Chapter 28: Magnetic Fields

- $\vec{F_B} = q\,\vec{v} \times \vec{B}$ "A magnetic force on moving charged particle"

  $[B] = T$

- A charged particle Circulating in a magnetic field

  $$q\,v_\perp B = m\frac{v^2}{R}$$

  $$r = \frac{m\,v_\perp}{|q|B}$$

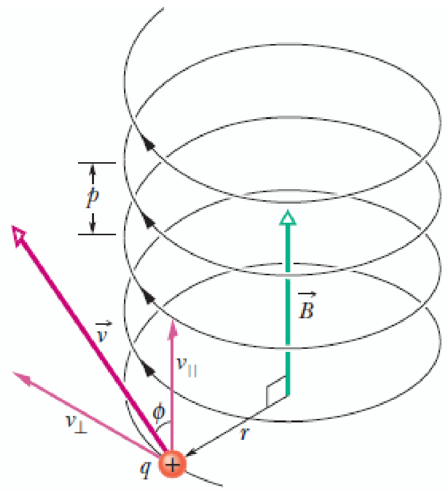
  $$\Rightarrow T = \frac{2\pi r}{v} = \frac{2\pi m}{|q|\,B} \quad \text{"periodic Time"}$$

  $$f = \frac{1}{T} = \frac{|q|B}{2\pi m}\,, \quad \omega = 2\pi f = \frac{|q|B}{m} \quad \text{"Angular frequency"}$$

- Helical path

  pitch ≡ the distance the particle travel parallel to the magnetic field $\vec{B}$ during one period T of circulation

  $$p = v_{\parallel}\,T = v\cos\theta\,\frac{2\pi m}{qB}$$



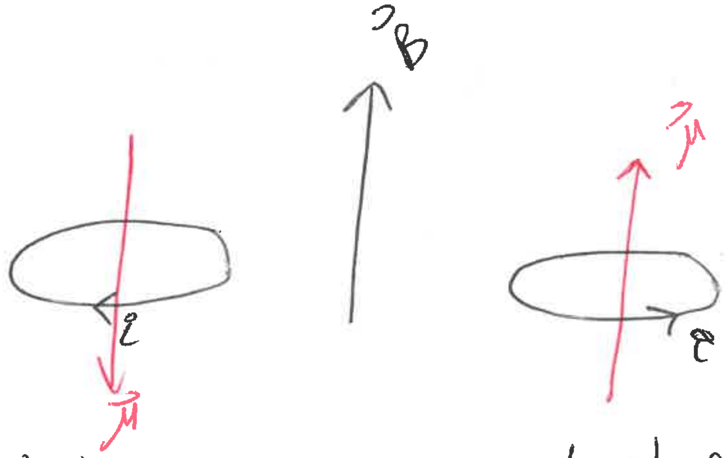
- The magnetic dipole moment

  $\vec{\mu} = N i A$ "perpendicular and directed outward"

  $\Rightarrow \vec{\tau} = \vec{\mu} \times \vec{B}$ Torque on a current-carrying coil

  $\Rightarrow U(\theta) = -\vec{\mu} \cdot \vec{B}$

  $[\mu] = A.m^2 = J/T$

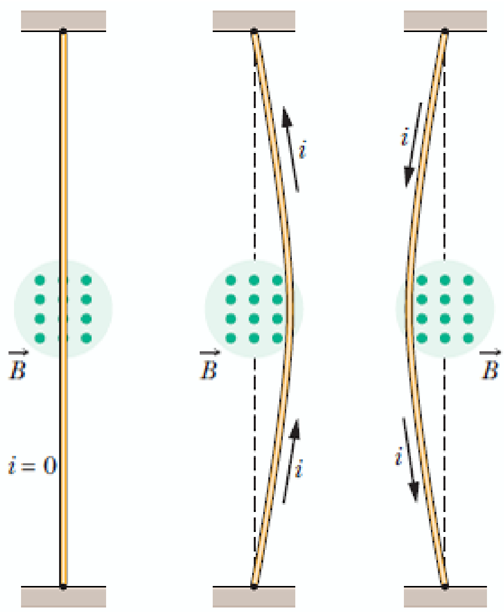


Highest energy Lowest energy

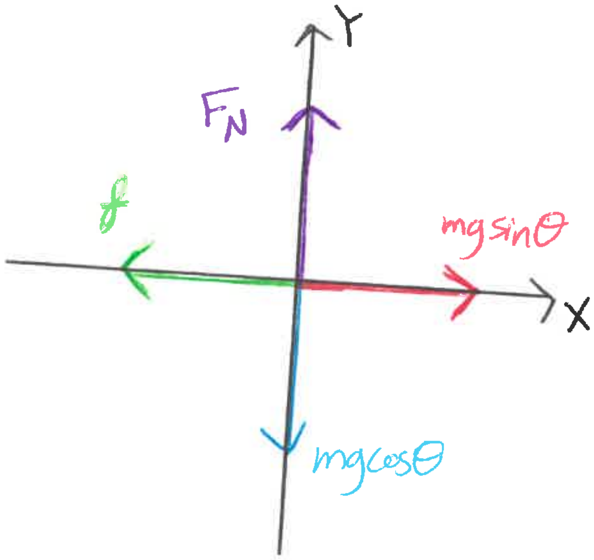
$$\Rightarrow U = \begin{cases} 0 , & \theta = 90 \\ \mu B & \theta = 180^\circ \quad \text{antiparallel} \\ -\mu B & \theta = 0^\circ \quad \text{parallel} \end{cases}$$

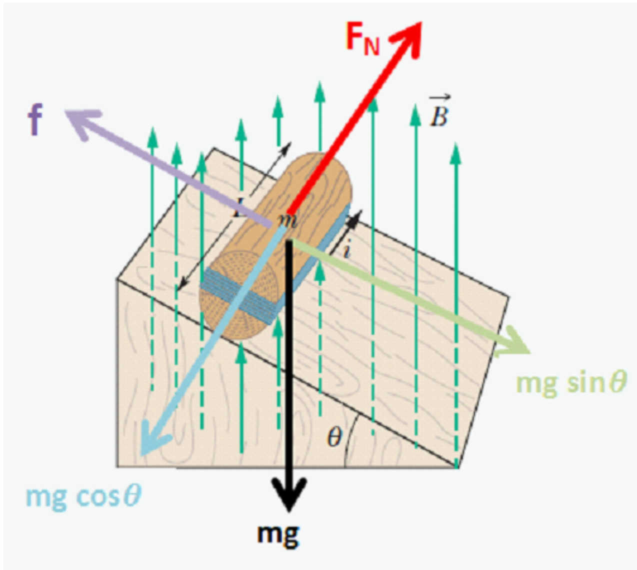
- Magnetic Force on a current - carrying wire

$$\vec{F}_B = i \vec{L} \times \vec{B}$$



28-7 The below Figure shows a wood cylinder of mass $m = 0.150$ kg and Length $L = 0.100$ m, with $N = 13.0$ turns of wire wrapped around it Longitudinally, so that the plane of the wire coil contains the Long central axis of the cylinder. The cylinder is released on a plane inclined at an angle θ to the horizontal, with the plane of the coil parallel to the incline plane. If there is a vertical uniform magnetic field of magnitude 0.92 T, What is the Least current i through the coil that keeps the cylinder from rolling down the plane?





- Newton's Second Law for the COM of cylinder

$$mg\sin\theta - f = ma$$

- Newton's Second Law for rotation about the center of the cylinder

$$\tau = I\alpha$$

→ The magnetic field produces a torque on the current-carrying coil

$\vec{\tau} = \vec{\mu} \times \vec{B}$, $\tau = \mu B \sin\theta$ "Counterclockwise angular acceleration"

→ Friction force produces a torque $(\tau = fr)$, $r \equiv$ cylinder radius.

"clockwise angular acceleration"

$\Rightarrow \tau = I\alpha \Rightarrow fr - \mu B\sin\theta = I\alpha$

- Keeps the cylinder from rolling down the plane

$$a = \alpha = 0$$

Translational and Rotational Equilibrium

- $mg\sin\theta = f$
- $fr = \mu B\sin\theta$

$\rightarrow mg\sin\theta \, r = \mu B\sin\theta \Rightarrow mgr = \mu B$

The Loop is rectangular, $A = 2rL$ and the magnitude of magnetic dipole moment $\mu = NiA = Ni2rL$

$$mgr = 2NirLB$$

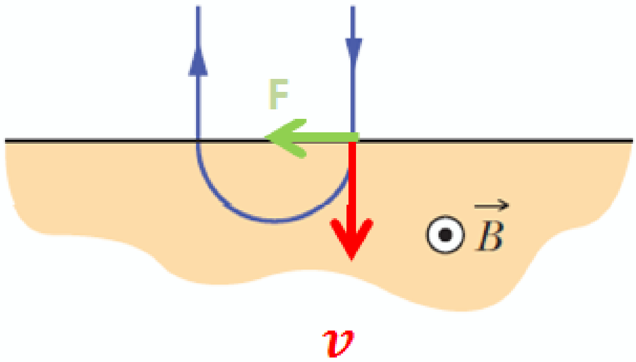
$$i = \frac{mg}{2NLB}$$

$$i = \frac{0.150 \times 9.8}{2 \times 13 \times 0.1 \times 0.92} = 0.61\,A$$

Note The Friction force tries to make the cylinder to roll down while the current creates a torque that tries to make it go uphill

28-8 In the below Figure, A charged particle moves into a region of uniform magnetic field $\vec{B}$, goes through half a circle, and then exists that region. The particle is either a proton or an electron (You must decide which). It spends 160 ns in the region. (a) What is the magnitude of $\vec{B}$? (b) If the particle is sent back through the magnetic field (along the same initial path) but with 2.00 times its previous kinetic energy, how much time does it spend in the field during this trip?

- At point A, which it enters the field-filled region.



→ v = velocity vector downward
$\vec{B}$ = out of the page
F = Leftward
⇒ $q>0$ "proton"

a) $T = \dfrac{2\pi m}{|q| B}$

proton ⇒ $T = \dfrac{2\pi m_p}{e B}$, $2 \times 160 \times 10^{-9} = \dfrac{2\pi (1.67 \times 10^{-27})}{1.6 \times 10^{-19} \times B}$

$\boxed{B = 0.205\,T}$

b) The period time T does not depend on speed $[T = 2\pi m / |q| B]$, So it remains the same. ⇒ $t = 160\,nsec$

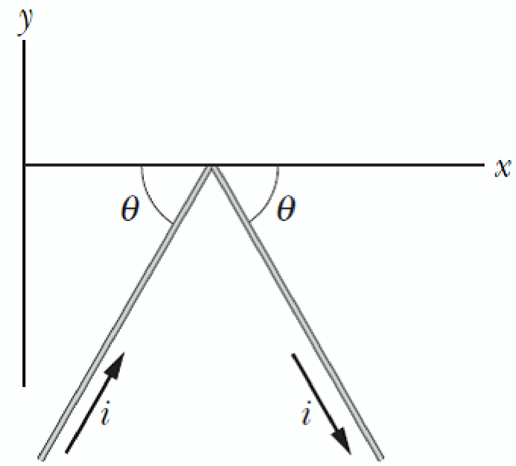
But doupling the kinetic energy will increase the speed by $v' = \sqrt{2}\, v$

$KE' = 2KE \Rightarrow \frac{1}{2} m v'^2 = 2\left(\frac{1}{2} m v^2\right) \Rightarrow v' = \sqrt{2}\, v$

So the radius of the path will be $r' = \sqrt{2}\, r$

$\Rightarrow r = \dfrac{m v}{q B}$

| 28-10 | The bent wire shown in the below Figure Lies in a uniform magnetic field. Each straight section is 2.0m long and makes an angle of $\theta = 60°$ with the x-axis, and the wire carries a current of 3.5A. What is the net magnetic force on the wire in unit vector notation if the magnetic field is given by a) $4.0\hat{k}$ T and b) $4.0\hat{i}$ T?



- Magnetic Force on a current carrying wire

$$\vec{F}_B = i \vec{L} \times \vec{B}$$

$$\rightarrow \vec{L}_1 = L(\cos\theta \hat{i} + \sin\theta \hat{j})$$

$$\vec{L}_2 = L(\cos\theta \hat{i} - \sin\theta \hat{j})$$

- Bent wire

$$\vec{L} = 2L \cos\theta \hat{i}$$

a) $\vec{B} = 4.0\hat{k}$

$$\vec{F}_B = i\vec{L} \times \vec{B} = i\, 2L(\cos\theta\, \hat{i} \times B_o \hat{k})$$

$$= 3.5 \times 2 \times 2 \cos 60 \times 4 (\hat{i} \times \hat{k})$$

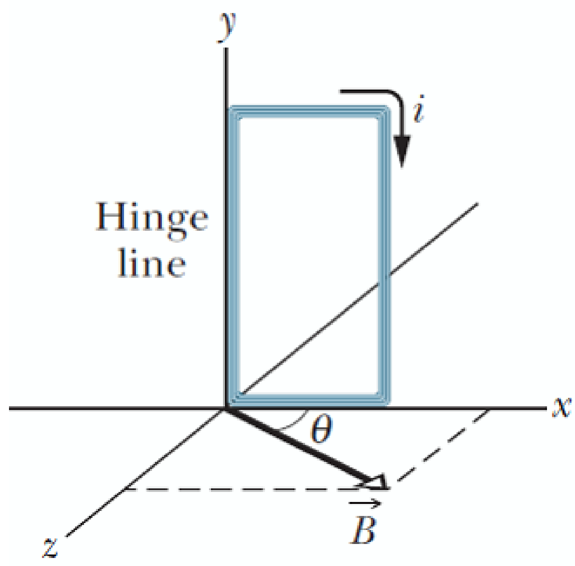
$$\boxed{\vec{F}_B = -28 N \hat{j}}$$

b) $\vec{B} = 4.0\hat{i}$ T

$$\hat{i} \times \hat{i} = \text{zero}$$

$$\vec{F}_B = \text{zero}$$

28–19 The below figure shows a rectangular 28-turn coil of wire, of dimensions 10.0 cm by 5.0 cm. It carries a current of 0.80A and is hinged along one long side. It is mounted in the xy plane, at angle $\theta = 25°$ to the direction of a uniform magnetic field of magnitude 0.50 T. In unit vector notation, what is the torque acting on the coil about the hing Line?



- Torque on a current-carrying loop

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

- $\vec{\mu}$ = magnetic dipole moment

$\mu = NiA$ in negative z-direction

$$\vec{\mu} = 28 \times 0.8 \times 5 \times 10 \times 10^{-4} (-\hat{k})$$

$$\vec{\mu} = -(0.112\ A.m^2)\hat{k}$$

$\Rightarrow$ let $\vec{B} = B\cos\theta\,\hat{i} + B\sin\theta\,\hat{k}$

$$\vec{\mu} = -\mu\hat{k}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$= -\mu\hat{k} \times [B\cos\theta\,\hat{i} + B\sin\theta\,\hat{k}]$$

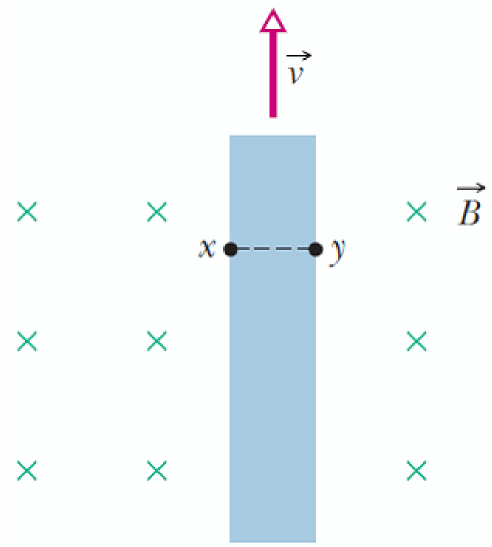
$$\boxed{\vec{\tau} = -\mu B\cos\theta\,\hat{j}}$$

$$\vec{\tau} = -0.112 \times 0.5\cos(25°)\hat{j}$$

$$= -(0.0508\ N.m)\hat{j}$$

$$\vec{\tau} = (-51\ m\,N.m)\hat{j}$$

28-22 A metal strip 6.50 cm long, 0.850 cm wide, and 0.760 mm thick moves with constant velocity $\vec{v}$ through a uniform magnetic field $B = 1.20$ mT directed perpendicular to the strip, as shown in the below figure. A potential difference of 3.30 μV is measured between points x and y across the strip. Calculate the speed v?



- The force on a free charge q inside the metal strip with velocity $\vec{v}$

$$\vec{F} = q[\vec{E} + \vec{v} \times \vec{B}]$$

By Newton's Second Law

$$\vec{F} = m\vec{a}$$

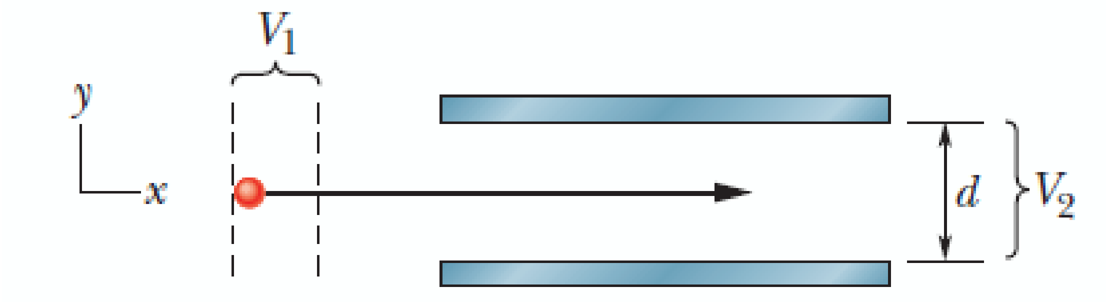
$\Rightarrow$ moves with constant velocity

$$\vec{a} = 0 \Rightarrow \vec{F}_B = \vec{F}_e$$

$$v = \frac{E}{B} \quad , \quad E = \frac{V_x - V_y}{d_{xy}}$$

$$v = \frac{3.3 \times 10^{-6}}{0.85 \times 10^{-2} \times 1.2 \times 10^{-3}} = 0.324 \text{ m/s}$$

$\boxed{28-31}$ In the below figure, an electron accelerated from rest through potential difference $V_1 = 2.50\,kV$ enters the gap between two parallel plates having separation $d = 16.0\,mm$ and potential difference $V_2 = 100\,V$. The lower plate is at the lower potential. Neglect fringing and assume that the electron's velocity vector is perpendicular to the electric field vector between the plates. a) In unit-vector notation, What uniform magnetic field allows the electron to travel in a straight line in the gap? b) If the potential difference is increased slightly, in What direction does the electron veer from straight line motion?



a) Electron travels in a straight line in the gap $\Rightarrow \vec{F}_{net} = 0$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = 0 \qquad \text{" } \vec{v} \perp \vec{B} \text{ given "}$$

$E = vB$ $\Rightarrow$ By using conservation of a mechanical energy

$$qV = \frac{1}{2} m v^2$$

Electric potential energy $\rightarrow$ Kinetic energy

$$v = \sqrt{\frac{2eV}{m_e}}$$

$$\cdot\; B = \frac{E}{v} = \frac{100/(16 \times 10^{-3})}{\sqrt{2 \times 1.6 \times 10^{-19} \times 2.5 \times 10^{3} / 9.11 \times 10^{-31}}} = 0.211 \times 10^{-3}\,T$$

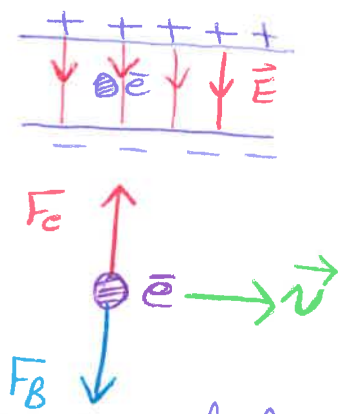
$\Rightarrow$ Lower plate is at the Lower potential

Electric Force on electron upward

magnetic Force on electron downward

$\Rightarrow \vec{B}$ into the page

$\left.\right\} F_{net} = \text{zero}$



$$\vec{B} = -(0.211\,mT)\hat{k}$$

b) Potential difference increases $\Rightarrow$ velocity increases $\rightarrow$ Electric field increases $\Rightarrow$ Electric Force increases $\Rightarrow$ Electron veer towards upper plate.

28-37 An electron moves through a uniform magnetic field given by $\vec{B} = B_x\hat{i} + (-3.0B_x)\hat{j}$. At a particular instant, the electron has velocity $\vec{v} = (2.0\hat{i} + 4.0\hat{j})$ m/s and the magnetic force acting on it is $(6.4\times10^{-19}\,N)\hat{k}$. Find B_x?

$$\vec{F}_B = q\,\vec{v}\times\vec{B}$$

$$(6.4\times10^{-19}N)\hat{k} = -e(2.0\hat{i} + 4.0\hat{j})\times(B_x\hat{i} + (-3.0B_x)\hat{j})$$

$$= -e\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2.0 & 4.0 & 0 \\ B_x & -3.0B_x & 0 \end{vmatrix}$$

$$6.4\times10^{-19}\hat{k} = -e[(-6B_x - 4B_x)\hat{k}]$$

$$6.4\times10^{-19} = e(10B_x)$$

$$B_x = \frac{6.4\times10^{-19}}{1.6\times10^{-19}\times10} = 0.4\,T$$

$$\boxed{B_x = 0.4\,T}$$

28-41 An electron follows a helical path in a uniform magnetic field of magnitude 1.30T. The pitch of the path is 6.00 μm, and the magnitude of the magnetic force on the electron is 2.00×10^{-14} N. What is the electron speed?

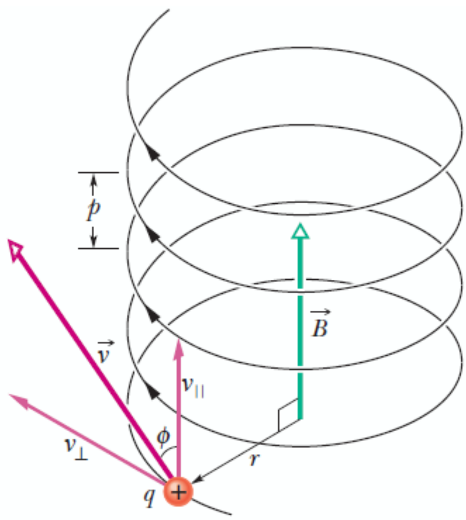
$$v_{\parallel} = v\cos\theta \quad , \quad v_{\perp} = v\sin\theta$$

- P = Pitch of the path $= v_{\parallel} T = v_{\parallel} \dfrac{2\pi m_e}{|e|B}$
- Magnetic Force on the electron $\Rightarrow \vec{F}_B = -e\,\vec{v}\times\vec{B}$

$$F_B = e\,v_{\perp} B$$

$$\Rightarrow v_{\parallel} = \frac{PeB}{2\pi m_e} = \frac{6.0\times10^{-6}\times1.6\times10^{-19}\times1.3}{2\pi\,(9.11\times10^{-31})}$$

$$\boxed{v_{\parallel} = 2.18\times10^{5}\ m/s}$$



$$\Rightarrow v_{\perp} = \frac{F_B}{eB} = \frac{2\times10^{-14}}{1.6\times10^{-19}\times1.3}$$

$$\boxed{v_{\perp} = 0.96\times10^{5}\ m/s}$$

- The electron speed

$$v = \sqrt{(v_{\perp})^2 + (v_{\parallel})^2}$$

$$= \sqrt{(2.18\times10^{5})^2 + (0.96\times10^{5})^2} = 2.38\times10^{5}\ m/s$$

$$\boxed{v = 238\ Km/s}$$

$\boxed{28-63}$ An electron that has an instantaneous velocity of $\vec{v} = (-5.0 \times 10^6\ m/s)\hat{i} + (3.0 \times 10^6\ m/s)\hat{j}$ is moving through the uniform magnetic field $\vec{B} = (0.030T)\hat{i} - (0.15T)\hat{j}$. a) Find the force on the electron due to the magnetic field. b) Repeat your calculation for a proton having the same velocity?

a) $\vec{F_B}$ on the electron $= -e\ \vec{v} \times \vec{B}$

$$= -e \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 \times 10^6 & 3 \times 10^6 & 0 \\ 0.03 & -0.15 & 0 \end{vmatrix}$$

$$= -e\left[(-5 \times 10^6 \times -0.15) - (0.03 \times 3 \times 10^6)\right]\hat{k}$$

$$\boxed{\vec{F_B}\ (\text{electron}) = -(1.056 \times 10^{-13} N)\hat{k}}$$

b) $\vec{F_B}$ on the proton $= e\ \vec{v} \times \vec{B}$

$$\boxed{\vec{F_B}\ (\text{proton}) = (1.056 \times 10^{-13} N)\hat{k}}$$