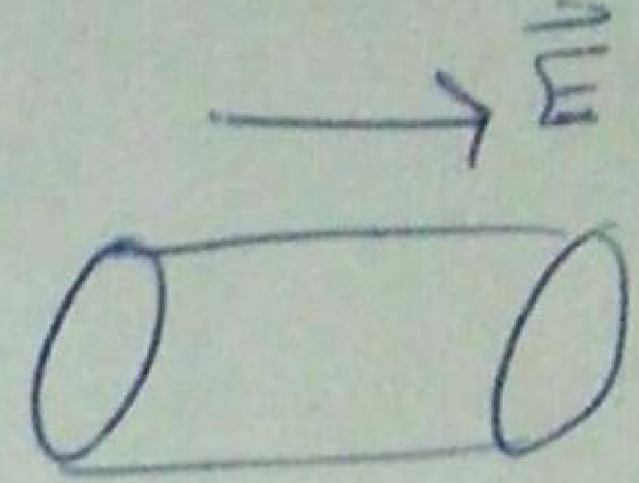
Ch. 23 Gauss' Law

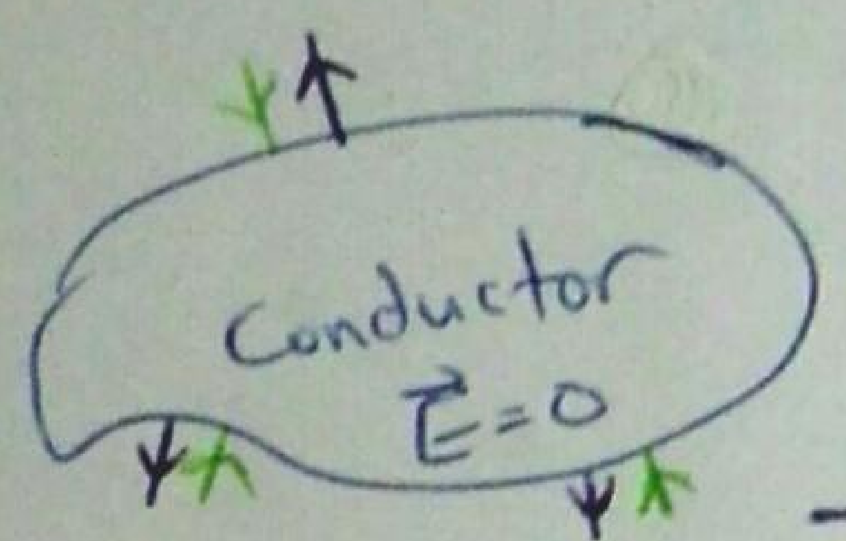
- $\varepsilon_0 \Phi = q_{enc}$

  $\Phi = \oint \vec{E} \cdot d\vec{A}$



<u>Note</u> $\vec{E}$ is uniform, net flux $\Phi_{net} = 0$

- Isolated conductor



$E = \frac{\sigma}{\epsilon_0}$ conducting surface

$\perp$ surface

– (q positive) $\Rightarrow$ outward <u>OR</u> Inward (q-negative)

- non-conducting sheet of charge $\boxed{E = \frac{\sigma}{2\epsilon_0}}$

- charged sphere (non-conducting, uniform volume charge distribution)

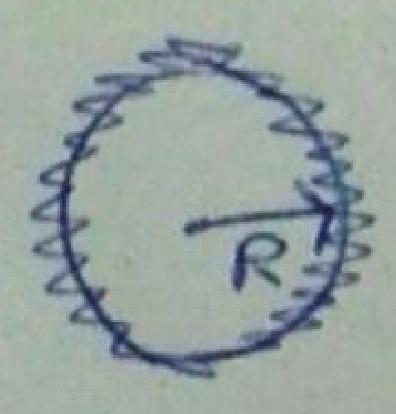
$$E_{sphere} = \begin{cases} kq \dfrac{r}{R^3} , & r \leq R \\ \dfrac{kq}{r^2} , & r > R \end{cases}$$

ρ is constant

$\rho = \frac{Q}{\text{Volume}}$

- spherical shell distribution

$$E = \begin{cases} \dfrac{Kq}{r^2} , & r \geq R \\ 0 , & r < R \end{cases}$$



Electric flux Φ

the electric flux Φ through a surface is the amount of electric field that pierces the surface

$$d\Phi = \vec{E} \cdot d\vec{A}$$

$d\vec{A} \equiv$ area vector , $d\vec{A} \perp$ surface area

- The total flux through a surface is given by

$$\Phi = \int_{\text{over the surface}} \vec{E} \cdot d\vec{A}$$

→ closed surface $\Phi = \oint \vec{E} \cdot d\vec{A}$

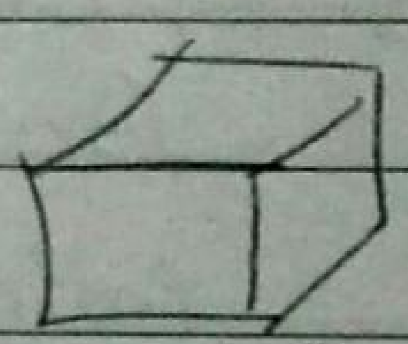
⇛ Gauss' Law relates the electric field at points on a (closed) Gaussian surface to the net charge enclosed by that surface.

$$\Phi = \int \vec{E} \cdot d\vec{A} \quad \text{total flux}$$

⇛ An inward piercing field is negative flux ($\theta = 180°$)
An outward piercing field is positive flux ($\theta = 0$)
A skimming field is zero flux $\theta = 90$

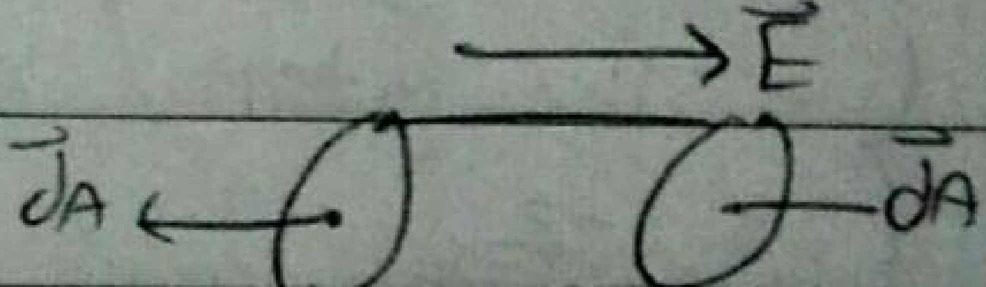
$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad \text{net flux}$$

$$[\Phi] = N \cdot m^2/C$$



uniform $\vec{E}$

net flux = zero



$\Phi = -EA$ $\Phi = EA$

net flux = 0

* Gauss' Law

Gauss' Law relates the net flux Φ penetrating a closed surface to the net charge q_{enc} enclosed by the surface.

$$\epsilon_0 \Phi = q_{enc} = \epsilon_0 \oint \vec{E} \cdot d\vec{A} \quad \text{(space, air)}$$

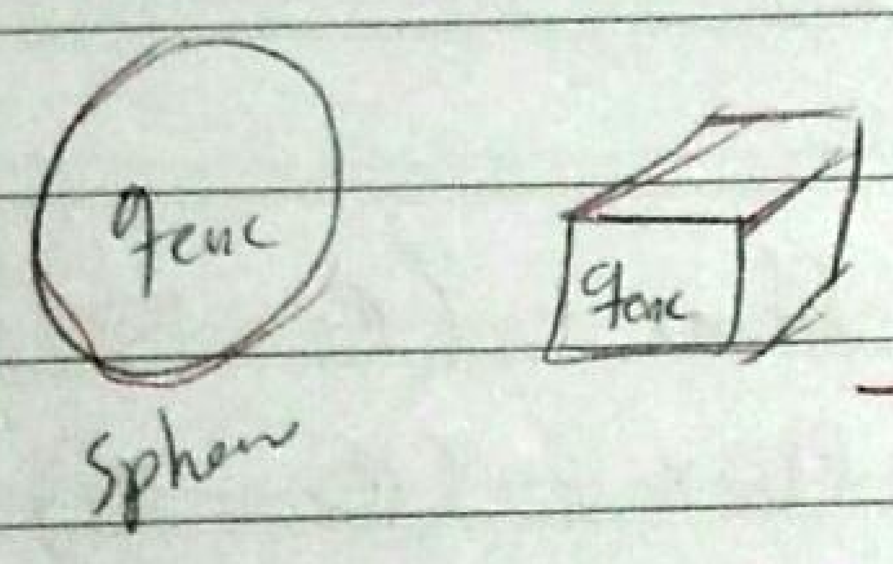
- q_{enc} ≡ Algebric sum of all the enclosed +ve and −ve charge (0, −ve, +ve)

⇒ q_{enc} is +ve, the net flux is outward

q_{enc} is −ve, the net flux is inward.

(net enclosed charge 8> magnitude + sign)

- $\vec{E}$ from all charges both those inside and those outside the Gaussian surface.



⇒ net flux is the same

— Gaussian surface.

* A charged isolated conductor

- An excess charge on an isolated conductor is located entirely on the outer surface of the conductor. None of the excess charge will be found within the body of the conductor.

- The internal electric field of a charged, isolated conductor is zero, and the external field (at near by points) is perpendicular to the surface and has a magnitude that depends on the surface charge density σ ⇒ $E = \dfrac{\sigma}{\epsilon_0}$ (conducting surface)

$E \perp$ surface of the conductor

$$\Rightarrow \epsilon_0 \Phi = q_{enc} \Rightarrow \epsilon_0 E A = \sigma A$$

- The sign of the charge gives the direction of $\vec{E}$
  - → q (−ve), $\vec{E}$ toward the conductor
  - → q (+ve), $\vec{E}$ directed away from the conductor.

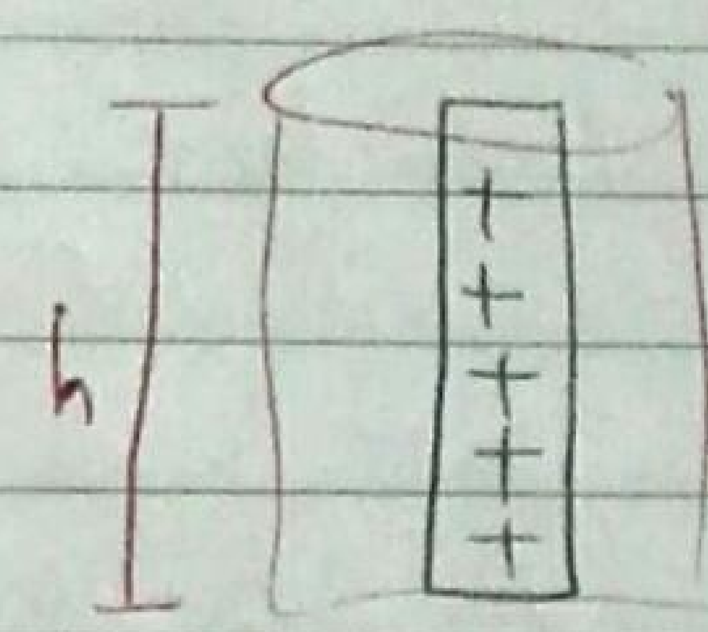
✱ Applying Gauss' law: Cylindrical symmetry

- The electric field at a point near an infinite line of charge (or charged rod) with uniform linear charge density λ is perpendicular to the line and has magnitude

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$r \equiv$ perpendicular distance from the line to the point

$$\left(\epsilon_0 \Phi = q_{enc} = \epsilon_0 E (2\pi r) h = \lambda h \right)$$

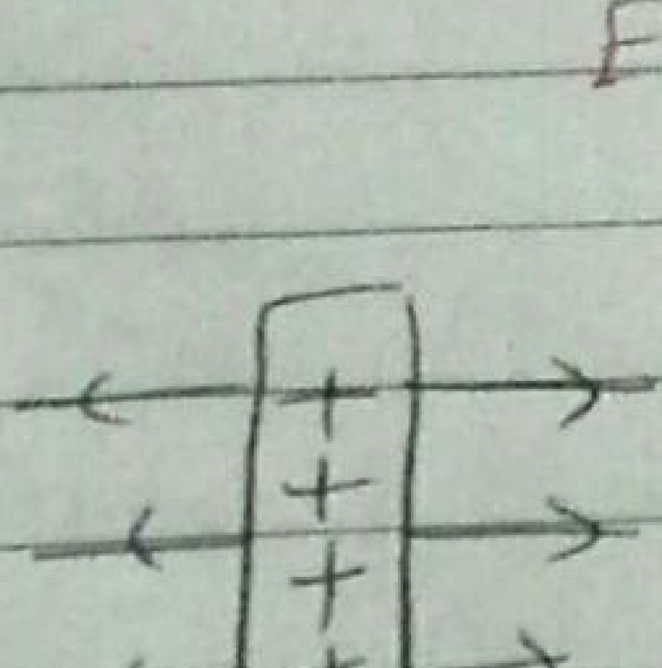


- radially outward from the line of charge if the charge is +ve
- radially inward if it is negative.

✱ Applying Gauss' law: Planar Symmetry

- The electric field due to an infinite nonconducting sheet with uniform surface charge density σ is perpendicular to the plane of the sheet and has magnitude

$$E = \frac{\sigma}{2\epsilon_0} \quad \text{(non-conducting sheet of charge)}$$



uniform $\vec{E}$

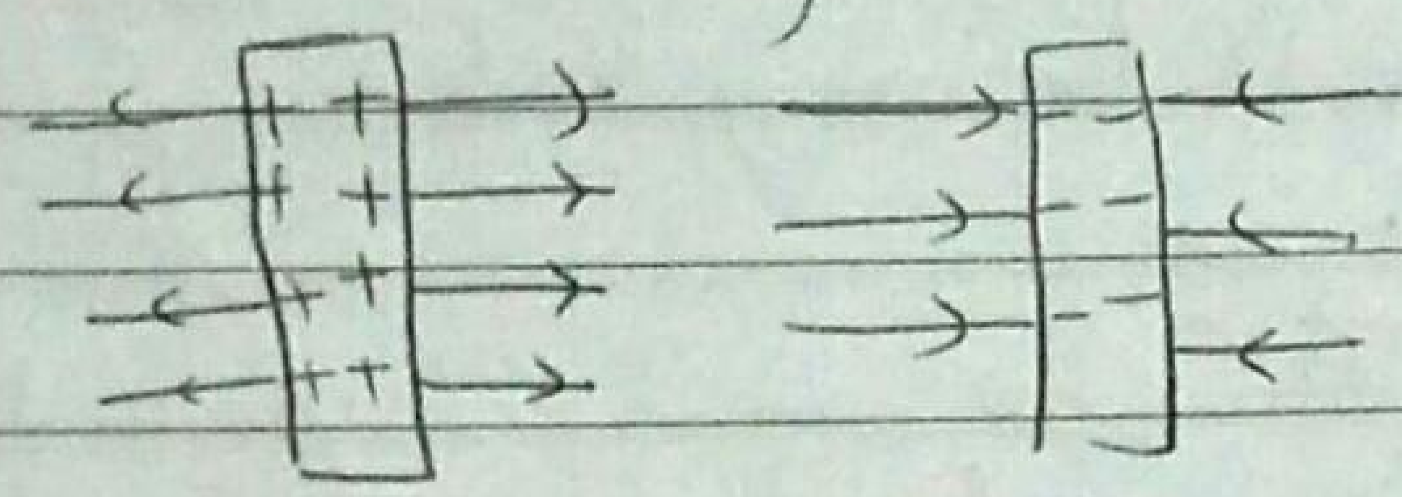
قيم ثابتة بغض النظر عن الموقع

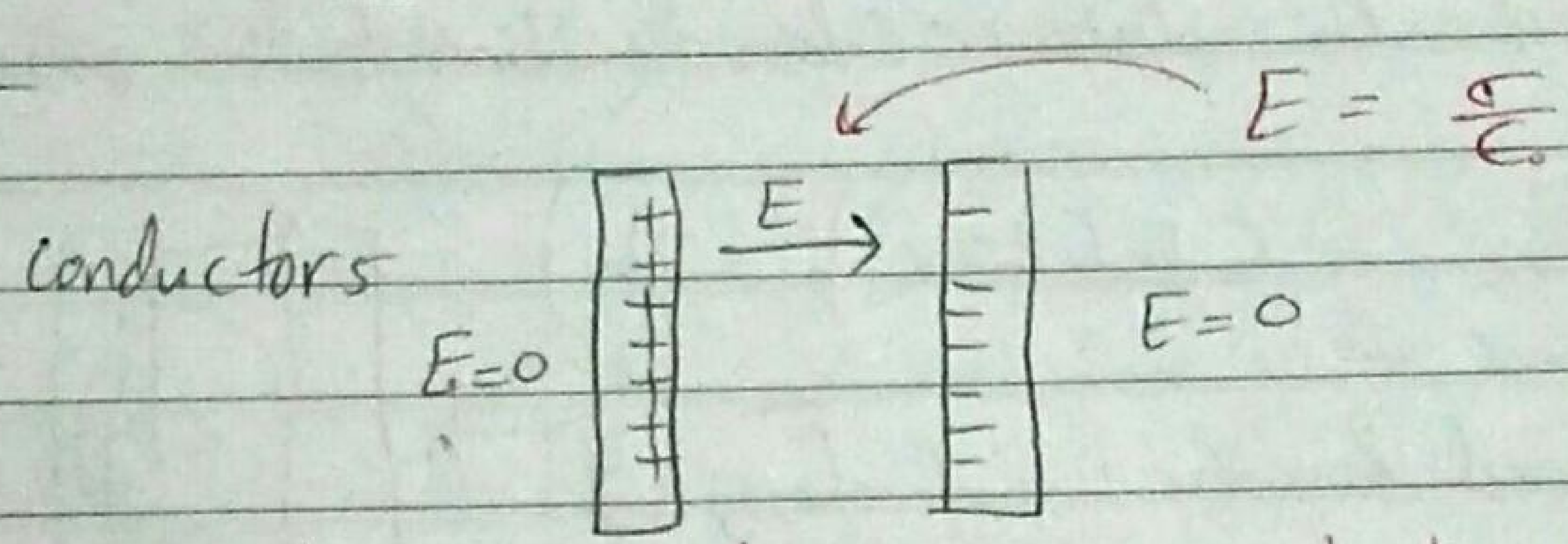
- The external electric field just outside the surface of an isolated charged conductor with surface charge density σ is perpendicular to the surface and has magnitude

$$E = \frac{\sigma}{\epsilon_0} \quad (\text{External, charged conductor})$$

$$E = 0 \quad (\text{Inside the conductor})$$

Two conducting plates





All the excess charge moves on to the interface of the plates

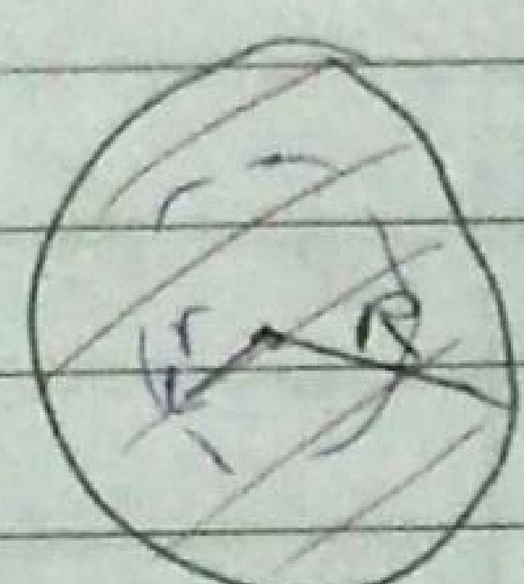
* Applying Gauss' Law: Spherical Symmetry

$$E = kq \begin{cases} \frac{1}{r^2}, & r \geq R \ (\text{outside spherical shell}) \\ \frac{r}{R^3}, & r < R \ (\text{inside sphere of charge}) \end{cases}$$

$\boxed{\text{I}}$ Spherical distribution

$\rightarrow r < R$

$$\rho_{constant} = \frac{q}{\frac{4}{3}\pi R^3} = \frac{q'}{\frac{4}{3}\pi r^3} \quad \text{"uniform distribution"}$$



volume charge distribution

$$q' = \frac{q r^3}{R^3}$$

$\bullet\ E = \frac{kq'}{r^2}, \quad r \le R$

$= \frac{kqr^3}{r^2 R^3} = \frac{kqr}{R^3}$

$\bullet$ field at $r \le R$ $\quad E = \frac{q}{4\pi\epsilon_0 R^3} r \quad$ (uniform charge)

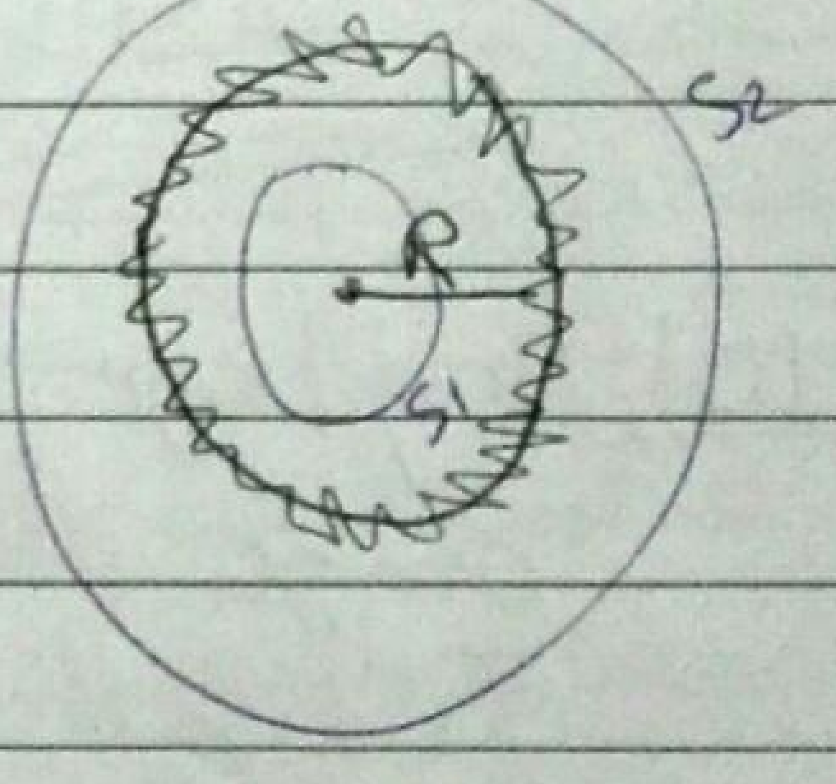
$r > R, \quad E = \frac{kq}{r^2}$

$$\boxed{E_{\text{spherical distribution}} = \begin{cases} kq\frac{r}{R^3}, & r \le R \\ \frac{kq}{r^2}, & r > R \end{cases}}$$

[2] Spherical shell distribution

$\Rightarrow$ Shell theorem: ① A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.

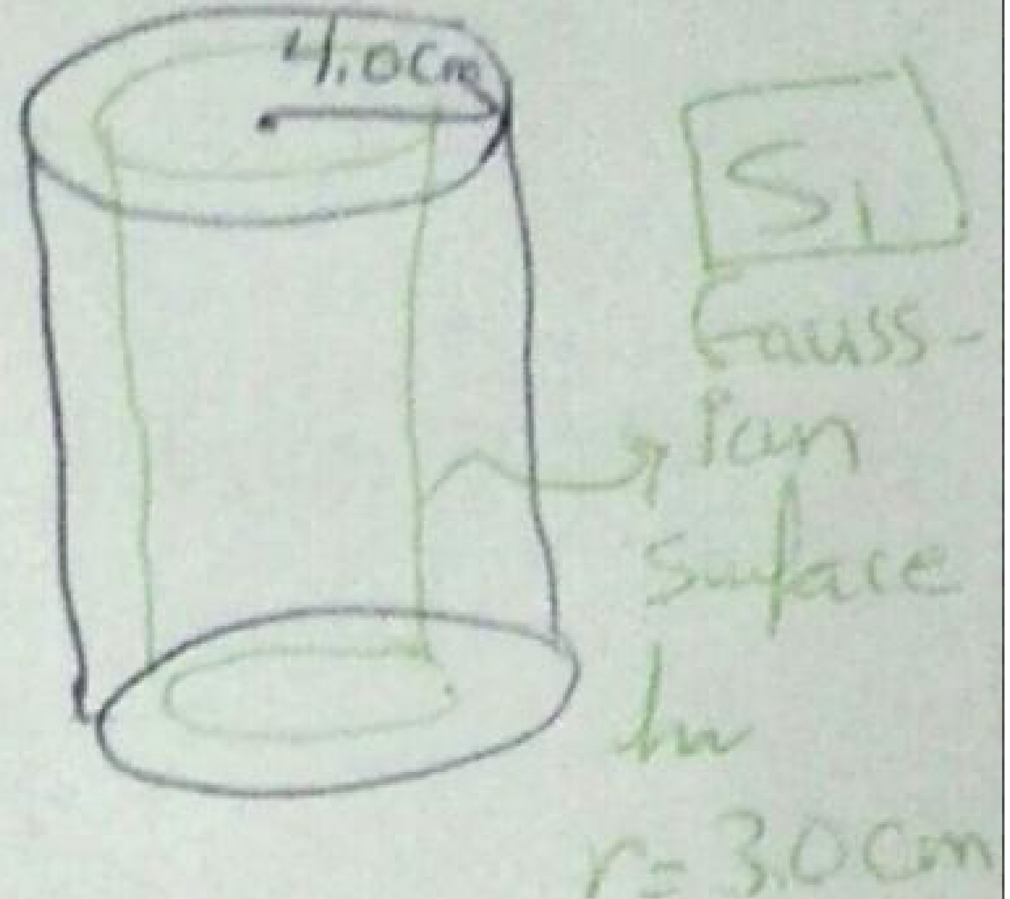
② If a charged particle is located inside a shell of uniform charge, there is no electrostatic force on the particle from the shell.



$$\Rightarrow E = \begin{cases} \frac{kq}{r^2} & \text{spherical shell, } r \ge R \\ 0 & \text{spherical shell, } r < R \end{cases}$$

$S_1 \Rightarrow$ this surface encloses no charge, $q_{en} = 0$

$\boxed{23\text{-}18}$ A Long, non-conducting, solid cylinder of radius 4.0 cm has a non-uniform volume charge density ρ that is a function of radial distance r from the cylinder axis $\rho = \grave{A} r^2$, $\grave{A} = 6.3\,\mu C/m^5$. What is the magnitude of the electric field at r = 3.0 cm and r = 5.0 cm ?



$\Rightarrow$ non-conducting
non-uniform volume charge density

a) $r = 3.0$ cm

use Gauss' Law

$$\epsilon_0 \Phi = q_{enc} \quad , \quad \Phi = \oint \vec{E} \cdot d\vec{A}$$

use $q_{enc} = \int \rho \, dV$, volume - charge density $\rho = \frac{Q}{V}$

cylinder $\Rightarrow \begin{cases} V = \pi r^2 h \\ dV = 2\pi r h dr \end{cases}$

$$q_{enc} = \int \grave{A} r^2 \, 2\pi r h \, dr$$

$$q_{enc} = \frac{2\pi h \grave{A} r^4}{4}$$

$$\boxed{q_{enc} = \frac{\pi h \grave{A} r^4}{2}}$$

• Applying Gauss' Law to S_1

$$\epsilon_0 \Phi = q_{enc}$$

$$\epsilon_0 E (2\pi r h) = \frac{\pi h \grave{A} r^4}{2}$$

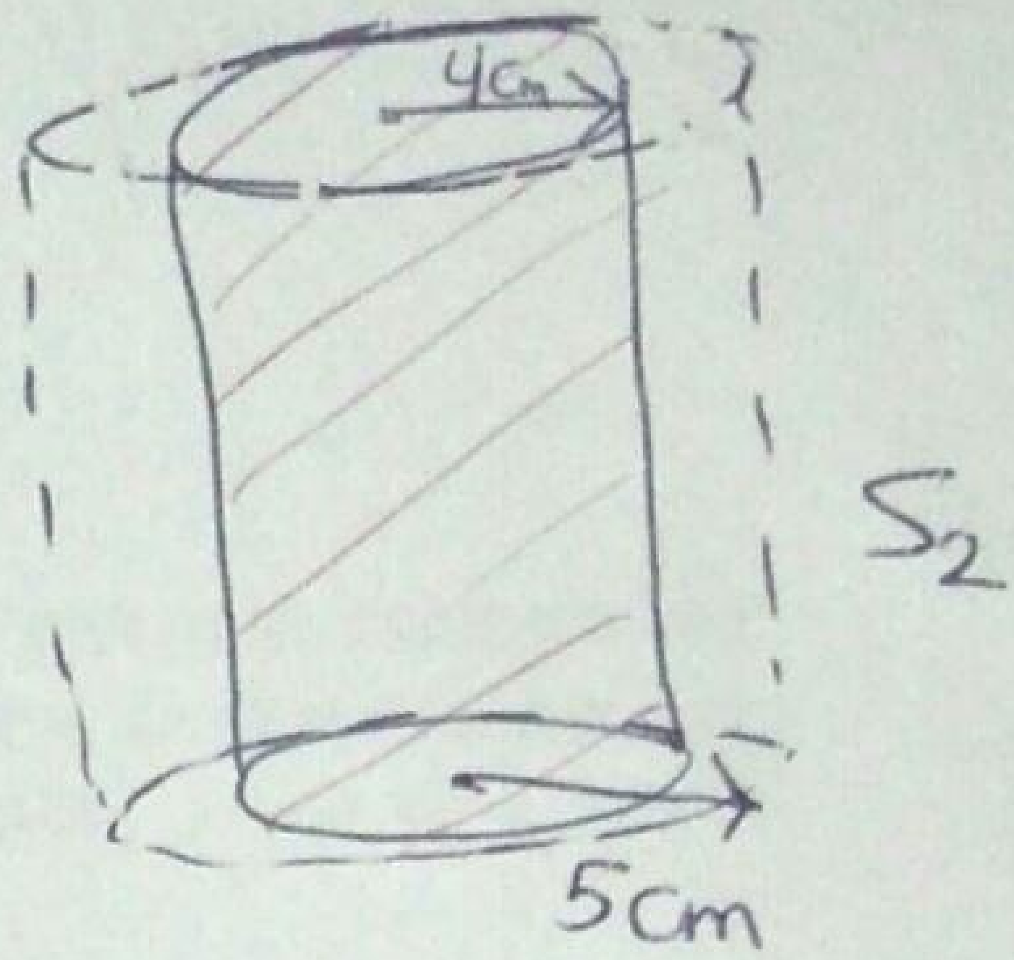
$$E = \frac{\grave{A} r^3}{4 \epsilon_0}$$

$$E(r=3\text{cm}) = \frac{A' r^3}{4\epsilon_0} = \frac{6.3\times10^{-6}(0.03)^3}{8.85\times10^{-12}\times4} \sim 4.8\,N/C$$

(b) E at $r = 5.0$ cm > 4.0 cm = radius of the cylinder

- Applying Gauss' Law to S_2

$$\epsilon_0 \Phi = q_{enc}$$



$$q_{enc} = \frac{\pi h A'}{2} R^4$$

"No charge at $r > R \equiv$ Cylinder radius"

$\Rightarrow$ Gauss' Law $\epsilon_0 \Phi = q_{enc}$

$$\epsilon_0 E(2\pi r h) = \frac{\pi h A' R^4}{2}$$

$$E = \frac{A' R^4}{4\epsilon_0 r} = \frac{6.3\times10^{-6}\times(4\times10^{-2})^4}{4\times8.85\times10^{-12}\times5\times10^{-2}}$$

$$\boxed{E = 9.1\,N/C}$$

$\Rightarrow$ Charged Rod (λ- linear charge density)

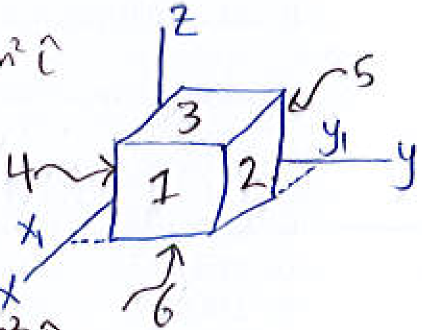
$$E = \frac{\lambda}{2\pi\epsilon_0 r}, \quad \lambda = \frac{q_{enc}}{h} = \frac{\pi h A' R^4}{2h}$$

$$E = \frac{\pi A' R^4}{4\pi\epsilon_0 r} = \frac{A' R^4}{4\epsilon_0 r}$$ #

Scanned by TapScanner

23-23 A closed Gaussian surface in the shape of a cube of edge Length 2.00 m, with one corner at $x_1 = 5.00$ m, $y_1 = 4.00$ m. the cube lies in a region where the electric field vector is given by $\vec{E} = +23.0\hat{i} - 2.00y^2\hat{j} - 16.0\hat{k}$ N/C, with y in meters. What is the net charge contained by the cube?

$q_{enc} = \varepsilon_0 \Phi$, $\Phi = \oint \vec{E} \cdot d\vec{A}$



$\Rightarrow$ Cube face (1) (Located at $x = 5.00$ m), $d\vec{A} = 4\,m^2\,\hat{i}$

$\Phi_1 = (+23.0\hat{i} - 2.00y^2\hat{j} - 16.0\hat{k}) \cdot 4\hat{i}$

$= +92$ N.m²/C

$\Rightarrow$ cube face (2), Located at $y = 4.00$ m, $d\vec{A} = 4\,m^2\,\hat{j}$

$\Phi_2 = -2(4)^2 \times 4 = -128$ N.m²/C

$\Rightarrow$ cube face (3), Located at $z = 2.0$ m, $d\vec{A} = 4\,m^2\,\hat{k}$

$\Phi_3 = -16 \times 4 = -64$ N.m²/C

$\Rightarrow$ cube face (4), Located at $y = 2$ m, $d\vec{A} = -4\,m^2\,\hat{j}$

$\Phi_4 = -2(2)^2 \times -4 = +32$ N.m²/C

$\Rightarrow$ cube face (5), Located at $x = 3.0$ m, $d\vec{A} = -4\,m^2\,\hat{i}$

$\Phi_5 = 23 \times -4 = -92$ N.m²/C

$\Rightarrow$ cube face (6), Located at $z = 0$, $d\vec{A} = -4\,m^2\,\hat{k}$

$\Phi_6 = -16 \times -4 = +64$ N.m²/C

$\Rightarrow \Phi_{total} = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6 = -96\ \frac{N.m^2}{C}$

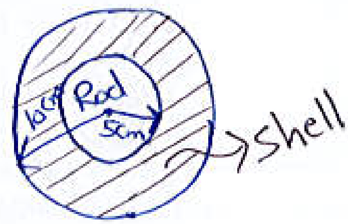
$q_{enc} = \varepsilon_0 \cdot \Phi = 8.85 \times 10^{-12} \times -96 = -8.496 \times 10^{-10}$ C

$$\boxed{q_{enc} = -0.85\,nC}$$

23-4 A charge of uniform linear density 1.5 nC/m is distributed along a long, thin, nonconducting rod. The rod is coaxial with a long conducting cylindrical shell (inner radius = 5.0 cm, outer radius = 10.0 cm). The net charge on the shell is zero. a) What is the magnitude of the electric field 15.0 cm from the axis of the shell? What is the surface charge density on the b) inner and c) outer surface of the shell?

⇒ a) $E = \frac{\lambda}{2\pi\epsilon_0 r}$ (Line of charge)

$$E = \frac{1.5\times10^{-9}}{2\times\pi\times8.85\times10^{-12}\times0.15} = 180\ N/C$$



b) the net charge on the shell is zero and the shell is conducting. So the electric field is zero inside the conductor

⇒ inner surface has negative linear density $(-\lambda)$
outer surface has positive linear density $(+\lambda)$
where λ is the linear density of the Rod = 1.5 nC/m

* Surface charge density on the inner of the shell

$$\sigma_{inner} = \frac{-q}{area} = \frac{-q}{2\pi r L} = \frac{-\lambda}{2\pi r}$$

$$\boxed{\lambda = \frac{q}{L}}$$

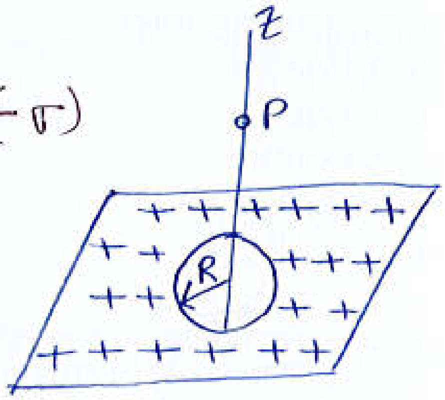
$$\sigma_{inner} = \frac{-1.5\times10^{-9}}{2\times\pi\times0.05} = -4.78\frac{nC}{m^2}$$

* Surface charge density of the outer of the shell

$$\sigma_{outer} = \frac{+q}{Area} = \frac{+q}{2\pi r L} = \frac{+\lambda}{2\pi r}$$

$$\sigma_{outer} = \frac{+1.5\times10^{-9}}{2\pi\times0.1} = +2.39\frac{nC}{m^2}$$

23-8 A small circular hole of radius $R = 1.30$ cm has has been cut in the middle of an infinite, flat, non-conducting surface that has uniform charge density $\sigma = 4.50 pC/m^2$. A z axis, with its origin at the hole's center, is perpendicular to the surface. In unit vector notection, what is the electric field at point P at $Z = 2.56$ cm?



Let the circular hole has charge density $(-\sigma)$

$\Rightarrow \vec{E}_1$ (infinite, flate, non-Conducting sheet) $= \frac{\sigma}{2\varepsilon_0} \hat{k}$

$\Rightarrow \vec{E}_2$ (circulu hole) $= \frac{-\sigma}{2\varepsilon_0}\left[1 - \frac{Z}{\sqrt{Z^2+R^2}}\right]\hat{k}$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\sigma Z}{2\varepsilon_0}\left[\frac{1}{\sqrt{Z^2+R^2}}\right]\hat{k}$$

$$\vec{E} = \frac{4.5 \times 10^{-12} \times 2.56 \times 10^{-2}}{2 \times 8.85 \times 10^{-12}}\left[\frac{1}{\sqrt{(2.56\times10^{-2})^2 + (1.3\times10^{-2})^2}}\right]\hat{k}$$

$$\boxed{\vec{E} = 0.227 \frac{N}{C}\hat{k}}$$

23-50 Flux and conducting shells. A charged particle is held at the center of two concentric conducting spherical shells. The below figure gives the net flux Φ through a Gaussian sphere centered on the particle, as a function of the radius r of the sphere. The scale of the vertical axis is set by $\Phi_s = 10 \times 10^5 \ N.m^2/C$. What are a) the charge of the ~~central~~ central particle and the net charges of shell A and B?

$\Rightarrow$ Gauss' Law $q_{enc} = \varepsilon_0 \Phi$

a) $\Phi = -18 \times 10^5 \ \frac{N.m^2}{C}$ for small r "Gaussian surface surrounds only the particle"

$q_{enc} = 8.85 \times 10^{-12} \times -18 \times 10^5 = -1.593 \times 10^{-5} \ C$

$$\boxed{q_{central\ particle} = -16 \mu C}$$

b) $\Phi = +8.0 \times 10^5 \ \frac{N.m^2}{C} \Rightarrow q'_{enc} = +7.08 \mu C$ "Gaussian surface surrounds the particle and shell A"

$q_{shell\ A} = q'_{enc} - q_{central\ particle}$

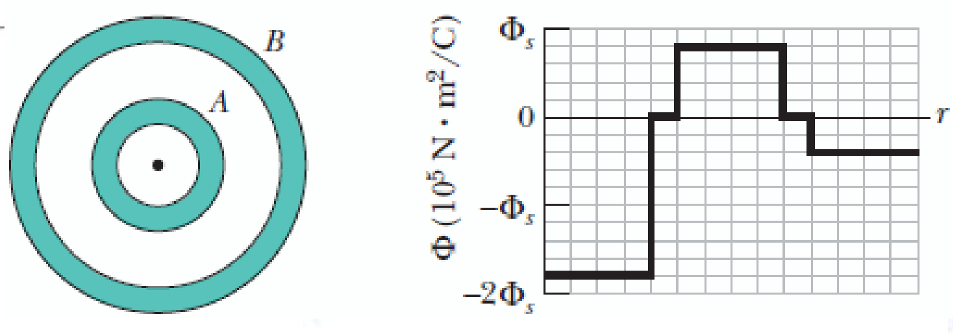
$= (+7.08 + 16) \mu C = \boxed{23.08 \mu C = q_{shell\ A}}$

c) $\Phi = -4.0 \times 10^{-5} \ \frac{N.m^2}{C} \Rightarrow q''_{enc} = -3.54 \mu C \Rightarrow$ {Gaussian surface surrounds the particle and shell A and B

$q''_{enc} \equiv$ total enclosed charge

$q_{shell\ B} = (-3.54 - 23.08 - -16) \mu C$

$$\boxed{q_{shell\ B} = -10.62 \mu C}$$



[6] Magnitude of the electric field inside and outside a sphere with a positive charged distributed uniformly throughout its volume. The scale of the vertical axis is set by

$E_s = 10 \times 10^7 \, N/C$

a) What is the charge on the sphere?

$E(r = 2\,cm \equiv \text{radius of sphere}) = 10 \times 10^7 \, N/C$

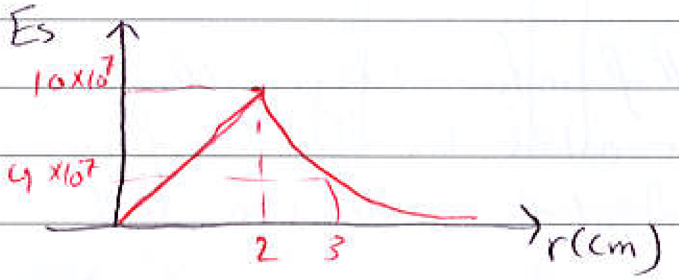
$E = K \frac{q}{R^2} \Rightarrow q = \frac{10 \times 10^7 \times (2 \times 10^{-2})^2}{9 \times 10^9}$

$q \sim 4.44 \, \mu C$

b) What is the field magnitude at $r = 8.0\,m$?

$r = 8.0\,m$ outside the sphere

$E = \frac{kq}{r^2} = \frac{9 \times 10^9 \times 4.44 \times 10^{-6}}{(8)^2} = 6.25 \times 10^2 \, N/C$



$r \le R$, E is a straight line

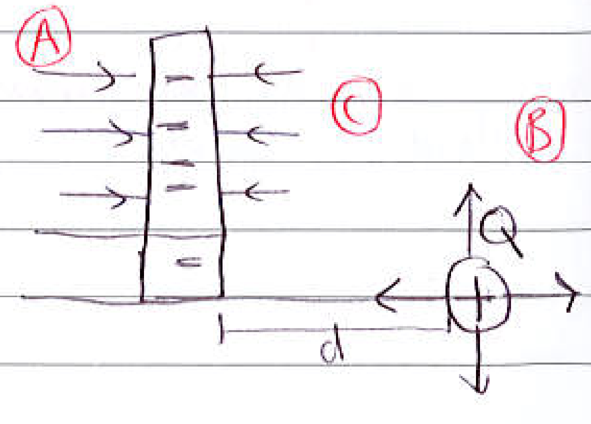
$E = \frac{kq}{R^3} r$

[10] Very Large non-conducting sheet that has a uniform surface charge density $\sigma = -2.00\,\mu C/m^2$ with a particle of charge $Q = 8\,\mu C$ at distance d from the sheet. Both are fixed in place. if $d = 0.2\,m$

a) at what +ve and b) -ve coordinate on x-axis (other than infinity) is the net $\vec{E}_{net}$ of the sheet and particle is zero?

$\vec{E}_{net}$ = zero in A or B regions

impossible $\vec{E}_{net}$ = zero in C region



(5)

$$\vec{E}_{shell} = \frac{\sigma}{2\epsilon_0} = \frac{-2\times10^{-6}}{2\epsilon_0} = \frac{-10^{-6}}{\epsilon_0} \; \frac{C}{m^2}$$

$$\Rightarrow \vec{E}_{sheet} = \vec{E}_{sphere}$$

$$\frac{\sigma}{2\epsilon_0} = \frac{kQ}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x^2} \quad , Q = 8\mu C$$

$$x^2 = \frac{2}{\pi} \quad , \quad x = \mp\sqrt{\frac{2}{\pi}} = \mp 0.798$$

$$d = 0.2\,m \quad , \quad |\bar{0}.798| > 0.2$$

So a) 0.798 m

b) −0.798 m

c) if d = 0.950 m at what coordinate on the x-axis is $\vec{E}_{net}$ = zero?

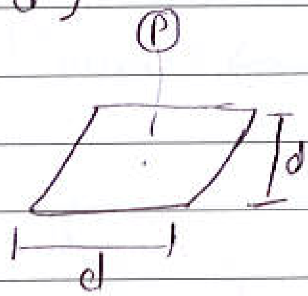
if d = 0.950

$d > |-0.798|$

$0.950 > 0.798$ it is not possible to get a negative coordinate.

So the only coordinate in x-axis is $\underline{\underline{0.798m}}$

[17] A proton is a distance $\frac{d}{2}$ directly above the center of a square of side d. What is the electric flux through a square? (Think of the square as one of a cube with edge d)



$$\Phi = \Phi_{total}/6$$

$$\Phi_{total,\,cube} = q/\epsilon_0 = 1.6\times10^{-19}/8.85\times10^{-12}$$
$$= 1.807\times10^{-8}\ \frac{N.m^2}{C}$$

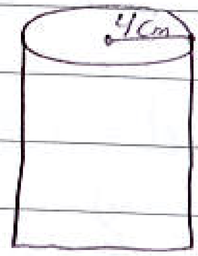
$$\Phi_{\text{one surface of cube}} = 3.01\times10^{-9}\ \frac{N.m^2}{C}$$

[18] A long, nonconducting, solid cylinder of radius 4.0 cm has a non uniform volume charge density ρ that is a function of radial distance r from the cylinder axis

$$\rho = Ar^2 \quad , A = 6.3\ \mu C/m^5$$

What is the magnitude of the electric field at $r = 3$ cm and 5 cm ?

$$\Phi = \frac{q_{enc}}{\epsilon_0}$$



$R = 0.04$ m

$\rho = Ar^2 \quad , A = 6.3\ \mu C/m^5$

$\ast \quad r = 3.00$ cm $= 0.03$ m

$V = \pi r^2 h$

$dv = 2\pi r h\, dr$

$$Q_{total} = \int_0^R \rho\, dV$$

$$Q = \int \rho\, dV = \int A r^2\, 2\pi r h\, dr = 2\pi h A \frac{r^4}{4}$$

$$\text{if } Q_{total} = \int_0^R \rho\, dV = 2\pi h A \frac{R^4}{4} = 2.5 \times 10^{-11} h$$

$$\Rightarrow q_{enc} = \frac{\pi h A r^4}{2}$$

$\rightarrow$ Gauss' law $\Rightarrow \quad \Phi = \frac{q_{enc}}{\epsilon_0} = E(A)$

$$E = \frac{\pi h A r^4}{2\epsilon_0} \frac{1}{2\pi r h}$$

$$E = \frac{A r^3}{4\epsilon_0}$$

$$E(3cm) = \frac{6.3\times10^{-6}(0.03)^3}{8.85\times10^{-12}\times 4} \sim 4.8\ N/C$$

- $r = 5cm\ ,\quad r > R$

$$q_{total} = 2.5\times10^{-11}\ h$$

$$E(2\pi r h) = \frac{q_{tot}}{\epsilon_0} \Rightarrow \frac{2.5\times10^{-11}\ h}{2\pi r h \epsilon_0}$$

$$E = 8.496\ N/C$$

$$\lambda = \frac{q}{h} = 2.5\times10^{-11}$$

$$\lambda = \frac{\pi}{2} A r^4\ ;\ (r=R)\ ,\quad \lambda = 2.5\times10^{-11}\ C/m$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}\ \#$$

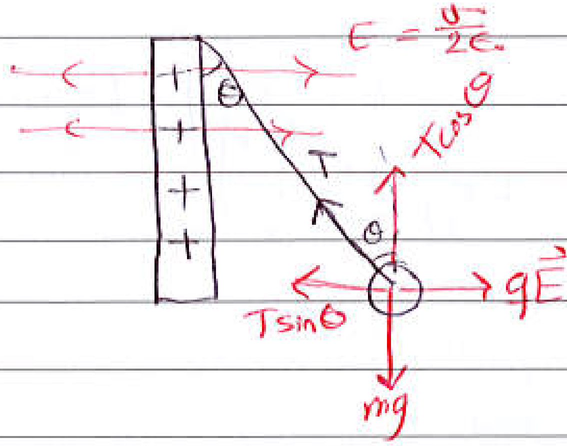
$\boxed{21}$ A small non conducting ball of mass $m = 7.3\,mg$ and charge $q = 2\times10^{-8}C$ (distributed uniformly through its volume) hangs from an insulating thread that makes an angle $\theta = 30$ with a vertical, uniformly charged non-conducting sheet. Considering the gravitational force on the ball and assuming the sheet extends far vertically and into and out ~~to~~ of the page, calculate the surface charge density σ of the sheet? $\sigma = ??$

$$T\sin\theta = qE$$

$$T\cos\theta = mg \Rightarrow T = \frac{mg}{\cos\theta}$$

$$\frac{mg}{\cos\theta}\sin\theta = qE$$

$$E = \frac{mg}{q}\tan\theta = \frac{\sigma}{2\epsilon_0}$$



$$\sigma = \frac{mg \tan\theta \, 2\epsilon_0}{q}$$

$$= \frac{7.3\times10^{-6} \times 9.8 \times \tan 30 \times 2 \times 8.85 \times 10^{-12}}{2\times10^{-8}}$$

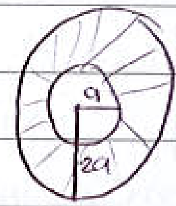
$$= 3.66\times10^{-8} \; C/m^2 = 37 \; nC/m^2$$

$\boxed{28}$ A spherical shell with uniform volume charge density $\rho = 1.56 \; nC/m^3$, inner radius $a = 10.0$ cm and outer radius $b = 2.00\, a$. What is the magnitude of the $\vec{E}$ at radial distances ?

a) $r = 0$

spherical shell $E = \begin{cases} \frac{kq}{r^2} & r \geq R \\ 0 & r < R \end{cases}$

$E(r=0) = \text{zero}$ inside the spherical shell.



b) $r = a/2.00$, $E(r = \frac{a}{2}) = \text{zero}$

$\frac{a}{2} < a$ inner radius

c) $r = a$, $E(r=a) = \text{zero}$

For $a \leq r \leq b \Rightarrow E = \frac{k\, q_{enc}}{r^2}$, $\rho = \frac{q_{enc}}{V}$

$$E = \frac{k\rho}{r^2}\left[\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right], \; a \leq r \leq b \qquad q_{enc} = \rho V$$

$$\text{at } r = a \Rightarrow E = \frac{k\rho}{a^2}\left[\frac{4\pi a^3}{3} - \frac{4\pi a^3}{3}\right] = \text{zero}$$

d) $r = 1.5a$

$$E = \frac{k\rho}{r^2}\left[\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right] = \frac{\rho}{4\pi\epsilon_0 r^2}\,\frac{4\pi}{3}\left[r^3 - a^3\right]$$

$$E = \frac{\rho}{3\epsilon_0 r^2}\left[r^3 - a^3\right]$$

$$E(r=1.5a) = \frac{\rho}{3\epsilon_0 (1.5)^2 a^2}\left[a^3(1.5)^3 - a^3\right] = 6.2 \text{ N/C}$$

e) $r = b = 2a$

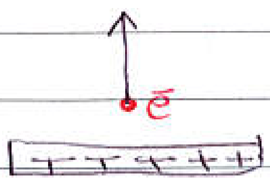
$$E = \frac{\rho}{3\epsilon_0 (2a)^2}\left[(2a)^3 - a^3\right] = \frac{7\rho a}{3\epsilon_0 4} = 10.28 \text{ N/C}$$

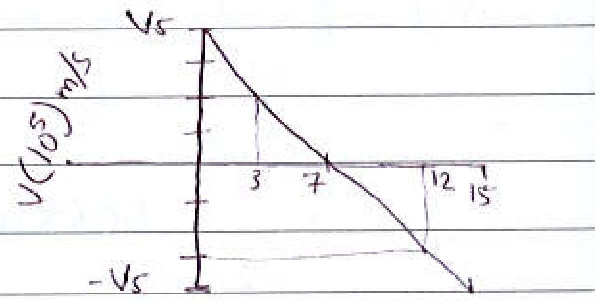
f) $r = 3b$, $r \geq b$ $\quad E = \frac{q_{total}}{4\pi\epsilon_0 r^2}$, $r = 6a$

$$E = \frac{\rho}{3\epsilon_0}\left[\frac{b^3 - a^3}{r^2}\right] = \frac{\rho}{3\epsilon_0}\left(\frac{(2a)^3 - a^3}{(6a)^2}\right)$$

$$= \frac{\rho}{3\epsilon_0}\,\frac{7a^3}{36a^2} = \frac{7\rho a}{3\times 36\,\epsilon_0} = 1.14 \text{ N/C}$$

$\boxed{2}$ An electron is shot directly away from a uniformly charged plastic sheet, at speed $v_s = 1.6\times10^5$ m/s. The sheet is nonconducting flat and very large.





Sheet's surface charge density?

electron's vertical velocity component versus t until the return to launch point.

a = slope (V vs t) $(0, 1.6\times10^5)$, $(7, 0)$

$$a = \frac{0 - 1.6\times10^5}{(7-0)\times10^{-12}} = -2.29\times10^{16}\ m/s^2$$

$$F = qE = ma$$

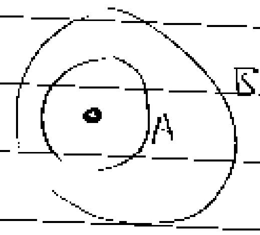
$$q\frac{\sigma}{2\epsilon_0} = ma$$

$$\sigma = \frac{2ma\,\epsilon_0}{q} = \frac{2(9.1\times10^{-31}kg)(2.29\times10^{16})(8.85\times10^{-12})}{1.6\times10^{-19}}$$

$$= 2.3\times10^{-6}\ C/m^2$$

[54] Flux and nonconducting shells. A charged particle is suspended at the centre of two concentric spherical shells that are very thin and made of non conducting material.

a) What is the charge of the central particle?
b) What are the net charges of shell A and B?



$$\epsilon_0 \Phi = q_{enc}$$

a) for small r, $\Phi = 4\times10^5\ N.m^2/C$

$$q_{enc} = 8.85\times10^{-12}\times4\times10^5 = 3.54\times10^{-6}\ C$$

b) $\Phi = -8\times10^5\ N.m^2/C$, $q_{enc} = -7.08\times10^{-6}\ C$

$$q_A = q_{enc} - q_{central\ particle} = -7.08\times10^{-6} - 3.54\times10^{-6}$$

$$q_A = -10.62\ \mu C$$

c) $\Phi = 12\times10^5\ N.m^2/C$, $q_{total\ enclosed} = 1.06\times10^{-5}\ C$

$$q_B = q_{total\ enc} - q_A - q_{central\ particle} =$$

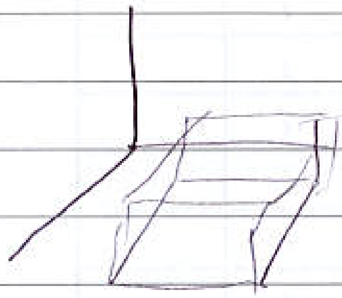
$$1.06\times10^{-5} + 7.08\times10^{-6} = 1.77\times10^{-5}\ C$$

$$17.7\ \mu C$$

#

$\boxed{34}$ The box like Gaussian surface encloses a net charge of $+32\epsilon_0$ C and lies in an electric field $\vec{E} = [(10+2x)\hat{i} - 3\hat{j} + bz\hat{k}]$ N/C x, z in meters. The bottom face is in the xz plane; the top face is in the horizontal plane passing through $y_2 = 1.00$ m. For $x_1 = 1$m, $x_2 = 4$m, $z_1 = 1$m $z_2 = 3$m. What is b?

$\vec{E}$ is constant in Y- direction , $\oint \vec{E} \cdot dA = \frac{q_{enc}}{\epsilon_0}$



- Φ through the two faces parallel to the xz plane = zero, since $\vec{E}$ is constant in Y- direction

$$\Phi_{xz} = \iint [E_y(y_2) - E_y(y_1)]\, dx\, dz$$

$$= \int_1^4 dx \int_1^3 dz\, (-3-(-3)) = 0$$

$$\Phi_{yz} = \iint [E_x(x_2) - E_x(x_1)]\, dy\, dz = \int_0^1 dy \int_1^3 dz\, [10+2(4)-10-2(1)]$$

$$= 6 \int_0^1 dy \int_1^3 dz = 6(1)(2) = 12$$

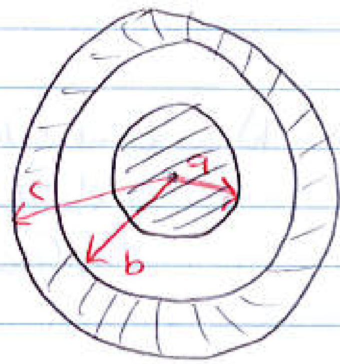
$$\Phi_{xy} = \iint [E_z(z_2) - E_z(z_1)]\, dx\, dy = \int_1^4 dx \int_0^1 dy\, (3b - b)$$

$$= (2b)(3)(1) = 6b$$

$$\Rightarrow \Phi_{total} = \frac{q_{enc}}{\epsilon_0} \rightarrow 0 + 12 + 6b = +\frac{32\epsilon_0}{\epsilon_0}$$

$$b = 3.33 \text{ N/C.m}$$

$\boxed{29}$ A solid sphere of radius $a = 2\,cm$ is concentric with a spherical conducting shell of inner radius $b = 2a$ and outer radius $c = 2.4a$. The sphere has a net uniform charge $q_1 = +2fC$. The shell has a net charge $q_2 = -q_1$. What is the magnitude of the electric field at radial distances a) $r = 0$, b) $r = \frac{a}{2}$ c) $r = a$, d) $r = 1.5a$, e) $r = 2.3a$, f) $r = 3.5a$? What is the net charge on the g) inner and h) outer surface of the shell?



$\vec{E}$ is radially outward.

$\Rightarrow \oint \vec{E} \cdot d\vec{A} = 4\pi r^2 E$

For $r<a$, $E = \dfrac{q_1 r}{4\pi\epsilon_0 a^3}$

a) $r=0$, $E=0$

b) $r = a/2$, $E = \dfrac{q_1 \; a/2}{4\pi\epsilon_0 a^3} = \dfrac{9\times10^{9} \times 2\times10^{-15}}{2\,(2\times10^{-2})^2} = 0.0225\, N/C$

c) $r = a$, $E = \dfrac{q_1}{4\pi\epsilon_0 a^2} = \dfrac{9\times10^{9} \times 2\times10^{-15}}{(2\times10^{-2})^2} = 0.045\, N/C$

For $a<r<b$, $E = \dfrac{q_1}{4\pi\epsilon_0 r^2}$

d) $r = 1.5a$, $E = \dfrac{kq_1}{r^2} = \dfrac{9\times10^{9}\times 2\times10^{-15}}{(1.5\times2\times10^{-2})^2} = 0.02\; N/C$

e) In the region $b<r<c$, since the shell is conducting, the Electric field is zero, $E(2.3a) = 0$

f) $r>c$, the charge enclosed by the Gaussian surface is zero
$E(r = 3.5a) = 0$

g) Gaussian surface that lies completely within the conducting shell
$\vec{E} = 0$ every where on the surface
$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} = 0$$

So $q_{enc} = 0$, but $Q_{inner\,shell}$

$q_1 + Q_{inner\,shell} = 0$, $Q_{inner} = -q_1$
$= -2\mu C$

h) $Q_{out\,shell} = 0$
net charge on the shell $= -q_1$
$Q_{outer} = -q_1 - Q_{inner} = 0$

$\boxed{23-44}$ The electric field just above the surface of the charged conducting drum of a photocopying machine has a magnitude E of 1.9×10^{5} N/C. What is the surface charge density on the drum?

$\Rightarrow$ charged conducting surface

$$E = \frac{\sigma}{\epsilon_0}$$

$$\sigma = \epsilon_0 E = \left(8.85 \times 10^{-12} \frac{C^2}{N.m^2}\right)\left(1.9 \times 10^{5} \frac{N}{C}\right)$$

$$\boxed{\sigma = 1.68 \times 10^{-6} \frac{C}{m^2}}$$