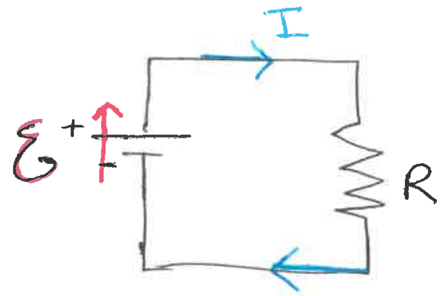
Chapter 27: Circuits

- Single loop circuit

Electromotive force of the power supply "$\mathcal{E}$"



- emf device "Battery" does work on charges to maintain a potential difference between its output terminals.

⇒ dW is the work the device does to force (move) positive charge dq from the negative to the positive terminal [inside the source].

emf (work per unit charge) of the device is

$$\mathcal{E} = \frac{dW}{dq} \quad , \quad [\mathcal{E}] = \frac{J}{C} = \text{Volt}$$

emf device

Ideal emf device

Lacks any internal resistance

The potential difference between its terminals is equal to the emf.

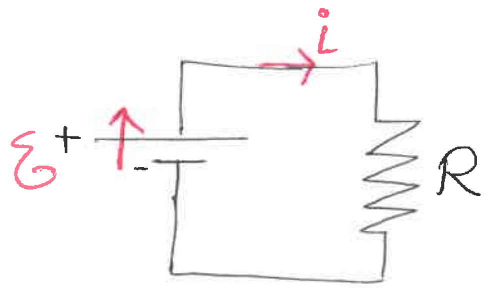
Real emf device

- Has internal Resistance
- The potential difference between its terminals is equal to the emf only if there is no current through the device.

• Calculating current in a single – loop circuit "Energy method"

$\Rightarrow P = i^2 R$ "Resistive Dissipation"

$i^2 R\,dt \equiv$ Amount of energy will appear in the resistor in a time interval dt.



• During the same interval, a charge $dq = i\,dt$ will have moved through battery, and the work that the battery will have done on this charge is

$$\boxed{dW = \mathcal{E}\,dq = \mathcal{E}\,i\,dt}$$

$P_{\mathcal{E}} = i\mathcal{E}$ "The amount of power supplied by emf device to the circuit"

By using Conservation of Energy principle $\Rightarrow$

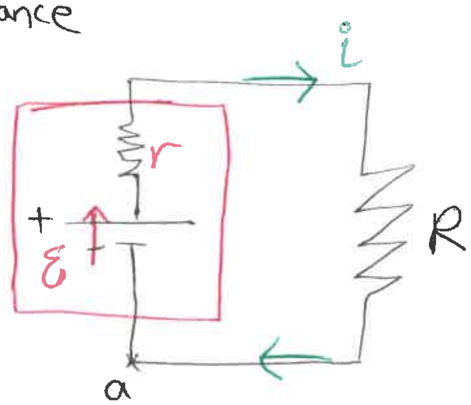
$$\mathcal{E}\,i\,dt = i^2 R\,dt \qquad \text{"}P_{\mathcal{E}}(\text{supplied}) = P_R(\text{consumed})\text{"}$$

$$\boxed{i = \frac{\mathcal{E}}{R}}$$

Single – loop circuit current for ideal power supply

* Real power supply "Has internal resistance"

$$\boxed{i = \frac{\mathcal{E}}{R+r}}$$



$$\mathcal{E} - ir - iR = 0$$

$$i = \frac{\mathcal{E}}{r+R}$$

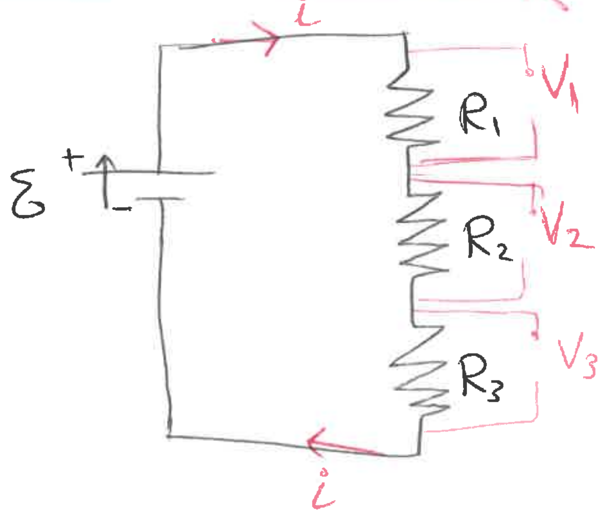
• Potential difference between two points ($V_{ab} = V_a - V_b$)

Loop Rule ⇒ The algebric sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero

Resistance Rule ⇒ For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$

Emf Rule ⇒ For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathcal{E}$, in the opposite direction it is $-\mathcal{E}$.

• Resistances in Series



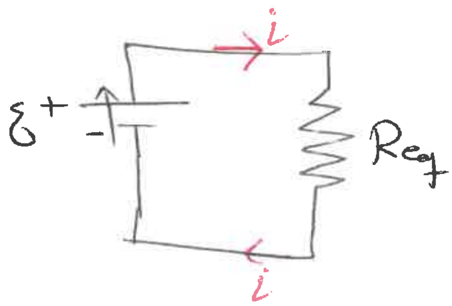
1) The same current is passing through each resistor.

$i = i_1 = i_2 = i_3$

2) By using loop Rule

$$\mathcal{E} - iR_1 - iR_2 - iR_3 = 0$$

$$i = \frac{\mathcal{E}}{R_1 + R_2 + R_3}$$



$\Rightarrow i R_{eq} = \mathcal{E} \Rightarrow i = \frac{\mathcal{E}}{R_{eq}}$

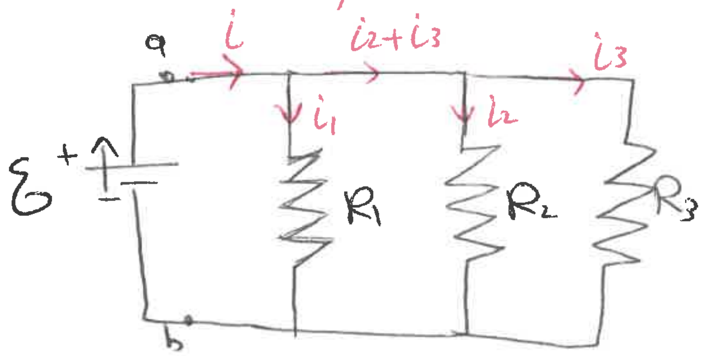
$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

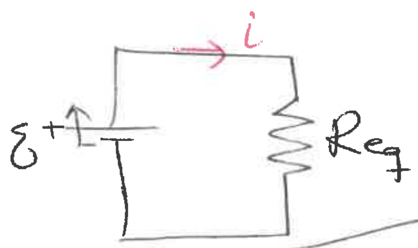
Ohm's law ⇒ $R = \frac{V}{i}$

$$\boxed{R_{eq} = \sum_{j=1}^{N} R_j}$$ "N-resistances in series"

• Resistances in Parallel — Multiloop Circuit

Junction Rule ⇒ The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction





$$\boxed{V/R_{eq} = i}$$

① $V = V_1 = V_2 = V_3$

$$\boxed{V_{ab} = V}$$

$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad i_3 = \frac{V}{R_3}$$

② $i = i_1 + i_2 + i_3$

$$i = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\boxed{\frac{1}{R_{eq}} = \sum_{j=1}^{N} \frac{1}{R_j} \quad \text{N-Resistors in parallel}}$$

• Multiloop circuits

⇒ Kirchhoff's rules

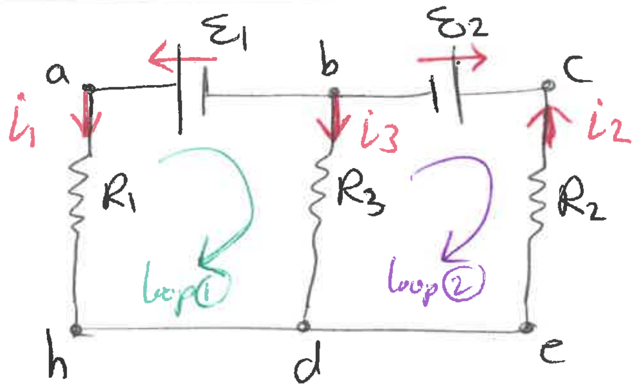
[1] Junction Rule ⟶ Conservation of charge

The sum of the currents entering any junction must be equal to the sum of currents leaving that junction.

[2] Loop Rule ⇒ Conservation of energy

The algebric sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero $\sum_{\text{closed loop}} V = \text{zero}$

Example



① Junction Rule

$$i_2 = i_3 + i_1$$

② Loop Rule

$$\sum V_{abdha} = -\mathcal{E}_1 - i_3 R_3 + i_1 R_1 = \text{zero} \quad \text{"loop ①"}$$

$$\sum V_{bcedb} = +\mathcal{E}_2 + i_2 R_2 + i_3 R_3 = \text{zero} \quad \text{"loop ②"}$$

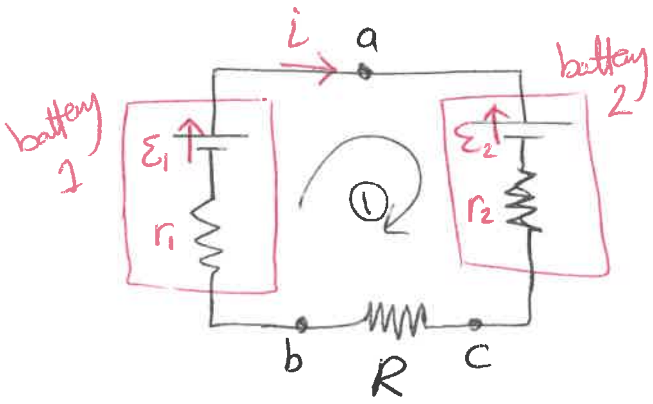
$$\Rightarrow \sum V_{aceha} = \text{zero} = -\mathcal{E}_1 + \mathcal{E}_2 + i_2 R_2 + i_1 R_1 = \text{zero}$$

$$= \text{loop ①} + \text{loop ②}$$

- Sample problem 27.01 : Single loop circuit with two Real batteries

$\mathcal{E}_1 = 4.4V$, $\mathcal{E}_2 = 2.1V$

$r_1 = 2.3\Omega$, $r_2 = 1.8\Omega$, $R = 5.5\Omega$



(a) What is the current in the circuit?

loop 1 $\Rightarrow$ $\mathcal{E}_1 - \mathcal{E}_2 - ir_2 - iR - ir_1 = 0$

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R + r_1 + r_2} = \frac{(4.4 - 2.1)V}{(5.5 + 2.3 + 1.8)\Omega} = 0.24A$$

$$\boxed{i = 240mA}$$

(b) What is the potential difference between the terminals of battery 1?

$V_{ab} = V_a - V_b$

$V_{ab} = V_a - \mathcal{E}_1 + ir_1 - V_b$

$V_a - V_b = +\mathcal{E}_1 - ir_1 = 4.4V - 0.24A(2.3\Omega)$

$$\boxed{V_a - V_b = +3.8V}$$

$$\boxed{V_{ab} = -V_{ba}}$$

- $V_b - V_a = V_b - ir_1 + \mathcal{E}_1 - V_a$

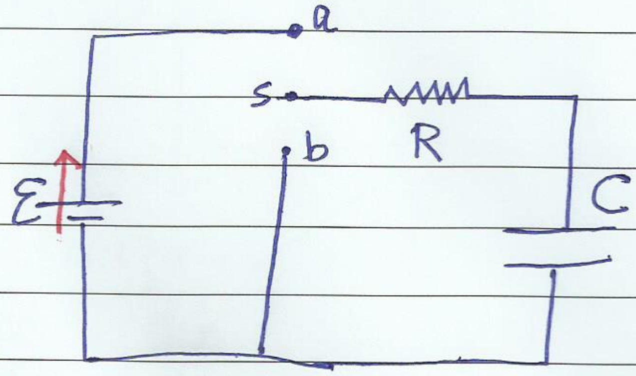
$V_b - V_a = ir_1 - \mathcal{E}_1 = -3.8V$

- $V_{ac} = V_a - V_c = V_a - \mathcal{E}_2 - ir_2 - V_c$

$V_a - V_c = \mathcal{E}_2 + ir_2 = (2.1) + (0.24 \times 1.8) = 2.53V$

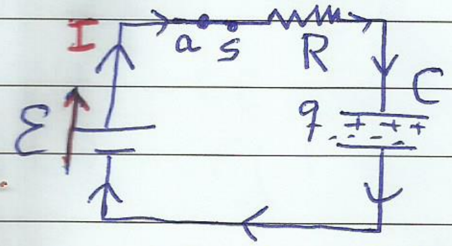
potential difference across battery 2

RC — Circuits:-



Charging a Capacitor:

Connecting (s + a)



remember $\rightarrow V_c = \frac{q}{C}$

$\rightarrow V_R = RI$

$q_0 = 0$ intial chargeon the Capacitor,

the Charge will increase gradually on the Capacitor to reac q_{max} after a long time

$q_{max} = C\mathcal{E}$

To find the Charge on the Capacitor at any time .....

$\sum V = 0$ around the closed loop

$$\mathcal{E} + {}^{-}RI + \frac{-q}{C} = 0 \quad , \; I = \frac{dq}{dt}$$

$$\mathcal{E} = \frac{q}{C} + R\frac{dq}{dt}$$

$$\mathcal{E} - \frac{q}{C} = R\frac{dq}{dt} \Rightarrow \frac{\mathcal{E}C - q}{C} = R\frac{dq}{dt}$$

$$\frac{\mathcal{E}C - q}{RC} = \frac{dq}{dt} \Rightarrow \frac{q - C\mathcal{E}}{RC} = (-)\frac{dq}{dt}$$

$$\frac{dq}{q - C\mathcal{E}} = (-)\frac{dt}{RC}$$ integrate from $t = 0 \rightarrow t = t$

$$\int_0^q \frac{dq}{q - C\mathcal{E}} = (-)\frac{1}{RC}\int_0^t dt$$

$$\ln\left[q - C\mathcal{E}\right]_0^q = \frac{-t}{RC}$$

$$\ln(q - C\mathcal{E}) - \ln(-C\mathcal{E}) = \frac{-t}{RC}$$

$$\ln\left(\frac{Q-C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$

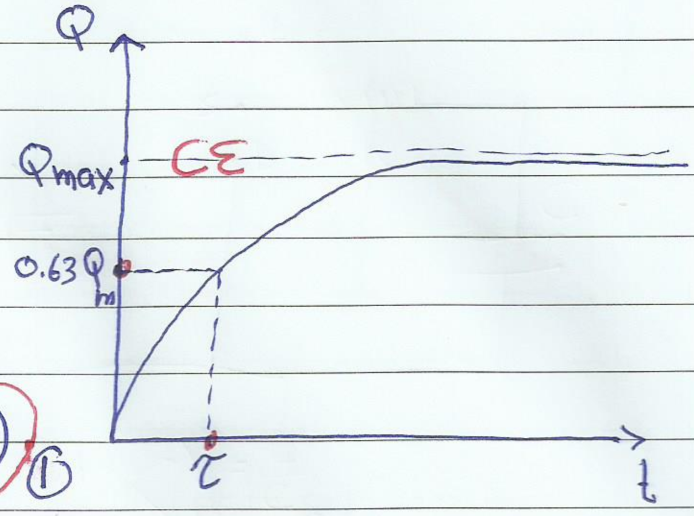
$$\frac{Q-C\mathcal{E}}{-C\mathcal{E}} = e^{-t/RC}$$

$$Q - C\mathcal{E} = -C\mathcal{E}\, e^{-t/RC}$$

$$Q(t) = C\mathcal{E} - C\mathcal{E}\, e^{-t/RC}$$

$$Q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right) \quad (1)$$

(charging a capacitor)



The time constant of the circuit $= RC$

$\Omega \; F = \text{sec.}$

$$\tau = RC$$

$$Q(t) = C\mathcal{E}\left(1 - e^{-t/\tau}\right) \quad (1)$$

1) to find Q_{max}, put $t \to \infty$ $\quad Q_{max} = C\mathcal{E}$

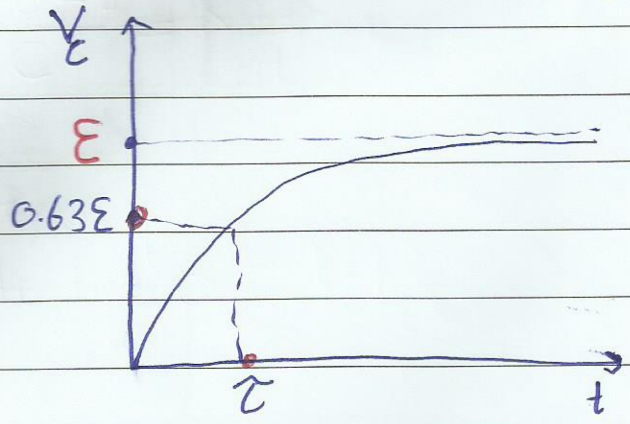
2) Find Q at $t = \tau \Rightarrow Q(\tau) = C\mathcal{E}(1 - e^{-1})$

$$= C\mathcal{E}(1 - 0.37)$$

$$Q(\tau) = 0.63\, C\mathcal{E} = 0.63\, Q_{max}$$

$\Rightarrow$ At $t = \tau \Rightarrow$ the charge on the capacitor will reach $0.63\, Q_{max}$

$$V_c(t) = \frac{Q(t)}{C} = \mathcal{E}\left(1 - e^{-t/\tau}\right)$$



Find the current at any time?

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left[C\mathcal{E}\left(1 - e^{-t/RC}\right)\right]$$

$$= 0 - C\mathcal{E}\left(\frac{-1}{RC}\right) e^{-t/RC}$$

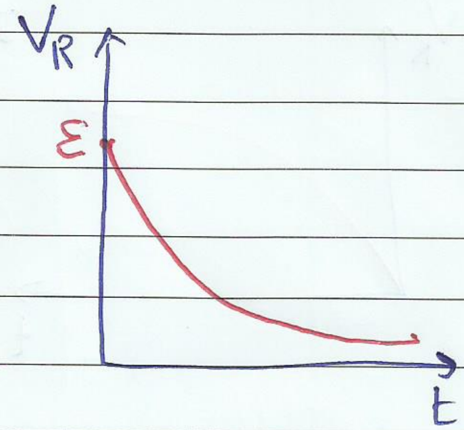
$$I(t) = \frac{\mathcal{E}}{R}\, e^{-t/RC} \;, \quad \frac{\mathcal{E}}{R} = I_0 \text{ at } t = 0$$

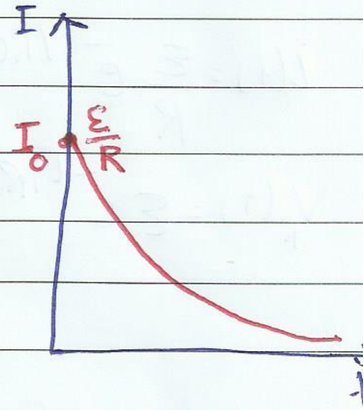
(charging C)

$$V_R = RI$$

$$V_R(t) = \mathcal{E}\, e^{-t/RC}$$

$$I(t) = \frac{\mathcal{E}}{R}\left(e^{-t/\tau}\right)$$





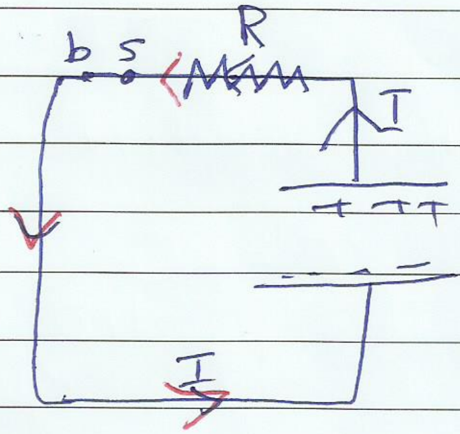
$$V_R(t) = \mathcal{E} e^{-t/\tau}$$

as $t \to \infty$ $\quad I = 0$

$V_R = 0$

Discharging a Capacitor

Connect b & S



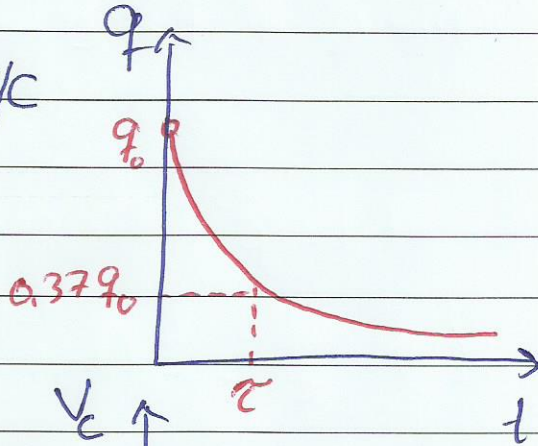
The charge will move from (+) plate $\to$ (−) plate through R

$q_0 = C\mathcal{E} = q_m$

$q_{finally} = 0$ as $t \to \infty$

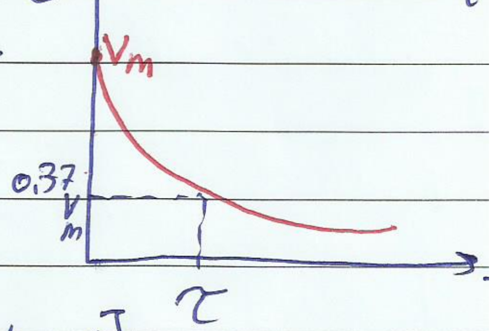
find q at any time.

$$RI + \frac{q}{C} = 0 \Rightarrow R\frac{dq}{dt} = -q/C$$



$$\frac{dq}{q} = -\frac{dt}{RC} \qquad \text{integrate}$$

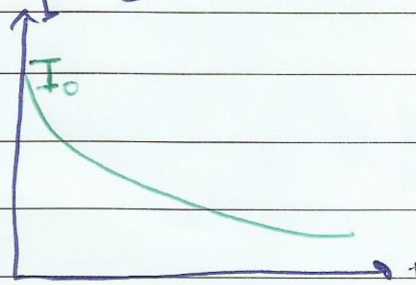
$$\int_{q_0}^{q} \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$



$$\ln\left(\frac{q}{q_0}\right) = \frac{-t}{RC} \Rightarrow \frac{q}{q_0} = e^{-t/RC}$$

$$q(t) = q_0 e^{-t/RC} \Rightarrow q(t) = q_0 e^{-t/\tau}$$

$$i(t) = \frac{dq}{dt} = q_0\left(\frac{-1}{RC}\right)e^{-t/RC} = \left(-\frac{q_0}{RC}\right)e^{-t/\tau}$$



$$I_0 = \frac{q_0}{RC}$$

$$I_{finally} = 0$$

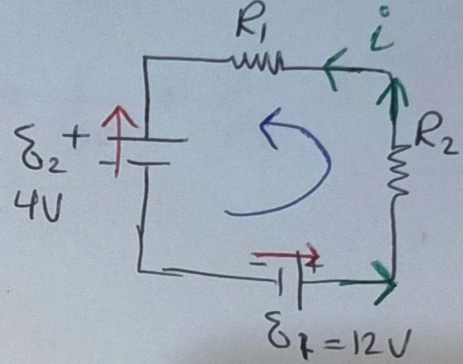
27–61 In the below figure, the ideal batteries have emfs $\mathcal{E}_1 = 12\ V$ and $\mathcal{E}_2 = 4.0\ V$. What are (a) the current, the dissipation rate in (b) resistor 1 ($4.0\ \Omega$) and (c) resistor 2 ($8.0\ \Omega$), and the energy transfer rate in (d) battery 1 and (e) battery 2? Is energy being supplied or absorbed by (f) battery 1 and (g) battery 2?

Supplied   absorbed

(a). Loop Rule ⇒

$$+\mathcal{E}_1 - iR_2 - iR_1 - \mathcal{E}_2 = 0$$

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{(12-4)\,V}{(4+8)\,\Omega} = 0.67\,A$$



(b) $P_{R_1} = i^2 R_1 = (0.67)^2(4)$

$i_1 = i_2 = i$

R_1 and R_2 in Series

$P_{R_1} = 1.78$ watt (thermal power)

(c) $P_{R_2} = i^2 R_2 = (0.67)^2(8) = 3.56$ watt (thermal power)

(d) $P_{\mathcal{E}_1} = i\mathcal{E}_1 = (0.67)(12) = 8.04$ watt

(e) $P_{\mathcal{E}_2} = -i\mathcal{E}_2 = -(0.67)(4) = -2.67$ watt

$8\,\frac{J}{s}$ Chemical energy lost by $\mathcal{E}_1$

Supplied to the circuit from $\mathcal{E}_1$

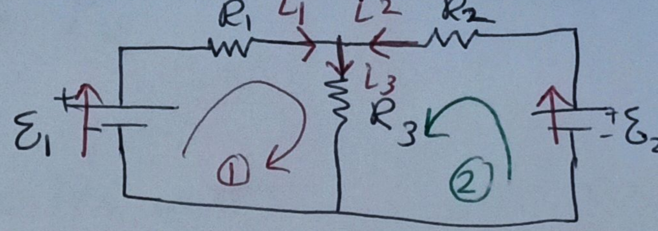
- Thermal energy produced by R_1 1.78 J/s
- Thermal energy produced by R_2 3.56 J/s
- chemical energy stored in $\mathcal{E}_2$ 2.67 J/s

8 J/s consumed in the circuit

Battery
- Supplier (the current in the same direction of emf $\mathcal{E}$
- Consumer (absorber) (the current in the opposite direction of $\mathcal{E}$

$\boxed{27-3}$ $\mathcal{E}_1 = 1.00V$, $\mathcal{E}_2 = 3.00V$, $R_1 = 4\Omega$, $R_2 = 2\Omega$, $R_3 = 5\Omega$

Both battries are ideal.



Junction Rule $\Rightarrow$ $i_1 + i_2 = i_3$

loop ① $\Rightarrow$ $\mathcal{E}_1 - i_1 R_1 - i_3 R_3 = 0$

loop ② $\Rightarrow$ $\mathcal{E}_2 - i_2 R_2 - i_3 R_3 = 0$

use $i_2 = i_3 - i_1$ $\Rightarrow$ $\mathcal{E}_2 - (i_3 - i_1) R_2 - i_3 R_3 = 0$

$$\boxed{\mathcal{E}_2 - i_3(R_2 + R_3) + i_1 R_2 = 0}$$

$\mathcal{E}_1 = i_1 R_1 + i_3 R_3$

$\mathcal{E}_2 = -i_1 R_2 + i_3 (R_3 + R_2)$

$1 = 4 i_1 + 5 i_3$

$(3 = -2 i_1 + 7 i_3) \times 2$

$1 = 4 i_1 + 5 i_3$

$+6 = -4 i_1 + 14 i_3$

Add $7 = 19 i_3$ $\Rightarrow$ $\boxed{i_3 = 0.368 A}$

use $1 = 4 i_1 + 5 i_3$ $\Leftrightarrow$ $1 = 4 i_1 + 5(0.368)$

$$\boxed{i_1 = -0.21 A}$$

$i_2 = i_3 - i_1 = 0.368 - -0.21 = 0.578A$

$i_1 = 0.21 A$ leftward $\leftarrow$

$i_2 = 0.578A$ leftward $\leftarrow$

$i_3 = 0.368 A$ downward $\downarrow$

$\Rightarrow$ Rate of energy dissipated in R $\equiv P_R$

$$P_{R_1} = i_1^2 R_1 = (0.21)^2(4) = 0.176 \text{ watt}$$

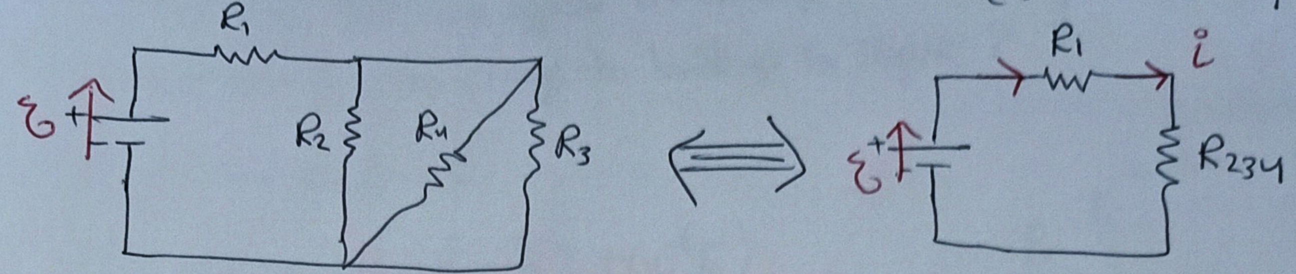
$$P_{R_2} = i_2^2 R_2 = (0.578)^2(2) = 0.668 \text{ watt}$$

$$P_{R_3} = i_3^2 R_3 = (0.368)^2(5) = 0.677 \text{ watt}$$

$$P_{\mathcal{E}_1} = i_1 \mathcal{E}_1 = (-0.21)(1) = \underbrace{-0.21 \text{ watt}}_{\text{Absorbed energy}}$$

$$P_{\mathcal{E}_2} = i_2 \mathcal{E}_2 = (0.578)(3) = \underbrace{+1.734 \text{ watt}}_{\text{supplied}}$$

27-8 $R_1 = 100\,\Omega$, $R_2 = R_3 = 50\,\Omega$, $R_4 = 75\,\Omega$, and the ideal battery has emf $\mathcal{E} = 12\,V$. (a) What is the equivalent resistance? What is i in (b) resistance 1, (c) resistance 2, (d) resistance 3, and (e) resistance 4?



R_2, R_3 and R_4 are all in parallel

$$\frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{50} + \frac{1}{50} + \frac{1}{75} = \frac{1.5+1.5+1}{75}$$

$$R_{234} = \frac{75}{4} = 18.75\,\Omega$$

R_1 and R_{234} are in Series

$$R_{eq} = R_1 + R_{234} = 100 + 18.75 = 118.75\,\Omega$$

$$i = \frac{\mathcal{E}}{R_1 + R_{234}} = \frac{12}{118.75} = 0.101\text{ A} \quad \text{in } R_1 \text{ and } R_{234}$$

$$\Rightarrow V_{R_{234}} = i R_{234} = (0.101)(18.75) = 1.89 \text{ volt}$$

R_2, R_3 and R_4 in parallel

$$V_{R_{234}} = V_{R_2} = V_{R_3} = V_{R_4}$$

$$1.89 \text{ volt} = i_2 R_2 = i_3 R_3 = i_4 R_4$$

$$\Rightarrow i_2 = \frac{1.89}{R_2} = \frac{1.89}{50} = 0.0378 = 37.8\text{ mA}$$

$$i_3 = \frac{1.89}{R_3} = 37.8\text{ mA}$$

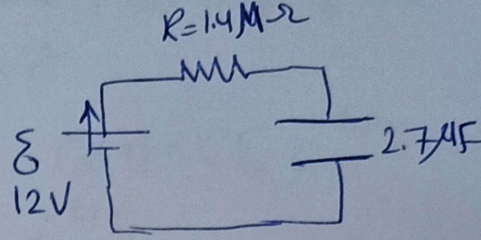
$$i_4 = \frac{1.89}{75} = 0.0252 = 25.2\text{ mA}$$

$i_2 + i_3 + i_4 = i$

Junction Rule

27-40 In an RC series circuit, emf $\mathcal{E} = 12.0V$, resistance $R = 1.4M\Omega$ and capacitance $C = 2.70\mu F$. (a) Calculate the time constant. (b) Find the maximum charge that will appear on the capacitor during charging. (c) How Long does it take for the charge to build up to $16.0\mu C$?

(a) τ = time constant



$$\tau = RC = (1.4\times10^{6}\,\Omega)(2.7\times10^{-6}F)$$

$$\boxed{\tau = 3.78\ sec}$$

(b) Q_{max} at $t\to\infty$, $Q(t) = C\mathcal{E}(1-e^{-t/\tau})$ charging

$$Q_{max}(t\to\infty) = C\mathcal{E} = (2.7\times10^{-6}F)(12V) = 32.4\times10^{-6}C$$

$$\boxed{Q_{max} = 32.4\mu C}$$

(c) Find t when $Q = 16\mu C$? [Build up $\Leftrightarrow$ charging]

$$Q(t) = Q_{max}(1-e^{-t/\tau})$$

$$16\times10^{-6} = 32.4\times10^{-6}(1-e^{-t/3.78})$$

$$0.494 = 1-e^{-t/\tau}$$

$$e^{-t/\tau} = 1-0.494 = 0.506$$

$$\frac{-t}{\tau} = \ln(0.506)$$

$$t = -\tau\ln(0.506) = -\tau(-0.68)$$

$$t = 0.68(3.78)$$

$$\boxed{t = 2.575\ sec}$$

27-18 A Capacitor with an initial potential difference of 80.0 V is discharged through a resistor when a switch between them is closed at $t=0$. At $t=10.0$ sec, the potential difference across the Capacitor is 1.00 volt. (a) What is the time constant of the circuit? (b) What is the potential difference across the capacitor at $t=17.0$ sec?

Discharging of the capacitor $V_c = \mathcal{E}\, e^{-t/\tau}$, $Q(t) = C\mathcal{E}\, e^{-t/\tau}$

(a) at $t=0$, $V_c = \mathcal{E} = 80.0$ Volts. $\Rightarrow V_c = 80\, e^{-t/\tau}$

at $t = 10\,s$, $V_c = 1$ volt $\Rightarrow$ $1 \text{ volt} = 80 \text{ volt}\; e^{-10/\tau}$

$$\frac{1}{80} = e^{-10/\tau} \iff \frac{-10}{\tau} = -\ln 80$$

$$\boxed{\tau = 2.28\,s}$$

$$\tau = \frac{10}{\ln(80)} = 2.28 \text{ sec}$$

(b) $V_c = 80\, e^{-t/2.28}$

$$V_c(t=17.0\text{sec}) = 80\, e^{-17/2.28} = 0.0462 \text{ Volts}$$

$$\boxed{V_c(t=17.0\text{sec}) = 46.2 \text{ mV}}$$

27-15 What multiple of the time constant τ gives the time taken by an initially uncharged capacitor in an RC series circuit to be charged to 89.0% of its final charge?

$$Q(t) = Q_0(1 - e^{-t/\tau}) \quad , \; Q_0 \equiv \text{maximum Charge} = C\mathcal{E}$$

$$0.89Q_0 = Q_0(1 - e^{-t/\tau})$$

$$0.89 = 1 - e^{-t/\tau}$$

$$e^{-t/\tau} = 1 - 0.89 = 0.11$$

$$-\frac{t}{\tau} = \ln(0.11) = -2.207$$

$$\boxed{t = 2.207\,\tau}$$

Find t for $Q = 99\%\, Q_0$

$$0.99\,Q_0 = Q_0(1 - e^{-t/\tau})$$

$$1 - 0.99 = e^{-t/\tau}$$

$$t = -\tau \ln(0.01) = 4.6\,\tau$$

27-30 A Capacitor with initial charge q_0 is discharged through a resistor. What multiple of the time constant τ gives the time the capacitor takes to lose (a) the first 25% of its charge and (b) 50% of its charge?

Discharging $\Rightarrow$ $Q = Q_0 e^{-t/\tau}$

(a) Lose the first 25% of its charge; the remaining charge 75% of its charge

$$Q(t) = Q_0 e^{-t/\tau}$$

$$0.75 Q_0 = Q_0 e^{-t/\tau}$$

$$-\frac{t}{\tau} = \ln(0.75) \Rightarrow t = -\tau \ln(0.75)$$

$\boxed{t = 0.29\tau}$ to lose the first 25% of its charge

(b) Lose 50% of its charge

$$0.5 Q_0 = Q_0 e^{-t/\tau}$$

$$\boxed{t = -\tau \ln(0.5) = 0.69\tau}$$

$$t_{\frac{1}{2}} = \tau \ln 2$$