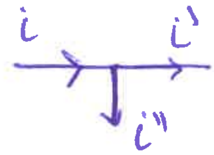
Chapter 27: Circuits

- $\mathcal{E} = \dfrac{dW}{dq}$, 1 Volt = 1 J/sec $\Rightarrow$ { ideal battery ($r_{in} = 0$); Real battery (r_{in}) }

$\Rightarrow$ Battery
- supplies energy → current with emf
- absorbs energy → current & emf are in opposite direction

- Loop Rule $\Rightarrow$ Conservation of energy $\Rightarrow$ closed loop $\sum V = 0$
- Junction Rule $\Rightarrow$ Conservation of charge



$i = i' + i''$

$\Rightarrow$ Series Resistances $R_{eq} = \sum_{i=1}^{N} R_i$

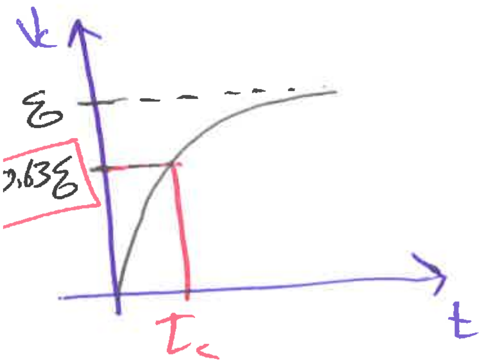
Parallel Resistances $\dfrac{1}{R_{eq}} = \sum_{i=1}^{N} \dfrac{1}{R_i}$

* RC - Circuits , $\tau_c \equiv$ capacitive time constant $= RC$

- charging a capacitor

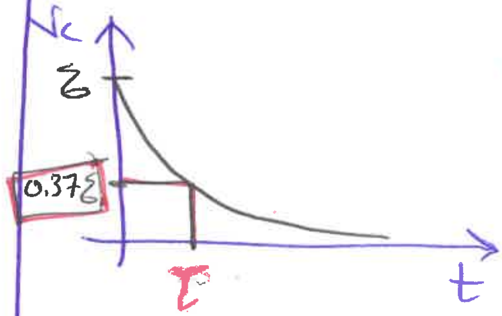
$q = C\mathcal{E}(1 - e^{-t/RC})$

$V_c = \mathcal{E}(1 - e^{-t/RC})$

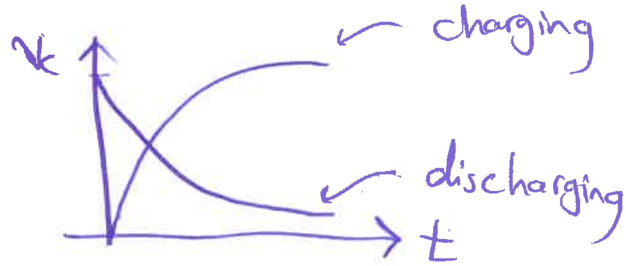


- Discharging a capacitor

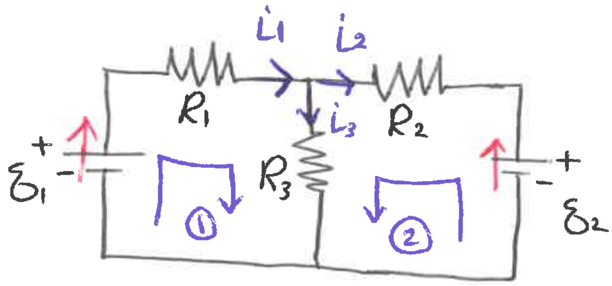
$V_c = \mathcal{E}\, e^{-t/RC}$



$\boxed{t_{1/2} = \tau \ln 2}$



27–2 In the below Figure, the ideal batteries have emfs $\mathcal{E}_1 = 12V$ and $\mathcal{E}_2 = 0.500\,\mathcal{E}_1$, and the resistances are each 4.00Ω. What is the current in a) resistance 2 and b) resistance 3?



- Junction Rule $\Rightarrow i_1 = i_2 + i_3$
- Loop Rule $\Rightarrow$

① $\mathcal{E}_1 - i_1 R_1 - i_3 R_3 = 0$

② $\mathcal{E}_2 + i_2 R_2 - i_3 R_3 = 0$

$\longrightarrow i_1 = i_2 + i_3$

$12 - i_2 R_1 - i_3 R_1 - i_3 R_3 = 0$

$6 + i_2 R_2 - i_3 R_3 = 0$

$\longrightarrow$

$12 - 4 i_2 - 8 i_3 = 0$

$6 + 4 i_2 - 4 i_3 = 0$

$\longrightarrow$

$18 = 12 i_3$

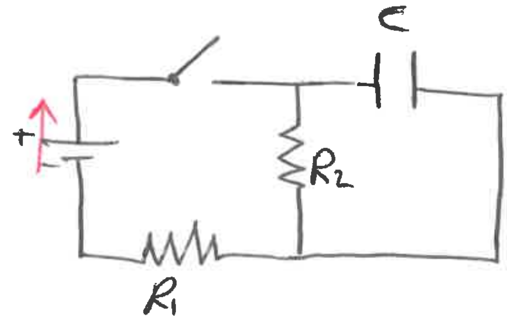
$\boxed{i_3 = 1.5\,A}$ downward

$\Rightarrow 6 + 4 i_2 - 4(1.5) = 0$

$\boxed{i_2 = 0\,A}$

$\boxed{i_1 = 1.5A, \text{ rightward}}$

27-9 In the below figure, $R_1 = 10.0\,k\Omega$, $R_2 = 15.0\,k\Omega$, $C = 0.40\,\mu F$ and the ideal battery has emf $\mathcal{E} = 20.0V$. First, the switch is closed a long time so that the steady state is reached. Then the switch is opened at time $t=0$. For resistor 2 and at time $t = 4.0ms$, what are a) the current, b) the rate at which the current is changing, and c) the rate at which the dissipation rate is changing?



a) $V_{capacitor} = V_{R_2}$ "parallel"

steady state

(open circuit)

$$\Rightarrow V_{capacitor} = \frac{\mathcal{E}\, R_2}{R_1 + R_2}$$

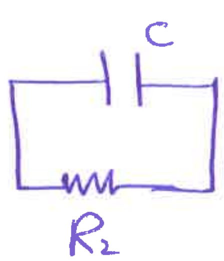
$$i = \frac{\mathcal{E}}{R_1 + R_2}$$

$$= \frac{20\,(15)}{10 + 15}$$

$$V_{R_2} = i\, R_2$$

$$\boxed{V_{capacitor} = 12\,V}$$

• Switch is opened



Discharging a capacitor

$$i = \frac{dq}{dt} = \left(-\frac{q_0}{RC}\right) e^{-t/RC}$$

OR $V_{Discharging} = V_0 e^{-t/RC}$

$$= 12\, e^{-(4\times10^{-3}/[15\times10^{3}\times 0.4\times10^{-6}])}$$

$$V = 6.16\,V$$

• Using ohm's Law $\Rightarrow i = \frac{V}{R_2} = \frac{6.16}{15\times10^{3}} = 4.11\times10^{-4}\,A$

b) The rate at which the current is changing

$$\frac{di}{dt} = \frac{d^2q}{dt^2} \Rightarrow \left.\begin{aligned} q(t) &= CV_0\, e^{-t/\tau} \\ i = \frac{dq(t)}{dt} &= \frac{-CV_0}{\tau}\, e^{-t/\tau} \\ \frac{d^2q}{dt^2} &= \frac{CV_0}{\tau^2}\, e^{-t/\tau} \end{aligned}\right\}$$

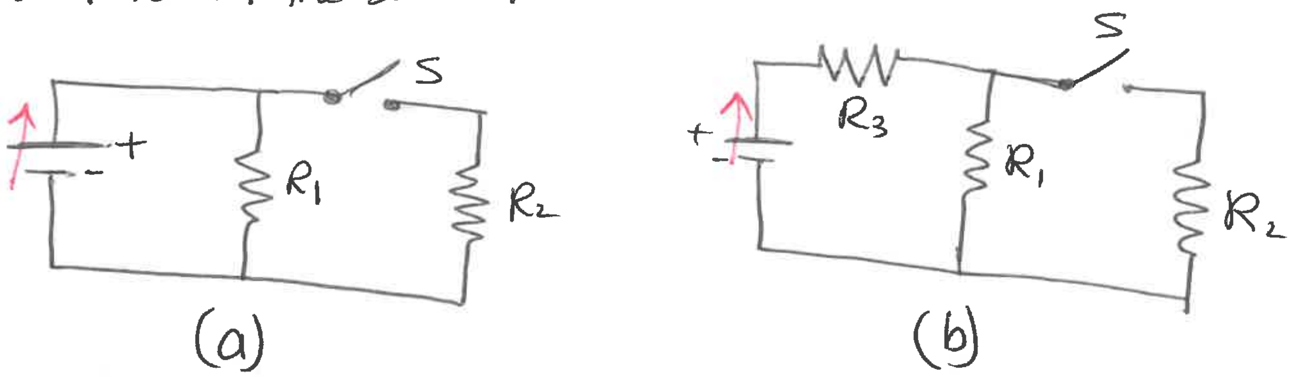
$$\frac{di}{dt} = \frac{0.4\times10^{-6}\times 12}{(15\times10^{3}\times0.4\times10^{-6})^2}\, e^{-(4\times10^{-3}/((15\times10^{3}\times0.4\times10^{-6}))}$$

$$\boxed{\frac{di}{dt} = 0.0685\,A/s = 68.5\,mA/s.}$$

c) $P = i^2R \Rightarrow \frac{dP}{dt} = \frac{d}{dt}(i^2R) = 2iR\frac{di}{dt}$

$$\frac{dP}{dt} = 2\times4.11\times10^{-4}\times15\times10^{3}\times68.5\times10^{-3} = 0.845\,\frac{W}{Sec}$$

27-14 The resistances in the below figure are all 4.0 Ω, and the batteries are ideal 12 V batteries. a) when switch S in Figure (a) is closed, what is the change in the electric potential V_1 across resistor 1, or does V_1 remain the same? b) when switch S in figure part b is closed, what is the change in V_1 across resistor 1, or does V_1 remain the same?



a) By loop Theorem, it remains the same
R_1 in parallel with R_2

b) Switch is closed R_1, R_2 parallel and their combination is in series with R_3

$$R_{123} = R_3 + \frac{R_1 * R_2}{R_1 + R_2} = 6.0\,\Omega$$

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2} = 2\,\Omega \Rightarrow i = \frac{12\,V}{6\,\Omega} = 2\,A$$

$$V_{R_3} + V_{R_{12}} = 12 \Rightarrow$$

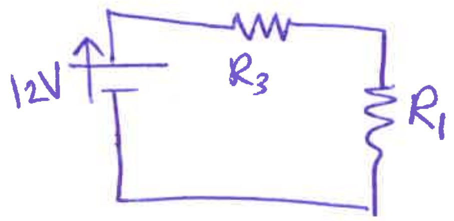
$$V_{R_3} = i R_3 = 2 \times 4 = 8\,V$$

$$V_{R_{12}} = i R_{12} = 2 \times 2 = 4\,V$$

$\Rightarrow$ R_1 and R_2 in parallel

$$V_{R_1} = V_{R_2} = 4\,V$$

- Switch is open



$$12 = V_{R_3} + V_{R_1}$$

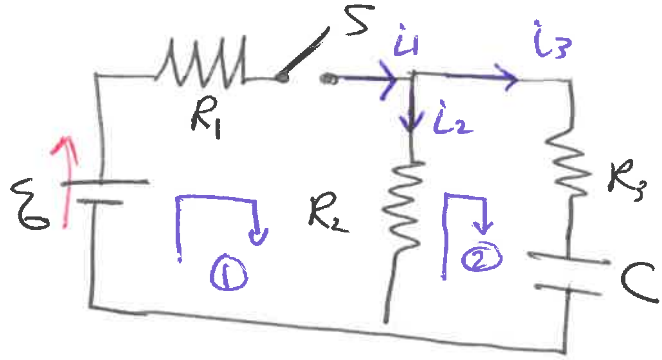
$$\Rightarrow V_{R_3} = V_{R_1} = 6V$$

Since $R_1 = R_3$

$\Rightarrow V_{R_1}$ "switch is open" $= 6V$

V_{R_1} "switch is closed" $= 4V$

27-47 In the below circuit, $\mathcal{E} = 1.2\,kV$, $C = 6.5\,\mu F$, $R_1 = R_2 = R_3 = 0.73\,M\Omega$. With C completely uncharged, switch S is suddenly closed (at $t=0$). At $t=0$, what are a) current i_1 in resistor 1 b) current i_2 in resistor R_2, and c) current i_3 in resistor 3? At $t=\infty$ (that is, after many time constants), what are i_1, i_2 and i_3? What is the potential difference V_2 across resistor 2 at $t=0$ and $t=\infty$?



* Junction Rule

$$i_1 = i_2 + i_3$$

* Loop Rule

① $1.2\,kV = 0.73M\, i_1 + 0.73M\, i_2$

② $0 = 0.73M\, i_3 - 0.73M\, i_2$ → use $i_3 = i_1 - i_2$

$$1.2\,kV = 0.73M\, i_1 + 0.73M\, i_2$$

$$-(\; 0 = 0.73M\, i_1 - 2(0.73)M\, i_2 \;)$$

$$1.2\,kV = 3(0.73)M\, i_2$$

$t=0$:

$$\boxed{i_2 = 0.55\,mA}$$

$$\boxed{i_1 = 2 i_2 = 1.1\,mA}$$

$$\boxed{i_3 = 0.55\,mA}$$

$V_{R_2} = i R_2$

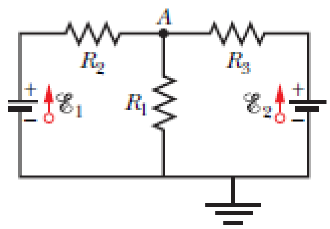
at $t=0$, $V_{R_2} = 402\,V$

at $t=\infty$, $V_{R_2} = 598.6\,V$

At $t=\infty$, $i_3 = 0$ ((Capacitor is fully charged current $=0$ "open circuit"))

$$i_1 = i_2 = \frac{1.2\,kV}{2(0.73M)} = 0.82\,mA$$

P20-27) In Fig, ε_1 = =.00 V, ε_2 =12.0 V, $R1$ = 100 Ω, $R2$ =200 Ω, and $R3$= 300 Ω. One point of the circuit is grounded (V = 0). What are the (a) size and (b) direction (up or down) of the current through resistance 1, the (c) size and (d) direction (left or right) of the current through resistance 2, and the (e) size and (f) direction of the current through resistance 3? (g) What is the electric potential at point A?



20. (a) Using the junction rule ($i_1 = i_2 + i_3$) we write two loop rule equations:

$$\varepsilon_1 - i_2 R_2 - (i_2 + i_3) R_1 = 0$$
$$\varepsilon_2 - i_3 R_3 - (i_2 + i_3) R_1 = 0.$$

Solving, we find i_2 = 0.0182 A (rightward, as was assumed in writing the equations as we did), i_3 = 0.02545 A (leftward), and $i_1 = i_2 + i_3$ = 0.04365 A (downward).

(b) The direction is downward. See the results in part (a).

(c) I_2 = 0.0182 A. See the results in part (a).

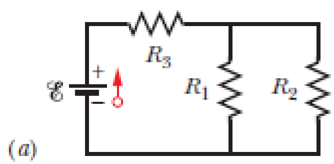
(d) The direction is rightward. See the results in part (a).

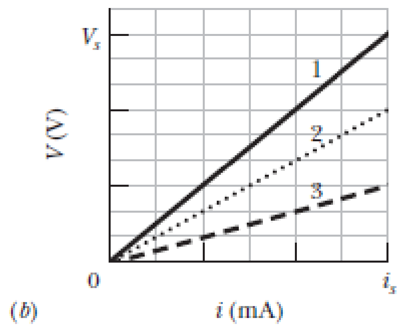
(e) i_3 = 0.0254 A. See the results in part (a).

(f) The direction is leftward. See the results in part (a).

(g) The voltage across R_1 equals V_A: (0.0382 A)(100 Ω) = +4.37 V.

P56-27) The ideal battery in Fig. 27-39*a* has emf ε_1 = 10.0 V. Plot 1 in Fig. 27-39*b* gives the electric potential difference *V* that can appear across resistor 1 of the circuit versus the current *i* in that resistor. The scale of the *V* axis is set by V_s = 18.0 V, and the scale of the *i* axis is set by i_s = 3.00 mA. Plots 2 and 3 are similar plots for resistors 2 and 3, respectively.What is the current in resistor 2?





56. Line 1 has slope R_1 = 6.0 kΩ. Line 2 has slope R_2 = 4.0 kΩ. Line 3 has slope R_3 = 2.0 kΩ. The parallel pair equivalence is $R_{12} = R_1R_2/(R_1+R_2)$ = 2.4 kΩ. That in series with R_3 gives an equivalence of

$$R_{123} = R_{12} + R_3 = 2.4\ \text{k}\Omega + 2.0\ \text{k}\Omega = 4.4\ \text{k}\Omega.$$

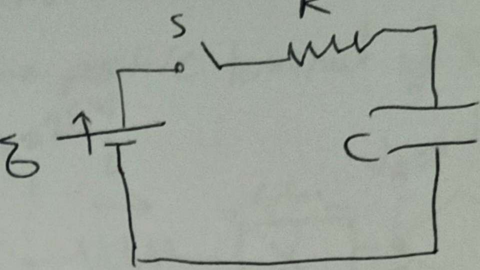
The current through the battery is therefore $i = \varepsilon / R_{123}$ = (10 V)/(4.4 kΩ) = 2.27 mA and the voltage drop across R_3 is $V_3 = iR_3 = (2.27 \times 10^{-3}\text{A})(2.0\ \text{k}\Omega) = 4.55$ V. Subtracting this (because of the loop rule) from the battery voltage leaves us with the voltage across R_2:

$$V_2 = \varepsilon - V_3 = 10.0\ \text{V} - 4.55\ \text{V} = 5.45\ \text{V}.$$

Then Ohm's law gives the current through R_2:

$$i_2 = \frac{V_2}{R_2} = \frac{5.45\ \text{V}}{4.0\ \text{k}\Omega} = 1.4\ \text{mA}.$$

27-21) Switch S in the below circuit is closed at $t = 0$, to begin charging an initially uncharged capacitor of capacitance $C = 49.0\,\mu F$ through a resistor of resistance $R = 32.0\,\Omega$. At what time is the potential across the capacitor equal to that across the resistor?



$$V_R = V_C$$

$\Rightarrow t_{\frac{1}{2}}$ "Half time"

$$t_{\frac{1}{2}} = \tau \ln 2 = RC \ln 2$$

$$t_{\frac{1}{2}} = (32\,\Omega)(49 \times 10^{-6} F) \ln 2 = 1.09 \times 10^{-3} \text{ sec}$$

$$\boxed{t = 1.09 \text{ ms}}$$

& $V_C = V_R$

$$\varepsilon(1 - e^{-t/\tau}) = \varepsilon e^{-t/\tau}$$

$$1 = 2 e^{-t/\tau}$$

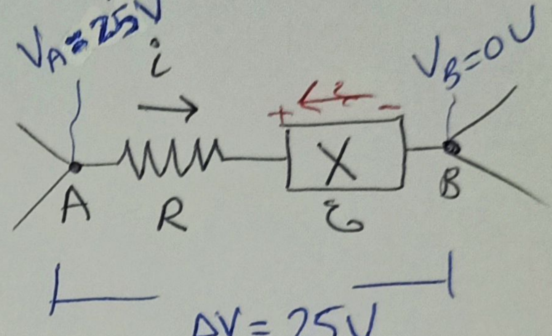
$$\frac{1}{2} = e^{-t/\tau}$$

$$\ln \frac{1}{2} = -t/\tau$$

$$-\ln 2 = -\frac{t}{\tau}$$

$$t = \tau \ln 2$$

27–33 In the below figure, circuit section AB absorbs energy at a rate of 50W when current $i = 2.0A$ through it is in the indicated direction. Resistance $R = 2.0\Omega$. (a) What is the potential difference between A and B? Emf device X lacks internal resistance. (b) What is its emf? (c) Is point B connected to the positive terminal of X or to the negative terminal?



(a) $P = i\,\Delta V$

$50\text{ watt} = 2.0A\,\Delta V$

$$\boxed{\Delta V = 25.0V}$$

$V_A - V_B = 25V$

$A \equiv$ Higher V
$B \equiv$ Lower V

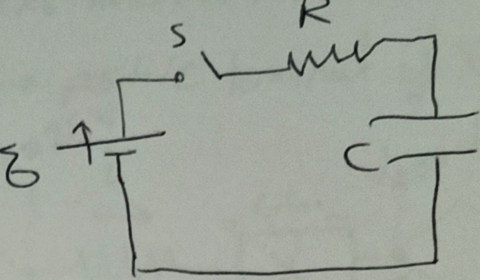
(b) $V_A - V_B = 25.0$ volts

$-iR - \mathcal{E} = 25$

$\mathcal{E} = 25 - iR = 25 - [2 \times 2] = 21V$

(c) Point B connected to the negative terminal of X

27-21) Switch S in the below circuit is closed at $t=0$, to begin charging an initially uncharged capacitor of capacitance $C = 49.0\,\mu F$ through a resistor of resistance $R = 32.0\,\Omega$. At what time is the potential across the capacitor equal to that across the resistor?



$$V_R = V_C$$

$\Rightarrow t_{\frac{1}{2}}$ "Half time"

$$t_{\frac{1}{2}} = \tau \ln 2 = RC \ln 2$$

$$t_{\frac{1}{2}} = (32\,\Omega)(49 \times 10^{-6} F) \ln 2 = 1.09 \times 10^{-3} \text{ sec}$$

$$\boxed{t = 1.09 \text{ ms}}$$

& $V_C = V_R$

$$\varepsilon(1 - e^{-t/\tau}) = \varepsilon e^{-t/\tau}$$

$$1 = 2 e^{-t/\tau}$$

$$\frac{1}{2} = e^{-t/\tau}$$

$$\ln \frac{1}{2} = -t/\tau$$

$$-\ln 2 = -\frac{t}{\tau}$$

$$t = \tau \ln 2$$