $R \rightarrow \infty$; infinite sheet

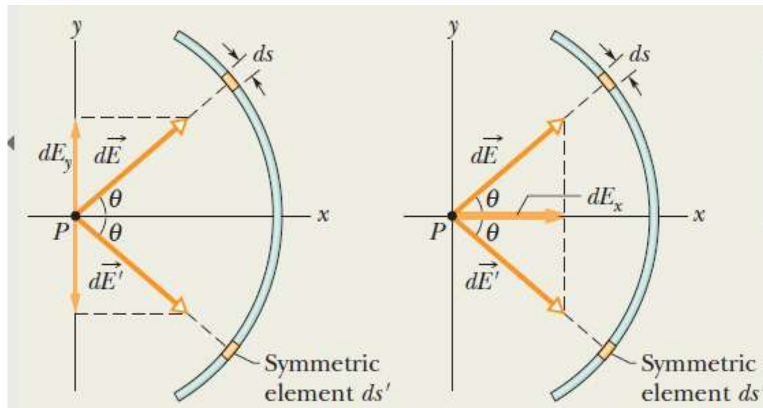
$$E = \sigma / 2\epsilon_0$$

Sample Problem 22.03 Electric field of a charged circular rod

Figure 22-13*a* shows a plastic rod with a uniform charge $-Q$. It is bent in a $120°$ circular arc of radius r and symmetrically paced across an x axis with the origin at the center of curvature P of the rod. In terms of Q and r, what is the electric field $\vec{E}$ due to the rod at point P?







$$dE_x = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}\cos\theta\, ds.$$

$$\begin{aligned}
E &= \int dE_x = \int_{-60°}^{60°} \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}\cos\theta\, r\, d\theta \\
&= \frac{\lambda}{4\pi\varepsilon_0 r}\int_{-60°}^{60°}\cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-60°}^{60°} \\
&= \frac{\lambda}{4\pi\varepsilon_0 r}[\sin 60° - \sin(-60°)]
\end{aligned}$$

$$= \frac{1.73\lambda}{4\pi\varepsilon_0 r}.$$

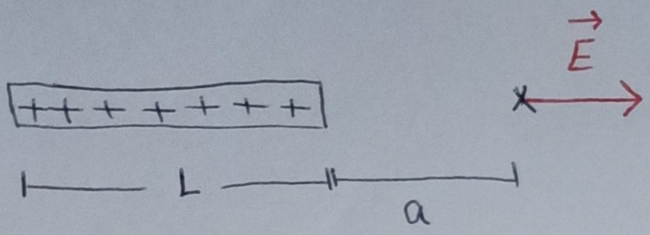
$$\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{2\pi r/3} = \frac{0.477Q}{r}.$$

$$\begin{aligned}
E &= \frac{(1.73)(0.477Q)}{4\pi\varepsilon_0 r^2} \\
&= \frac{0.83Q}{4\pi\varepsilon_0 r^2}.
\end{aligned}$$

$$\vec{E} = \frac{0.83Q}{4\pi\varepsilon_0 r^2}\,\hat{\mathrm{i}}.$$

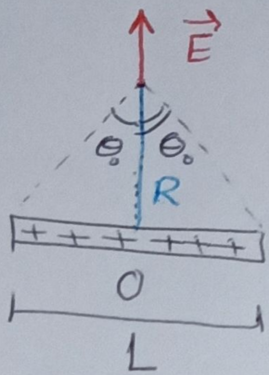
- Uniformly charged rod

[1]



$$E = \frac{q}{4\pi\epsilon_0 a(L+a)}$$

[2]

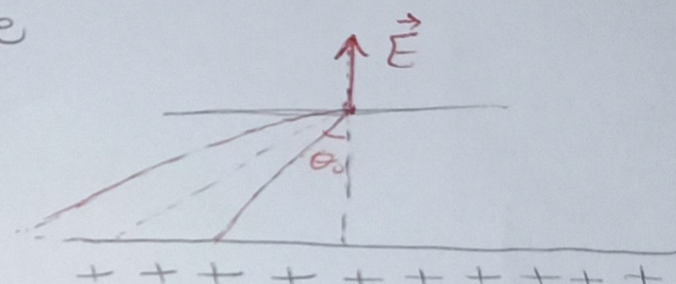


$$E = \frac{\lambda}{2\pi\epsilon_0 R}\sin\theta_0$$

[3] $\vec{E}$ due to an infinite line of charge

very long wire $L \to \infty$

$\theta_0 \to \pi/2$

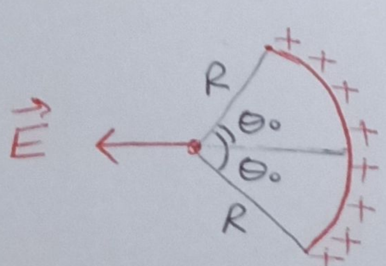


$$E = \frac{\lambda}{2\pi\epsilon_0 R}\sin(\pi/2)$$

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 R}}$$ For very long line of charge

$L >>> R$

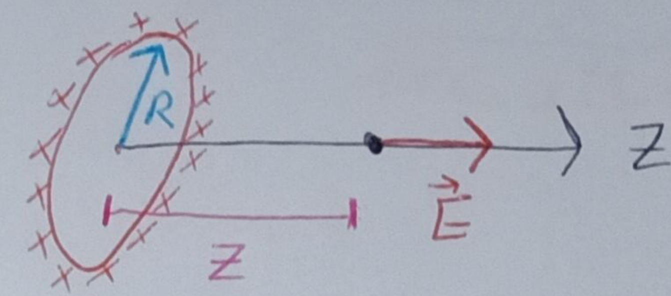
[4] Uniformly charged circular arc



$$E = \frac{\lambda}{2\pi\epsilon_0 R}\sin\theta_0$$

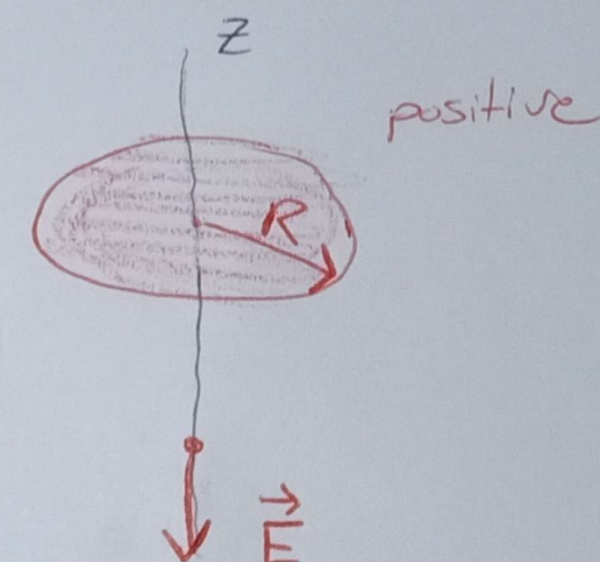
[5] Uniformly charged Ring

$$E = \frac{qz}{4\pi\epsilon_0 (z^2 + R^2)^{3/2}}$$



[6] Uniformly charged disk

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$



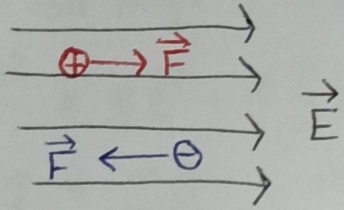
- A Point charge in an Electric field

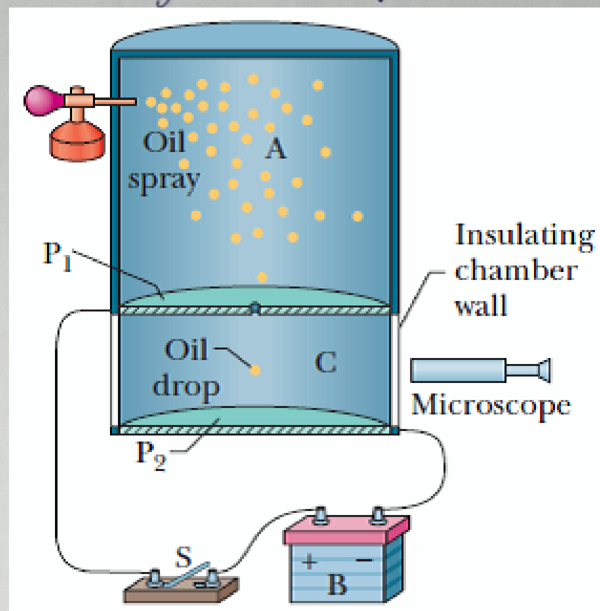
⇒ If a particle with charge q is placed in an external electric field $\vec{E}$, an electrostatic force $\vec{F}$ acts on the particle

$$\vec{F} = q\vec{E}$$

⇒ if q is positive → $\vec{F}$ in the same direction of $\vec{E}$ (Parallel)

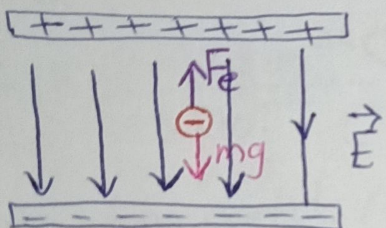
if q is negative → $\vec{F}$ in the Opposite direction of $\vec{E}$ (anti-parallel)





- Millikan's oil-drop experiment

For measuring the elementary charge e



- A negatively charged oil drop is suspended between the plates
- The oil drop is at rest under the influence of two forces ⇒ { mg, downward; F_e, Upward }

$$\vec{F}_{\text{oil drop}} = \text{Zero} \Rightarrow qE = mg$$

$$q = \frac{mg}{E} \; ; \; q = ne \; ; \; n = 0, \mp 1, \mp 2 \ldots$$

$$e = 1.6 \times 10^{-19}\,C$$

example In Millikan's experiment, an oil drop of radius 1.64 μm and density 0.851 g/cm^3 is suspended in champer C of figure (22-16) when a downward electric field of 1.92×10^5 N/C is applied. Find the charge on the drop in terms of e?

The oil drop is suspended

$$\vec{F} = q\vec{E} + m\vec{g} = \text{Zero}$$

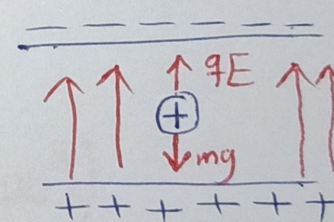
$$q = \frac{mg}{E} \quad , \quad m = \rho V = \rho\left(\frac{4}{3}\pi r^3\right) = 851 \frac{kg}{m^3}\left(\frac{4}{3}\pi\right)\left(1.64 \times 10^{-6}\right)^3 m^3$$

$$m = 1.572 \times 10^{-14} \text{ kg}$$

$$q = \frac{mg}{E} = \frac{(1.572 \times 10^{-14})(9.8)}{1.92 \times 10^5} = 8.02 \times 10^{-19} \text{ C}$$

$$n = \frac{q}{e} = \frac{8.02 \times 10^{-19} \text{ C}}{1.6 \times 10^{-19} \text{ C}} \quad \boxed{n = 5} \quad q = 5e$$

22-60 An alpha particle (the nucleus of a helium atom) has a mass of 6.64×10^{-27} kg and a charge of +2e. What are the (a) magnitude (b) and direction of the electric field that will balance the gravitational force on the particle? (c) if the field magnitude is then doubled, what is the magnitude of the particle's acceleration?



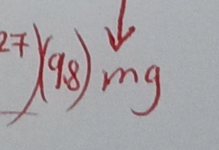
An alpha particle $\Rightarrow$ $m = 6.64 \times 10^{-27}$ kg, $q = +2e = +3.2 \times 10^{-19}$ C

$\vec{E}$ must be upward

$$E = mg/q = 6.64 \times 10^{-27}(9.8)/3.2 \times 10^{-19} = 2.03 \times 10^{-7} \text{ N/C upward}$$

$$\vec{E} = 2.03 \times 10^{-7} \frac{N}{C} \hat{k}$$

(c) E is doubled, $E = 4.06 \times 10^{-7}$ N/C upward

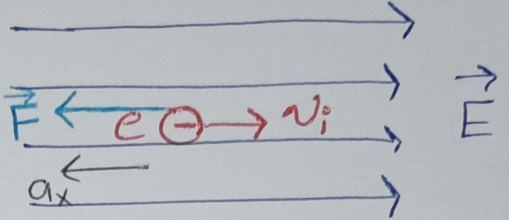


$$\vec{F}_{net} = q\vec{E} + m\vec{g} \Longleftrightarrow ma = qE - mg$$

$$a = \frac{qE - mg}{m} = \frac{(3.2 \times 10^{-19})(4.06 \times 10^{-7}) - (6.64 \times 10^{-27})(9.8)}{6.64 \times 10^{-27}}$$

$$\boxed{a = 9.77 \text{ m/s}^2 \text{ upward}}$$

22-38 An electron enters a region of uniform electric field with an initial velocity of 30 km/s in the same direction as the electric field, which has magnitude E = 50 N/C. (a) What is the speed of the electron 1.5 ns after entering this region? How far does the electron travel during the 1.5 ns interval?



- $\vec{v}_f = \vec{v}_i + \vec{a}t$

$\vec{v}_i = {}^{+}30 \ \frac{km}{s} \ \hat{i}$

$v_{ix} = 30 \ km/s$

- $\vec{E} = {}^{+}50 \ N/C \ \hat{i}$ (right ward)

$\vec{F} = q\vec{E} \iff m_e \vec{a} = -e\vec{E} \iff \boxed{a_x = \frac{-eE}{m_e}}$

$$a_x = -\frac{eE}{m_e} = \frac{(-1.6 \times 10^{-19} C)(50 \frac{N}{C})}{9.11 \times 10^{-31} kg} = -8.78 \times 10^{12} \frac{m}{s^2}$$

$v_{fx} = v_{ix} + a_x t$

$v_{fx} = (3 \times 10^4 \frac{m}{s}) + (-8.78 \times 10^{12} \frac{m}{s^2})(1.5 \times 10^{-9} s)$

$\boxed{v_{fx} = 1.64 \times 10^4 \ m/s}$ $\quad \vec{v}_f = +1.64 \times 10^4 \ \frac{m}{s} \ \hat{i}$

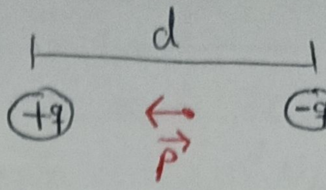
(b) $\Delta x = ?$ $\quad \Delta x = v_{ix} t + \frac{1}{2} a_x t^2$

$= (3 \times 10^4)(1.5 \times 10^{-9}) + \frac{1}{2}(-8.78 \times 10^{12})(1.5 \times 10^{-9})^2$

$\boxed{\Delta x = 3.51 \times 10^{-5} \ m}$

$\Delta x = \left(\frac{v_i + v_f}{2} \right) t$

• A dipole in An electric field



$\vec{p}$ = electric dipole moment

$p = qd$

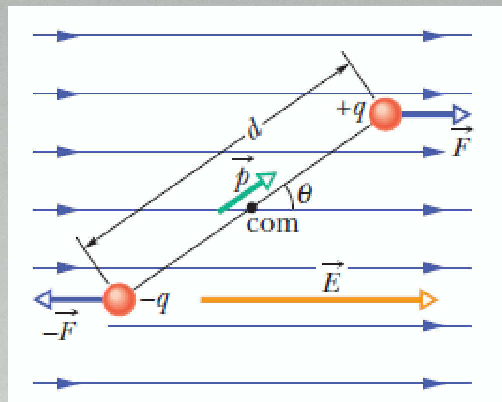
$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau_+ = -\frac{d}{2} qE \sin\theta \quad \text{"clock wise"}$$

$$\tau_- = -\frac{d}{2} qE \sin\theta \quad \text{"clock wise"}$$

$$\vec{\tau}_{net} = \vec{\tau}_+ + \vec{\tau}_- = -2\frac{d}{2} qE \sin\theta$$

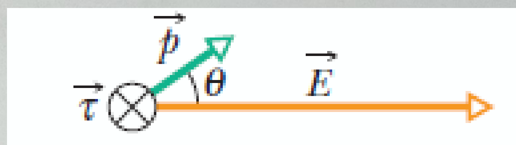
$$= -qd\, E \sin\theta$$



$$\tau = pE\sin\theta$$

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

Tourque on a dipole by the field $\vec{E}$



• A potential energy U is associated with the orientation of the dipole moment in the field

$$\boxed{U = -\vec{p}\cdot\vec{E}}$$

$$U = \begin{cases} pE \text{ (greatest value)}, & \vec{p} \text{ and } \vec{E} \text{ antiparallel } (\theta = 180^\circ) \\ \text{Zero}, & \vec{p} \text{ and } \vec{E} \text{ perpendicular } (\theta = \pi/2) \\ -pE \text{ (Least value)}, & \vec{p} \text{ and } \vec{E} \text{ parallel } (\theta = 0^\circ) \end{cases}$$

• If the dipole orientation changes ⇒

The work done by the electric field ⇒ $W_E = -\Delta U$

if the change in the orientation is due to an external agent

$$W_{\text{external agent}} = +\Delta U$$

$$W_{\text{required}} = +\Delta U$$

22-40 An electric dipole consists of charges +2e and -2e separated by 0.85 nm. It is in an electric field of strength 3.4×10^6 N/C. Calculate the magnitude of the torque on the dipole when the dipole moment is a) parallel to, b) perpendicular to, and a) antiparallel to the electric field $\vec{E}$?

$$\vec{\tau} = \vec{P} \times \vec{E} = PE \sin\theta$$

a) $\vec{P} \parallel \vec{E}$, θ = Zero $\sin 0 = 0$, $\vec{\tau} = 0$

b) $\vec{P} \perp \vec{E}$, $\theta = \pi/2$, $\sin \pi/2 = 1$ use $p = qd$

$$\vec{\tau} = 2edE = 2 \times 1.6 \times 10^{-19} \times 0.85 \times 10^{-9} \times 3.4 \times 10^{6}$$

$$\vec{\tau} = 9.248 \times 10^{-22} \text{ N.m} \quad [J]$$

c) $\vec{P}$ antiparallel $\vec{E}$, $\theta = 180$, $\sin(180) = 0$, $\vec{\tau} = 0$

22-41 How much work required to turn an electric dipole 180° in a uniform electric field of magnitude $E = 46.0$ N/C if the dipole moment has a magnitude f $p = 3.02 \times 10^{-25}$ C.m and the initial angle is 23° ?

$$W_{\text{external agent}} = +\Delta U \quad , U = -\vec{P} \cdot \vec{E} = -PE\cos\theta$$

$$W = -PE\left[\cos(180° + 23°) - \cos(23°)\right]$$

$$= -PE\left[\cos 23° \cos 180° - \sin 23° \overset{\to 0}{\cancel{\sin 180}} - \cos(23°)\right]$$

$$= 2pE\cos(23°) = 2 \times 3.02 \times 10^{-25} \times 46 \cos(23°)$$

$$\boxed{W = 2.56 \times 10^{-23} \text{ J}}$$

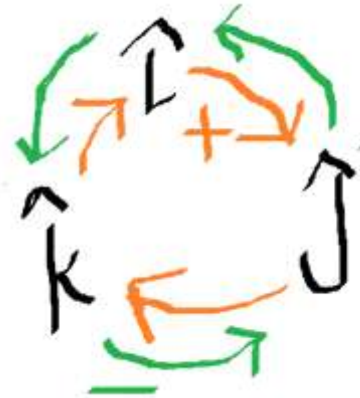
Products of Vectors

Let $\hat{\mathrm{i}}$, $\hat{\mathrm{j}}$, and $\hat{\mathrm{k}}$ be unit vectors in the x, y, and z directions. Then

$$\hat{\mathrm{i}}\cdot\hat{\mathrm{i}} = \hat{\mathrm{j}}\cdot\hat{\mathrm{j}} = \hat{\mathrm{k}}\cdot\hat{\mathrm{k}} = 1, \quad \hat{\mathrm{i}}\cdot\hat{\mathrm{j}} = \hat{\mathrm{j}}\cdot\hat{\mathrm{k}} = \hat{\mathrm{k}}\cdot\hat{\mathrm{i}} = 0,$$

$$\hat{\mathrm{i}}\times\hat{\mathrm{i}} = \hat{\mathrm{j}}\times\hat{\mathrm{j}} = \hat{\mathrm{k}}\times\hat{\mathrm{k}} = 0,$$

$$\hat{\mathrm{i}}\times\hat{\mathrm{j}} = \hat{\mathrm{k}}, \qquad \hat{\mathrm{j}}\times\hat{\mathrm{k}} = \hat{\mathrm{i}}, \qquad \hat{\mathrm{k}}\times\hat{\mathrm{i}} = \hat{\mathrm{j}}$$



Any vector $\vec{a}$ with components a_x, a_y, and a_z along the x, y, and z axes can be written as

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}.$$

Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be arbitrary vectors with magnitudes a, b, and c. Then

$$\vec{a}\times(\vec{b}+\vec{c}) = (\vec{a}\times\vec{b}) + (\vec{a}\times\vec{c})$$

$$(s\vec{a})\times\vec{b} = \vec{a}\times(s\vec{b}) = s(\vec{a}\times\vec{b}) \qquad (s = \text{a scalar}).$$

Let θ be the smaller of the two angles between $\vec{a}$ and $\vec{b}$. Then

$$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a} = a_x b_x + a_y b_y + a_z b_z = ab\cos\theta$$

$$\vec{a}\times\vec{b} = -\vec{b}\times\vec{a} = \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$= \hat{\mathrm{i}}\begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{\mathrm{j}}\begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{\mathrm{k}}\begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

$$= (a_y b_z - b_y a_z)\hat{\mathrm{i}} + (a_z b_x - b_z a_x)\hat{\mathrm{j}}$$

$$+ (a_x b_y - b_x a_y)\hat{\mathrm{k}}$$

$$|\vec{a}\times\vec{b}| = ab\sin\theta$$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \vec{b}\cdot(\vec{c}\times\vec{a}) = \vec{c}\cdot(\vec{a}\times\vec{b})$$

$$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$$