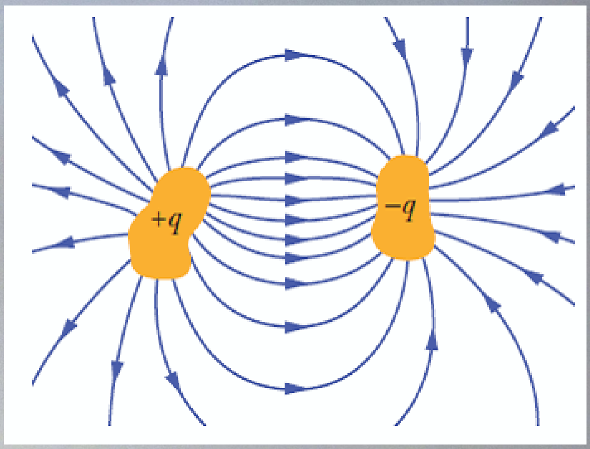
Chapter 25: Capacitance

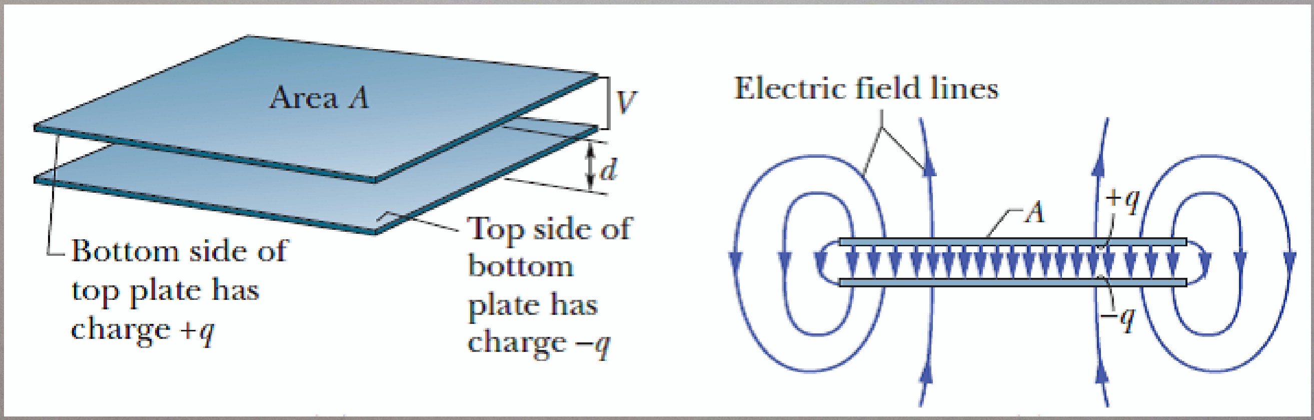
- Capacitor ⇛
  A device in which electrical energy can be stored.

Capacitor consists of two isolated conductors (the plates) with charge $+q$ and $-q$.



- Capacitor symbol (⊣⊢) used for all capacitors of all geometries
- Capacitance is a measure of how much charge must be put on the plates to produce a certain potential difference between them.
- Capacitance depends only on the geometry of the plates.
- Capacitance defined from $\boxed{q = CV}$

$$1 \text{ farad} = 1\,\text{F} = 1\,\text{C/V}$$



⇛ Calculating the capacitance:

[1] Assume a charge q to have been placed on the plates.

[2] Find the electric field $\vec{E}$ due to this charge by using Gauss' Law

$$q = \epsilon_0 \Phi = \epsilon_0 \oint \vec{E} \cdot d\vec{A}$$

[3] Evaluate the potential difference V between the plates

$$V = -\int_i^f \vec{E} \cdot d\vec{s}$$

"choose a path that follows an electric field, from the negative plate to the positive plate

$\vec{E}$ and $d\vec{s}$ will be in opposite direction

$$\vec{E} \cdot d\vec{s} = -E\,ds$$

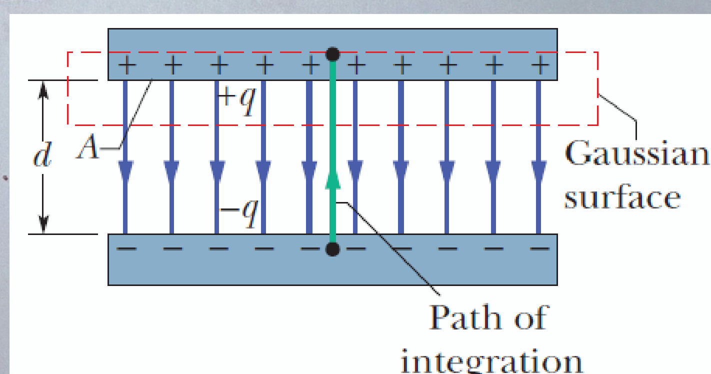
$$V = \int_-^+ E\,ds$$

[4] Calculate C from $q = CV$

[1] A parallel plate capacitor

plates are so Large and so close together that we can neglect the fringing of the electric field at the edges of the plates.

$\vec{E}$ is constant between the plates.

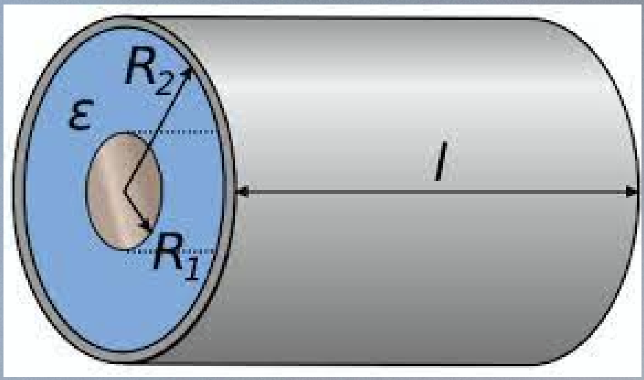


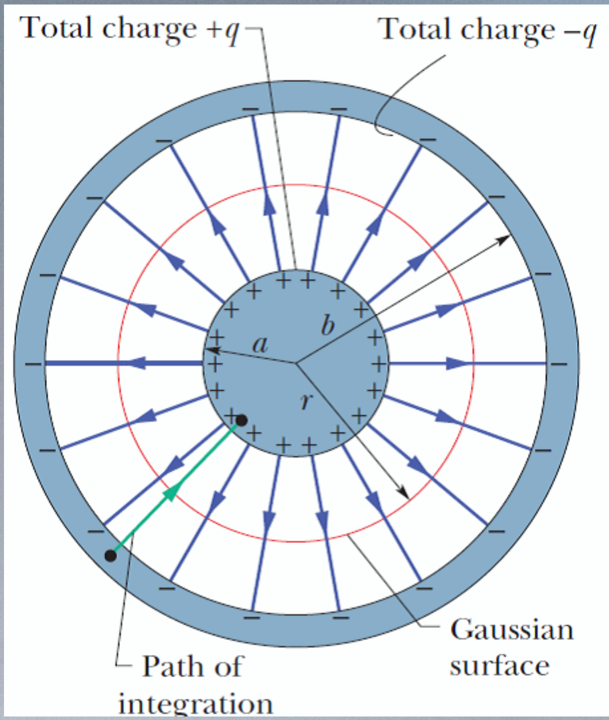
- Apply Gauss' Law to the Red-dashed Gaussian surface that encloses just q on the positive plate. $q = \epsilon_0 E A$; A is the area of the plate
- $V = \int_-^+ E\,ds = E \int_0^d ds = Ed$; d is the plate separation
- $C = \frac{q}{V} = \frac{\epsilon_0 E A}{Ed} = \frac{\epsilon_0 A}{d}$

$$\boxed{C = \frac{\epsilon_0 A}{d} \text{ (Parallel plate capacitor)}}$$

2 A cylindrical Capacitor

- Formed by two coaxial cylinders of radii a and b and length L.





- Find E by using Gauss' Law

$$q = \epsilon_0 E A = \epsilon_0 E(2\pi r L)$$

$$E = \frac{q}{2\pi\epsilon_0 L r} \; ; \quad a \le r \le b \quad \text{(radially outward)}$$

- $$V = -\int_{-}^{+} E\,ds = \frac{-q}{2\pi\epsilon_0 L}\int_b^a \frac{dr}{r}$$ " $ds = -dr$ " integrate radially inward

$$= \frac{-q}{2\pi\epsilon_0 L}\left[\ln(a) - \ln(b)\right]$$

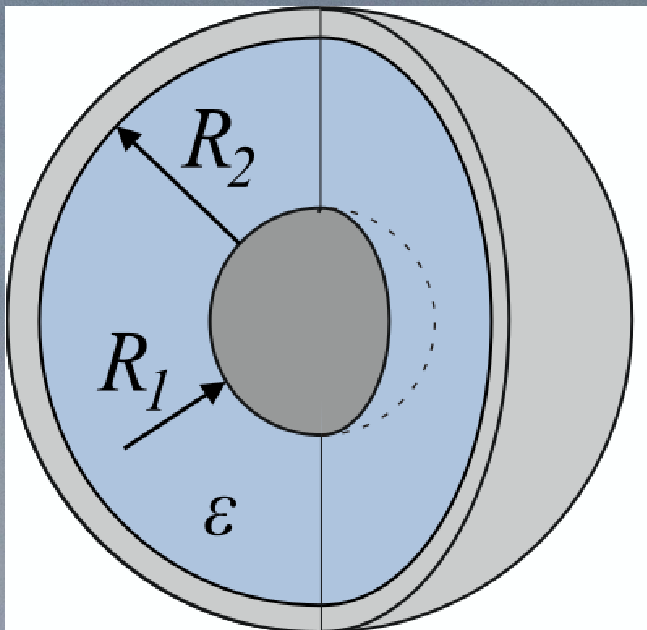
$$V = \frac{q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right)$$

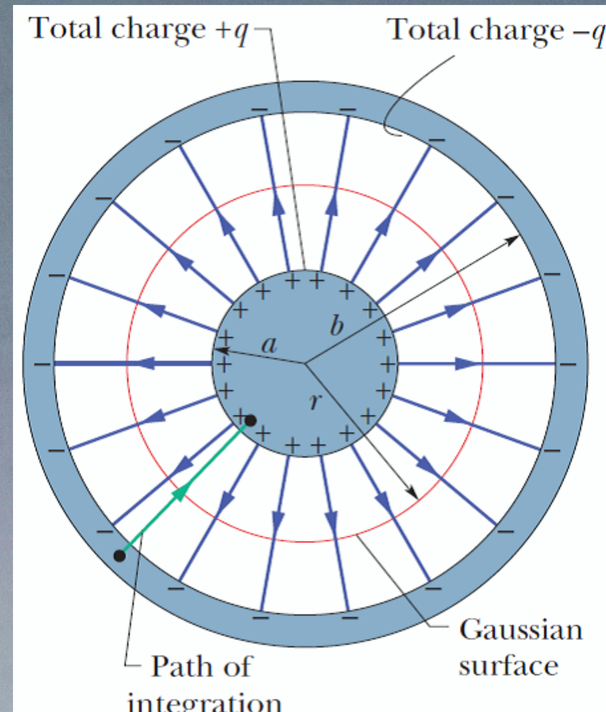
- $$C = \frac{q}{V} = \frac{q}{\frac{q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right)} = \frac{2\pi\epsilon_0 L}{\ln(b/a)}$$

$$\boxed{C = \frac{2\pi\epsilon_0 L}{\ln(b/a)} \quad \text{(cylindrical capacitor)}}$$

$\Rightarrow$ Capacitance per unit length $= \frac{C}{L} = \frac{2\pi\epsilon_0}{\ln(b/a)} = \frac{1}{2k\ln(b/a)}$

3 A spherical Capacitor





⇒ Central cross section of a capacitor that consists of two concentric spherical shells, of radii a and b.

- E by using Gauss' Law

$$q = \epsilon_0 E A = \epsilon_0 E (4\pi r^2)$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

- $V = \int_-^+ E\,ds = -\frac{q}{4\pi\epsilon_0} \int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$

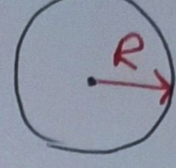
$$V = \frac{q}{4\pi\epsilon_0} \frac{b-a}{ab}$$

- $C = \frac{q}{V} = 4\pi\epsilon_0 \frac{ab}{b-a}$

$$\boxed{C = 4\pi\epsilon_0 \frac{ab}{b-a} \quad (\text{spherical capacitor})}$$

4 An Isolated sphere

Single isolated spherical conductor of radius R by assuming the "missing plate" is a conducting sphere of infinite radius



Spherical capacitor $C = 4\pi\epsilon_0 \dfrac{ab}{b-a}$

$$C = 4\pi\epsilon_0 \frac{a}{1-\frac{a}{b}} \quad [b \to \infty ,\ a = R]$$

$$\boxed{C = 4\pi\epsilon_0 R \quad (\text{Isolated sphere})}$$

You can use the electric potential for the spherical conductor $V = \dfrac{kq}{R}$

$$\Rightarrow C = \frac{q}{V} = \frac{R}{k} = 4\pi\epsilon_0 R$$

example Capacitance of the earth

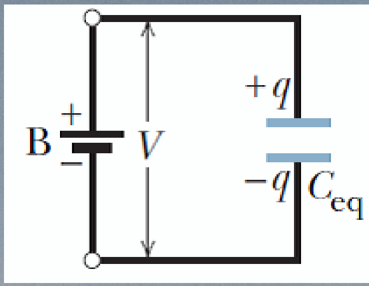
$$R \simeq 6400\,\text{Km} = 64 \times 10^5\,\text{m}$$

$$C = 4\pi\epsilon_0 R = \frac{1}{9\times10^9}\, 64\times10^5 = 0.711\times10^{-3} = 711\,\mu F$$

All formulas that we have derived for the Capacitance involve the constant ϵ_0 multiplied by a quantity that has the dimension of a length.

Capacitors in Parallel and in Series

1 Capacitors in parallel



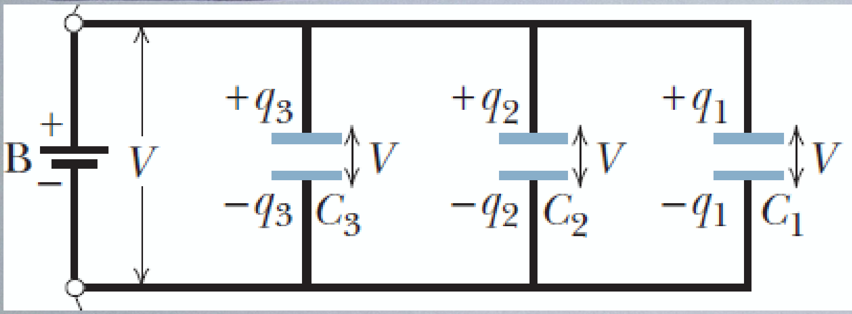
$q_1 = C_1 V$, $q_2 = C_2 V$, $q_3 = C_3 V$

q (total charge) $= q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V$

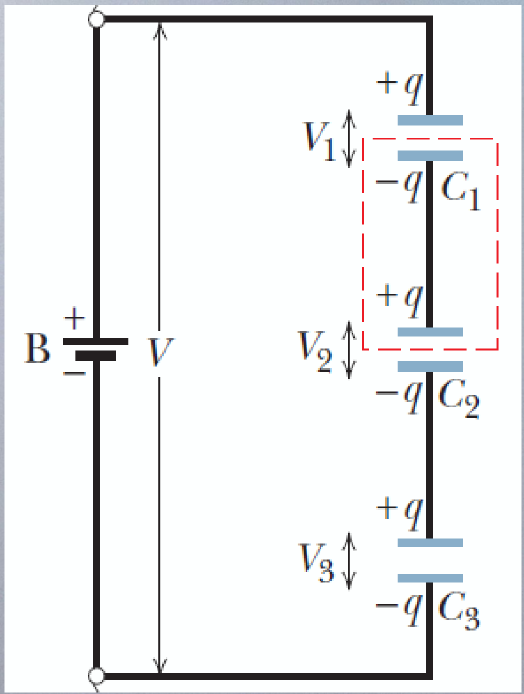
$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3$$

"Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same total charge q and the same potential V as the actual capacitors"

$$\boxed{C_{eq} = \sum_{j=1}^{N} C_j \quad (N\text{- Capacitors in parallel})}$$



2 Capacitors in Series



$V_1 = \frac{q}{C_1}$, $V_2 = \frac{q}{C_2}$, $V_3 = \frac{q}{C_3}$

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

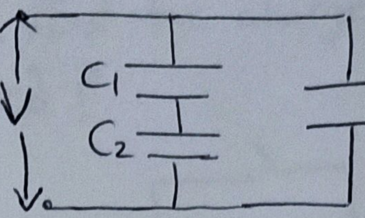
$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\boxed{\frac{1}{C_{eq}} = \sum_{j=1}^{N} \frac{1}{C_j} \quad (N\text{- Capacitors in Series})}$$

Capacitors that are connected is series can be replaced with an equivalent capacitor that has the same charge q and the same total potential difference V as the actual series capacitors

25-6 A potential difference $V = 100\,V$ is applied across a capacitor arrangement with capacitances $C_1 = 10.0\,\mu F$, $C_2 = 5.0\,\mu F$ and $C_3 = 2.00\,\mu F$. What are the q, V and U for each capacitor?



- Find C_{eq}

$\Rightarrow$ C_1 and C_2 in series

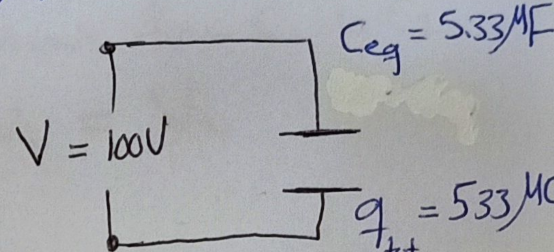
$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10} + \frac{1}{5} = \frac{3}{10}$$

$$\boxed{C_{12} = \frac{10}{3} = 3.33\,\mu F}$$

$\Rightarrow$ C_{12} and C_3 in parallel

$$C_{123} = C_{12} + C_3 = 3.33 + 2 = 5.33\,\mu F$$

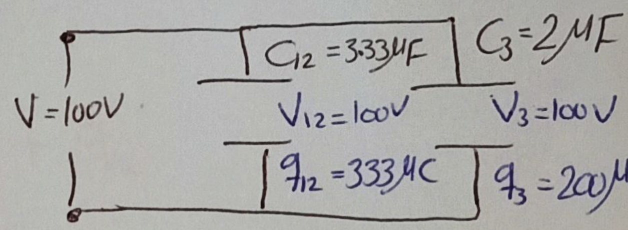
$$\boxed{C_{eq} = 5.33\,\mu F}$$



$U_{tot} = 26.6\,mJ$

- To Find charges

$$q_{tot} = C_{eq} V = 5.33\,(100) = 533\,\mu C$$

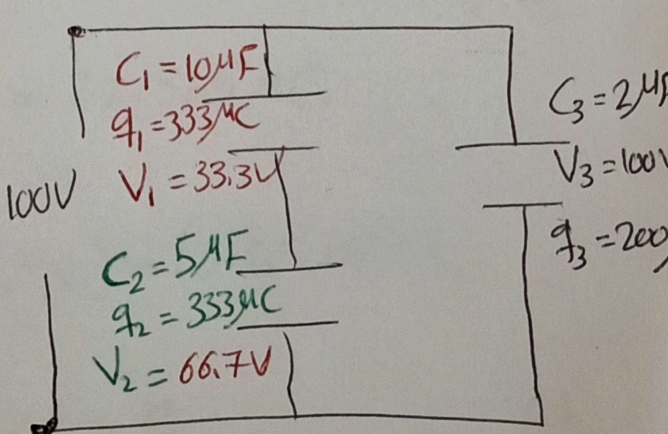


- $V_3 = V_{12} = V = 100\,V$

$\Rightarrow$ $q_3 = C_3 V_3 = 200\,\mu C$

$$U_3 = \frac{1}{2} q_3 V_3 = \frac{1}{2}(200 \times 10^{-6})(100)$$

$$\boxed{U_3 = 0.01\,J = 10\,mJ}$$



$\Rightarrow$ C_1 and C_2 in Series

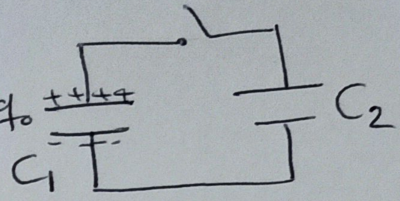
$$q_{12} = q_1 = q_2 = 333\,\mu C$$

$$V_1 = \frac{q_1}{C_1} = \frac{333\,\mu C}{10\,\mu F} = 33.3\,V$$

$$V_2 = \frac{q_2}{C_2} = \frac{333\,\mu C}{5\,\mu F} = 66.7\,V$$

$$\left.\begin{aligned} U_1 &= 5.54\,mJ \\ U_2 &= 11\,mJ \\ U_3 &= 10\,mJ \end{aligned}\right\} U_{tot} = \underline{26.5\,mJ}$$

Sample problem 25.3: Capacitor 1, with $C_1 = 3.55\,\mu F$, is charged to a potential difference $V_0 = 6.3V$, using a 6.3V battery. The battery is then removed, and the capacitor is connected as the below figure to an uncharged capacitor 2, with $C_2 = 8.95\,\mu F$. When switch S is closed, charge flows between the capacitors. Find the charge on each capacitor when equilibrium is reached?



- C_1 is connected to a battery

$$q_0 = C_1 V_0 = (3.55 \times 10^{-6})F\,(6.3V)$$

$$\boxed{q_0 = 22.365\,\mu C}$$

- C_1 is removed from the battery and then connected to C_2

C_1 begins to charge C_2; the charge will move from C_1 to C_2 until the equilibrium is reached $\Rightarrow V_1 = V_2$

$$V_1 = V_2$$

$$\frac{q_1}{C_1} = \frac{q_2}{C_2}$$

By conservation of charge $\Rightarrow q_0 = q_1 + q_2$

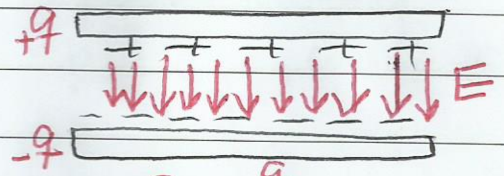
$$\frac{q_1}{C_1} = \frac{q_2}{C_2}$$

$$\frac{q_1}{3.55\,\mu F} = \frac{q_0 - q_1}{8.95\,\mu F} \quad ; \quad q_0 = 22.365\,\mu C$$

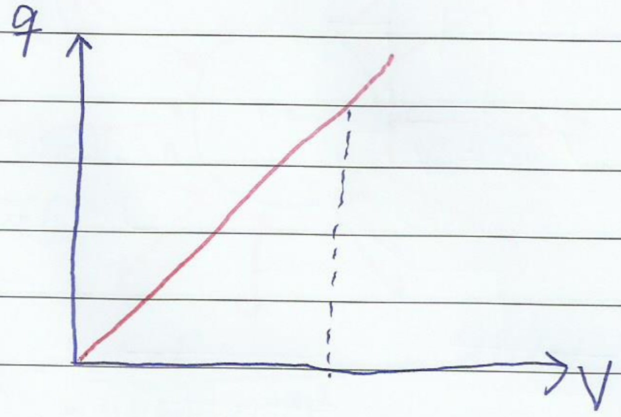
$$(8.95\,\mu F)\,q_1 = (3.55\,\mu F)(22.365\,\mu C) - (3.55\,\mu F)\,q_1$$

$$\left.\begin{aligned} q_1 &= 6.35\,\mu C \\ q_2 &= 16.0\,\mu C \end{aligned}\right\} \quad q_1 + q_2 = q_0$$

Energy Stored in an Electric Field:-



$$C = \frac{q}{V}$$



$$\text{Slope} = \frac{\Delta q}{\Delta V} = C$$

U = Area Under the curve of q versus V

$$= \frac{1}{2} V q \text{ Joul}$$

$$U = \frac{1}{2} q V$$

$$= \frac{1}{2}(CV)V = \frac{1}{2} C V^2 \text{ Joul}$$

$$U = \begin{cases} \frac{1}{2} q V \\ \frac{1}{2} C V^2 \\ \frac{1}{2} \frac{q^2}{C} \end{cases}$$

$$U = \frac{1}{2} q \left(\frac{q}{C}\right) = \frac{1}{2} \frac{q^2}{C}$$

$$\text{Energy Density} = \frac{\text{Energy}}{\text{Volume}} = \frac{U}{\text{Volume}}$$

$$u = \frac{U}{\text{Volume}}$$

$$= \frac{\frac{1}{2} C V^2}{A d}$$

$$= \frac{\epsilon_0 A / d}{2 A d} V^2$$

for parallel plate Capacitor

$$C = \epsilon_0 A / d$$

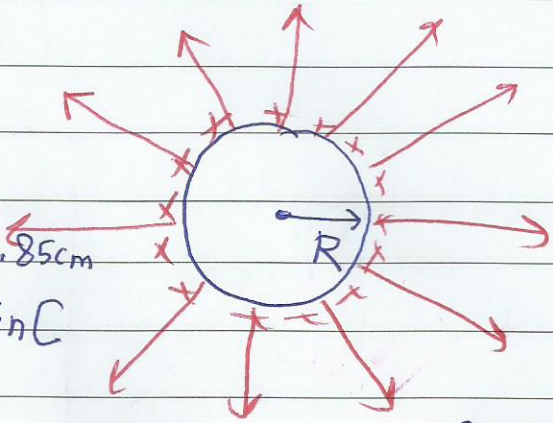
$$\text{Volume} = A d$$

$$U = \frac{1}{2} C V^2$$

$$= \frac{1}{2} \epsilon_0 \frac{V^2}{d^2} = \frac{1}{2} \epsilon_0 \left(\frac{V}{d}\right)^2 = \frac{1}{2} \epsilon_0 E^2 \text{ J/m}^3$$

$$u = \frac{1}{2} \epsilon_0 E^2 \text{ J/m}^3$$

Sample Problem 25.04:



An isolated Conducting sphere

→ radius $= R = 6.85\,cm$

→ Charge $= 1.25\,nC$

1) → $\rho = 0$

2) → $\sigma = \dfrac{q}{4\pi R^2} = \dfrac{1.25\,nC}{4\pi(6.85\times10^{-2})^2} = 21.2\,nC/m^2$

3) → $E = 0$, inside the Conducting sphere

4) $E = 0$, $r < R$

5) → $E_s = \dfrac{q}{4\pi\epsilon_0 R^2} = \dfrac{9\times10^9\times1.25\times10^{-9}}{(6.85\times10^{-2})^2}$

$= 2.4\times10^3\ N/C$

6) → $E = \dfrac{q}{4\pi\epsilon_0 r^2}$, $r \geq R$

→ $V_s = \dfrac{q}{4\pi\epsilon_0 R} = \dfrac{9\times10^9\times1.25\times10^{-9}}{6.85\times10^{-2}}$

$= 164\ V$

→ $V_{center} = 164\,V$

مراجعة

→ $C = 4\pi\epsilon_0 R = \dfrac{6.85\times10^{-2}}{9\times10^9}$

$= 7.61\times10^{-12}\ F$

$= 7.61\ pF$

هنا مطلوب → $U = \dfrac{q^2}{2C} = \dfrac{(1.25\times10^{-9})^2}{2(7.61\times10^{-12})}$

$= 1.026\times10^{-7}\ J$

$= 102.6\ nJ = 103\ nJ$

هنا مطلوب → $u_s = \frac{1}{2}\epsilon_0 E_s^2$

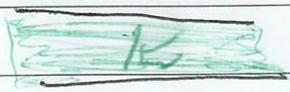
$= \frac{1}{2}\epsilon_0\left(\dfrac{q}{4\pi\epsilon_0 R^2}\right)^2 = 2.54\times10^{-5}\ J/m^3$

Capacitor With a Dielectric:

If you fill the space between the plates of a Capacitor with a dielectric, Which is an insulating material, the Capacitance Will increase by a factor of (K): the Dielectric Constant

$$C_0 = \frac{\epsilon_0 A}{d}$$ air

$$C = K C_0 \quad , \quad K > 1$$



For each Dielectric (insulating material)

- Dielectric Constant $K = \frac{\epsilon}{\epsilon_0}$
- Dielectric strength is the maximum Electric field (E_{max}) the material can withstand

In Dielectric material replace in all equation $\epsilon_0 \rightarrow K\epsilon_0$

$$E = \frac{q}{4\pi\epsilon_0 r^2} \longrightarrow E = \frac{q}{4\pi K\epsilon_0 r^2}$$

$$V = \frac{q}{4\pi\epsilon_0 r} \longrightarrow V = \frac{q}{4\pi (K\epsilon_0) r}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0} \longrightarrow \oint \vec{E} \cdot d\vec{A} = \frac{q}{K\epsilon_0} \Rightarrow \oint K\vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0} \longrightarrow E = \frac{\sigma}{2(K\epsilon_0)}$$

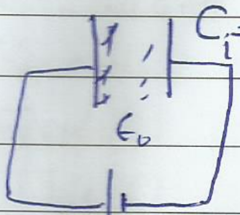
$$u = \frac{1}{2}\epsilon_0 E^2 \longrightarrow u = \frac{1}{2}(K\epsilon_0)E^2$$

Sample Problem 25.05

Work and Energy when a dielectric is inserted into a Capa

$C_i = 13.5\,pF$

(1)



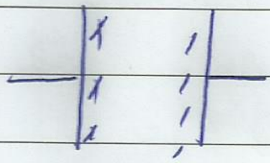
$V_i = 12.5\,V$

a)

$$q_i = C_i V_i = (13.5\,pF)(12.5\,V)$$
$$= 168.75\ pC$$

$$U_i = \frac{1}{2} C_i V_i^2 = \frac{1}{2}(13.5\times10^{-12})(12.5)^2$$
$$= 1.055\times10^{-9}\,J$$
$$= 1.055\ nJ$$

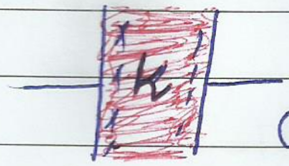
(2)



U_i, q_i, V_i, C_i

(3)

b)



C_f, q_f, V_f, U_f ? $\quad k = 6.5$

$$C_f = k C_i = (6.5)(13.5\,pF)$$
$$= 87.75\,pF$$

$q_f = q_i = 168.75\,pC$ (q does not change because No Source (Battery) for the Capacitor to gain Charge)

$$V_f = \frac{q_f}{C_f} = \frac{168.75\,pC}{87.75\,pF}$$

$$V_f = 1.92\,V$$

$$U_f = \frac{1}{2}\frac{q_f^2}{C_f} = \frac{1}{2}\left[\frac{[168.75\times10^{-12}]^2}{87.7\times10^{-12}}\right.$$

$$U_f = 1.62\times10^{-10}\,J = 0.162\,nJ$$

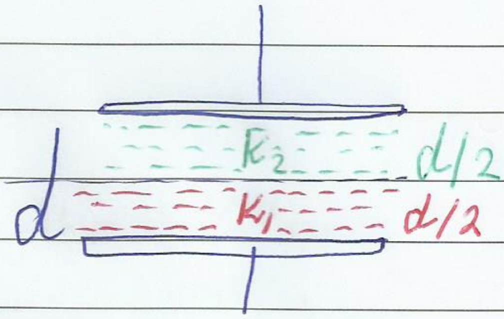
Find the Work done by E during inserting the slab?

$$W_E = -\Delta U = -[U_f - U_i] = -[0.162 - 1.055] = -[-0.893]$$
$$= 0.893\,nJ$$
$$= 893\,pJ$$

, the slab will be Sucked in Not pushed

(12)

Problem (25-31)



$A = 7.89\,cm^2$

$d = 4.62\,mm$

$K_1 = 11$,, $K_2 = 4$

Find C?

This Capacitor will be considered to be 2 Capacitors in series

$$C_1 = \frac{K_1\epsilon_0 A}{d/2} = 2\frac{K_1\epsilon_0 A}{d} = 2K_1\left(\frac{\epsilon_0 A}{d}\right) = 2(11)\left[1.51\times10^{-12}\right] = 33.25\,pF$$

$$C_2 = \frac{K_2\epsilon_0 A}{d/2} = 2\frac{K_2\epsilon_0 A}{d} = 2(4)\left(\frac{\epsilon_0 A}{d}\right) = 2(4)(1.51\times10^{-12}) = 12.08\,pF$$

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K_1\epsilon_0 A} + \frac{d}{2K_2\epsilon_0 A} = \frac{d}{2\epsilon_0 A}\left[\frac{1}{K_1} + \frac{1}{K_2}\right]$$

$$\frac{1}{C_{12}} = \frac{d}{2\epsilon_0 A}\left[\frac{K_2 + K_1}{K_1 K_2}\right]$$

$$C_{12} = \frac{2\epsilon_0 A}{d}\left[\frac{K_1 K_2}{K_1 + K_2}\right] = \frac{\epsilon_0 A}{d}\left[\frac{2K_1 K_2}{K_1 + K_2}\right]$$

$$C_{12} = (1.51\,pF)\left[\frac{2(4)(11)}{11+4}\right] = 1.51\,pF\left[\frac{88}{15}\right] = 1.51\,pF(5.8667)$$

$$C_{12} = 8.86\,pF$$

Problem (25–37)

Dielectric material $\to K = 5.6$

Dielectric material $\to$ Dielectric strength $= 18\,MV/m$

$E_{max} = 18\,MV/m$

$C = 3.9\times10^{-2}\,\mu F$

A_{min} ? For C to withstand a potential difference of 4 kV.

$C \to 3.9\times10^{-2}\,\mu F$

$C \to K = 5.6$

$C \to E_{max} = 18\,MV/m$

$C \to V_{max} = 4\,kV$

$C \to A_{min}$?

$C \to d$?

$$C = \frac{K\epsilon_0 A}{d} \quad ,, \quad \vec{E}\cdot\vec{d} = \Delta V \quad ,, \quad d = \frac{V}{E}$$

$$A = \frac{C.d}{K\epsilon_0} = \frac{C}{K\epsilon_0}\left(\frac{V_{max}}{E_{max}}\right)$$

(A is min. When E is max)

$$A_{min} = \frac{3.9\times10^{-2}\times10^{-6}}{(5.6)(8.85\times10^{-12})}\left(\frac{4\times10^{3}}{18\times10^{6}}\right)$$

$$= \frac{3.9\times10^{-8}}{49.56\times10^{-12}}\left(2.222\times10^{-4}\right)$$

$$A_{min} = 0.175\ m^2$$