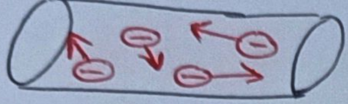
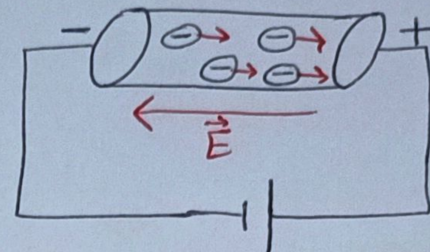
# Chapter 26: Current and Resistance

- Electric Current



Conductor (overall motion zero)



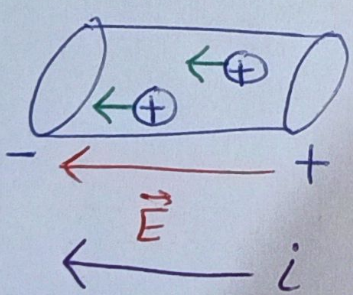

Directionality of charges "flow of electrons"

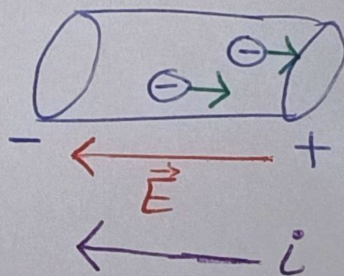
- $i = \dfrac{\Delta Q}{\Delta t}$ "average current"

$[i] = \dfrac{C}{\text{Sec}} = A \equiv \text{Ampere}$

- $i = \dfrac{dq}{dt}$ "An electric current $i$ in a conductor"

[The time rate of positive charges that crossing the Conductor]



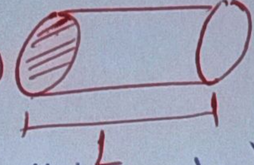

- Current density

$$i = \int \vec{J} \cdot d\vec{A} \Rightarrow$$

$i$ is a scalar quantity
$\vec{J}$ is a vector quantity

J has the same direction as the velocity of the moving charges if they are positive and the opposite direction if the moving charges are negative.

$$i = \int \vec{J} \cdot d\vec{A}$$




If $\vec{J}$ is constant ($\vec{J}$ is uniform and parallel to $dA$)

$$i = JA \implies J = \frac{i}{A} \quad ; \quad [J] = \frac{A}{m^2}$$

$$i = \begin{cases} \vec{J} \cdot \vec{A} \ , & \text{constant } J \\ \int \vec{J} \cdot d\vec{A} \ , & \text{Variable } J \end{cases}$$

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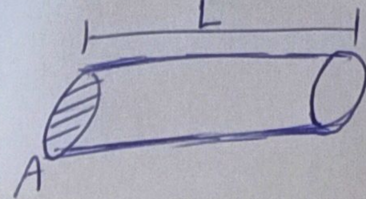
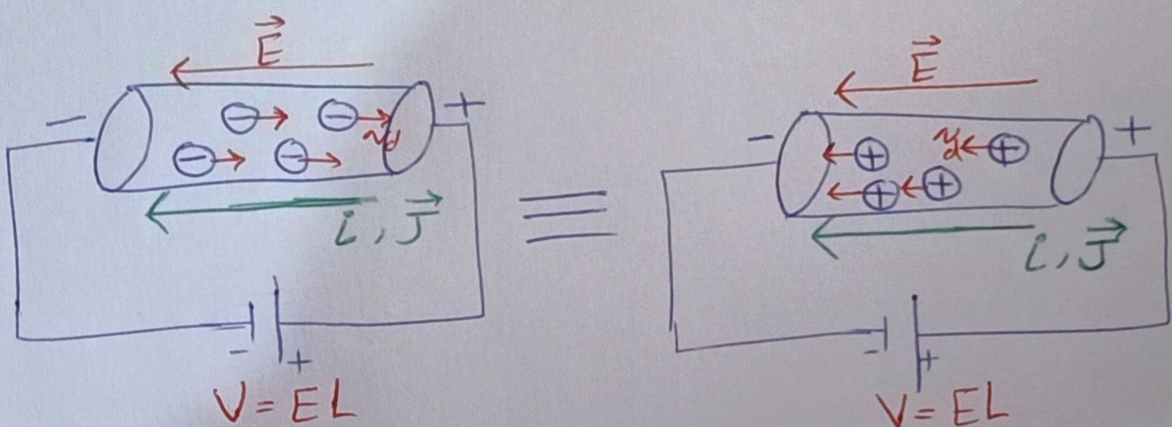
Conductor of length $L$ and cross-sectional area $A$




- $n$ ≡ The number of free electrons per unit volume ; $[n] = \frac{1}{m^3}$
- $ne$ ≡ Carrier charge density
  $[ne] = \frac{C}{m^3}$

$n$ ≡ depends on the metal
Al, Cu, Fe ...




$V = EL$ $\qquad\qquad$ $V = EL$

- The number of charge carriers in the wire (conductor) = $nAL$
- The total charge of the carriers in the wire = $(nAL)e$

$$\boxed{q = nALe}$$

- The electric current $\frac{q}{t} = \frac{nALe}{t}$ , use $\frac{L}{t} = v_d$

$$\boxed{i = nAev_d}$$

$v_d$ ≡ drift speed of each charge carrier

$$J = \frac{i}{A} \implies \boxed{\vec{J} = ne\vec{v}_d}$$

• The current in the conductor depends on

[1] Geometry of the conductor [A, L]

$$\boxed{i = nAe\,v_d}$$

[2] The type of the conductor

[3] The applied voltage V or the applied electric field E $[V = EL]$

## Ohm's Law

The current density in the conductor is directly proportional to applied electric field $J \propto E$

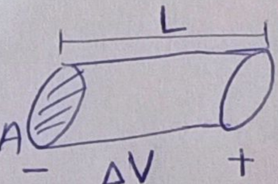
$$\boxed{\vec{E} = \rho \vec{J} \quad ; \text{ Ohm's Law}}$$

$\rho$ is the resistivity of the conductor

$\sigma$ is the conductivity of the material

$$\boxed{\sigma = \frac{1}{\rho}}$$

$[\rho_{Ag} = 1.62 \times 10^{-8}\ \Omega.m, \ \rho_{Cu} = 1.69 \times 10^{-8}\ \Omega.m, \ldots]$




$\Rightarrow E = \rho J$

$$E = \frac{V}{L}$$

$$J = \frac{i}{A}$$

$$\frac{V}{L} = \rho \frac{i}{A}$$

$$V = \left(\frac{\rho L}{A}\right) i$$

$$\boxed{V = R i} \quad \text{Ohm' Law}$$

$\Rightarrow$ The Resistance R of a conductor is defined as $R = \frac{V}{i}$

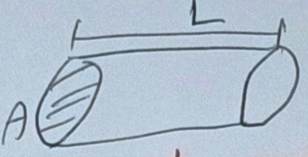
V is the potential difference across the conductor and i is the current through it.

$$[R] = \frac{\text{Volt}}{\text{Ampere}} = \frac{V}{A} = \Omega$$

Resistivity is property of a material and the Resistance is property of an object

- Conducting wire of Length L and uniform cross-section

$$R = \rho \frac{L}{A}$$




L ≡ parallel to the current

The resistor of a conducting wire depends on:

[1] Geometry of a wire [ L and A ]

[2] The type of the conductor $\rho$

$$[\rho] = \Omega \cdot m$$

$$[\sigma] = (\Omega \cdot m)^{-1}$$

- The resistivity ($\rho$) for most materials changes with temperature

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

$$\boxed{\rho = \rho_0 [1 + \alpha (T - T_0)]}$$

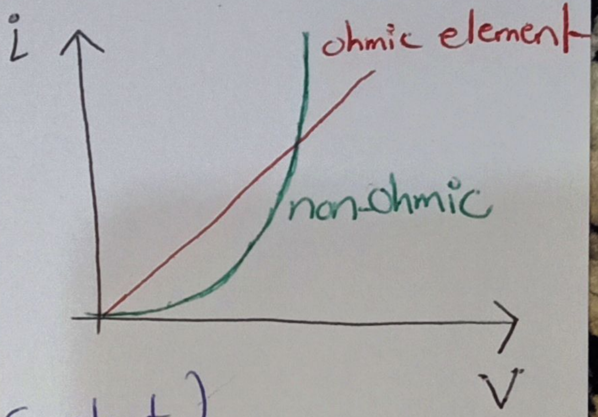
$$\boxed{R = R_0 [1 + \alpha (T - T_0)]}$$

$\alpha$ ≡ Temperature coeffient of resistivity $[\alpha] = K^{-1}$

| Insulators | Semiconductor | Conductor | Super conductor |
|---|---|---|---|
| المواد العازلة | شبه الموصلة<br>$\alpha$ = negative<br>$R\downarrow, T\uparrow$ | الموصلة<br>$\alpha$ = positive<br>$R\uparrow, T\uparrow$ | فائقة الموصلية<br>Lose all electrical resistance at low temperature. |

# Ohm's Law $\quad R = \frac{V}{i}$

⇒ Ohmic element ⇒ An electrical device (conductor, resistor) obeys ohm's Law if its resistance $R$ is independent of the applied potential difference $V$ and if its resistivity is independent of the magnitude and direction of the applied electric field $\vec{E}$.




⇒ ohmic element ⇒ linear (constant R)

The ratio $\frac{i}{V}$ is constant (the same for all values of V)

⇒ non-ohmic element ⇒ non-linear

The ratio $\frac{i}{V}$ is not constant (R isn't constant)

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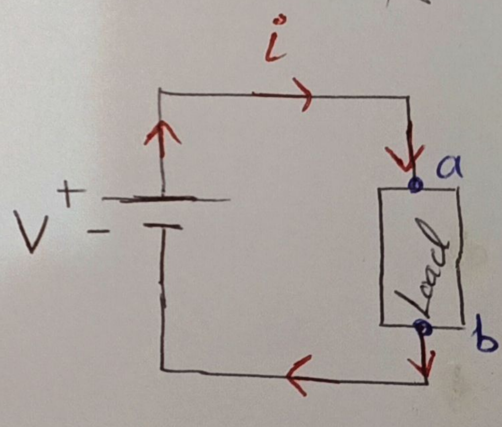
• Power in electric circuit

When charge ($dq$) moves through the Load from a to b, its electrical potential energy decreases in magnitude by the amount

$$\boxed{dU = dq\, V = i\, dt\, V}$$




$V_a > V_b$

The principle of conservation of energy ⇒

the decrease in the potential energy from (a) to (b) is accompained by a transfer of energy to some other form.

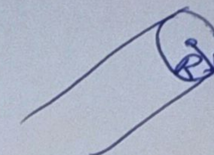
Power ≡ Rate of energy transfer in an electrical device across which a potential difference V is maintained

$$P = \frac{\text{Energy}}{\text{time}} = iV \Rightarrow$$ If the device is a resistor

$$P = i^2 R = \frac{V^2}{R} \qquad [P] = \frac{J}{s} = W$$

Watt

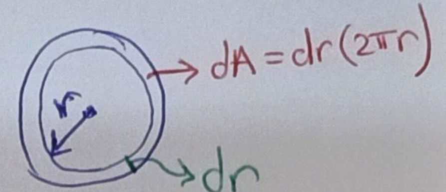
**26-54** The magnitude J of the current density in a certain lab wire with a circular cross section of radius $R = 2.5\,mm$ is given by $J = (3.0\times10^8)r^2$, with J in Amperes per square meter and radial distance r in meters. What is the current through the outer section bounded by $r = 0.9R$ and $r = R$?





- $J = 3\times10^8 r^2 = Cr^2 \; ; \; C = 3\times10^8$

$$\Rightarrow i = \int \vec{J}\cdot\vec{dA}$$





$$= \int_{r_1}^{r_2} Cr^2(2\pi r\,dr)$$

$$= 2\pi C \int_{0.9R}^{R} r^3\,dr = 2\pi C\left[\frac{r^4}{4}\right]_{0.9R}^{R}$$

$$i = \frac{2\pi C}{4}\left[R^4 - (0.9R)^4\right] = \frac{2\pi C R^4}{4}\left[1-(0.9)^4\right]$$

$$i = 0.344\left[\frac{\pi C}{2}R^4\right]$$

$$i = 0.344\left[\frac{3.14\times3\times10^8}{2}(2.5\times10^{-3})^4\right] = 6.33\,mA$$

$\Rightarrow$ The total current in the wire

$$i = \int_0^R \vec{J}\cdot\vec{dA} = \frac{2\pi C}{4}\left[r^4\right]_0^R$$

$$i = \frac{2\pi C R^4}{4} = \frac{2\times3.14\times3\times10^8(2.5\times10^{-3})^4}{4}$$

$$\boxed{i_{total} = 18.4\,mA}$$

26-37 A 120V potential difference is applied to a space heater that dissipates 1500 watt during operation. (a) What is its resistance during operation? (b) At what rate do electrons flow through any cross section of the heater element?

$V = 120\,V$

$P = 1500$ watt

(a) $R = ?$

$$P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{(120\,V)^2}{1500 \text{ watt}} = 9.6\,\Omega$$

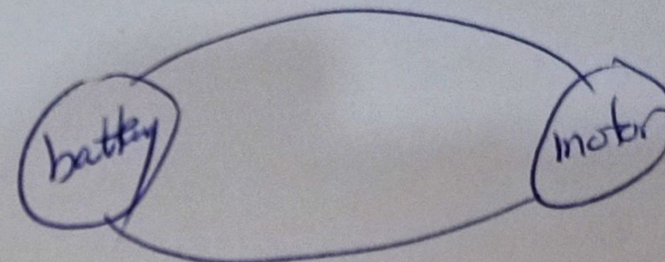
(b) Rate of electrons flow $= i/e$

$$P = iV \Rightarrow i = \frac{P}{V} = \frac{1500}{120} = 12.5\frac{C}{sec}$$

$$\frac{i}{e} = \frac{12.5\frac{C}{sec}}{1.6\times10^{-19}C} = 7.8\times10^{19}\frac{\text{electron}}{\text{sec}}$$

How many electrons flow in one second?

26-43 How Long does it take electrons to get from a car battery the starting motor? Assume the current is 285 A and the electrons travel through a copper wire with cross-sectional area 0.17 cm$^2$ and length 0.43 m. The number of charge carriers per unit volume is $8.49 \times 10^{28}\ m^{-3}$.




$\rightarrow i = neA\, v_d$

$\rightarrow v_d = \dfrac{i}{neA} = \dfrac{285A}{(8.49 \times 10^{28}\ m^{-3})(1.6 \times 10^{-19} C)(0.17 \times 10^{-4} m^2)}$

$\boxed{v_d = 1.23 \times 10^{-3}\ m/s}$

$\rightarrow t = \dfrac{L}{v_d} = \dfrac{0.43m}{1.23 \times 10^{-3}\ m/s} = 348\ sec = 5.8\ min$