Chapter 24: Electric potential

V = Electric potential "in Volts"
U = Electric potential Energy "in Joule"
$U = qV$
Scalar

- $\Delta U = q\Delta V = q(V_f - V_i)$

⇒ Conservation of Mechanical Energy

$\Delta K + \Delta U = 0 \Rightarrow K_f + U_f = K_i + U_i$

$$\boxed{\Delta K = -q\Delta V}$$

$\boxed{W_{\text{done by the field}} = -\Delta U}$

If there is applied Force ⟶ W_{app}

$$\boxed{\Delta K = -q\Delta V + W_{app}}$$

Note $\Delta K = 0$, No changing in Kinetic energy $K_f = K_i$
$W_{app} = q\Delta V$

- Equipotential Surfaces ⇒ All points have the same Electric potential

$\boxed{V \text{ from } \vec{E}}$

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

Note
1. $V_f - V_i = -\int_{y_i}^{y_f} E_y \, dy$
2. $\vec{E}$ is uniform "Ex ⇒ $V_f - V_i = -E\Delta X$

$\boxed{\vec{E} \text{ from } V}$

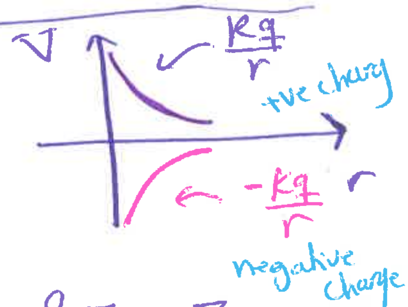
$$E_s = -\frac{\partial V}{\partial s}$$

"Note 1. $E_y = -\frac{\partial V}{\partial y}$"

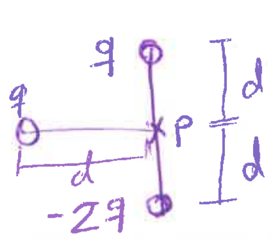
2. $\vec{E}$ is uniform
$E = -\frac{\Delta V}{\Delta S}$

- Potential due to a charged particles

$$V = K \sum_{i=1}^{N} \frac{q_i}{r_i}$$



Note



$$V_p = K\left[\frac{q}{d} + \frac{-2q}{d} + \frac{q}{d}\right] = \text{Zero}$$

• potential due to a continuous charge distribution

$$V = K \int \frac{dq}{r}$$

use $\lambda = Q/\text{Length}$

$\sigma = Q/\text{Area}$

$\rho = Q/\text{Volume}$

• Electric Potential Energy of a system of charged particles

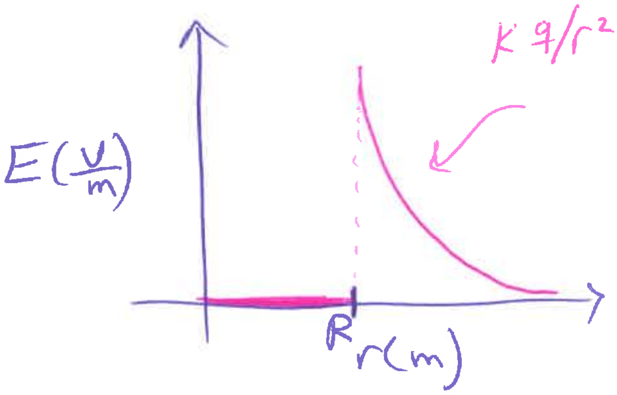
$$U = W = K \frac{q_1 q_2}{r}$$ "Two particles at separation r"

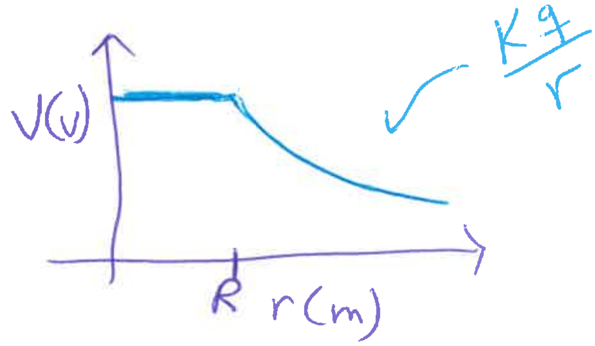
"The electric potential energy of a system of charged particles is equal to the work needed to assemble the system with the particles initially at rest and infinitely distant from each other"

• Potential of a charged conductor

⇒ charges on the outer surface of the conductor

Note spherical Shell "conductor"





$$V_{\text{interior point inside the conductor}} = V_{\text{at the surface of the conductor}}$$

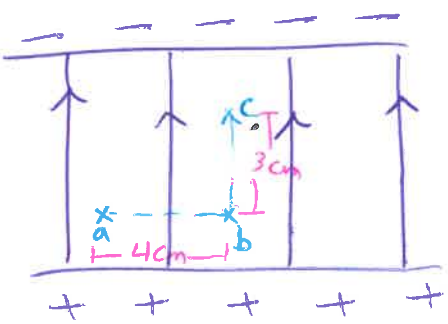
- $\Delta K = -q\Delta V + W_{app}$

$$W_{app} = \begin{cases} 0 \\ \text{negative} \\ \text{positive} \end{cases}$$

$\Rightarrow$ the energy of the system can be increase OR decrease OR Remain the same.

- V = path independent

Note



$q = 4\mu C$

$\vec{E} = 200 \frac{V}{m} \hat{j}$

$W_{abc} = W_{ac}$, use $W = qE \cdot d$

- $W_{ac} = 200 \times 4 \times 10^{-6} \times (5 \times 10^{-2}) \times \frac{3}{5}$ $\qquad \cos\theta = \frac{3}{5}$

$= 24 \mu J$

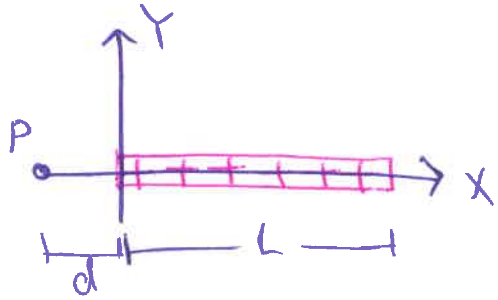
- $W_{abc} = W_{ab} + W_{bc}$

$= \cancel{qE(4\times10^{-2})\cos(90)}^{\,0} + qE(3\times10^{-2})\cos 0$

$= 4\times10^{-6} \times 200 \times 3\times10^{-2} \times 1$

$= 24 \mu J$

24-3 The thin plastic rod as shown in the below figure has Length $L = 24.0\,cm$ and nonuniform linear charge density $\lambda = cx$, where $c = 28.9\,pC/m^2$. With $V = 0$ at infinity, find the electric potential at point P on the axis, at distance $d = 3.00\,cm$ from one end?



• Use $V = K \int \frac{dq}{r}$

$q = \lambda x \rightarrow dq = \lambda\, dx$

$$V = K \int_0^L \frac{dq}{d+x} \Rightarrow \left[\begin{array}{ll} r = d+x, & dr = dx \\ x = 0 \rightarrow r = d \\ x = L \rightarrow r = d+L \end{array} \right] (*)$$

$$\Rightarrow V = K \int_0^L \frac{cx\, dx}{d+x} \xrightarrow{\text{Use } (*)} V = Kc \int_d^{d+L} \frac{r-d}{r}\, dr$$

$$V = Kc \left[\int_d^{d+L} dr - \int_d^{d+L} d\, r^{-1}\, dr \right]$$

use $\int \frac{dr}{r} = \ln(r)$

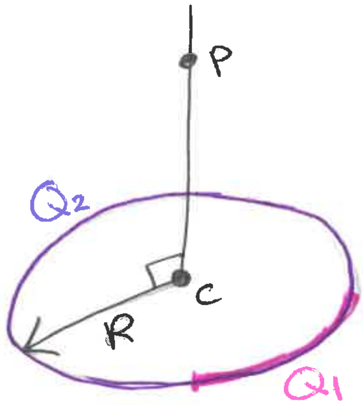
$$V = Kc \left[d+L-d - d\left(\ln(d+L) - \ln(d)\right) \right]$$

$$V = Kc \left[L + d \ln\left(\frac{d}{d+L}\right) \right]$$

$$V = 9\times10^{9} \times 28.9\times10^{-12} \left[0.24 + 0.03 \ln\left(\frac{0.03}{0.03+0.24}\right) \right]$$

$$\boxed{V = 4.5\times10^{-2}\ \text{volts} = 45.2\ mV}$$

24-9 A plastic rod has been bent into a circle of radius $R = 8.2\,cm$. It has a charge $Q_1 = +7.07\,pC$ uniformly distributed along one-quarter of its circumference and a charge $Q_2 = -6Q_1$ uniformly distributed along the rest of the circumference. With $V = 0$ at infinity, what is the electric potential at (a) the center C of the circle and (b) point P, on the central axis of the circle at distance $D = 2.05\,cm$ from the center?



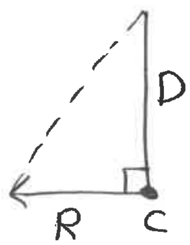
- Use Potential due to a continuous charge distribution $V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$

(a) $$V_c = \frac{1}{4\pi\epsilon_0}\left[\int \frac{dq_1}{R} + \int \frac{dq_2}{R}\right]$$

$$V_c = \frac{1}{4\pi\epsilon_0} \cdot \frac{1}{R}\left[Q_1 - 6Q_1\right] = \frac{1}{4\pi\epsilon_0}\left[-\frac{5Q_1}{R}\right]$$

$$V_c = \frac{-9\times10^9 \times 5 \times 7.07\times10^{-12}}{8.2\times10^{-2}} = -3.88 \text{ Volts.}$$

(b) $$V_p = \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{R^2+D^2}}\left[Q_1 - 6Q_1\right]$$

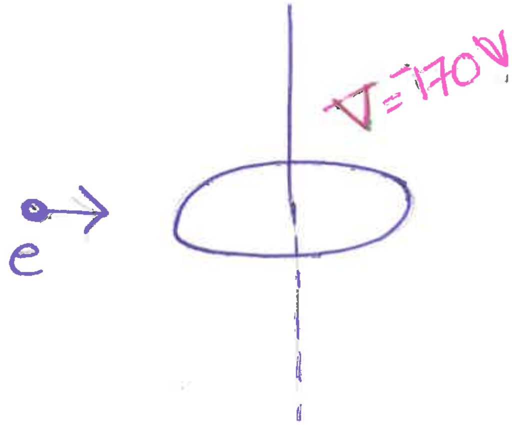


$$V_p = -\frac{1}{4\pi\epsilon_0}\frac{5Q_1}{\sqrt{R^2+D^2}}$$

$r = \sqrt{R^2+D^2}$

$$V_p = -\frac{9\times10^9 \times 5 \times 7.07\times10^{-12}}{\sqrt{(8.2\times10^{-2})^2 + (2.05\times10^{-2})^2}} = -3.76 \text{ volts}$$

$\boxed{24\text{-}10}$ A thin, spherical, conducting shell of radius R is mounted on an isolating support and charged to a potential of $-170V$. An electron is then fired directly toward the center of the shell, from point P at distance r from the center of the shell $(r \gg R)$. What initial speed v_0 is needed for the electron to just reach the shell before reversing direction?



$\Rightarrow$ conservation of mechanical energy

$$\Delta U + \Delta K = 0 \Rightarrow \Delta U = -\Delta K$$

$$U_i + K_i = U_f + K_f$$

$$0 + \frac{1}{2} m v_i^2 = U_f + 0$$

$\left\{ \begin{array}{l} U_i = 0 \text{ "electron initially at distance } (r)\,;\ r \gg R \text{"} \\ K_f = 0\,, \text{ 'Reversing its direction} \end{array} \right.$

$$v_i = \sqrt{\frac{2U_f}{m}}$$

$\Rightarrow \left\{ \Delta U = q \Delta V \right.$

$$\Rightarrow U_f = qV$$

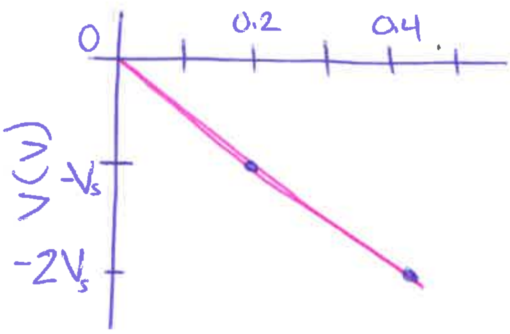
$$= -e(V)$$

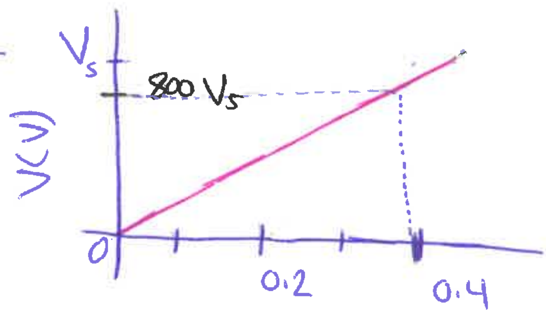
$$U_f = -1.6 \times 10^{-19} \times -170 \text{ J} = 1.6 \times 10^{-19} \times 170 \text{ J}$$

$$v_i = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 170}{9.1 \times 10^{-31}}}$$

$$v_i = 7.73 \times 10^{6} \frac{m}{s}$$

$\boxed{24-11}$ An electron is placed in an xy plane where the electric potential depends on x and y as shown, for the coordinate axes in the below Figures (the potential does not depend on z). The scale of the vertical axis is set by $V_s = 1000\,V$. In unit-vector notation, what is the electric force on the electron?





$\Rightarrow \vec{F} = q\vec{E}$, $q = -e$ "electron"

- $E_x = -\dfrac{\partial V}{\partial x} = -$ slope of (V vs x) curve

$$E_x = -\frac{-2V_s}{0.4} = \frac{2\times 1000}{0.4} = 5\times 10^3\ \frac{N}{C}$$

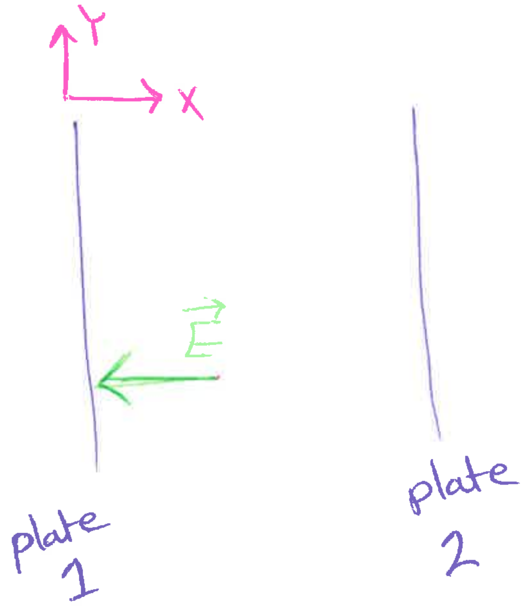
- $E_y = -\dfrac{\partial V}{\partial y} = -$ slope of (V vs y) curve

$$E_y = -\frac{V_s}{0.4} = -\frac{800}{0.4} = -2.0\times 10^3\ N/C$$

$$\Rightarrow \vec{F} = -e\left[5\hat{i} - 2.0\hat{j}\right]\times 10^3\ N \quad , \quad e = 1.6\times 10^{-19}\,C$$

$$\vec{F} = (-8.0\times 10^{-16}\,N)\hat{i} + (3.2\times 10^{-16}\,N)\hat{j}$$

24-12 The electric potential V in the space between two flat parallel plates 1 and 2 is given (in volts) by $V = 1500x^2$, where x (in meters) is the perpendicular distance from plate 1. At $x = 1.8$ cm, (a) what is the magnitude of the electric field and (b) is the field directed toward or away from plate 1?



- $E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(1500x^2)$

$$E_x = -3000x$$

$$\vec{E} = (-3000\,x)\,\hat{\imath}\ \text{N/C}$$

$$\Rightarrow \vec{E}(x = 1.8\,\text{cm}) = -3000 \times 0.018\,\hat{\imath}$$

$$\vec{E} = -54\ \frac{\text{V}}{\text{m}}\,\hat{\imath}$$

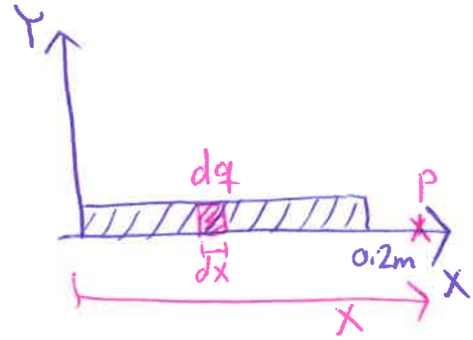
$\therefore$ Toward plate 1

24-14 A nonuniform Linear charge distribution given by $\lambda = bx$, where b is a constant, is located along an x axis from $x=0$ to $x=0.2$m. If $b = 15\,nC/m^2$ and $V =$ zero at infinity, what is the electric potential at (a) the origin and (b) the point $y = 0.15$m on the y axis?

$$V = \int dV = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

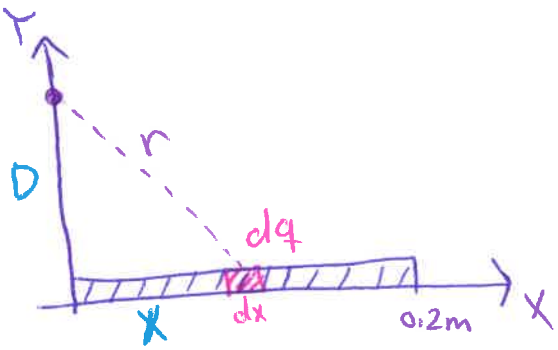
Linear charge density $dq = \lambda\, dx = bx\, dx$ $[0 \leqslant x \leqslant 0.2\text{m}]$

a) $$V = \frac{1}{4\pi\epsilon_0}\int_0^{0.2\text{m}} \frac{bx\,dx}{x} = \frac{b}{4\pi\epsilon_0}\, x\Big]_0^{0.2\text{m}}$$



$$V = 9\times10^9 \times 15\times10^{-19}\,(0.2) = 27\,V$$

b) $$r = \sqrt{x^2 + D^2}$$



$$V = \frac{1}{4\pi\epsilon_0}\int_0^{0.2\text{m}} \frac{bx\,dx}{\sqrt{x^2 + D^2}}$$

$$= \frac{b}{4\pi\epsilon_0}\int_0^{0.2\text{m}} \frac{x\,dx}{\sqrt{x^2+D^2}}$$

let $u = x^2 + D^2 \Rightarrow du = 2x\,dx \Rightarrow \int \frac{x\,dx}{\sqrt{x^2+D^2}} = \frac{1}{2}\int \frac{du}{u^{1/2}}$

$$= \frac{1}{2}\int u^{-1/2}\,du = u^{\frac{1}{2}}$$

$$V = \frac{b}{4\pi\epsilon_0}\int_0^{0.2} \frac{x\,dx}{\sqrt{x^2+D^2}} = \frac{b}{4\pi\epsilon_0}\sqrt{x^2+D^2}\,\Big|_0^{0.2\text{m}}$$

$$V = \frac{b}{4\pi\epsilon_0}\left[\sqrt{(0.2)^2 + (0.15)^2} - 0.15\right] = 13.5\,V$$

$$* \quad V = \frac{b}{4\pi\epsilon_0}\left[\sqrt{D^2 + L^2} - D\right] \quad *$$

{ V at point in y-axis by a nonuniform rod with $\lambda = bx$ on x axis from $x=0$ to $x=L$ }

24-17 What is the magnitude of the electric field at the point $(-1.00\hat{i} - 2.00\hat{j} + 4.00\hat{k})$ m if the electric potential in the region is given by $V = 2.00\,xyz^2$, where V is in volts and coordinates x, y, z are in meters?

$\Rightarrow$ Use $E_s = -\dfrac{\partial V}{\partial s}$

- $E_x = -\dfrac{\partial V}{\partial x} = -\dfrac{\partial}{\partial x}(2xyz^2) = -2yz^2$

  $E_x[(-\hat{i} - 2\hat{j} + 4\hat{k})m] = 64$ volt/m

- $E_y = -\dfrac{\partial V}{\partial y} = -\dfrac{\partial}{\partial y}(2xyz^2) = -2xz^2$

  $E_y[(-\hat{i} - 2\hat{j} + 4\hat{k})] = 32$ volt/m

- $E_z = -\dfrac{\partial V}{\partial z} = -\dfrac{\partial}{\partial z}(2xyz^2) = -4xyz$

  $E_z[(-\hat{i} - 2\hat{j} + 4\hat{k})] = -32$ volt/m

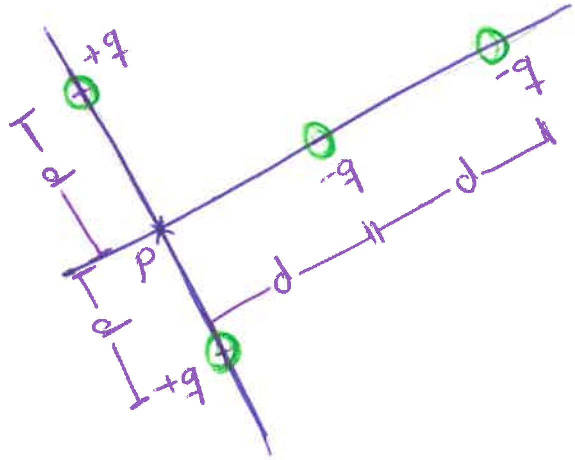
$\Rightarrow \vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

$\vec{E} = (64\hat{i} + 32\hat{j} - 32\hat{k})$ volt/m

$|\vec{E}| = \sqrt{E_x^2 + E_y^2 + E_z^2}$

$|\vec{E}| = 78.4$ volt/m

24-31 What is the net electric potential at point p due to the four particles if $V=0$ at infinity, $q = 7.5$ fC, and $d = 1.6$ cm?



$$V_p = K \sum \frac{q_i}{r_i}$$

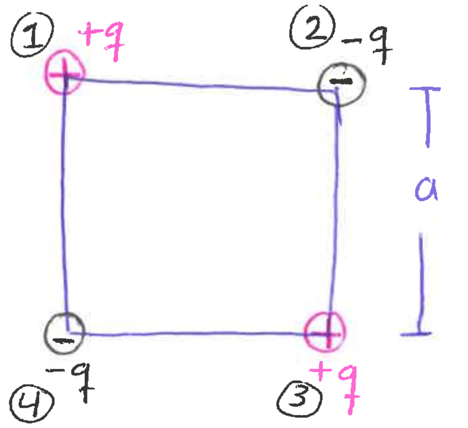
$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q}{d} + \frac{q}{d} + \frac{-q}{d} + \frac{-q}{2d}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{2q - q}{2d}\right] = 9\times10^9 \left[\frac{q}{2d}\right]$$

$$V_p = \frac{9\times10^9 \times 7.5\times10^{-15}}{2\times1.6\times10^{-2}}$$

$$\boxed{V_p = 2.11\,mV}$$

$\boxed{24\text{-}33}$ Each particle have charge magnitude $q = 5.00\ pC$ and were initially infinitely far apart. To form the square with edge Length $a = 64.0\ cm$, (a) how much work must be done by an external agent, (b) how work must be done by the electric force, and (c) What is the potential energy of the system?



(a) $W_{\text{external agent}} = +\Delta U = U_f - U_i$

$\Rightarrow U_i$ (each particle) $= 0$ "They were initially far apart"

$$W_{\text{external agent}} = U_{12} + U_{13} + U_{23} + U_{14} + U_{24} + U_{34}$$

Use $U = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{r}\right]$ "Two-particle system"

$$W_{\text{external agent}} = \frac{1}{4\pi\epsilon_0}\left[\frac{(+q)(-q)}{a} + \frac{(+q)(+q)}{\sqrt{2}\,a} + \frac{(-q)(+q)}{a} + \frac{(+q)(-q)}{a} + \frac{(-q)(-q)}{\sqrt{2}\,a} + \frac{(+q)(-q)}{a}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\cdot\frac{q^2}{a}\left[-1 + \frac{1}{\sqrt{2}} - 1 - 1 + \frac{1}{\sqrt{2}} - 1\right]$$

$$\Rightarrow W_{\text{external}} = \frac{9\times10^9 \times (5\times10^{-12})^2}{0.64}\left[-4 + \frac{2}{\sqrt{2}}\right] = -9.09\times10^{-13}\ J$$

(b) $W_{\text{Electric}} = -W_{\text{external}} = 0.909\times10^{-12} = 0.909\ pJ$

(c) $U = -0.909\ pJ$

The total potential energy of a system of particles is the sum of the potential energies for every pair of particles in the system.

$\boxed{24-39}$ (a) What is the escape speed for an electron initially at rest on the surface of a sphere with a radius of 20 cm and a uniformly distributed charge of 1.6×10^{-15} C? That is, what initial speed must the electron have in order to reach an infinite distance from the sphere and have zero kinetic energy when it gets there? (b) If its initial speed is twice the escape speed, what is its kinetic energy at infinity?

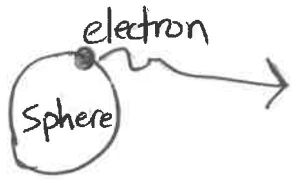
$\Rightarrow$ Initial kinetic energy (used to escape) must equal to the initial potential energy.

(a) • Use Electric Potential Energy of the system $U = K \frac{q_1 q_2}{r}$

$$\frac{1}{2} m_e v^2 = \frac{K e \, q_{sphere}}{r}$$

$$\frac{1}{2}(9.1 \times 10^{-31}) v^2 = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-15}}{20 \times 10^{-2}}$$

$$v = 5031.8 \ \frac{m}{s}$$



$$\boxed{v = 5.0 \ Km/s}$$

Conservation of mechanical energy

$$\Delta K + \Delta U = 0$$

$$(K_f - K_i) + (U_f - U_i) = 0$$

$U_f = 0$, electron reach an infinite distance from the sphere

$K_f = 0$, given in the problem

$$K_i + U_i = 0$$

$$\frac{1}{2} m_e v^2 + \frac{K(-e)\, q_{sphere}}{r} = 0$$

(b) $\Delta U + \Delta K = 0$

$K_i + U_i = K_f$, $v_i = 2v_{escape} = 2 \times 5 \frac{km}{s} = 10 \frac{km}{s}$

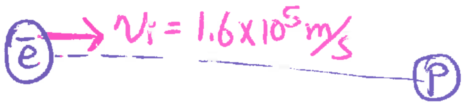
$$\frac{1}{2} m_e v_i^2 - \frac{K e q_{sphere}}{r} = K_f$$

$$K_f = \frac{1}{2}(9.1\times10^{-31})(10\times10^{3})^2 - \frac{9\times10^{9}\times1.6\times10^{-19}\times1.6\times10^{-15}}{0.2}$$

$$K_f = 3.398\times10^{-23}\ \text{J}$$

$$\boxed{K_f = 3.4\times10^{-23}\ \text{J}}$$

24-47 An electron is projected with an initial speed of $1.6 \times 10^5 \frac{m}{s}$ directly toward a proton that is fixed in place. If the electron is initially a great distance from the proton, at what distance from the proton is the speed of the electron instantaneously equal to twice the initial value?



$q_1 = q_e = -e$

$q_2 = q_p = +e$

- By using conservation of a mechanical energy

$$\Delta K + \Delta U = 0$$

$$K_i + U_i = K_f + U_f$$

$U_i = 0$ "the electron is initially a great distance from the proton"

$$\frac{1}{2} m v_i^2 + 0 = \frac{1}{2} m (2v_i)^2 + \frac{K(e)(-e)}{r}$$

$$\frac{4}{2} m v_i^2 - \frac{1}{2} m v_i^2 = \frac{K e^2}{r}$$

$$\frac{3}{2} m v_i^2 = \frac{K e^2}{r}$$

$$r = \frac{2Ke^2}{3 m v_i^2} = \frac{2 \times 9 \times 10^9 \times (1.6 \times 10^{-19})^2}{3 (9.1 \times 10^{-31})(1.6 \times 10^5)^2}$$

$$r = 6.59 \times 10^{-9} \text{ m}$$

$$\boxed{r = 6.6 \text{ nm}}$$

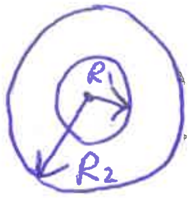
24-48 Two isolated, concentric, conducting spherical shells have radii $R_1 = 0.5m$ and $R_2 = 1.0m$, uniform charges $q_1 = +3.0\mu C$ and $q_2 = +1.0\mu C$, and negligible thicknesses. What is the magnitude of the electric field E at radial distance

(a) $r = 4.0\,m$

$\Rightarrow$ spherically symmetric $E(r) = \frac{1}{4\pi\epsilon_0} \frac{q_{enc}}{r^2}$

$r = 4.0m > R_2 > R_1$, $q_{enc} = (+3+1)\mu C$

$E(r=4.0m) = \frac{9\times10^9 \times 4\times10^{-6}}{(4)^2} = 2.25\times10^3 \frac{N}{C}$



(b) $r = 0.7m$, $R_1 < 0.7m < R_2$, $q_{enc} = +3\mu C$

$E(r=0.7m) = \frac{9\times10^9 \times 3\times10^{-6}}{(0.7)^2} = 5.51\times10^4\ N/C$

(c) $r = 0.2\,m$, $0.2m < R_1 < R_2$, $q_{enc} = 0$

$E(r=0.2m) = 0$

With $V = 0$ at infinity, What is V at

d) $r = 4.0m$

Electric potential $V(r) - V(r') = -\int_{r'}^{r} E(r)\, dr$

$r = 4.0m > R_2 > R_1$

$V(r) = \frac{1}{4\pi\epsilon_0} \frac{(q_1+q_2)}{r} = \frac{9\times10^9 \times 4\times10^{-6}}{4} = 9\times10^3 V$

e) $r = 1.0m = R_2 > R_1$

$V(1.0m) = \frac{9\times10^9 \times 4\times10^{-6}}{1} = 3.6\times10^4\ V$

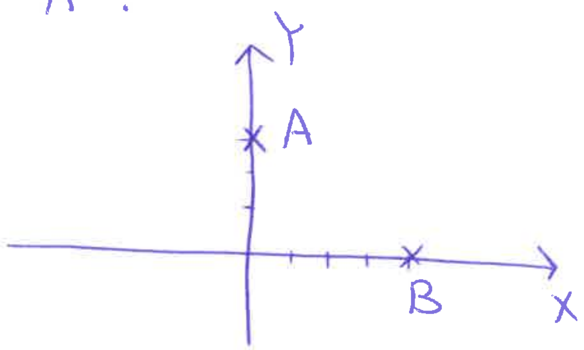
(f) $r = 0.7m$

$R_2 > r = 0.7m > R_1$

$$V(r) = K\left[\frac{q_1}{r} + \frac{q_2}{R_2}\right] = 9\times10^9\left[\frac{3\times10^{-6}}{0.7} + \frac{1\times10^{-6}}{1.0}\right]$$

$$V(0.7m) = 4.75\times10^4\ V$$

$\boxed{24\text{-}59}$ The electric field in a region of space has the components $E_y = E_z = 0$ and $E_x = (4.00\ N/C)x^2$. Point A is on the y-axis at $y = 3.00m$, and point B is on the x-axis at $x = 4.00m$. What is the potential difference $V_B - V_A$?



- $E_y, E_z = 0$
- $V_B - V_A = -\int_{X_A}^{X_B} E_x\, dx$

$$= -\int_0^4 4x^2\, dx$$

$$\left\{ \begin{array}{l} X_A = 0 \\ X_B = 4 \end{array} \right\}$$

$$= -4 \frac{x^3}{3} \Big]_0^4$$

$$= -4 \frac{(4)^3}{3} - 0 = -85.3 \text{ Volts}$$

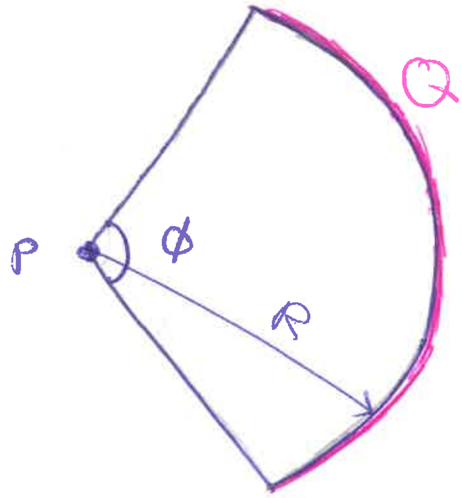
$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

if $\vec{E} = E_x \hat{\imath} + E_y \hat{\jmath} + E_z \hat{k}$

$$V_f - V_i = -\int_i^f (E_x \hat{\imath} + E_y \hat{\jmath} + E_z \hat{k}) \cdot (dx\,\hat{\imath} + dy\,\hat{\jmath} + dz\,\hat{k})$$

$$V_f - V_i = -\int_{x_i}^{x_f} E_x\, dx + -\int_{y_i}^{y_f} E_y\, dy + -\int_{z_i}^{z_f} E_z\, dz$$

$\boxed{24-60}$ A plastic Rod having a uniformly distributed charge $Q = -28.9\,pC$ has been bent into a circular arc of radius $R = 3.71$ cm and central angle $\phi = 120°$. With $V = 0$ at infinity what is the electric potential at P, the center of curvature of the Rod?



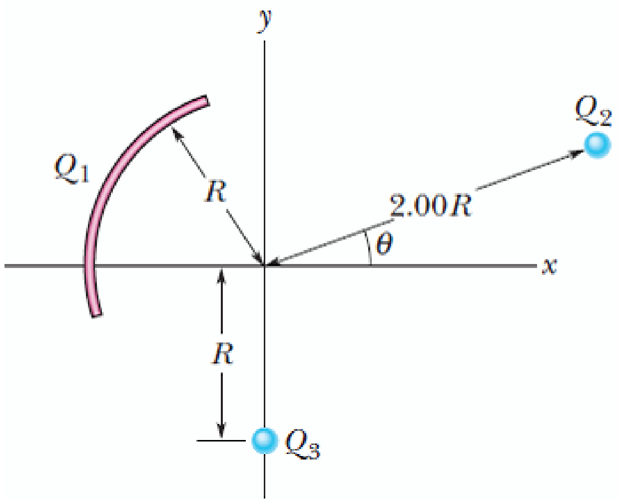
$$V = \frac{KQ}{R}$$

$$= \frac{9\times10^{9} \times (-28.9\times10^{-12})}{3.71\times10^{-2}}$$

$$\boxed{V_p = -7.003 \text{ Volts}}$$

24-61 (a) In the below figure, what is the net electric potential at the origin due to the circular arc of charge $Q_1 = +7.21\,pC$ and the two particles of charges $Q_2 = 4.00\,Q_1$ and $Q_3 = -2Q_1$? The arc's center of curvature is at the origin and its radius is $R = 2.00\,m$; the angle indicated is $\theta = 35.0°$. (b) What is the net electric potential at the origin if both Q_1 and R are doubled?

$$V_{arc} = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{R}$$



Since the charge distribution on the arc is equidistance from the point where V is evaluated; its contribution is identical to that of a point charge at that distance.

(a) $$V = \frac{1}{4\pi\epsilon_0}\left[\frac{Q_1}{R} + \frac{4Q_1}{2R} - \frac{2Q_1}{R}\right] = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{R}$$

$$V = \frac{9\times10^{9}\times7.21\times10^{-12}}{2.00} = 3.24\times10^{-2}\,V = 32.4\,mV$$

b) If both Q_1 and R are double; the net electric potential at the origin is still unchanged.

24-63 The Electric potential at points in an xy plane is given by $V = (2.00\frac{V}{m^2})x^2 - (3.00\frac{V}{m^2})y^2$. What are (a) the magnitude and (b) angle (relative to +x) of the Electric field at the point (4.00 m, 2.00 m)?

• use $E_s = -\frac{\partial V}{\partial s}$

$$\Rightarrow E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(2x^2 - 3y^2) = -4x$$

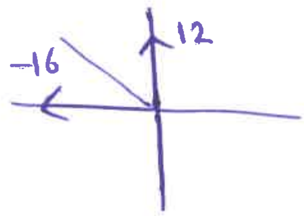
$$E(x=4m) = -16 \text{ V/m}$$

$$\Rightarrow E_y = -\frac{\partial V}{\partial y} = +6y \quad , \quad E(y=2m) = +12 \text{ V/m}$$

$$\boxed{\vec{E} = -16\hat{i} + 12\hat{j}}$$

$$\vec{E} = \sqrt{(16^2) + (12)^2} \Rightarrow \vec{E} = 20\frac{N}{C} = 20\frac{V}{m}$$

$$\tan\theta \simeq \frac{E_y}{E_x}$$

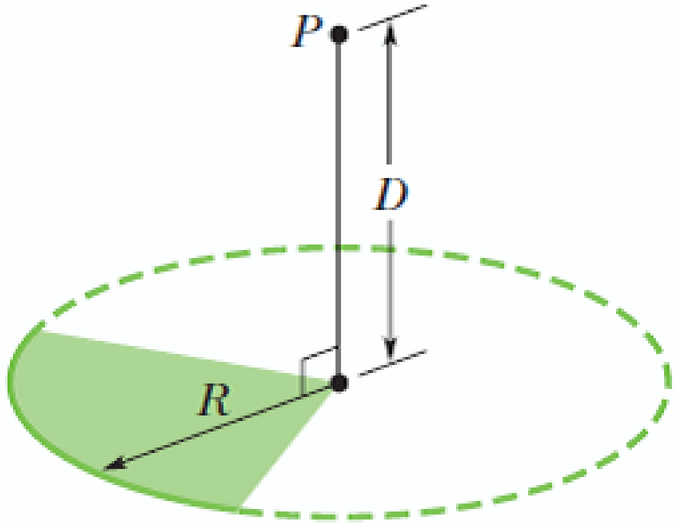


$\theta \sim 36.9$ ignore

$$\boxed{\theta = 143^\circ}$$, E_x = negative

E_y = positive

24-67 A plastic disk of radius $R = 64.0$ cm is charged on one side with a uniform surface charge density $\sigma = 7.73\ fC/m^2$, and then three quadrants of the disk are removed. The remaining quadrant is shown in the below figure. With $V = 0$ at infinity, what is the potential due to the remaining quadrant at point P, which is on the central axis of the origional disk at distance $D = 45.0$ cm from the origional center?



The potential at point P due to a single quadrant is one-forth the potential due to the entire disk since the disk is uniformly charged

$$V_{Disk} = \frac{\sigma}{2\epsilon_0}\left[\sqrt{R^2+D^2} - D\right]$$

$$V_{one\ quadrant} = \frac{V_{Disk}}{4} = \frac{\sigma}{8\epsilon_0}\left[\sqrt{R^2+D^2} - D\right]$$

$$V_{one\ quadrant} = \frac{7.73\times10^{-15}}{8\times 8.85\times10^{-12}}\left[\sqrt{(0.64)^2+(0.45)^2} - 0.45\right]$$

$$= 3.63\times10^{-5}\ V$$

$$\boxed{V_{one\ quadrant} = 36.3\ \mu V}$$