Ch. 26: Current and Resistance

- $i = \frac{dq}{dt}$, $[i] = A = \frac{C}{sec}$ $\qquad$ $i \equiv$ scalar quantity

- $\vec{J} \equiv$ current density

$$i = \int \vec{J} \cdot d\vec{A}$$

- Resistance R

$R = \frac{V}{i}$ "ohm's Law"

$\Rightarrow$ $\boxed{\text{Conducting wire } R = \rho \frac{L}{A}}$

- Resistivity $\equiv \rho$ المقاومية

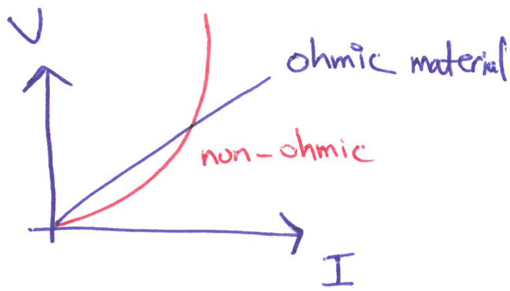
$\rho = \frac{1}{\sigma} = \frac{E}{J}$

$\sigma \equiv$ conductivity الموصلية

$$\boxed{\vec{E} = \rho \vec{J}}$$

- Resistivity ρ for most material changes with temperature

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$
$$\rho = \rho_0 [1 + \alpha (T - T_0)]$$

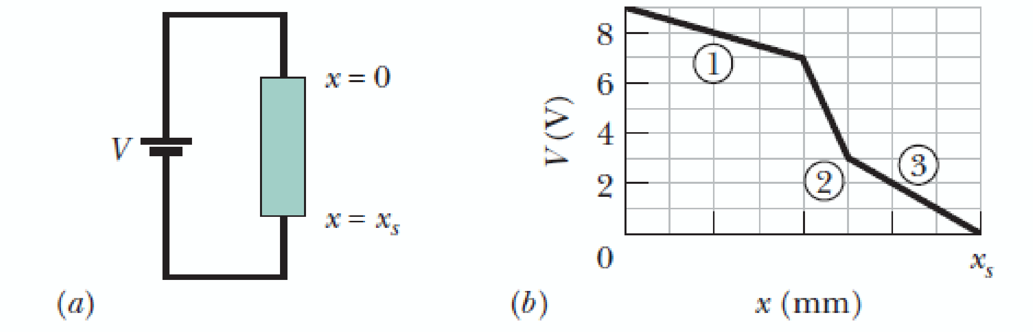


- Power $\equiv$ Rate of energy transfer in an electrical device across which a potential difference V is maintained.

$$P = \frac{\text{Energy}}{\text{time}} = iV = i^2 R = \frac{V^2}{R} \quad , [P] = W = \frac{J}{Sec}$$

المواد العازلة	شبه الموصلة	الموصلة	فائقة الموصلية
Insulator	Semiconductor	conductor	Superconductor
	$\alpha = -$	$\alpha = +$	Lose all electrical resistance at low Temp.
	$R\downarrow, T\uparrow$	$R\uparrow, T\uparrow$	

26-4 In Fig. 26-15a, a 9.00V battery is connected to a resistive strip that consists of three sections with the same cross-sectional areas but different conductivities. Fig 26-15b gives the electric potential $V(x)$ versus position x along the strip. The horizontal scale is set by $x_s = 8.00$ mm. Section 3 has conductivity $4.00 \times 10^7 (\Omega \cdot m)^{-1}$! What is the conductivity of section 1 and 2?



By using Fig. 26-15b, We can find the $\vec{E}$ from V

$$E_s = -\frac{\partial V}{\partial s} \Rightarrow E_x = -\frac{\partial V}{\partial x}$$

- $E_1 = -\frac{\partial V_1}{\partial x}$ [slope of the first segment] $= 0.5 \times 10^3 \frac{V}{m}$
- $E_2 = 4.0 \times 10^3 \frac{V}{m}$ · $E_3 = 1 \times 10^3 \frac{V}{m}$

$\Rightarrow \vec{J} = \sigma \vec{E}$, $J = \frac{i}{A}$ "current density"

Section (1) $\Rightarrow J_1 = \sigma_1 E_1$
Section (2) $\Rightarrow J_2 = \sigma_2 E_2$
Section (3) $\Rightarrow J_3 = \sigma_3 E_3$

The three sections have the same cross-sectional area (A) and the same current passes through them. $J_1 = J_2 = J_3$

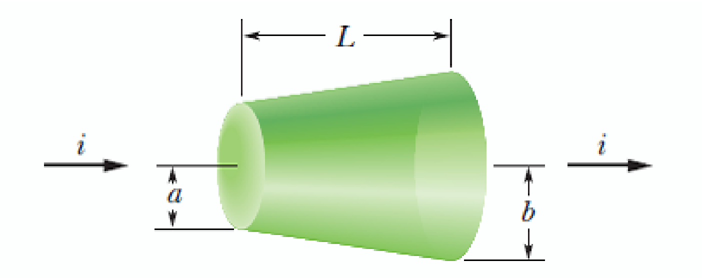
$\Rightarrow \sigma_3 = 4.00 \times 10^7 (\Omega \cdot m)^{-1}$, $J = \sigma_3 E_3 = 4.00 \times 10^7 \times 1 \times 10^3$

$J = 4.0 \times 10^{10}$ A/m²

$$\sigma_1 = \frac{J}{E_1} = \frac{4 \times 10^{10}}{0.5 \times 10^3} = 8.00 \times 10^7 (\Omega \cdot m)^{-1}$$

$$\sigma_2 = \frac{J}{E_2} = \frac{4.0 \times 10^{10}}{4.0 \times 10^3} = 1.0 \times 10^7 (\Omega \cdot m)^{-1}$$

$\boxed{26-5}$ In Fig. 26-16, current is set up through a truncated right circular cone of Resistivity 731 $\Omega \cdot m$, Left radius $a = 1.70mm$, right radius $b = 2.30\,mm$, and Length $L = 3.50cm$. Assume that the current density is uniform across any cross-section taken perpendicular to the Length. What is the resistance of the cone?



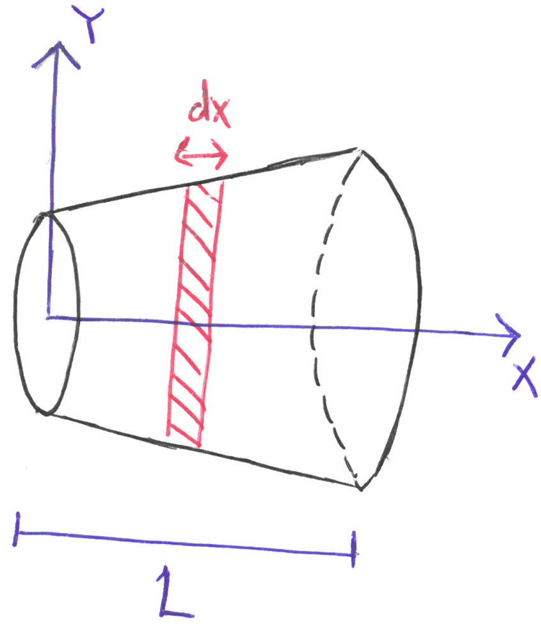
- $\vec{J}$, uniform current density $\Rightarrow$ J is constant along the cone length

$$J = \frac{i}{A} = \frac{i}{\pi r^2}$$

$\Rightarrow$ The radius of the cross-sectional area taken perpendicular to the cone length increases Linearly with X.

Let $r = C_1 X + C_2$

$$\text{when} \begin{cases} X = 0,\ r = a \Rightarrow C_2 = a \\ X = L,\ r = b \Rightarrow C_2 = \frac{b-a}{L} \\ \Rightarrow r = a + \left(\frac{b-a}{L}\right)X \end{cases}$$



$$\Rightarrow R = \frac{\rho}{\pi}\int_0^L \left[a + \left(\frac{b-a}{L}\right)x\right]^{-2} dx$$

$$R = \frac{-\rho}{\pi}\,\frac{L}{b-a}\left[a + \left(\frac{b-a}{L}\right)x\right]^{-1}\Bigg|_0^L$$

$$R = \frac{-\rho L}{\pi(b-a)}\left[b^{-1} - a^{-1}\right]$$

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2}$$

$$R = \frac{-\rho L}{\pi(b-a)}\left[\frac{1}{b} - \frac{1}{a}\right]$$

$$= \frac{-\rho L}{\pi(b-a)} \frac{(a-b)}{ab}$$

$$\boxed{R = \frac{\rho L}{\pi a b}}$$

Note if $a = b = r$

$$R = \frac{\rho L}{\pi r^2}$$

- $\rho = 731\ \Omega \cdot m$

$L = 3.50\ cm = 3.50 \times 10^{-2}\ m$

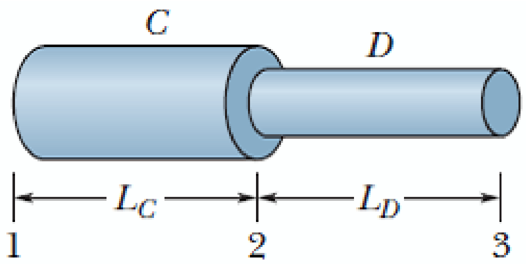
$a = 1.70\ mm = 1.70 \times 10^{-3}\ m$

$b = 2.30\ mm = 2.30 \times 10^{-3}\ m$

$$\Rightarrow R = \frac{731 \times 3.50 \times 10^{-2}}{\pi\ 1.70 \times 2.30 \times 10^{-6}} = 208.4 \times 10^{4}\ \Omega$$

$$\boxed{R = 2.08\ M\Omega}$$

26-25 Wire C and wire D are made from different materials and have Length $L_C = L_D = 1.0\,m$. The resistivity and radius of wire C are $2.0\times10^{-6}\ \Omega.m$ and $1.00\,mm$ and those of wire D are $1.0\times10^{-6}\ \Omega.m$ and $0.5\,mm$. The wires are joined as shown in the below Figure, and a current of 2.0 A is set up in them. What is the electric potential difference between (a) points 1 and 2 and (b) points 2 and 3? What is the rate at which energy is dissipated between (c) points 1 and 2 and (d) points 2 and 3?



$$\boxed{R = \rho \frac{L}{A}}$$

a) $V_{12} = i R_c$, $R_c \equiv$ resistance of wire C

$$R_c = \frac{\rho_c L_c}{\pi r_c^2} = \frac{2.0\times10^{-6} \times 1.0}{\pi (1\times10^{-3})^2} = 0.637\ \Omega$$

$$V_{12} = i R_c = 2.0\times0.637 = 1.3\ V$$

b) $V_{23} = i R_D$, $R_D \equiv$ resistance of wire D

$$R_D = \frac{\rho_D L_D}{\pi r_D^2} = \frac{1.0\times10^{-6}\times1.0}{\pi (0.5\times10^{-3})^2} = 1.27\ \Omega$$

$$V_{23} = i R_D = 2.0\times1.27 = 2.5\ V$$

c) $P_{12} = i^2 R_c = 2.5\ W$

d) $P_{23} = i^2 R_D = 5.1\ W$

$\boxed{26\text{-}32}$ The current-density magnitude in a certain circular wire is $J = (2.75 \times 10^{10}\ A/m^4)r^2$, where r is the radial distance out to the wire's radius of 3.00 mm. The potential applied to the wire (end to end) is 80.0 V. How much energy is converted to thermal energy in 1.00 h ?

$\Rightarrow P = iV$ = rate of energy transfer, in an electrical device across which a potential difference V is maintained.

$\Rightarrow i = \int \vec{J} \cdot d\vec{A}$, let $J = Cr^2$, $C = 2.75 \times 10^{10} \frac{A}{m^4}$

$$\boxed{\begin{aligned} A &= \pi r^2 \\ dA &= 2\pi r dr \end{aligned}}$$

$$i = \int_0^{3.0mm} Cr^2\, 2\pi r dr$$

$$= 2\pi C \int r^3 dr = 2\pi C \frac{r^4}{4} \Big]_0^{3.0mm}$$

$$i = \frac{\pi C}{2} r^4 \Big]_0^{3 \times 10^{-3} m}$$

$$i = \frac{\pi \times 2.75 \times 10^{10} (3 \times 10^{-3})^4}{2}$$

$$\boxed{i = 3.5 A}$$

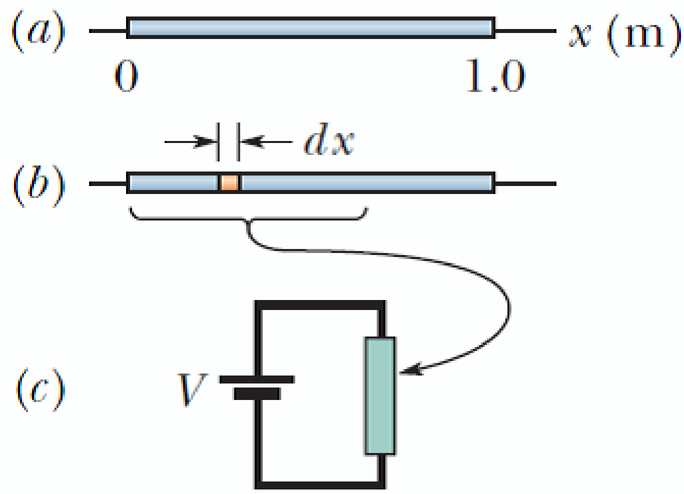
- Energy converted to thermal energy $= P * t$

$$E = P * t = iVt$$

$$E = 3.5 \times 80.0 \times 1.0 \cancel{h} \left[\frac{60 \times 60 \text{ sec}}{1 \cancel{h}}\right]$$

$$\boxed{E = 1.008 MJ}$$

26-40 The below Figure shows a rod of resistive material. The resistance per unit length of the rod increases in the positive direction of the x axis. At any position x along the rod, the resistance dR of a narrow (differential) section of width dx is given by $dR = 5.00x\,dx$, where dR is in ohms and x is in meters. Part b of the Figure shows such a narrow section. You are to slice off a length of the rod between $x = 0$ and some position $x = L$ and then connect that length to a battery with potential difference $V = 8.0\,V$ [Part c in the Figure]. You want the current in the length to transfer energy to thermal energy at the rate of 180W. At what position $x = L$ should you cut the Rod?



- $P = V^2/R$

  $R = V^2/P = (8.0)^2/180$

  $\boxed{R = 0.356\ \Omega}$

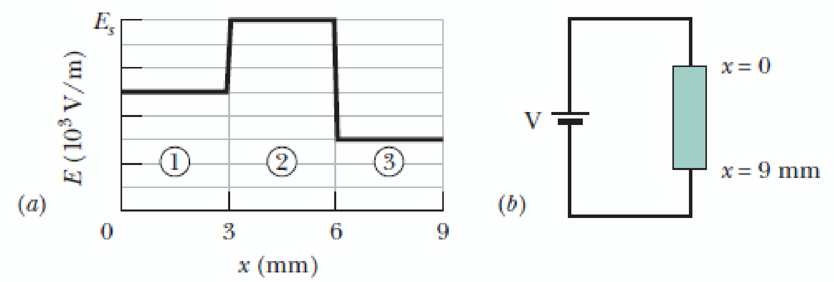
- $dR = 5.00x\,dx$

  $R = \int_0^L 5.00x\,dx = \frac{5x^2}{2}\Big]_0^L$

  $0.356 = \frac{5L^2}{2}$

  $\boxed{L = 0.38\text{m}}$

26-48 The below Figure part (a) gives the magnitude E(x) of the electric fields that have been set up by a battery along a resistive rod of length 9.00 mm (Part (b) of the below Figure). The vertical scale is set by $E_s = 8.00 \times 10^3$ V/m. The rod consists of three sections of the same material but with different radii. (The Schematic below diagram does not indicate the different radii.) The radius of section 3 is 1.70 mm. What is the radius of (a) section 1 and (b) section 2 ?



- Same material ⇒ Same resistivity for the three sections

$$\vec{E} = \rho \vec{J} \Rightarrow \rho = \frac{E}{J} \Rightarrow \frac{E_1}{J_1} = \frac{E_2}{J_2} = \frac{E_3}{J_3} \quad \text{——} (*)$$

From the Figure (a) ⇒ $E_1 = 5 \times 10^3$ V/m

$E_2 = 8 \times 10^3$ V/m

$E_3 = 3 \times 10^3$ V/m

- The currents in the rods must be the same, they are in series

$$J = \frac{i}{A} \quad (\text{substituted it in } *)$$

$$E_1 A_1 = E_2 A_2 = E_3 A_3 \quad , \quad A = \pi r^2$$

$$E_1 r_1^2 = E_2 r_2^2 = E_3 r_3^2$$

$$5 r_1^2 = 8 r_2^2 = 3 r_3^2$$

use $r_3 = 1.70\text{mm} = 1.70 \times 10^{-3}$ m

$$\Rightarrow r_2^2 = \frac{3}{8}(1.7 \times 10^{-3})^2 \quad , \quad \boxed{r_2 = 1.04 \text{ mm}}$$

$$r_1 = \sqrt{\frac{3}{5}}(1.7 \times 10^{-3}) \qquad \boxed{r_1 = 1.32 \text{ mm}}$$

26-13 Nichrome wire consists of a nickel-chromium-iron alloy, is commonly used in heating elements such as on a stove, and has conductivity $2.0 \times 10^6 (\Omega . m)^{-1}$! If a Nichrome wire with a cross-sectional area of $2.3\ mm^2$ carries a current of 5.5A when a 1.4V potential difference is applied between its ends, what is the wire length?

$$R = \frac{\rho L}{A} = \frac{V}{i} \qquad \text{we } \rho = \frac{1}{\sigma}$$

$$\frac{V}{i} = \frac{L}{\sigma A}$$

$$L = \frac{V \sigma A}{i} = \frac{1.4(2.0 \times 10^6)(2.3 \times 10^{-6})}{5.5}$$

$$\boxed{L = 1.2m}$$

$\{ 1m = 10^{-3}\ mm \}$

$1m^2 = 10^{-6}\ mm^2$

26-45 What is the current in a wire of radius $R = 2.67$ mm if the magnitude of the current density is given by (a) $J_a = J_o r/R$ and (b) $J_b = J_o(1 - r/R)$, in which r is the radial distance and $J_o = 5.5\times10^4 \frac{A}{m^2}$? (c) which function maximizes the current density near the wire's surface?

$$i = \int \vec{J} \cdot d\vec{A} \quad , dA = 2\pi r\, dr$$

(a) $$i_a = \int J_a \, 2\pi r\, dr = \int_0^R \frac{J_o r}{R} 2\pi r\, dr$$

$$i_a = \frac{2\pi J_o}{R} \int_0^R r^2\, dr = \frac{2\pi J_o}{R} \frac{r^3}{3}\Big]_0^R$$

$$i_a = \frac{2\pi J_o}{R} \frac{R^3}{3} = \frac{2\pi J_o R^2}{3}$$

$$i_a = \frac{2\pi}{3} (5.5\times10^4)(2.67\times10^{-3})^2$$

$$\boxed{i_a = 0.821 A}$$

(b) $$i_b = \int J_b \, 2\pi r\, dr = \int_0^R J_o\left(1 - \frac{r}{R}\right) 2\pi r\, dr$$

$$= 2\pi J_o \left[\int_0^R r\, dr - \frac{1}{R} \int_0^R r^2\, dr\right]$$

$$= 2\pi J_o \left[\frac{R^2}{2} - \frac{1}{R} \frac{R^3}{3}\right]$$

$$= \frac{2\pi J_o R^2}{6}$$

$$i_b = \frac{\pi J_o R^2}{3} = \frac{\pi}{3} (5.5\times10^4)(2.67\times10^{-3})^2$$

$$\boxed{i_b = 0.422 A}$$

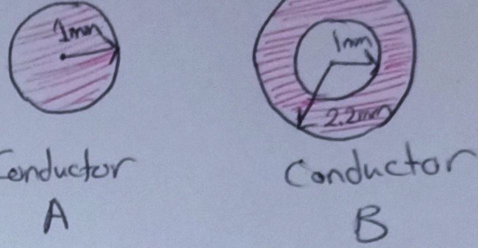
(c) At the surface $(r=R)$

$$J_a(r=R) = J_0$$

$$J_b(r=R) = \text{Zero}$$

$\Rightarrow$ The current density of wire (a) has its maximum at the surface.

26-49 Two Conductors are made of the same material and have the same length. Conductor A is a solid wire of radius 1.0 mm. Conductor B is a hollow tube of outside radius 2.2 mm and inside radius 1.0 mm. What is the resistance ratio R_A / R_B, measured between their ends?



• $R = \frac{\rho L}{A}$

$\Rightarrow A_A = \pi r_A^2$

$A_B = \pi r^2_{B,outside} - \pi r^2_{B,inside}$

$\rho_A = \rho_B = \rho$

$L_A = L_B = L$

$\Rightarrow R_A = \frac{\rho L}{\pi r_A^2}$

$R_B = \frac{\rho L}{\pi r^2_{B,outside} - \pi r^2_{B,inside}}$

$\Rightarrow \frac{R_A}{R_B} = \frac{r^2_{B,outside} - r^2_{B,inside}}{r_A^2}$

$= \frac{(2.2 \times 10^{-3})^2 - (1 \times 10^{-3})^2}{(1 \times 10^{-3})^2}$

$$\boxed{\frac{R_A}{R_B} = 3.84}$$