Chapter 21: Coulomb's Law

- Electric charge q $[[q] = C]$ can be either positive or negative.
- Particles with the same sign of charge repel each other [Repulsion]
- Particles with opposite sign of charge attract each other [Attraction]

object

electrically neutral — "has equal amounts of the two kinds of charge"

electrically charged — "has an excess charge"

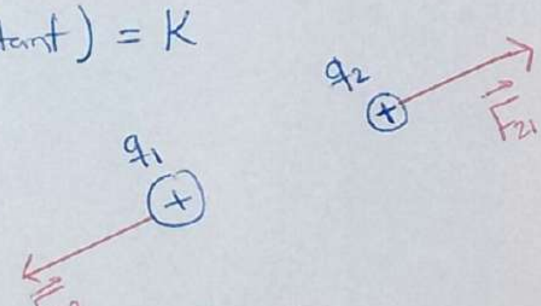
- Coulomb's Law

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{r^2}$$

ϵ_0 ≡ permittivity constant $= 8.85\times10^{-12}\ C^2/N.m^2$

$\frac{1}{4\pi\epsilon_0}$ ≡ Electrostatic constant (Coulomb constant) = K

$$K = 8.99\times10^{9}\ N.m^2/C^2$$



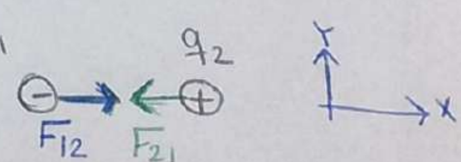
Repulsion

$F_{12} = F_{21}$

- The Elementary charge

Electric charge is quantized

$$q = ne,\ n = \mp1, \mp2, \mp3, \dots$$
$$e = 1.6\times10^{-19}\ C$$



Attraction

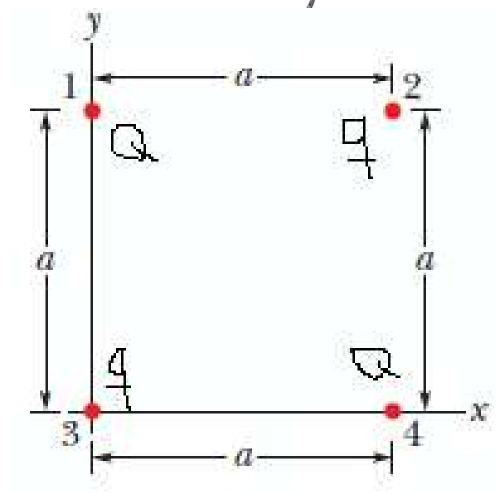
$\vec{F}_{21} = -F_{21}\ \hat{i}$

$\vec{F}_{12} = +F_{12}\ \hat{i}$

- Conservation of charge

The net electric charge of any Isolated system is always conserved.

21-2 In the below figure, Four particles form a square. The charges are $q_1 = q_4 = Q$ and $q_2 = q_3 = q$. (a) What is Q/q if the net electrostatic force on particles 2 and 3 is zero? (b) Is there any value of q that makes the net electrostatic force on each of the four particles zero? Explain.



a) Net electrostatic force on 2 and 3 is zero

$\vec{F}_2 = 0$, $\vec{F}_3 = 0$

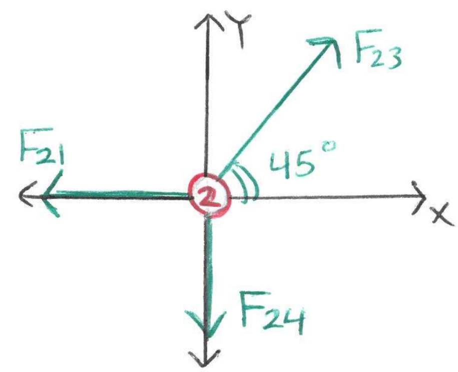
$\vec{F}_{21} + \vec{F}_{23} + \vec{F}_{24} = 0$, $\vec{F}_{31} + \vec{F}_{32} + \vec{F}_{34} = 0$

- $\vec{F}_2 = 0 \Rightarrow F_{2,x} = F_{2,y} = 0$

$F_{2,x} = 0 \Rightarrow F_{23}\cos(45°) = F_{21}$

$$\frac{Kq^2}{(\sqrt{2}a)^2}\frac{1}{\sqrt{2}} = \frac{KQq}{a^2}$$

$$\boxed{Q/q = 1/2\sqrt{2}}$$



$F_{2,y} = 0 \Rightarrow F_{23}\sin(45°) = F_{24}$

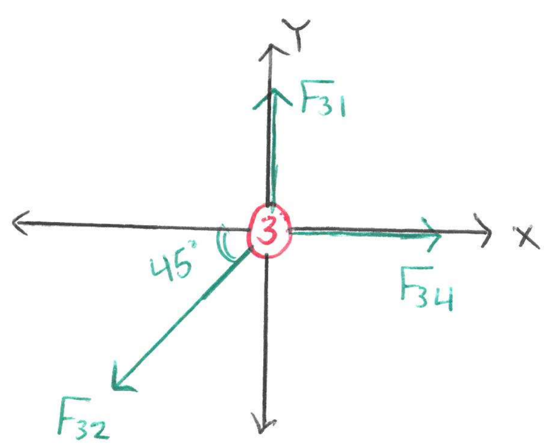
$\Rightarrow Q/q = 1/2\sqrt{2}$

- $F_3 = 0 \Rightarrow F_{3,x} = F_{3,y} = 0$

$F_{3,x} = 0 \Rightarrow F_{34} = F_{32}\cos(45°)$

$$\frac{KQq}{a^2} = \frac{Kq^2}{(\sqrt{2}a)^2}$$

$$\boxed{Q/q = 1/2\sqrt{2}}$$



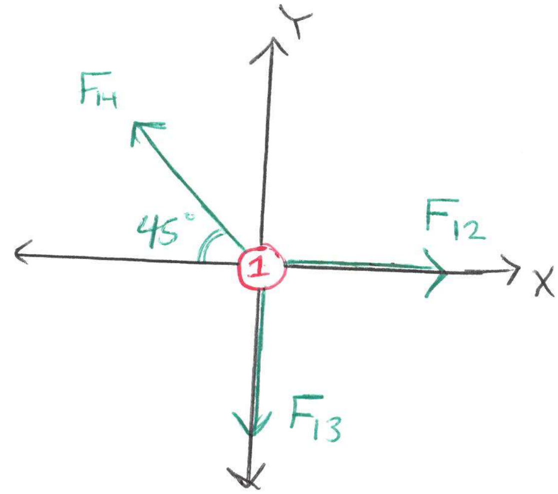
$F_{3,y} = 0$, $F_{31} = F_{32}\sin(45°)$

$$\frac{KqQ}{a^2} = \frac{Kq^2}{(\sqrt{2}a)^2} \Rightarrow Q/q = 1/2\sqrt{2}$$

(b) q that makes the net electrostatic force on each of the four particles zero?

$\vec{F}_1 = \vec{F}_4 = \vec{F}_2 = \vec{F}_3 = \text{Zero}$

- $\vec{F}_1 = 0$, $F_{1,x} = 0$



$$F_{12} = F_{14} \cos(45°)$$

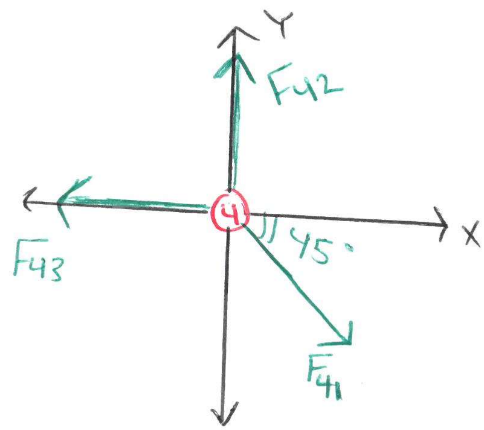
$$\frac{KqQ}{a^2} = \frac{KQ^2}{(\sqrt{2}\,a)^2}\left(\frac{1}{\sqrt{2}}\right)$$

$$\boxed{Q/q = 2\sqrt{2}}$$

$F_{1,y} = 0 \Rightarrow F_{14} \sin(45°) = F_{13} \Rightarrow Q/q = 2\sqrt{2}$

- $\vec{F}_4 = \text{Zero}$, $F_{4,x} = F_{4,y} = 0$

$F_{4,x} = 0 \Rightarrow F_{41} \cos(45°) = F_{43}$



$$\frac{KQ^2}{(\sqrt{2}\,a)^2}\frac{1}{\sqrt{2}} = \frac{KqQ}{a^2}$$

$$\boxed{Q/q = 2\sqrt{2}}$$

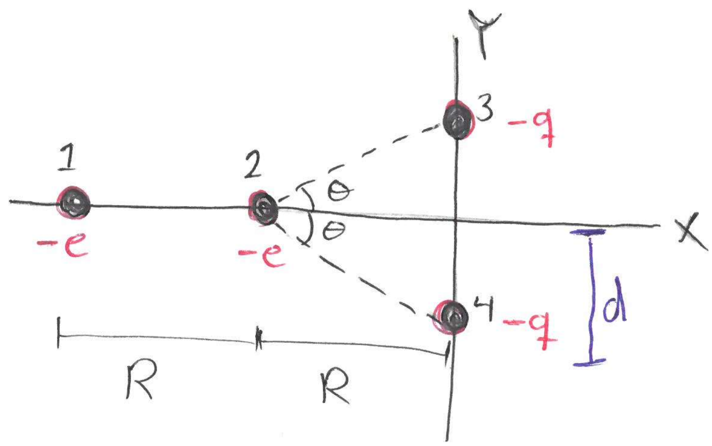
$F_{4,y} = 0 \Rightarrow F_{42} = F_{41} \sin(45°)$

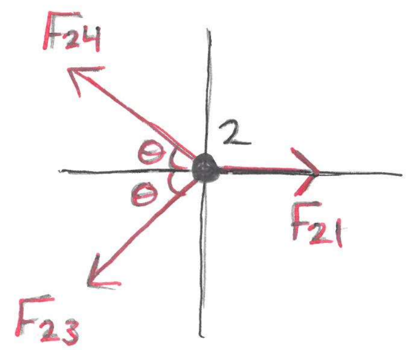
$$\frac{KqQ}{a^2} = \frac{KQ^2}{(\sqrt{2}\,a)^2}\frac{1}{\sqrt{2}}$$

$$Q/q = 2\sqrt{2}$$

There is no value of q that makes the net electrostatic force on all four particles zero.

|21-10| Electrons 1 and 2 on an X-axis and charged ions 3 and 4 of identical charge $-q$ and at identical angles θ. Electron 2 is free to move; the other three particles are fixed in place at horizental distances R from electron 2 and are intended to hold electron 2 in place. For physically possible values of $q \leqslant 5e$, what are the largest, second largest, and third largest values of θ for which electron 2 is held in place?





• $F_{2,net} = 0 = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{24}$

$|F_{23}| = |F_{24}|$

$\Rightarrow$ By symmetry

$F_{23} \sin\theta = F_{24} \sin\theta$

• $F_{2,net} = \left[F_{21} + F_{24} \cos\theta + F_{23} \cos\theta \right] \hat{i}$

$F_{2,net} = 0 \Rightarrow F_{21} = (F_{24} + F_{23}) \cos\theta$

$$\frac{Ke^2}{R^2} = 2\frac{Kqe}{R^2 + d^2} \cos\theta$$

$$\frac{e}{R^2} = \frac{2q\cos\theta}{R^2+d^2}$$

$$e - \frac{2q\cos\theta R^2}{R^2+d^2} = 0$$

$$\Rightarrow e - 2q\cos^3\theta = 0$$

$$q = \frac{e}{2\cos^3\theta} \leqslant 5e$$

$$\frac{e}{2\cos^3\theta} \leqslant ne, \quad n = 1,2,3,4,5$$

$$\cos^3\theta \geqslant \frac{1}{2n}$$

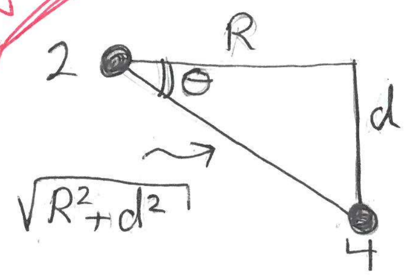
$$\cos\theta = \left[\frac{1}{2n}\right]^{1/3}$$

$n = 5 \rightsquigarrow \cos\theta = 0.46$, $\theta = 62.34°$

$n = 4 \rightarrow \cos\theta = 0.5$, $\theta = 60°$

$n = 3 \rightarrow \cos\theta = 0.55$, $\theta = 56.6°$

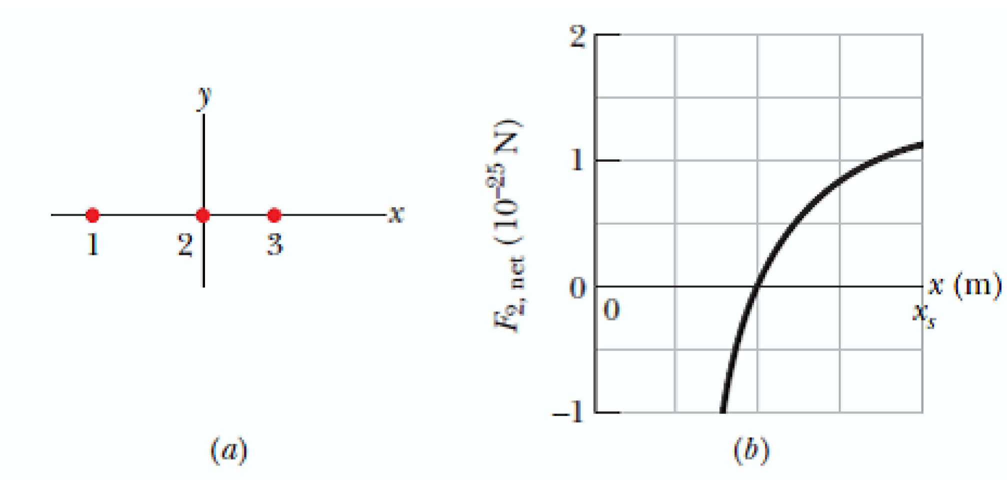
Note



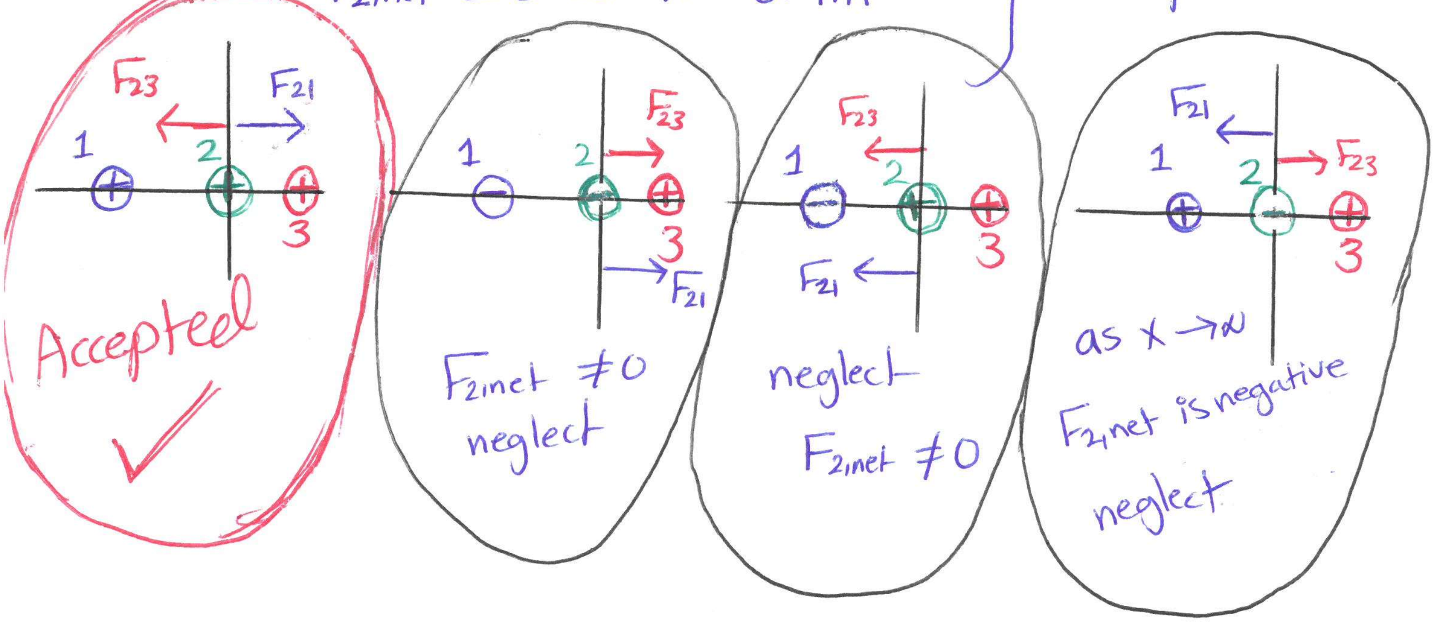
$$\cos\theta = \frac{R}{\sqrt{R^2+d^2}}$$

$$\cos^2\theta = \frac{R^2}{R^2+d^2}$$

21–20 In the below figure part (a) shows charged particles 1 and 2 that are fixed in place on an x axis. Particle 1 has a charge with a magnitude of $|q_1| = 8.00e$. Particle 3 of charge $q_3 = +7.00e$ is initially on the x axis near particle 2. Then particle 3 is gradually moved in the positive direction of the x axis. As a result, the magnitude of the net electrostatic force $\vec{F}_{2,net}$ on particle 2 due to particles 1 and 3 changes. Figure part (b) gives the x component of that net force as a function of the position x of particle 3. The scale of the x axis is set by $x_s = 0.80\,m$. The plot has an asymptote of $F_{2,net} = 1.5\times10^{-25}\,N$ as $x \to \infty$. As a multiple of e and including the sign, what is the charge q_2 of particle 2?

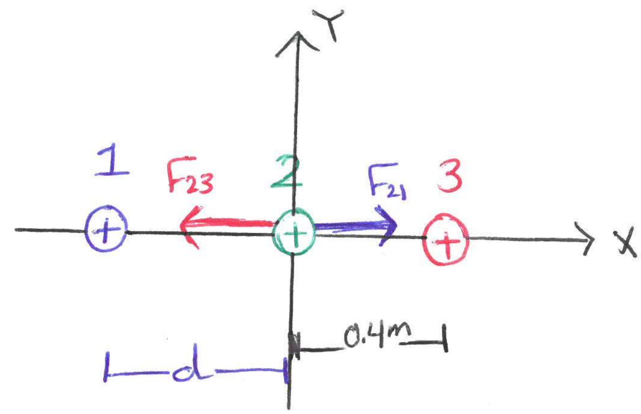


• Figure (b) ⇒ $F_{2,net} = 1.5\times10^{-25}\,N$ as $x \to \infty$
$F_{2,net} = 0$ at $x = 0.4\,m$
} x ≡ particle 3 position



$\Rightarrow$ q_1 and q_2 are postive

- F_{21net} = zero at $x = 0.4m$



$$F_{23} = F_{21}$$

$$\frac{K q_2 q_3}{(0.4)^2} = \frac{K q_2 q_1}{d^2}$$

$$d^2 = (0.4)^2 \frac{q_1}{q_3} = (0.4)^2 \frac{18.00e}{17.00e}$$

$$d^2 = 0.183 \ m^2$$

- $F_{2net} = 1.5\times10^{-25} N$ as $x \to \infty$ $\Rightarrow$ F_{23} = Zero

$$F_{21} = 1.5\times10^{-25} N$$

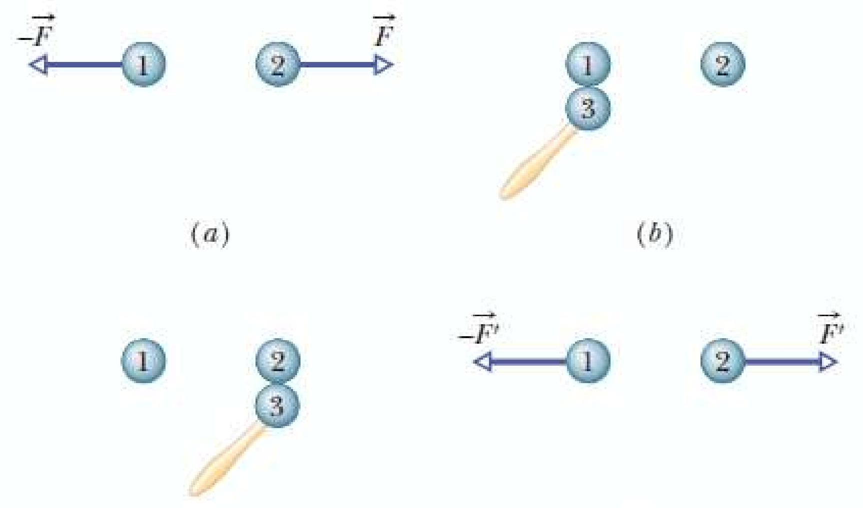
$$\frac{K q_2 q_1}{d^2} = 1.5\times10^{-25} N$$

$$q_2 = \frac{d^2 (1.5\times10^{-25})}{K q_1} = \frac{(0.4)^2 8 \times 1.5\times10^{-25}}{7\times9\times10^9 \times 8 \times 1.6\times10^{-19}}$$

$$q_2 = 2.381 \times10^{-18} C$$

$$q_2 = 15.0 \ e$$

21-24 Identical isolated conducting spheres 1 and 2 have equal charges and are separated by a distance that is Large compared with their diameters. The electrostatic force acting on sphere 2 due to sphere 1 is $\vec{F}$. suppose now that a third identical sphere 3, having an insulating handle and initially neutral, is touched first to sphere 1 as show in figure (b), then to sphere 2 (figure c), then to sphere 1 again (not shown), and then finally removed (figure d). The electrostatic force that now acts on sphere 2 has magnitude F'. What is the ratio F'/F ?



(a) (b) (c) (d)

(a) $F = \frac{K q^2}{r^2}$ "Repulsive"

b) $q_1 = \frac{q}{2}$, $q_2 = q$, $q_3 = \frac{q}{2} \Rightarrow \{(q+0)/2 = q/2\}$

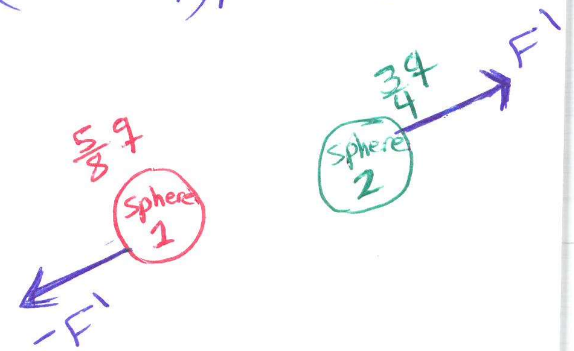
c) $q_1 = \frac{q}{2}$, $q_2 = q_3 = \frac{3q}{4} \Rightarrow \left[\left(\frac{q}{2}+q\right)/2 = \frac{3q}{4}\right]$

⇒ Sphere 3 is touched again sphere 1 $\Rightarrow \left(\frac{q}{2} + \frac{3q}{4}\right)/2 = \frac{5}{8}q$

$q_1 = q_3 = \frac{5}{8}q$, $q_2 = \frac{3q}{4}$

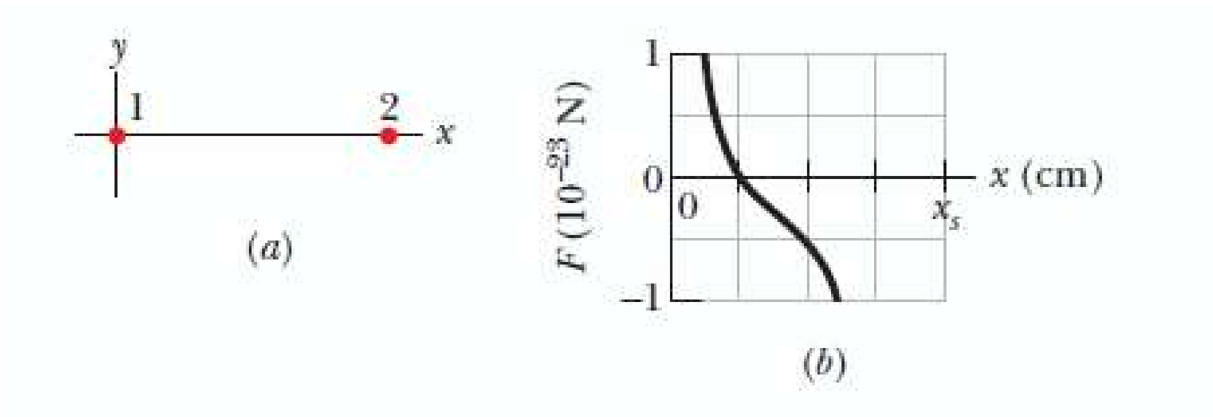
d) $F' = ?$

$$F' = \frac{K\left(\frac{5}{8}q\right)\left(\frac{3}{4}q\right)}{r^2}$$

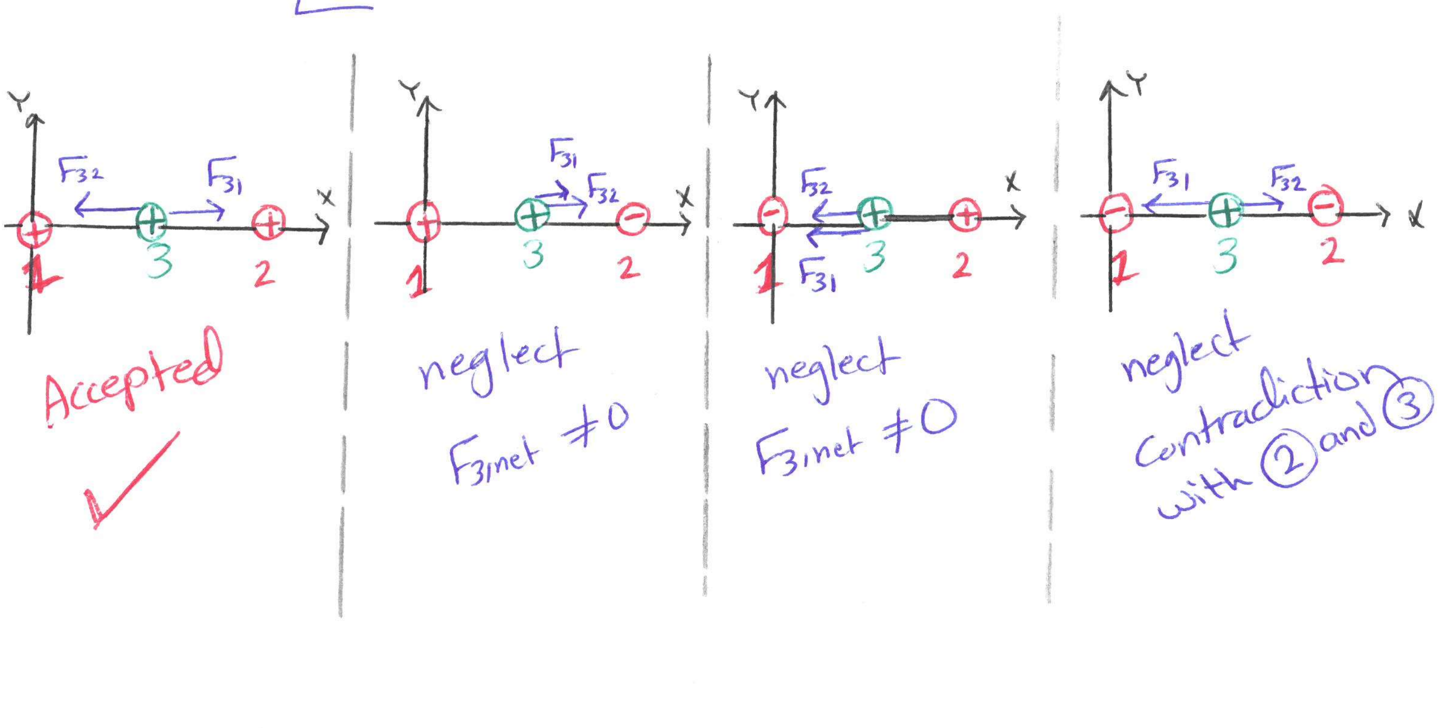


$$F' = \frac{15}{32}\frac{Kq^2}{r^2} \Rightarrow \boxed{\frac{F'}{F} = \frac{15}{32}}$$

21-32 The below figure (a), particle 1 (of charge q_1) and particle 2 (of charge q_2) are fixed in place on an x axis, 8.00 cm apart. particle 3 (of charge $q_3 = +6.00 \times 10^{-19}$ C) is to be placed on the line between particles 1 and 2 so that they produce a net electrostatic force $\vec{F}_{3net}$ on it. Figure (b) gives the x component of that force versus the coordinate x at which particle 3 is placed. The scale of the x axis is set by $x_s = 8.0$ cm. What are (a) the sign of charge q_1 and (b) the ratio q_2/q_1 ?

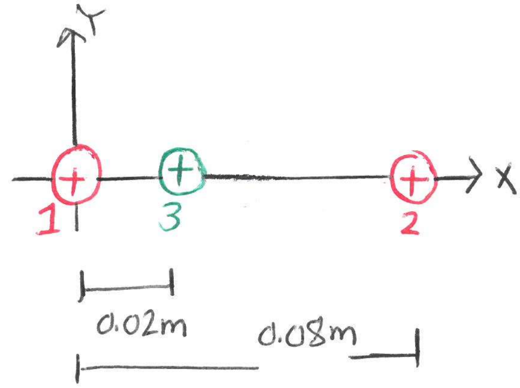


- Figure (b) ⇒
  1. $F_{3,net} = 0$ at $x = 2.0$ cm $= 0.02$ m
  2. $F_{3,net}$ is positive when q_3 is closer to q_1 rather than q_2
  3. F_{3net} is negative when q_3 is closer to q_2 rather than q_1



- q_1 and q_2 are postive charges

- $F_{3net} = 0$ at $x = 0.02\,m$



$$F_{31} = F_{32}$$

$$\frac{K q_3 q_1}{(0.02)^2} = \frac{K q_3 q_2}{(0.06)^2}$$

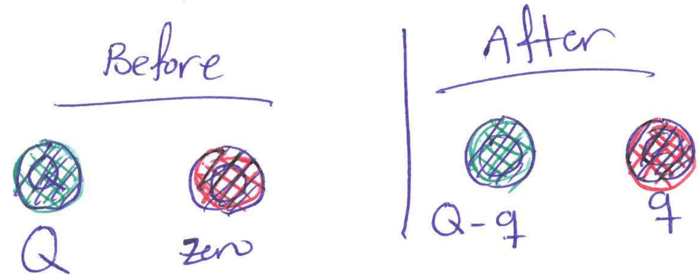
$$\frac{q_2}{q_1} = \left(\frac{0.06}{0.02}\right)^2 = 3^2 = 9$$

$$\boxed{q_2/q_1 = 9}$$

1-37 | Of the charge Q initially on a tiny sphere, a portion q is to be transferred to a second, nearby sphere. Both spheres can be treated as particles and are fixed with a certain separation.

a) For what value of $\frac{q}{Q}$ will the electrostatic force between the two spheres be maximized? What are the b) smaller and c) larger values of q/Q that give a force magnitude that is 75% of that maximum?

a)



$$\Rightarrow |F| = \frac{K(Q-q)\,q}{r^2}$$

$$\frac{dF}{dq} = \frac{K}{r^2}\frac{d}{dq}(Q-q)(q) = \frac{K}{r^2}\frac{d}{dq}(Qq-q^2)$$

• maximization of F $\Rightarrow \frac{dF}{dq} = 0 \Rightarrow \frac{d}{dq}(Qq-q^2) = 0$

$$\Rightarrow Q - 2q = 0$$

$$\boxed{\frac{q}{Q} = \frac{1}{2}}$$

b) $F_{max}\left[\text{when } \frac{q}{Q} = \frac{1}{2}\right] = \frac{KQ^2}{4r^2}$ $\left\{\text{use } q = \frac{Q}{2}\right\}$

$$75\% \text{ of } F_{max} = \frac{3}{4}\left[\frac{KQ^2}{4r^2}\right] = \frac{3}{16}\frac{KQ^2}{r^2}$$

$\Rightarrow$

- $F = \frac{K(Q-q)q}{r^2} = 75\%$ of F_{max}

$$\frac{K(Q-q)q}{r^2} = \frac{3}{16}\frac{KQ^2}{r^2}$$

$$(Q-q)q = \frac{3Q^2}{16}$$

$$Qq - q^2 = \frac{3Q^2}{16}$$

$$q^2 - Qq + \frac{3Q^2}{16} = 0$$

$$q = \frac{Q \mp \sqrt{Q^2 - \frac{4\times 3}{16}Q^2}}{2} = \frac{Q \mp \frac{Q}{2}}{2}$$

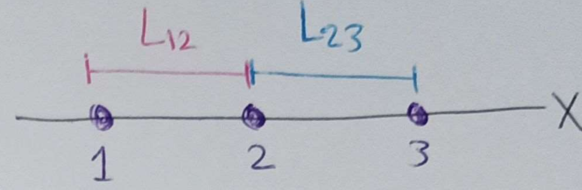
$$\boxed{q = \frac{1}{4}Q, \ \frac{3}{4}Q}$$

$\frac{q}{Q} = \frac{1}{4}$ "smaller value"

$\frac{q}{Q} = \frac{3}{4}$ "Larger value"

21-35 Three charged particles lie on x axis, particles 1 and 2 are fixed in place. Particle 3 is free to move, but the net electrostatic force on it from particles 1 and 2 happens to be zero. If $2L_{23} = L_{12}$, what is the ratio q_1/q_2?

$$\vec{F}_{net,3} = \text{Zero} = \vec{F}_{31} + \vec{F}_{32}$$



$$\Rightarrow 2L_{23} = L_{12}$$

* q_1 has an opposite sign of q_2 charge

$$F_{31} = F_{32}$$

$$\frac{k\, q_3 q_1}{(L_{12} + L_{23})^2} = \frac{k\, q_3 q_2}{L_{23}^2}$$

$$\frac{q_1}{(3L_{23})^2} = \frac{q_2}{(L_{23})^2}$$

$$q_1 / q_2 = 9$$

$$\Rightarrow \frac{q_1}{q_2} = -9$$

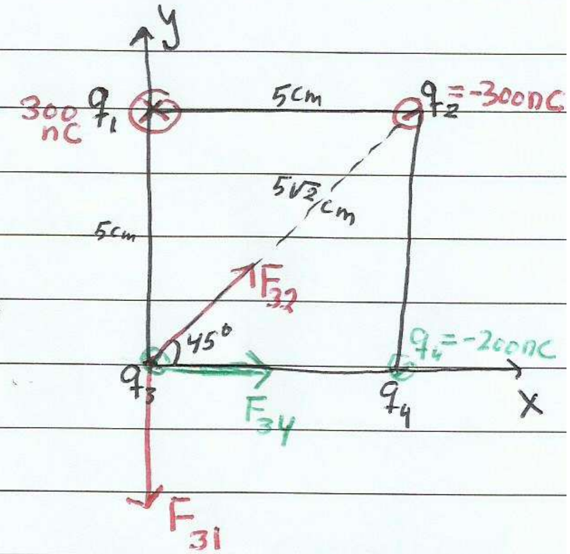
Chapter-21 Lecture Problems

(21-3) $q_1 = -q_2 = 300\,nC$

$q_3 = -q_4 = 200\,nC$

$a = 5\,cm$

$\vec{F_3}$?



All vectors (Forces) must Begin from q_3

$$F_{31} = \frac{9\times10^{9} \times 200\times10^{-9} \times 300\times10^{-9}}{(5\times10^{-2})^2}$$

$\vec{F}_{31} = 0.216\,N$ in $(-y)$ or $-\hat{j}$

$$F_{32} = \frac{9\times10^{9} \times 200\times10^{-9} \times 300\times10^{-9}}{(5\sqrt{2}\times10^{-2})^2}$$

$\vec{F}_{32} = 0.108\,N$ at 45° with +x

$$F_{34} = \frac{9\times10^{9} \times 200\times10^{-9} \times 200\times10^{-9}}{(5\times10^{-2})^2}$$

$\vec{F}_{34} = 0.144\,N$ in +x

$\vec{F_3} = \vec{F}_{31} + \vec{F}_{32} + \vec{F}_{34}$

$(F_3)_x = 0.144 + F_{32}\cos 45 = 0.144 + 0.108(0.707)$

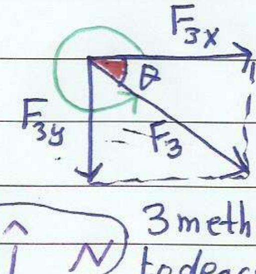
$= 0.22\,N$

$(F_3)_y = -F_{31} + F_{32}\sin 45 = -0.216 + 0.108 \sin 45$

$= -0.14\,N$

$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = \sqrt{(0.22)^2 + (-0.14)^2}$

$= 0.26\,N$



$\theta = \tan^{-1}\left(\frac{F_{3y}}{F_{3x}}\right) = \tan^{-1}\left(\frac{-0.14}{0.22}\right)$

$\theta = -32.5°$

3 methods to describe F_3

① $\vec{F_3} = 0.22\hat{i} - 0.14\hat{j}\ N$

② $\vec{F_3} = 0.26\,N$ at 32.5° with +x clockwise

③ $\vec{F_3} = 0.26\,N$ at 327.5° with +x Counter clockwise

(21-6)

q_1 at $x_1 = -a$

q_2 at $x_2 = +a$

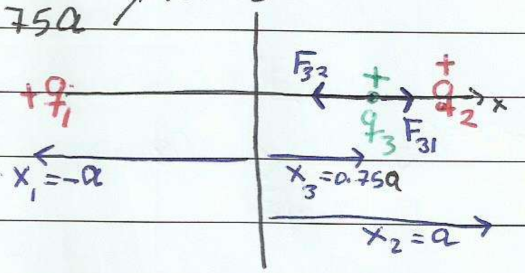
$\vec{F_3} = 0$ on $q_3 = +Q$

a) Find $\frac{q_1}{q_2}$? when q_3 at $x_3 = +0.75a$ ($q_3 = Q$) ↗ For $\vec{F_3} = 0$

both q_1 & q_2 must be possitive

or

both q_1 & q_2 must be negative



$\Rightarrow \frac{q_1}{q} = +?$

$\vec{F_3} = \vec{F_{31}} + \vec{F_{32}}$, $\vec{F_3} = 0 \Rightarrow \vec{F_{31}} = -\vec{F_{32}}$ opposite and equal

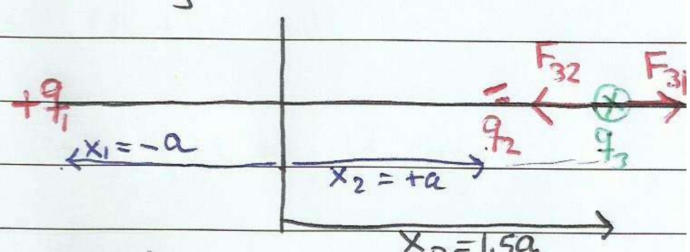
$0 = k\frac{q_1 Q}{(a+0.75a)^2}\hat{i} + \frac{k q_2 Q}{(0.25a)^2}(-\hat{i}) \Rightarrow \frac{k q_1 Q}{(1.75a)^2} = \frac{k q_2 Q}{(0.25a)^2}$

$\frac{q_1}{(1.75)^2} = \frac{q_2}{(0.25)^2} \Rightarrow \frac{q_1}{q_2} = \left(\frac{1.75}{0.25}\right)^2 = (7)^2$

$\boxed{\frac{q_1}{q_2} = +49}$

b) Find $\frac{q_1}{q_2} = ?$ When $q_3 = +Q$ at $x_3 = 1.5a$

For $\vec{F_3} = 0$



$\vec{F_3} = \vec{F_{31}} + \vec{F_{32}}$

For $\vec{F_3} = 0$

$\vec{F_{31}} + \vec{F_{32}} = 0 \Rightarrow$

$\vec{F_{31}} = -\vec{F_{32}}$ must be equal & opposite

let q_1 be positive, then q_2 must be negative $\Rightarrow \frac{q_1}{q_2} = -?$

$\frac{k q_1 Q}{(2.5a)^2} = \frac{k q_2 Q}{(0.5a)^2} \Rightarrow \frac{q_1}{(2.5)^2} = \frac{q_2}{(0.5)^2}$

$\frac{q_1}{q_2} = \left(\frac{2.5}{0.5}\right)^2 = (5)^2 = 25 \Rightarrow$

$\boxed{\frac{q_1}{q_2} = -25}$

(21-31)

$q_1 = q_2 = +4e$

$q_3 = +8e$

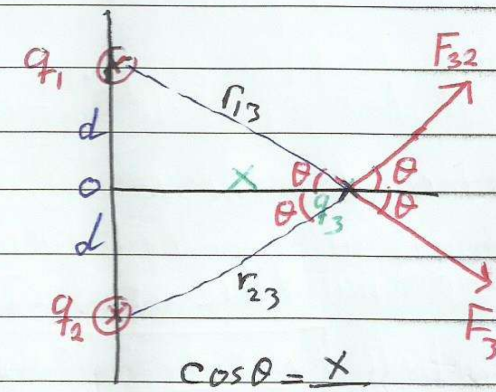
$d = 17.0\,cm$

$x\,(0 \longrightarrow 5m)$

Find x? for $\vec{F_3}$ is max

Find x? For $\vec{F_3}$ is min.

Find $(F_3)_{max}$? $(F_3)_{min}$?



$\cos\theta = \frac{x}{r_{13}}$

$\cos\theta = \frac{x}{\sqrt{d^2+x^2}}$

$r_{13} = r_{23} = \sqrt{d^2+x^2}$

$F_{31} = F_{32} = \frac{k\,q_1 q_3}{(r_{13})^2} = \frac{k(4e)(8e)}{[\sqrt{d^2+x^2}]^2} = \frac{32ke^2}{d^2+x^2}$, the direction for each Force on the figure.

$\vec{F_3} = \vec{F_{31}} + \vec{F_{32}}$

$(F_3)_x = \frac{32ke^2}{d^2+x^2}\cos\theta + \frac{32ke^2}{d^2+x^2}\cos\theta = 2\left(\frac{32ke^2}{d^2+x^2}\right)\left(\frac{x}{\sqrt{d^2+x^2}}\right)$

$(F_3)_x = 64ke^2\left(\frac{x}{(d^2+x^2)^{3/2}}\right)$

$(F_3)_{min} = 0$ at $x = 0$

$(F_3)_y = -F_{31}\sin\theta + F_{32}\sin\theta = 0$

$$F_3 = 64ke^2 \frac{x}{(d^2+x^2)^{3/2}}$$

to find F_{max} + F_{min} do $\frac{dF_3}{dx}$ must equal zero

$\frac{dF_3}{dx} = 64ke^2 \frac{d}{dx}\left[x(d^2+x^2)^{-3/2}\right]$

$0 = 64ke^2\left[(1)(d^2+x^2)^{-3/2} + x\left(-\frac{3}{2}\right)(2x)(d^2+x^2)^{-5/2}\right]$

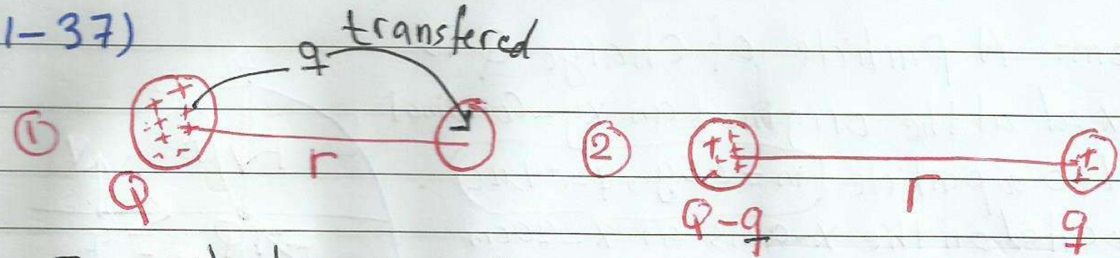
$\frac{3x^2}{(d^2+x^2)^{5/2}} = \frac{1}{(d^2+x^2)^{3/2}} \Longrightarrow \frac{3x^2}{\;} = \frac{(d^2+x^2)^{5/2}}{(d^2+x^2)^{3/2}}$

$3x^2 = d^2+x^2 \Rightarrow 2x^2 = d^2 \Rightarrow x = \frac{d}{\sqrt{2}}$ for $(F_3)_{max}$

$(F_3)_{max} = 64ke^2\left(\frac{d/\sqrt{2}}{[d^2+d^2/2]^{3/2}}\right)$

$= 64ke^2\left[\frac{d}{\sqrt{2}\,[1.5d^2]^{3/2}}\right]$ ← $k = 9\times10^9$, $d = 17\times10^{-2}\,m$, $e = 1.6\times10^{-19}\,C$

(21-37)



a) For what value $\frac{q}{Q}$? will maximaize the force between the 2 spheres

$$F = k\frac{q_1 q_2}{r^2} = k\frac{q(Q-q)}{r^2}$$

$\frac{dF}{dq}$ must be zero for F to be maximum

$$\frac{dF}{dq} = \frac{k}{r^2}\frac{d}{dq}(Qq - q^2) = \frac{k}{r^2}(Q-2q)$$

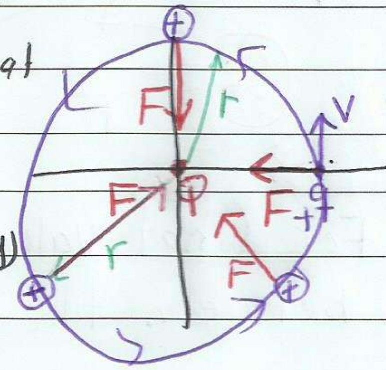
$$0 = \frac{k}{r^2}(Q-2q) \Longrightarrow Q - 2q = 0 \Rightarrow q = \frac{1}{2}Q$$

$\frac{q}{Q} = \frac{1}{2}$ for F_{max}

$$F_{max} = k\left(\frac{q_1 q_2}{r^2}\right) = \frac{k}{r^2}\left[\left(\frac{1}{2}Q\right)\left(\frac{1}{2}Q\right)\right] = \frac{kQ^2}{4r^2}$$

b) do this part.

Problem:- A particle of charge Q is fixed at the origin of an xy-Coordinat. At $t=0$ a particle ($m = 0.5g$, $q = +4\mu c$) is located on the x-axis at $x = 20cm$ moving with speed of $50 m/s$ in the $(+y)$. For what value of Q will the moving Particle execute circular motion [Neglect gravitational Force on the Particle



For the Particle to move circular motion Force on the Particle must be toward the center $\Longrightarrow$ Q must be negative

$F_q = k\frac{qQ}{r^2}$, $r = 20cm$, $q = +4\times10^{-6}$

$\frac{mv^2}{r} = \frac{kqQ}{r^2} \Longrightarrow$ $m = 0.5\times10^{-3}$ kg

$Q = \frac{mv^2 r}{kq}$

$Q = \frac{(0.5\times10^{-3})(50)^2(0.2)}{(9\times10^{9})(4\times10^{-6})} = 6.94\times10^{-6} C = 6.94 \mu C$

$Q = -6.94 \mu C$