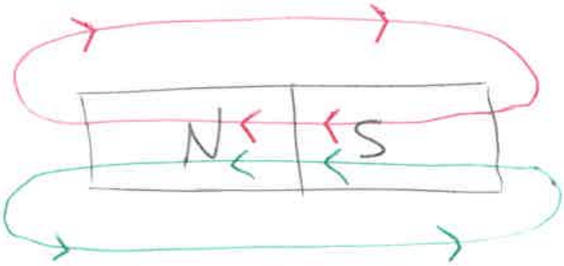
# Chapter 28: Magnetic Fields

- All magnets [Natural and Industrial magnets] have two poles (South and North pole) (NO magnetic monopole)
- Opposite magnetic poles attract each other (Attractive magnetic force), and like magnetic poles repel eachother (Repulsive magnetic force).

  ⇒ Attractive magnetic force N–S

  ⇒ Repulsive magnetic force N–N, S–S
- Magnetic field lines extend from North pole to South pole outside the magnet and from South pole to North pole inside the magnet.




- Magnetic field lines Form a closed Loop. Why !!
- Magnetic field lines do not cross each other Why !!
- Magnetic flux through a closed surface equals zero

$$\oint \vec{B} \cdot d\vec{A} = \text{zero}$$

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$\vec{B}$ = Magnetic field "Vector quantity"

$[\vec{B}]$ = Tesla

• Magnetic Force on a charged particle:

A charged particle moves through a magnetic field $\vec{B}$, a magnetic force acts on the particle as given by

$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

particle's charge [sign included]

particle's velocity

$$1 \text{ Tesla} = \frac{1 \text{ newton}}{1 \text{ Coulomb} \left(\frac{\text{meter}}{\text{second}}\right)} = \frac{1\ N}{A \cdot m}$$

$\Rrightarrow$ $q$ is positive particle ; $\vec{F}_B$ in the same direction of $\vec{v} \times \vec{B}$

$q$ is negative particle ; $\vec{F}_B$ in the opposite direction of $\vec{v} \times \vec{B}$

$F_B = |q|\, v B \sin\theta$ , $\theta$ = the angle between $\vec{v}$ and $\vec{B}$

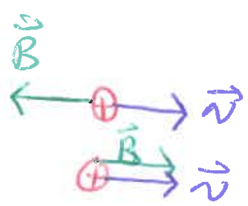
## Notes

$\boxed{1}$ $\vec{F}_B \perp \vec{v}$ and $\vec{F}_B \perp \vec{B}$

$\boxed{2}$ $\vec{F}_B$ = zero $\Rrightarrow$ ① $q = 0$ , neutral particles

② Rest particle $(v = 0)$

③ $\vec{v} \parallel \vec{B}$ , $\theta = 0°, 180°$




$\vec{F}_B$ exists $\Rrightarrow$ moving charged particle and it has a perpendicular velocity component to the magnetic field

$\boxed{3}$ $\vec{F}_B$ does not change the magnitude of the particle's velocity ($\vec{F}_B \perp \vec{v}$) but it change the direction of its velocity only.

$\boxed{4}$ $\vec{F}_B$ does not change the kinetic energy $\left(K = \frac{1}{2} m v^2\right)$ Scalar

$\boxed{5}$ $\vec{F}_B$ change the linear momentum of the charged particle by changing the direction of its velocity. $\vec{p} = m\vec{v}$

$\boxed{6}$ The charge will move in a circular motion if it has only perpendicular component of the velocity to a uniform magnetic field $\vec{B}$ $[\vec{v} \perp \vec{B}]$ $\theta = 90^\circ$ between $\vec{v}$ and $\vec{B}$

$\boxed{7}$ If the velocity of the particle has a component parallel to the magnetic field, the particle moves in a helical path about field vector $\vec{B}$ (Spiral motion) $\theta \neq 90^\circ$

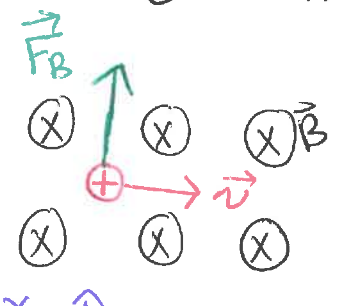
$\Rightarrow$ Charged particle
- $v_{\parallel}$ only $\longrightarrow$ No magnetic force
- $v_{\perp}$ only $\longrightarrow$ Circular motion
- $v_{\parallel}$ and $v_{\perp}$ $\longrightarrow$ helical path

• Determine the direction of $\vec{F}_B$ [By Right hand Rule]

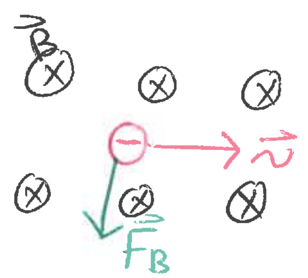
Note $\Rightarrow$ $\odot$ out of the page $+\hat{k}$

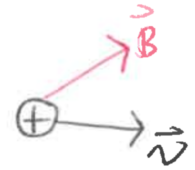
$\otimes$ into the page $-\hat{k}$

$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

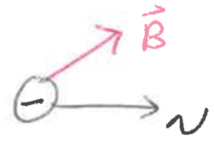



$+\hat{i} \times -\hat{k} = +\hat{j}$

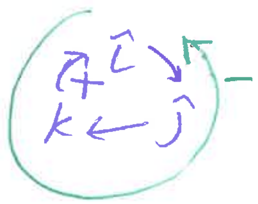



$F_B$ = outward $\odot$




$F_B$ inward $\otimes$



## Sample problem 28.01 : Magnetic force on a moving charged particle

A uniform magnetic field $\vec{B}$, with magnitude 1.2 mT, is directed vertically upward throughout the volume of laboratory chamber. A proton with kinetic energy 5.3 MeV enters the chamber, moving horizontally from south to North. What magnetic deflecting force acts on the proton as it enters the chamber? ($m_p = 1.67\times10^{-27}$ kg, Neglect Earth's magnetic field).

$\Rrightarrow$ $q_p = +e = +1.6\times10^{-19}\,C$

$m_p = 1.67\times10^{-27}\,kg$

$$\boxed{1\ eV = 1.6\times10^{-19}\,J}$$

$$K = 5.3\ MeV = 5.3\times10^{6}\,eV\left(\frac{1.6\times10^{-19}\,J}{1\,eV}\right) = 8.48\times10^{-13}\,J$$

$$K = \frac{1}{2}mv^2 \Longrightarrow v = \sqrt{\frac{2K}{m_p}} = 3.2\times10^{7}\ m/s$$

$$F_B = qvB\sin\theta$$

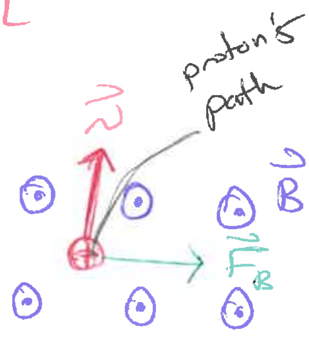
$$= {}^{+}1.6\times10^{-19}\,C\,(3.2\times10^{7}\ m/s)(1.2\times10^{-3}\,T)\sin 90^\circ$$

$$\boxed{\vec{F}_B = 6.1\times10^{-15}\,N}$$

$\vec{F} = m\vec{a}$ Newton's $2^{nd}$ Law

$$\vec{a} = \frac{\vec{F}_B}{m} = \frac{6.1\times10^{-15}\,N}{1.67\times10^{-27}\,kg} = {}^{+}(3.7\times10^{12}\ m/s^2)\,\hat{i}$$

the proton is deflected toward the east

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$$\vec{B} = (1.2\ mT)\,\hat{k}$$

$$\vec{v} = {}^{+}(3.2\times10^{7}\ m/s)\,\hat{j}$$

$$\vec{F}_B = q\,\vec{v}\times\vec{B}$$

$$= +1.6\times10^{-19}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 3.2\times10^{7} & 0\\ 0 & 0 & 1.2\times10^{-3}\end{vmatrix} = +1.6\times10^{-19}\,(3.2\times10^{7})(1.2\times10^{-3})\,\hat{i}$$

$$= 6.1\times10^{-15}\,\hat{i}$$

#

28-56 A proton moves through a uniform magnetic field given by $\vec{B} = (10\hat{i} - 20\hat{j} + 25\hat{k})\,mT$. At time $t_1$, the proton has a velocity given by $\vec{v} = v_x\hat{i} + v_y\hat{j} + (2\,km/s)\hat{k}$ and the magnetic force on the proton is $\vec{F}_B = (4\times10^{-17}\,N)\hat{i} + (2\times10^{-17}\,N)\hat{j}$. At that instant, what are (a) $v_x$ and (b) $v_y$?

$$\vec{F}_B = q\,\vec{v}\times\vec{B} = +1.6\times10^{-19}(\vec{v}\times\vec{B})$$

$$\Rightarrow \vec{v}\times\vec{B} = 10^{-3}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & 2000 \\ 10 & -20 & 25 \end{vmatrix} = \left[(25v_y + 4\times10^4)\hat{i} - (25v_x - 2\times10^4)\hat{j} + (-20v_x - 10v_y)\hat{k}\right]10^{-3}$$

$$\frac{\vec{F}_B}{1.6\times10^{-19}} = \vec{v}\times\vec{B}$$

1) $$\frac{4\times10^{-17}}{1.6\times10^{-19}} = 250 = (25v_y + 4\times10^4)10^{-3}$$

$$\boxed{v_y = 8.4\times10^3\ m/s}$$

2) $$\frac{2\times10^{-17}}{1.6\times10^{-19}} = 125 = (2\times10^4 - 25v_x)10^{-3}$$

$$\boxed{v_x = -4.2\times10^3\ m/s}$$

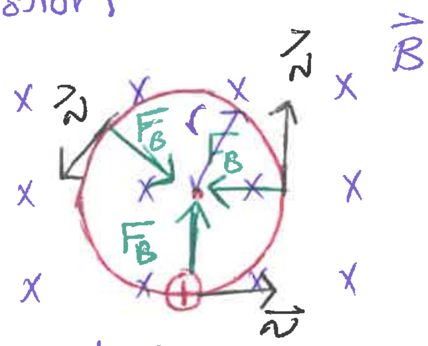
$$\vec{v} = (-4.2\,Km/s)\hat{i} + (8.4\,Km/s)\hat{j} + (2\,Km/s)\hat{k}$$

• A circulating charged particle:

⇒ A charged particle with mass $m$ and charge magnitude $|q|$ moving with velocity $\vec{v}$ perpendicular to a uniform magnetic field $\vec{B}$ will travel in a circle.

$\vec{v} \perp \vec{B} \Rightarrow$ magnetic force, circular motion

$\vec{F_B} = q\,\vec{v} \times \vec{B} \Rightarrow q v B \sin 90° = F_B$




Apply Newton's 2nd Law $\Rightarrow \vec{F} = m\vec{a}$

$q v B = m a \quad , \quad a = \frac{v^2}{r}$ ; Centripetal acceleration

$q v B = m \frac{v^2}{r}$

$\boxed{r = \frac{m v}{|q| B}}$ the radius of the circular path.

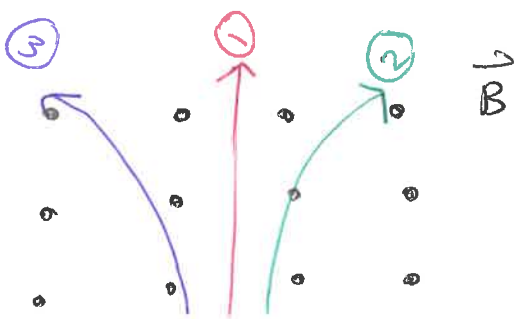
⇒ periodic time $T$ (the time for one full revolution)

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \frac{m v}{|q| B} = \frac{2\pi m}{|q| B} \quad \text{(period)}$$

$$f = \frac{1}{T} = \frac{|q| B}{2\pi m} \quad \text{[frequency]} \qquad [f] = sec^{-1} = Hz$$

$$\omega = 2\pi f = \frac{|q| B}{m} \quad \text{[Angular frequency]} \qquad [\omega] = rad/sec$$

Note



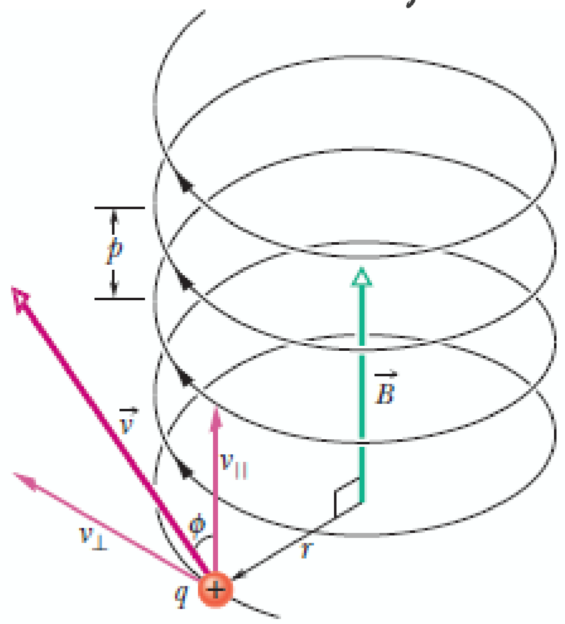

① ⇒ neutron

② ⇒ Proton

③ ⇒ electron.

• Helical motion (Spiral motion)

If the velocity of a charged particle has a component parallel to the magnetic field, the particle will move in a helical path about the direction of $\vec{B}$.




$\phi \neq 90°$ ; The angle between $\vec{B}$ and $\vec{v}$

⇒ The particle's velocity has two components

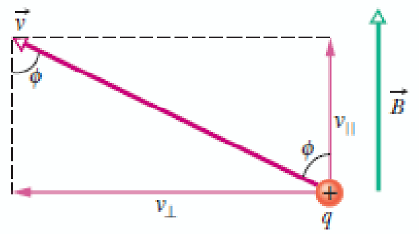
$v_{\parallel} = v\cos\phi$ ⇒ Linear motion along $\vec{B}$

$v_{\perp} = v\sin\phi$ ⇒ Circular motion around $\vec{B}$

⇒ $v_{\perp}$ ⇒ Circular motion with

$$r = \frac{m v_{\perp}}{|q|B} = \frac{m v \sin\phi}{|q|B}$$

$$T = \frac{2\pi r}{v_{\perp}} = \frac{2\pi m}{|q|B}$$




$f = \frac{1}{T} = \frac{|q|B}{2\pi m}$ ; frequency depends only on particle's properties q and m and $\vec{B}$

⇒ $v_{\parallel}$ ⇒ Linear motion along $\vec{B}$

Pitch (P) ⇒ $p = v_{\parallel} T = v\cos\phi \left[\frac{2\pi m}{|q|B}\right]$

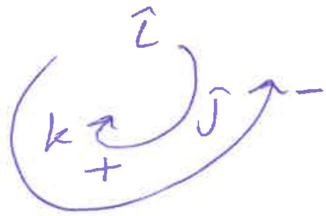
• Lorentz' Force ⇒

A charged particle moves in an Electric Field $\vec{E}$ and Magnetic field $\vec{B}$

$$\vec{F} = q\vec{E} + q\,\vec{v}\times\vec{B} = q[\vec{E} + \vec{v}\times\vec{B}] \quad \text{Lorentz' force}$$

28-50 | A proton travels through uniform magnetic field and electric fields. The magnetic field is $\vec{B} = -3.25\,\hat{i}$ mT. At one instant the velocity of the proton is $\vec{v} = 2000\,\hat{j}$ m/s. At that instant and in unit vector notation, what is the net force acting on the proton if the electric field (a) $4.00\,\hat{k}$ V/m, (b) $-4.00\,\hat{k}$ V/m, and $4.00\,\hat{i}$ V/m?

$\vec{B} = -3.25\,\hat{i}$ mT , $\vec{v} = 2000\,\hat{j}$ m/s




1) $\vec{E} = 4.00\,\hat{k}$ V/m

$$\vec{F} = q[\vec{E} + \vec{v}\times\vec{B}]$$

$$= q[4.00\,\hat{k} + (2000\,\hat{j}\text{ m/s} \times -3.25\,\hat{i} \times 10^{-3}\text{ T})]$$

$$= +1.6\times10^{-19}[4.00\,\hat{k} + {}^{-}6.5\,(\hat{j}\times\hat{i})]$$

$\hat{j}\times\hat{i} = -\hat{k}$

$$= 1.6\times10^{-19}[4.00\,\hat{k} + 6.5\,\hat{k}]$$

$$\boxed{\vec{F} = +1.68\times10^{-18}\,\hat{k}\text{ N}}$$

b) $\vec{E} = -4.00\,\hat{k}$ V/m

$$\vec{F} = 1.6\times10^{-19}[-4.00\,\hat{k} + 6.5\,\hat{k}]$$

$$\boxed{\vec{F} = 4\times10^{-19}\,\hat{k}\text{ N}}$$

c) $\vec{E} = 4.00\,\hat{i}$ V/m

$$\vec{F} = q[\vec{E} + \vec{v}\times\vec{B}]$$

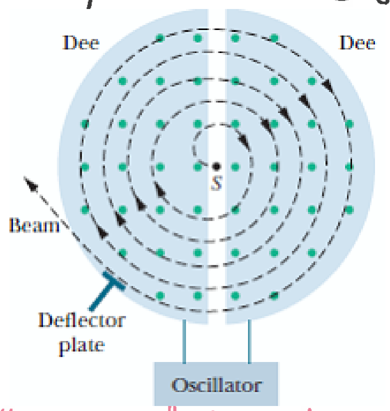
$$= 1.6\times10^{-19}[4.00\,\hat{i} + 6.5\,\hat{k}]$$

$$= 6.4\times10^{-19}\,\hat{i} + 1.04\times10^{-18}\,\hat{k}$$

$$\boxed{\vec{F} = (6.4\,\hat{i} + 10.4\,\hat{k})10^{-19}\text{ N}}$$

• Applications of a magnetic force on a charged particle

[1] Cyclotron ⇒ To accelerate a charged particle by electric forces as they circle in a magnetic field.




It consists of two hollow D-shaped objects

There is an oscillating electric potential difference between the two Dees across the gap.

The Oscillating frequency of V = The frequency of the Circulating charged particle in the magnetic field

$$f_{osc} = f_{charged\,particle} = \frac{|q|B}{2\pi m}$$

Notes

• The magnetic field is perpendicular to the particle's velocity $\vec{B} \perp \vec{v}$

⇒ The charged particle will move in a Circle

• The potential difference between the two Dees to accelerate the charged particle.

• In each cycle, the charged particle gains kinetic energy $K = 2qV$

$K_{max}$ [maximum kinetic energy] $= N(2qV)$ ; $N \equiv$ Circle's number

$$\frac{1}{2} m v_{max}^2 = N(2qV)$$

⇒ To find $v_{max}$

$$\frac{mv^2}{r} = |q| v B$$

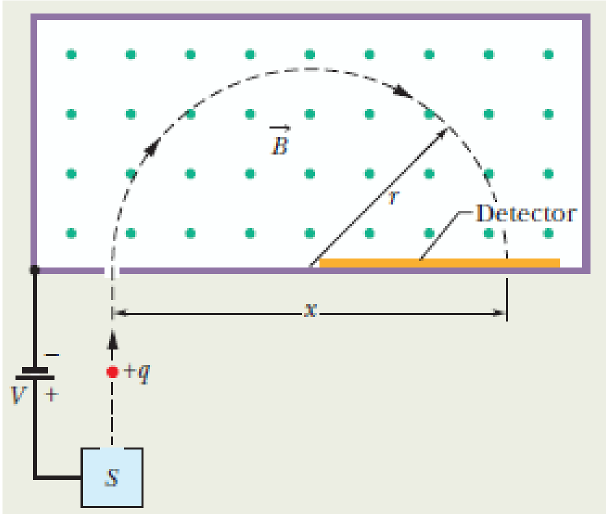
$$v = \frac{|q| B r}{m} \Rightarrow v_{max} = \left(\frac{|q|B}{m}\right) R \quad , R \equiv \text{Cyclotron radius}$$

[The velocity of the charged particle when it leave the cyclotron $v_{max}$ ]

## 2 Mass Spectrometer:

To measure the mass of an ion $(q, m)$

the initially stationary ion is accelerated by the electric field due to a potential difference $V$. The ion leaves S and enters a separator chamber in which a uniform magnetic field $\vec{B}$ is perpendicular to the path of the ion.




$\vec{B} \perp \vec{v} \Rightarrow$ the ion will move in a circle

By the conservation of mechanical energy

$$\Delta K + \Delta U = 0$$

$$\frac{1}{2} m v^2 = qV$$

$$\boxed{v = \sqrt{\frac{2qV}{m}}}$$

$\vec{B} \perp \vec{v} \Rightarrow$ Circular motion

$$r = \frac{mv}{|q|B} = \frac{m}{|q|B}\sqrt{\frac{2qV}{m}}$$ (The radius of the circle)

$$X = 2r$$ (The distance where the ion strikes a detector

$$X = \frac{2m}{|q|B}\sqrt{\frac{2qV}{m}}$$

$$X^2 = \frac{4m^2}{|q|^2B^2}\frac{2qV}{m} \Rightarrow \boxed{m = \frac{|q|BX^2}{8V}}$$

The mass of ion can be determined by measuring $B$, $X$ and $V$

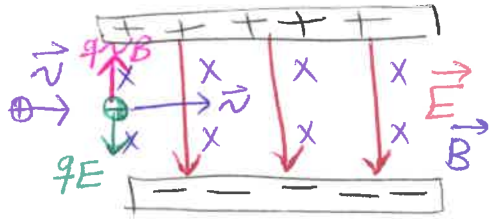
# 3 Crossed Fields ( velocity Selector )

⇒ The Electric and magnetic fields are perpendicular to each other

$$\vec{E} \perp \vec{B}$$

Velocity Selector is a device to select charged particles with a certain velocity.

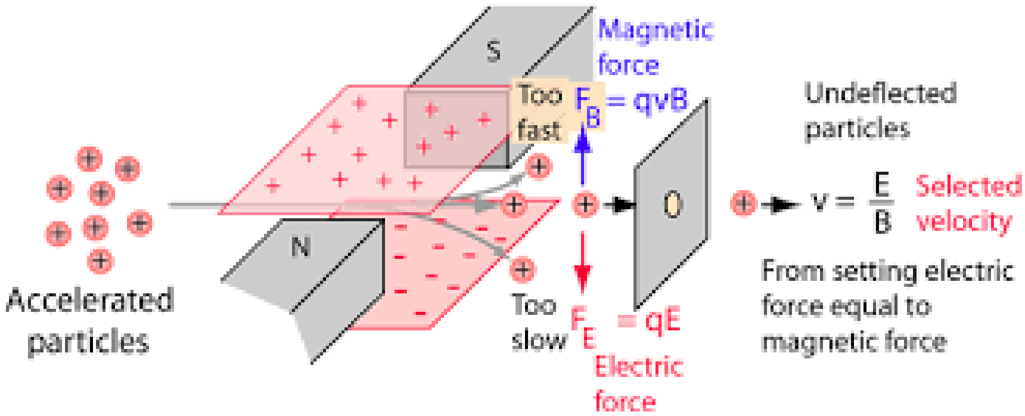
⇒ If the forces are in opposite directions, a particular speed will result in no deflection of the particle.



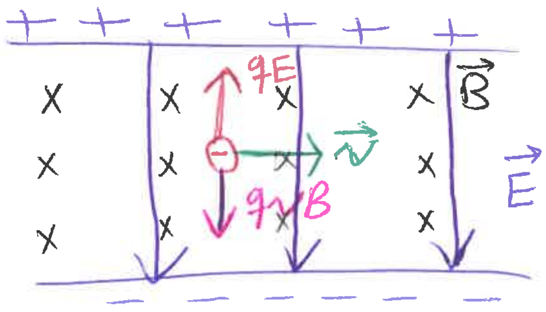

$$|q| E = |q| vB \sin(90°) = |q| vB$$

$\boxed{v = \frac{E}{B}}$ Opposite forces Cancelling ; No Deflection of the particle

if the particle has a velocity Larger than $v = \frac{E}{B}$ ; the particle will deflect up and if its velocity $v < \frac{E}{B}$ it will deflect down.



28-60 An electric field of 1.5 kV/m and a perpendicular magnetic field of 0.350 T act on a moving electron to produce no net force. What is the electron's speed?




$\vec{F}_{net, electron} = zero$

$$qE = qvB$$

$$v = \frac{E}{B} = \frac{1.5 \times 10^3 \ V/m}{0.35 \ T}$$

$$v = 4.3 \times 10^3 \ m/s = 4.3 \ km/s$$

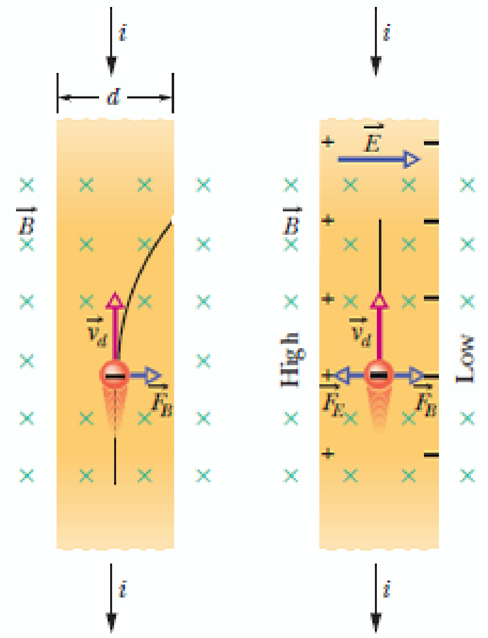
Suppose $\vec{v}$ rightward

$\vec{E}$ downward

So $\vec{B}$ must be inward to get crossed fields

# 4 The Hall effect (crossed fields)

- When a uniform magnetic field $\vec{B}$ is applied to a conducting strip carrying current $i$, with the field perpendicular to the direction of the current, a Hall-effect potential difference $V$ is set up across the strip.




- A Hall potential difference $V$

$$V_H = Ed$$

$\Rightarrow$ The electric force $\vec{F}_E$ on the charge carrier is then balanced by the magnetic force $\vec{F}_B$ on them $\boxed{eE = ev_d B}$

$v_d \equiv$ The drift speed $= \frac{J}{ne}$

where $n$ is the number density of the charge carrier

$$J = i/A = \frac{\text{Current}}{\text{Cross sectional area}}$$

$$v_d = \frac{J}{ne} = \frac{i}{Ane} \Rightarrow n = \frac{i}{Aev_d}$$

$\Rightarrow n = \frac{iB}{AeE}$, use $E = \frac{V_H}{d}$

$A = dL$, $L \equiv$ strip thickness (parallel to $\vec{B}$)

$$n = \frac{iB}{dLe\frac{V_H}{d}}, \quad \boxed{n = \frac{iB}{V_H Le}}$$

- When a conductor moves through a uniform magnetic field $\vec{B}$ at speed $\vec{v}$, the Hall effect potential difference $V$ across it is

$\boxed{V_H = v_d Bd}$, where $d$ is the width perpendicular to both velocity $\vec{v}$ and field $\vec{B}$.

**28-47** A strip of copper 75.0 $\mu m$ thick and 4.5 mm wide is placed in a uniform magnetic field $\vec{B}$ of magnitude 0.65 T, with $\vec{B}$ perpendicular to the strip. A current $i = 57A$ is then sent through the strip such that a Hall potential difference V appears across the width of the strip. Calculate V. (The number of charge carriers per unit volume for copper is $8.47 \times 10^{28}$ electrons/$m^3$?

$$V_H = E d$$
$$= v_d B d$$

$$v_d = \frac{i}{nAe}$$

OR You can use $n = \frac{iB}{V_H L e}$ ; $L$ = thickness (parallel to $\vec{B}$)

$$V_H = \frac{iB}{nLe} = \frac{(57A)(0.65T)}{\left(\frac{8.47 \times 10^{28}}{m^3}\right)(75 \times 10^{-6} m)(1.6 \times 10^{-19} C)}$$

$$V_H = 3.65 \times 10^{-5} \text{ Volts} = 36 \mu V$$

$$v_d = \frac{i}{nAe} = \frac{57}{(8.47 \times 10^{28})(75 \times 10^{-6})(4.5 \times 10^{-3})(1.6 \times 10^{-19})} = 0.0125$$

$$v_d = 12.5 \text{ mm/s}$$

OR

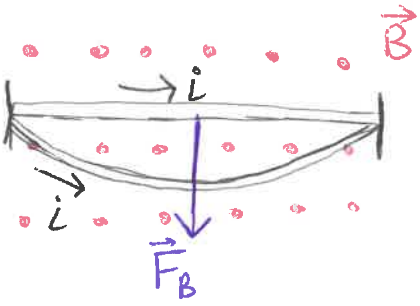
$$v_d = \frac{V_H}{Bd} = \frac{3.65 \times 10^{-5}}{(0.65)(4.5 \times 10^{-3})} = 12.5 \text{ mm/s}$$

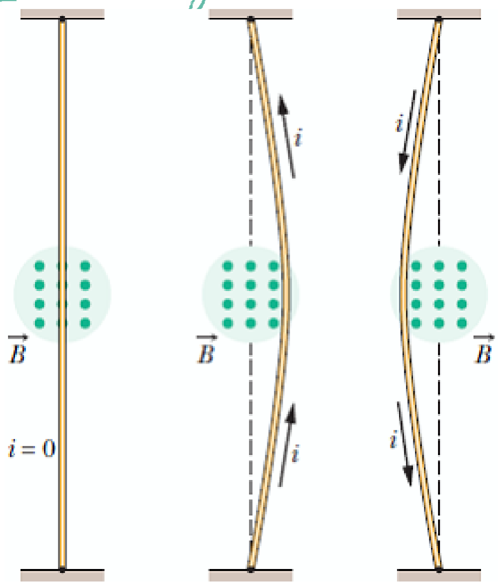
- Magnetic Force on a Current-carrying wire:

$\Rightarrow$ A straight wire carrying a current $i$ in a uniform magnetic field experiences a sideways force

$$\vec{F}_B = i\vec{L} \times \vec{B}$$

The direction of the length vector $\vec{L}$ is that of the current $i$



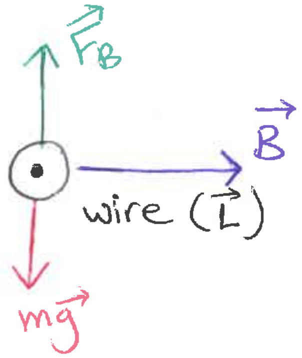




Sample problem 28.06: Magnetic force on a wire carrying current

A straight, horizontal length of copper wire has a current $i = 28A$ through it. What are the magnitude and direction of the minimum magnetic field $\vec{B}$ needed to suspend the wire – that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is 46.6 g/m.

$$\vec{F}_B = m\vec{g} \Rightarrow \Sigma F_{net} = Zero$$

$$iLB\sin\phi = mg \quad ; \phi = 90^\circ \;, \sin(90^\circ) = 1$$




$$B = \frac{mg}{iL} = \frac{(9.8\,m/s^2)}{28\,A}\left(\frac{46.6\,g}{m} * \frac{1\,kg}{10^3\,g}\right)$$

$$\boxed{B = 1.6 \times 10^{-2}\,T}$$

$$\vec{B} = 1.6 \times 10^{-2}\,T\,\hat{i}$$

• Torque on a Current Loop

$\Rightarrow$ The magnetic dipole moment $\vec{\mu}$ $\Rightarrow$ $\mu = N i A$

$N \rightarrow$ the number of turns in the coil

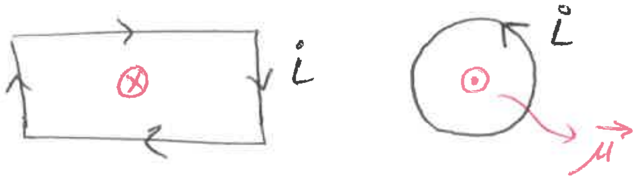
$i \rightarrow$ The current in the coil

$A \rightarrow$ The area of each turn

$\vec{\mu} \equiv$ vector quantity

The direction of the magnetic dipole moment is given by the right-hand Rule.

$[\vec{\mu}] = A.m^2$




• A coil (of area A and N turns, carrying current $i$) in a uniform magnetic field $\vec{B}$ will experience a torque given by

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$

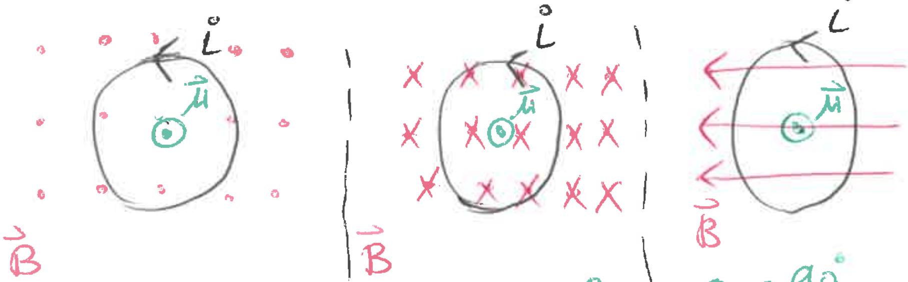
$[\vec{\tau}] = N.m$

• The orientation energy of a magnetic dipole in a magnetic field is

$$\boxed{U(\theta) = -\vec{\mu} \cdot \vec{B}}$$

$[U]$ = Joule

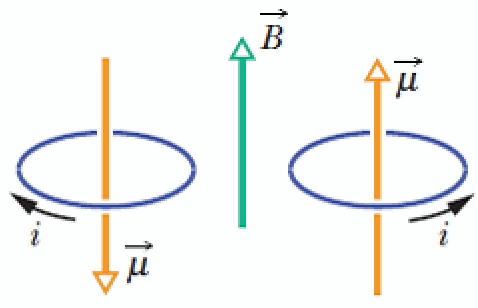
$$U(\theta) = \begin{cases} +\mu B \text{ (highest energy)}, & \theta = 180^\circ; \text{ antiparallel } \mu \,\&\, B \\ 0, & \vec{\mu} \perp \vec{B} \\ -\mu B \text{ (lowest energy)}, & \theta = 0^\circ; \ \vec{\mu} \text{ and } \vec{B} \text{ parallel} \end{cases}$$




$\theta = 0^\circ$
$U = -\mu B$
lowest energy

$\theta = 180^\circ$
$U = +\mu B$
Highest energy

$\theta = 90^\circ$
$U =$ Zero




Highest energy

Lowest energy

- If an external agent rotates a magnetic dipole from an initial orientation $\theta_i$ to some other orientation $\theta_f$ and the dipole is stationary both initially and finally, the work $W_a$ done on the dipole by the agent is

$$\boxed{W_a = \Delta U = U_f - U_i = (-\vec{\mu}\cdot\vec{B})_f - (-\vec{\mu}\cdot\vec{B})_i}$$

$W_a = W_{\text{external, applied}} = +\Delta U$

$W_{\vec{B}}$ = The work done by the field $= -\Delta U$

28-49 The below coil carries current $i = 4.6$ A in the direction indicated, is parallel to an XZ plane, has 30 turns and an area of $4.0\times10^{-3}\,m^2$, and lies in a uniform magnetic field $\vec{B} = (3\hat{i} - 3\hat{j} - 4\hat{k})$mT. What are (a) the orientation energy of the coil in the magnetic field and (b) the torque on the coil due to the magnetic field?

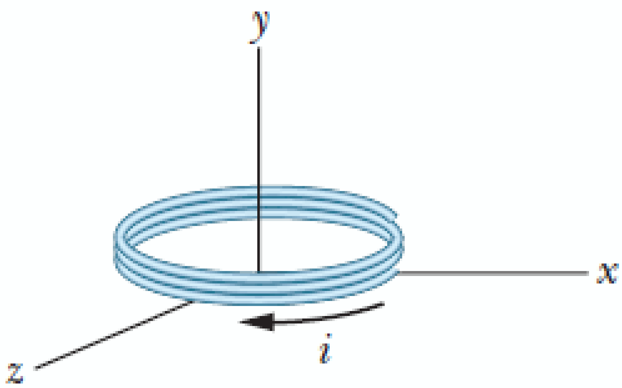
$$\vec{\mu} = -\mu\hat{j}$$

$$\mu = NiA = 3(4.6)(4\times10^{-3}) = -(0.0552\ A.m^2)\hat{j}$$

(a) $U = -\vec{\mu}\cdot\vec{B}$

$$= -(-0.0552\hat{j})\cdot(3\hat{i} - 3\hat{j} - 4k)\times10^{-3}$$

$$\boxed{U = -1.656\times10^{-4}\ J}$$




b) $\vec{\tau} = \vec{\mu}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -0.0552 & 0 \\ 3 & -3 & -4 \end{vmatrix} 10^{-3}$

$$\boxed{\vec{\tau} = (0.221\hat{i} + 0.166\hat{k})\ mN.m}$$

$$\vec{\tau} = (-0.0552\times-0.004)\hat{i} + [(3\times0.0552)\hat{k}]10^{-3}$$

$$= 2.208\times10^{-4}\hat{i} + 1.656\times10^{-4}\hat{k}$$