Chapter 24 Electric Potential

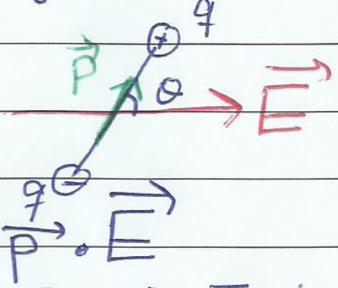
Conservative Forces & Potential Energy:

$\vec{F}_g = m\vec{g} \Longrightarrow U_g = mgy$ Joul

$F_{spring} = -kx \Longrightarrow U_{spring} = \frac{1}{2}kx^2$ Joul

Electric Force acting on electric dipole

$\vec{E}$ on $\vec{P} = q\vec{d}$



$\Longrightarrow U = -\vec{p} \cdot \vec{E}$

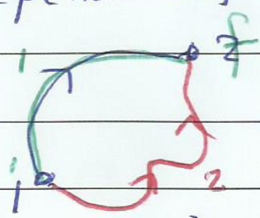
$= -pE\cos\theta$ Joul

Properties of conservative Forces:-

1) Work done Against Conservative Force is Stored as energy, Called Potential energy.

2) Work done by Conservative force is path independent.
Work done by conservative force depends on initial point & final point

$$\left(W^{cons.}_{i \to f}\right)_1 = \left(W^{cons.}_{i \to f}\right)_2$$



3) Work done by Conservative force $_{i \to f}$ $= -\Delta U$
$= -[U_f - U_i]$

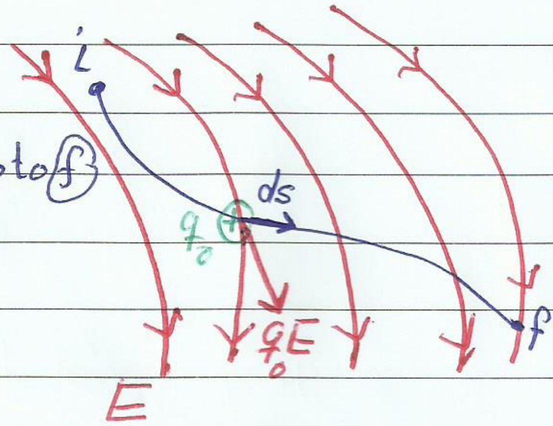
U is the Potential Energy depends on the State.

4) Mechanical Energy is Conserved Under the influence of conservative Force

$$(K+U)_i = (K+U)_f \quad , \quad K = \frac{1}{2}mv^2 \text{ Joul}$$

Electric Potential Energy U:

E is a nonuniform Electric Field



q_0 is a test charge moves from $i \to$ to f along the shown path

$$dW_E = q_0 \vec{E} \cdot \vec{ds}$$

$$(W_E)_{i \to f} = \int_i^f q_0 \vec{E} \cdot \vec{ds} \quad \text{Joul}$$

But: The electric Force $\vec{F}_E = q_0 \vec{E}$ is Conservative Force

$$W_{E\, i \to f} = -\Delta U = -[U_f - U_i]$$

$$U_f - U_i = -W_{E\, i \to f} = -\int_i^f q_0 \vec{E} \cdot \vec{ds}$$

$$U_f - U_i = (-) q_0 \int_i^f \vec{E} \cdot \vec{ds} \quad \text{Joul} \quad (1)$$

Electric

the Zero level for Potential Energy is when q_0 is at ∞

$$U_f - U_\infty = (-) q_0 \int_\infty^f \vec{E} \cdot \vec{ds} \quad , \text{ let } U_\infty = 0$$

(2)

$$U_f = (-) q_0 \int_\infty^f \vec{E} \cdot \vec{ds}$$ Electric Potential energy For q_0 in an Electric Field.

$U_f = (-)$ Work done by $\vec{E}$ in moving $q_0 (\infty \to f)$

Electric Potential V:

$$U_f - U_i = (-) q_0 \int_i^f \vec{E} \cdot \vec{ds} \quad J$$

$$\frac{U_f}{q_0} - \frac{U_i}{q_0} = -\int_i^f \vec{E} \cdot ds \quad J/C$$

$$V_f - V_i = -\int_i^f \vec{E} \cdot \vec{ds} \quad (1) \quad J/C = \text{Volt}$$

to find V at a certain point let $V_\infty = 0$

$$V_f - V_\infty = -\int_\infty^f \vec{E} \cdot \vec{ds}$$

$$V_f = -\int_\infty^f \vec{E} \cdot \vec{ds} \quad (2) \quad J/C = \text{Volt}$$

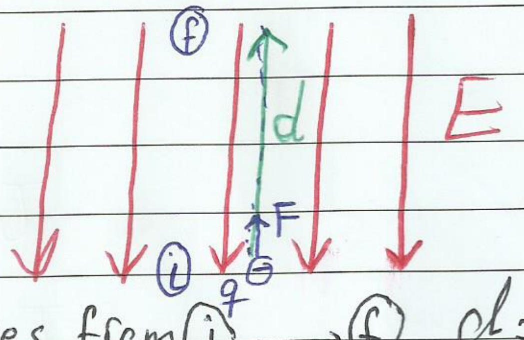
V at a certain point $= (-)$ Work done by $\vec{E}$ in moving $+1C$ from $\infty \rightarrow$ to the Point

$U = qV$ — U: Joul, q: C, V: Volt

In this Course we will find V from E

1) We will find V due to a point charge
2) We will find V due to a Set of Point charges
3) We will find V due to a a Continous charge distribution

Sample Problem 24.01



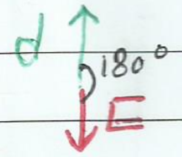
$E = 150\ N/C$ downward

$q = -e$

$= -1.6 \times 10^{-19}\ C$

due to $\vec{E}$, the election moves from (i) ⟶ (f) $d = 520\ m$ upward

1)



$$W_{\underset{i \to f}{E}} = \int_i^f q\vec{E} \cdot \vec{ds} = q\vec{E} \cdot \vec{d}$$

$$W_{\underset{i \to f}{E}} = (-1.6 \times 10^{-19})(150)(520)\cos 180^\circ$$

$$= 1.2 \times 10^{-14}\ J$$

2) find $\Delta U = U_f - U_i$?

$$\Delta U = U_f - U_i = -W_{\underset{i \to f}{E}} = -1.2 \times 10^{-14}\ J$$

3) find $\Delta V = V_f - V_i$?

$$\Delta U = q\,\Delta V$$

$$U_f - U_i = q(V_f - V_i)$$

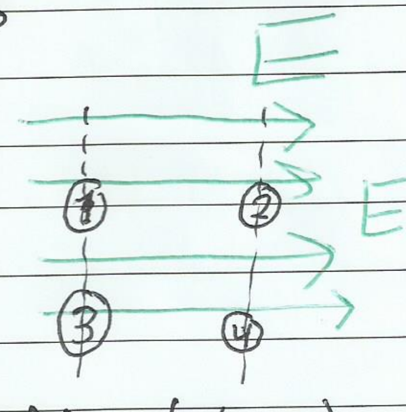
$$V_f - V_i = \frac{U_f - U_i}{q} = \frac{-1.2 \times 10^{-14}}{-1.6 \times 10^{-19}} = +7.5 \times 10^{4}\ V$$

$V_f > V_i$

Note: When you move with $\vec{E}$, V will decrease

When you move opposit to $\vec{E}$

V will increase.



$\Rightarrow \quad V_1 > V_2$

$V_1 = V_3$,, $V_2 = V_4$

$(V_1 = V_3) > (V_2 = V_4)$

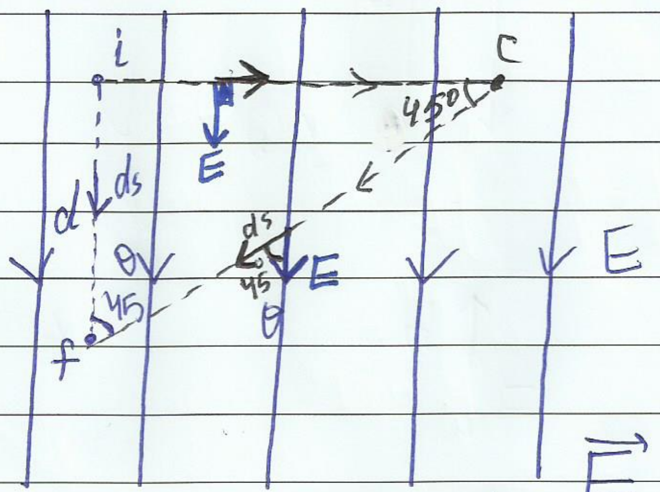
4

Sample Problem 24.02 ΔU is Path independent للاعتماد

ΔV is Path independent على المسار

$\vec{E}$ is Uniform downward

$d_{if} = d$



a) find $V_f - V_i$ by moving +1C directly from $i \to f$

$$V_f - V_i = -\int_i^f \vec{E} \cdot \vec{ds}$$

$$= (-)\, \vec{E} \cdot \vec{d}_{i \to f}$$

$$= (-)\, E d \cos\theta$$

$\cos\theta = \dfrac{d}{d_{cf}}$

$d = d_{cf} \cos\theta$

$$V_f - V_i = -Ed \Rightarrow V_i > V_f$$

b) find $V_f - V_i$ by moving ⊕ along the Path icf

$$V_f - V_i = -\int_i^c \vec{E} \cdot \vec{ds} - \int_c^f \vec{E} \cdot \vec{ds} \quad , \vec{E} = \text{constant}$$

$$= -\vec{E} \cdot \vec{d}_{i \to c} - \vec{E} \cdot \vec{d}_{c \to f}$$

$$= -E d_{i \to c} \cos 90 - E d_{c \to f} \cos 45$$

$$= 0 - E d_{c \to f} \cos 45$$

from the graph

$\cos 45 = \dfrac{d}{d_{cf}}$

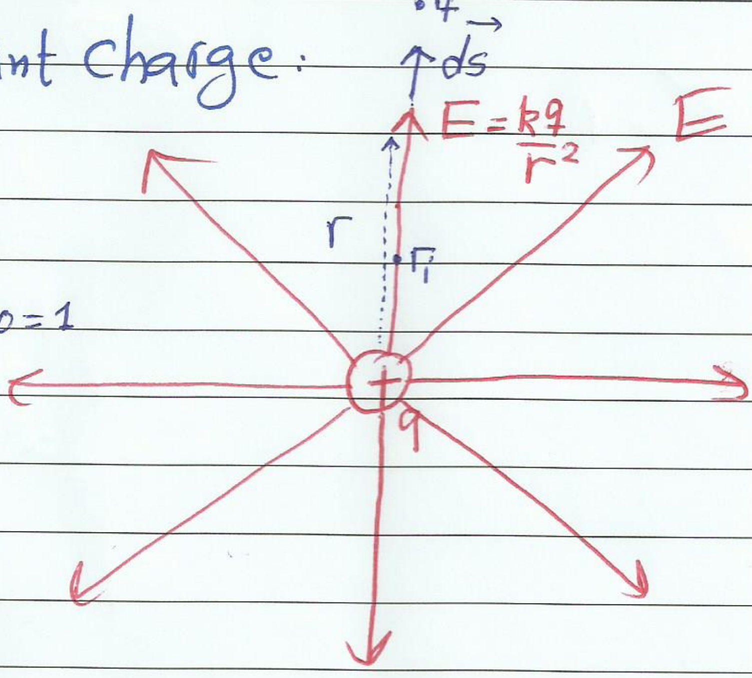
$d = d_{cf} \cos 45$

$$V_f - V_i = -Ed \Rightarrow V_i > V_f$$

as Part (a)

1) V due to a point charge:

$$E = k\frac{q}{r^2} = \frac{q}{4\pi\epsilon_0 r^2}$$



$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s} \quad , \cos 0 = 1$$

$$= (-)\int_{r_i}^{r_f} \frac{q}{4\pi\epsilon_0 r^2}\, dr$$

$$= \frac{-q}{4\pi\epsilon_0}\int_{r_i}^{r_f} \frac{dr}{r^2}$$

$$= \frac{-q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_{r_i}^{r_f} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_{r_i}^{r_f}$$

$$V_f - V_i = \frac{q}{4\pi\epsilon_0 r_f} - \frac{q}{4\pi\epsilon_0 r_i} \quad , \text{let } (i) \text{ at } \infty \; , V_\infty = 0$$

$$V_f - V_\infty = \frac{q}{4\pi\epsilon_0 r_f} - 0$$

$$V_f = \frac{q}{4\pi\epsilon_0 r_f}$$

for $(+q)$ $V_f \rightarrow +$

for $(-q)$ $V_f \rightarrow -$

2) V due to a Set of point charges

$$V = V_1 + V_2 + V_3 + \cdots = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n} \frac{q_i}{r_i}$$

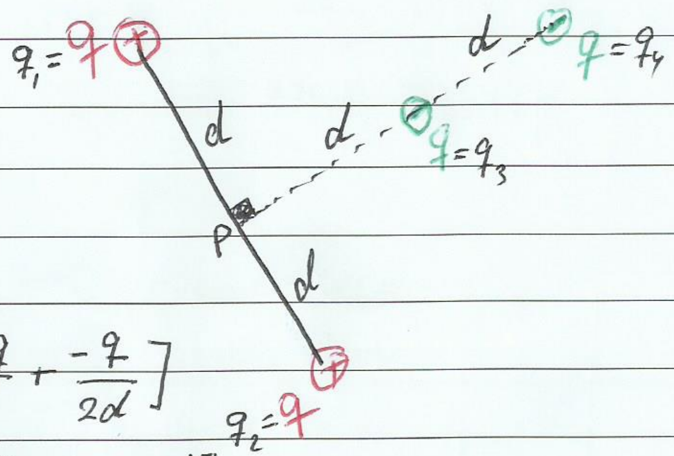
Solve Sample Problems 24.03 & 24.04

(24-31)

$q = 7.50\,fC$

$d = 1.60\,cm$

find V_p ?



$$V_p = V_1 + V_2 + V_3 + V_4$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{+q}{d} + \frac{+q}{d} + \frac{-q}{d} + \frac{-q}{2d}\right]$$

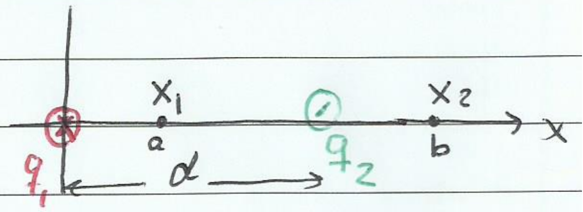
$$= \frac{1}{4\pi\epsilon_0}\left[\frac{+q}{2d}\right] = \frac{9\times10^9 \times 7.5\times10^{-15}}{2(1.6\times10^{-2})}$$

$$V_p = \frac{4.22\times10^{-3}}{2}\,V = \frac{4.22}{2}\,mV = 2.11\,mV$$

(24-7) $q_1 = +15e$

$q_2 = -5e$

$d = 2.40\,cm$



find Values of x ? At which $V = 0$

a) At x_1 between them ?

$$V_a = \frac{+15e}{4\pi\epsilon_0 x_1} + -\frac{5e}{4\pi\epsilon_0(d - x_1)} = 0$$

$$\frac{+15e}{4\pi\epsilon_0 x_1} + \frac{-5e}{4\pi\epsilon_0(d - x_1)} = 0 \Rightarrow \frac{3}{x_1} = \frac{1}{d - x_1}$$

$$x_1 = 3d - 3x_1 \Rightarrow 4x_1 = 3d \Rightarrow x_1 = \frac{3}{4}d = {}^{+}18\,cm$$

b) At x_2 to the the right of q_2

$$V_b = \frac{+15e}{4\pi\epsilon_0(x_2)} + \frac{-5e}{4\pi\epsilon_0(x_2 - d)} = 0 \Rightarrow \frac{15e}{4\pi\epsilon_0 x_2} - \frac{5e}{4\pi\epsilon_0(x_2 - d)}$$

$$\frac{3}{x_2} = \frac{1}{x_2 - d} \Rightarrow 3x_2 - 3d = x_2$$

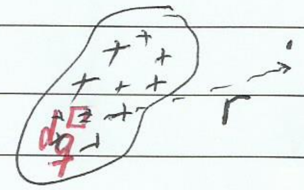
$$2x_2 = 3d \Rightarrow x_2 = 1.5d = 36\,cm$$

Problem Repeat problem 7 for $q_1 = +5e$, $q_2 = -15e$

3) Potential due to a Continous Charge distribution:

$$dV = \frac{dq}{4\pi\epsilon_0 r}$$

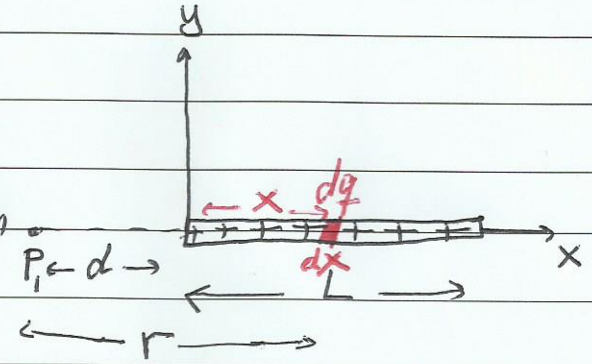
$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$



I] V due to a Uniformly charged rod (Line of charge)

Problem (24-02)

thin Plastic rod
- → length $= L = 12\,cm$
- → $Q = {}^{+}47.9\,fC$
- → $\lambda = \frac{Q}{L} = 400\,fC/m$
  $= +400\,fC/m$



Find V at point P_1, a distance $d = 2.5\,cm$ from the lef end of the rod?

$$dV = \frac{dq}{4\pi\epsilon_0 r} = \frac{\lambda\,dx}{4\pi\epsilon_0(d+x)}, \quad \lambda = 4.00\times10^{-13}\,C/m \text{ Constant}$$

$$V_{P_1} = \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{dx}{(d+x)} = \frac{\lambda}{4\pi\epsilon_0}\ln(d+x)\Big]_0^L$$

$$= \frac{\lambda}{4\pi\epsilon_0}\left[\ln(d+L) - \ln(d)\right]$$

$$V_{P_1} = \frac{\lambda}{4\pi\epsilon_0}\ln\left(\frac{d+L}{d}\right)$$

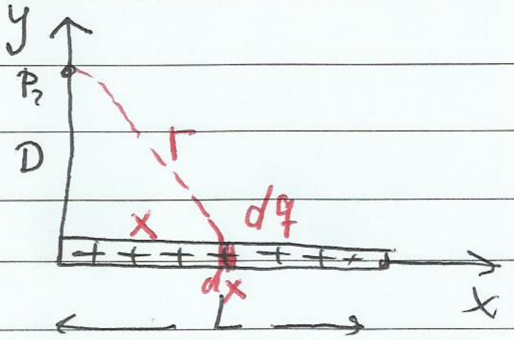
$$V_{P_1} = 9\times10^{9}\times4\times10^{-13}\ln\left(\frac{2.5+12}{2.5}\right)$$

$$= 36\times10^{-4}(1.75786)$$

$$= 6.33\times10^{-3}\,V$$

$$= 6.33\,mV$$

Problem (24-6) Nonuniform Linear charge density



The thin plastic rod → length = 12 cm
→ $\lambda = Cx$

$C = 49.9\ pC/m^2$

a) Find V at p_2; on the y-axis at $y = D = 3.56\ cm$

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r}$$

$$V_{P_2} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\int_0^L \frac{\lambda dx}{\sqrt{D^2+x^2}} = \frac{1}{4\pi\epsilon_0}\int_0^L \frac{(Cx)dx}{\sqrt{D^2+x^2}}$$

$$= \frac{C}{4\pi\epsilon_0}\int_0^L \frac{x\,dx}{\sqrt{D^2+x^2}} \quad , \text{let } D^2+x^2 = u$$
$$du = 2x\,dx$$

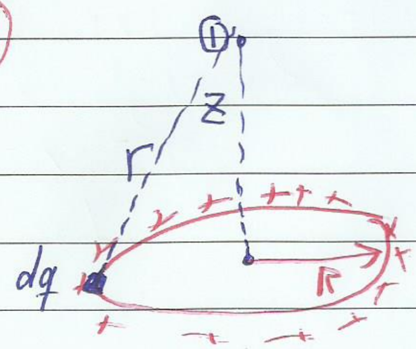
$$= \frac{C}{4\pi\epsilon_0}\int \frac{du/2}{u^{1/2}} = \frac{C}{8\pi\epsilon_0}\int u^{-1/2}du \qquad \frac{u^{1/2}}{1/2}$$

$$= \frac{2C}{8\pi\epsilon_0}\left[\sqrt{D^2+x^2}\right]_0^L = \frac{C}{4\pi\epsilon_0}\left[\sqrt{D^2+L^2} - D\right]$$

$$V_{P_2} = \frac{C}{4\pi\epsilon_0}\left[\sqrt{D^2+L^2} - D\right] = 9\times10^9\left[49.9\times10^{-12}\right]\left[\sqrt{(0.0356)^2+(0.12)^2} - 0.0356\right]$$
$$= 0.04\ \text{Volt.}$$

V due to a Uniformly Charged Ring

Ring → radius = R
→ charge = +Q
→ $\lambda = \frac{Q}{2\pi R}$



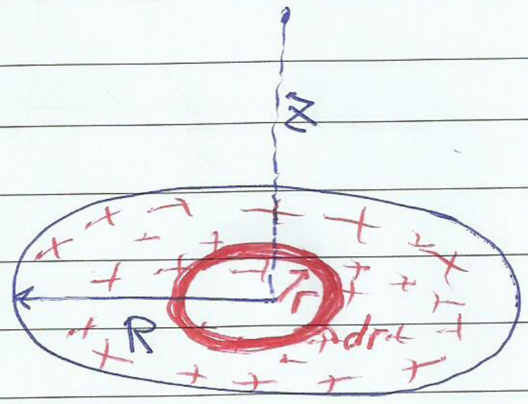
Find V at a point above the center a distance = z

$$dV = \frac{dq}{4\pi\epsilon_0 r} = \frac{dq}{4\pi\epsilon_0\sqrt{z^2+R^2}}$$

$$V_{ring} = \int \frac{dq}{4\pi\epsilon_0\sqrt{z^2+R^2}} = \frac{1}{4\pi\epsilon_0\sqrt{z^2+R^2}}\int dq = \frac{Q}{4\pi\epsilon_0\sqrt{z^2+R^2}}$$

V due to a Uniformly charged Disk

Disk:
- radius $= R$
- charge $= Q$ on one face
- $\sigma = \frac{Q}{\pi R^2}$



Find V at a point above the center a distance (Z) from the center

Divide the disk to rings

each ring of:
- radius $= r$
- width $= dr$
- Area $dA = 2\pi r dr$
- $dq = \sigma(2\pi r dr) \leftarrow$ charge on the ring
- $dV = \frac{dq}{4\pi\epsilon_0\sqrt{r^2+z^2}}$ due to the ring

$$dV = \frac{\sigma(2\pi r dr)}{4\pi\epsilon_0\sqrt{r^2+z^2}} \quad \text{(ring Potential)}$$

$$V_{disk} = \int dV_{ring} = \frac{\sigma\pi}{4\pi\epsilon_0}\int_0^R \frac{2r dr}{\sqrt{r^2+z^2}} , \quad \text{let } u = r^2+z^2, \quad du = 2r dr$$

$$= \frac{\sigma}{4\epsilon_0}\int \frac{du}{u^{1/2}} = \frac{\sigma}{4\epsilon_0}\int u^{-1/2} du = \frac{\sigma}{4\epsilon_0}\left[\frac{u^{1/2}}{1/2}\right]$$

$$= \frac{\sigma}{2\epsilon_0}\left[\sqrt{r^2+z^2}\right]_0^R = \frac{\sigma}{2\epsilon_0}\left[\sqrt{R^2+z^2} - z\right]$$

Calculating The Field E From the Potential V

$$dV = -\vec{E}\cdot\vec{ds} \Rightarrow -\frac{\partial V}{\partial s} = E \Rightarrow$$

$$E_x = -\frac{\partial V}{\partial x} \;,\; E_y = -\frac{\partial V}{\partial y} \;,\; E_z = -\frac{\partial V}{\partial y} \quad V/m$$

Sample Problem 24.05

$$V_{disk} = \frac{\sigma}{2\epsilon_0}\left[\sqrt{z^2+R^2} - z\right]$$

Find E_z?

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\epsilon_0}\frac{d}{dz}\left[\sqrt{z^2+R^2} - z\right]$$

$$= -\frac{\sigma}{2\epsilon_0}\left[\frac{1}{2}\frac{2z}{\sqrt{z^2+R^2}} - 1\right] = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

Problem:

$$V_{ring} = \frac{Q}{4\pi\epsilon_0(R^2+z^2)^{1/2}} \quad \text{Find } E_z$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{Q}{4\pi\epsilon_0}\frac{\partial}{\partial z}(R^2+z^2)^{-1/2}$$

$$= -\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{2}(R^2+z^2)^{-3/2} \cdot 2z\right]$$

$$E_z = \frac{Qz}{4\pi\epsilon_0(R^2+z^2)^{3/2}} \quad \text{Ring}$$

Example $V = 2x^2 + 3y^2$

Find $\vec{E}$ at the point $(4m, 2m)$?

$$E_x = -\frac{\partial V}{\partial x} = -2(2x) = -4x$$

$$E_y = -\frac{\partial V}{\partial y} = -3(2y) = -6y$$

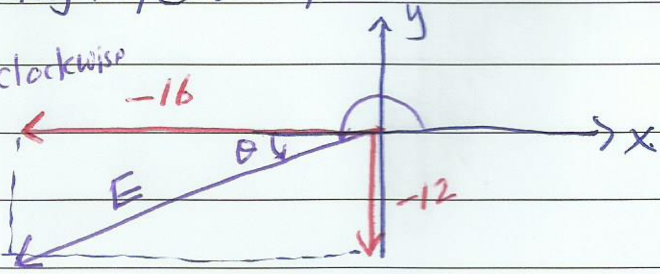
$$\vec{E} = -4x\hat{i} - 6y\hat{j}$$

$$\vec{E} = -4(4)\hat{i} - 6(2)\hat{j} = -16\hat{i} - 12\hat{j} \text{ N/C or V/m}$$

$\theta = \tan^{-1}\left(\frac{-12}{-16}\right) = 37°$ counter clockwise

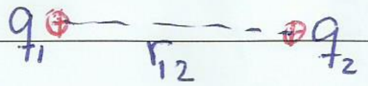
$\vec{E} = 20$ V/m, at $37°$ with $-x$

$= 20$ V/m, at $217°$ with $+x$ Counter Clockwise



Electric Potential Energy (U) of a System of Charged Particles

U between 2 point charges = Work must be done to put the 2 charges in their places by bringing them from ∞.



$$U_{12} = W_{\infty \to q_1} + W_{\infty \to q_2}$$

$$= 0 + q_2 V_1$$

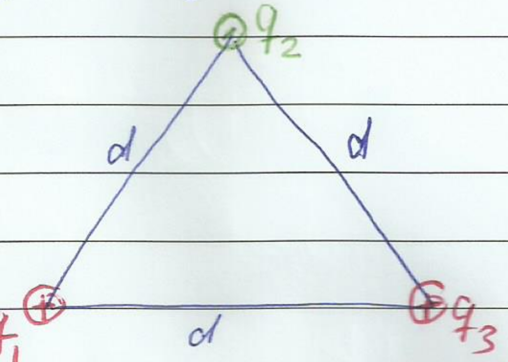
$$= 0 + q_2\left(\frac{q_1}{4\pi\epsilon_0 r_{12}}\right)$$

$$U_{12} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} \text{ Joul}$$

Sample Problem 24.06 Potential energy of a System of 3 charged Particles.

$q_1 = +q$, $q_2 = -4q$

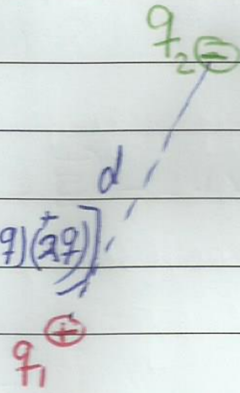
$q_3 = +2q$, $q = 150\,nC$, $d = 12\,cm$



$$U = W_{\infty \to q_1} + W_{\infty \to q_2} + W_{\infty \to q_3}$$

$$= 0 + q_2\left(\frac{q_1}{4\pi\epsilon_0 d}\right) + q_3\left(\frac{q_1}{4\pi\epsilon_0 d} + \frac{q_2}{4\pi\epsilon_0 d}\right)$$

$$= \underbrace{\frac{q_1 q_2}{4\pi\epsilon_0 d}}_{U_{12}} + \underbrace{\frac{q_1 q_3}{4\pi\epsilon_0 d}}_{U_{13}} + \underbrace{\frac{q_2 q_3}{4\pi\epsilon_0 d}}_{U_{23}}$$



$$= \frac{1}{4\pi\epsilon_0 d}\left[(+q)(-4q) + (+q)(+2q) + (-4q)(2q)\right]$$

$$= \frac{1}{4\pi\epsilon_0 d} q^2 \left[-4 + 2 - 8\right]$$

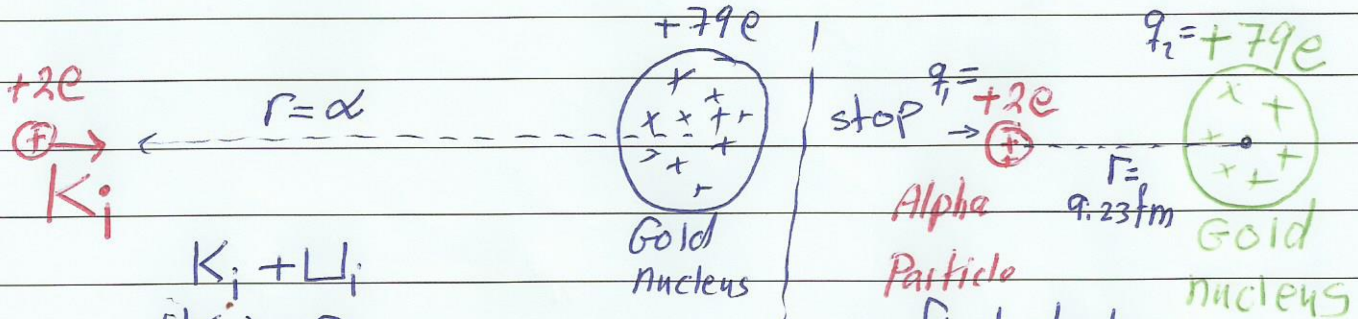
$$= \frac{-10\, q^2}{4\pi\epsilon_0 d} = \frac{(-10)\,9\times10^{9} \times (150\times10^{-9})^2}{0.12} = -0.017\, V.C$$

$$= -0.017\, J$$

$$= (-)17\, mJ$$

Sample Problem: 24.07

Conservation of Mechanical energy With electic Potential energy:



$$K_i + U_i = K_f + U_f$$

$$(K_i)_\alpha + 0 = 0_\alpha + \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

$$(K_i)_\alpha = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

$$= \frac{9\times10^{9} \times 2(1.6\times10^{-19})(+79)(1.6\times10^{-19})}{9.23\times10^{-15}}$$

$$(K_i)_\alpha = +3.94\times10^{-12}\,J$$

But $1eV = 1.6\times10^{-19}\,J$

$$(K_i)_\alpha = \frac{+3.94\times10^{-12}}{1.6\times10^{-19}} = 2.46\times10^{7}\ eV$$

$$= 24.6\ MeV$$

Units of Energy: Joul

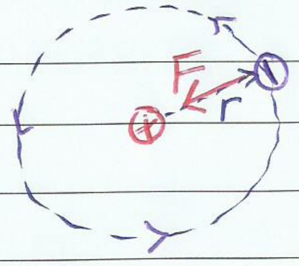
$$kWh = (1000)(3600) = (1000\tfrac{J}{s})(3600s) = 3.6\times10^{6}\,J$$

$$kWh = 3.6MJ$$

$$eV = (1.6\times10^{-19}C)(1V) = 1.6\times10^{-19}\,J$$

Problem: Hydrogen Atom

In the H-atom, the electron moves in a Uniform Circular motion around the nucleus, of radius $= 5.29 \times 10^{-11}$ m



$q_{electron} = -1.6 \times 10^{-19}$ C

$q_{nucleus} = +1.6 \times 10^{-19}$ C

$m_e = 9.11 \times 10^{-31}$ kg

1) Find the kinetic energy of the electron ?
2) Find the Potential energy of the electron ?
3) Find the Mechanical Energy of the electron?

1) $F = \dfrac{q_e q_n}{4\pi\epsilon_0 r^2} = \dfrac{mv^2}{r}$ Coulomb's Law + Newton's 2nd Law

$$\frac{1}{2}\left[mv^2 = \frac{q_e q_n}{4\pi\epsilon_0 r}\right]$$

$$\frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{q_e q_n}{4\pi\epsilon_0 r}\right) = \frac{1}{2}\left(\frac{(1.6\times10^{-19})^2}{5.29\times10^{-11}}(9\times10^9)\right)$$

$$K.E = 2.18\times10^{-18}\,J = 13.6\,eV$$

2) $$U = q_e\left(\frac{q_n}{4\pi\epsilon_0 r}\right) = -1.6\times10^{-19}\left(\frac{9\times10^9\times1.6\times10^{-19}}{5.29\times10^{-11}}\right)$$

$$= (-)\,4.36\times10^{-18}\,J = (-)\,27.2\,eV$$

3) Mechanical Energy $= K.E + U = {}^{+}13.6 + {}^{-}27.2$

$$= -13.6\,eV$$

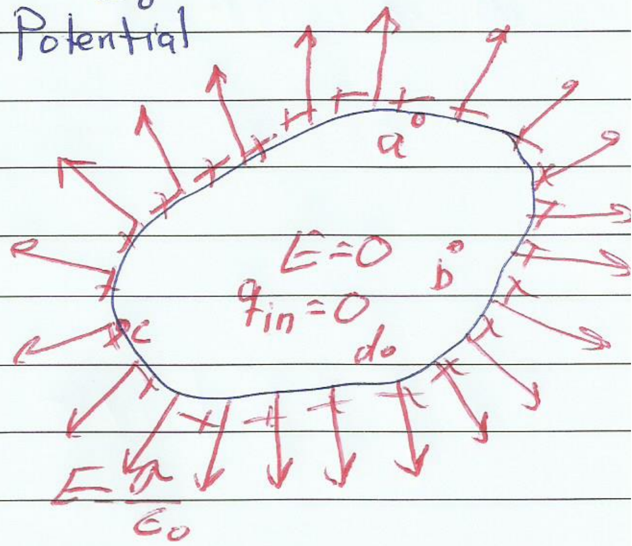
$$= -2.18\times10^{-18}\,J$$

Potential of a Charged Isolated Conductor.

Charged Isolated Conductor:

1) The charge sit at the outer surface
2) charge inside the Conductor $= 0$
3) E inside the Conductor $= 0$
4) E_{near} the outer surface $= \frac{\sigma}{\epsilon_0}$
5) All its points have the same Potential

$V_a = V_b = V_c = V_d$

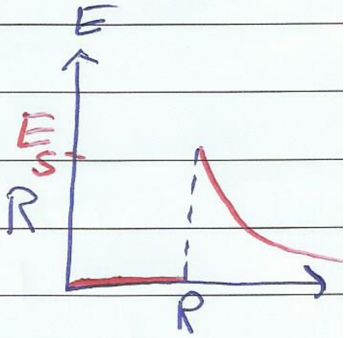


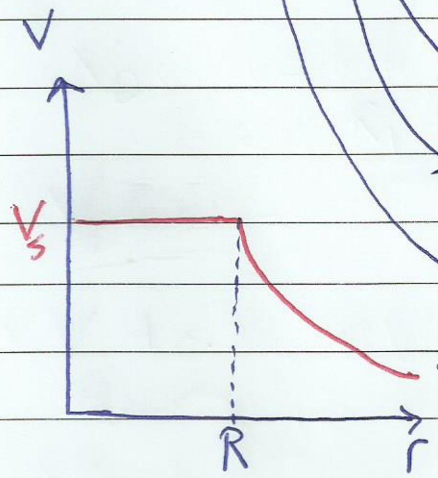
Charged Isolated Conducting sphere.

Conducting sphere:

- → radius $= R$
- → charge $= +Q$
- → $\sigma = \frac{+Q}{4\pi R^2}$ [All extra charge Q on the surfa
- → $\rho = 0$ [No charge inside charged Conducting sphere
- → $E_s = \frac{Q}{4\pi\epsilon_0 R^2}$ or $E_s = \frac{\sigma}{\epsilon_0}$
- → $E_{insid} = 0$, $r < R$
- → $E_{outside} = \frac{Q}{4\pi\epsilon_0 r^2}$, $r \geq R$
- → $V_s = \frac{+Q}{4\pi\epsilon_0 R}$
- → $V_{inside} = V_s = \frac{+Q}{4\pi\epsilon_0 R}$, $r \leq R$

$V_{outside} = \frac{+Q}{4\pi\epsilon_0 r}$, $r \geq R$





The voltage on the surface of a uniformly conducting sphere

To find the potential on the surface let's find it relative to ∞

Note V_R is the voltage at the surface of a sphere of radius R

$$V_\infty - V_R = -\int_R^\infty \vec{E}_{outside} . d\vec{r}$$

$$0 - V_R = -\int_R^\infty E_{outside}\, dr$$

$$V_R = \int_R^\infty E_{outside}\, dr$$

Note that the electric field outside the sphere is $E_{outside} = KQ/r^2$, thus

$$V_R = \int_R^\infty \frac{KQ}{r^2} dr$$

$$V_R = \frac{-KQ}{r}\bigg|_R^\infty$$

$$V_R = -KQ\left(\frac{1}{\infty} - \frac{1}{R}\right)$$

$V_R = \frac{KQ}{R}$ this is the potential at the surface of a uniformly charged conducting sphere

Now let's find the voltage inside the sphere

The difference in the voltage between the center of the sphere and the surface is

Note: V_0 is the voltage at the center of the sphere

$$V_R - V_0 = -\int_0^R \vec{E}_{inside} . d\vec{r}$$

$$V_R - V_0 = -\int_R^\infty E_{inside}\, dr$$

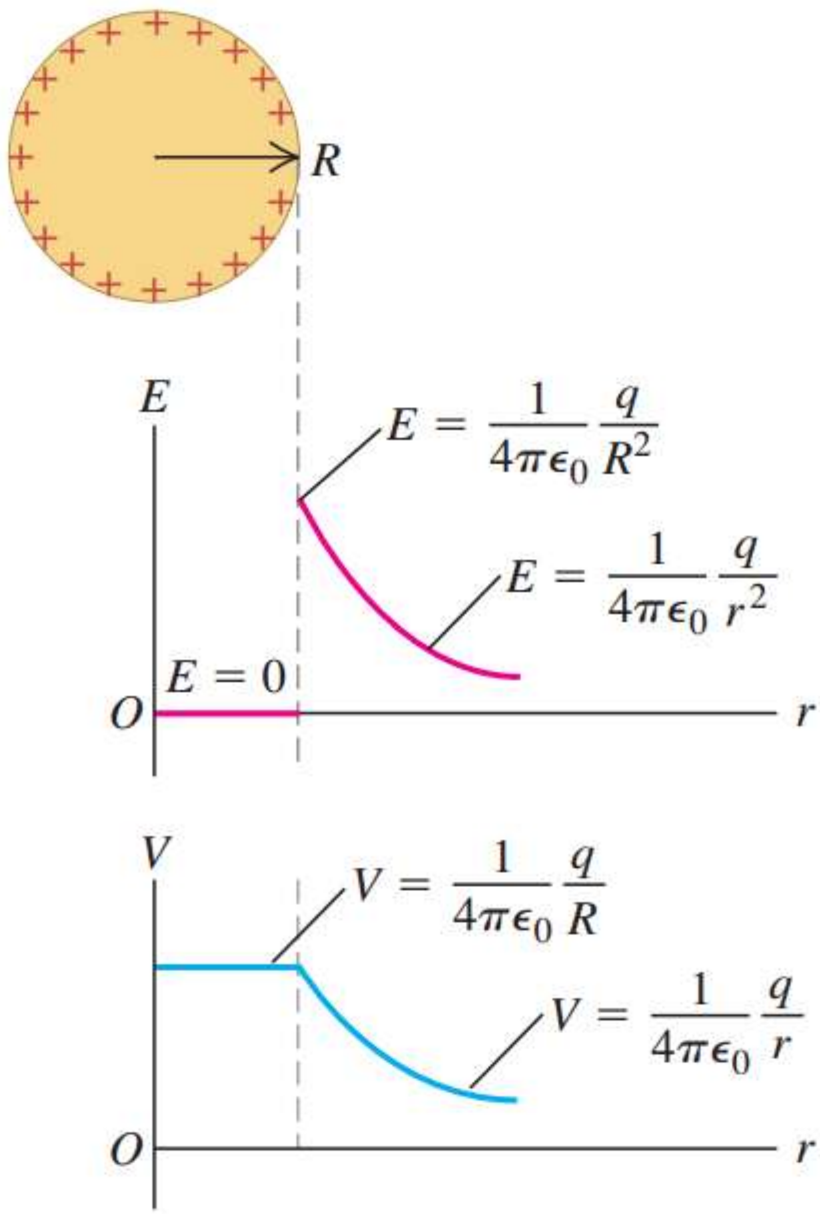
Note that the electric field inside the sphere is $E_{inside} = 0$, thus

$V_R - V_0 = 0$, Thus

$V_R = V_0$

This means that the potential at the center of a uniformly charged conducting sphere equals the voltage on its surface, thus the potential inside a uniformly conducting sphere is constant

See the figures below



As you can see from the figures the electric field inside a **conductor** is zero while the potential is constant