DOING PHYSICS

1

EXERCISES

Section 1.2 Measurements and Units

10. **INTERPRET** This problem involves converting between different multiples of the same SI-based unit (i.e, the watt).

DEVELOP Before we can express 1000 MW in different multiples of the Watt, we need to know the meaning of the various prefixes. From Table 1.1, we find that the prefix M = 10^6, k = 10^3, and G = 10^9. Notice that by dividing each of these equations by the right-hand side, we arrive at the following expressions for unity:

$$1=\frac{10^6}{\text{M}}, \quad 1=\frac{10^3}{\text{k}}, \quad 1=\frac{10^9}{\text{G}}$$

Because multiplying or dividing any quantity by unity does not change the quantity in question, we can convert between the different multiples simply by multiplying or dividing by these expressions for unity, then canceling symbols that appear in both the numerator and denominator, just as with any ordinary algebraic formula.

EVALUATE (**a**) To express 1000 MW in terms of W, we simply multiply 1000 MW by the first expression above for unity. Thus, we find

$$1000\text{ MW}=\left(1000\text{ M W}\right)\overbrace{\left(\frac{10^6}{\text{M}}\right)}^{=1}=\left(10^3\ \cancel{\text{M}}\ \text{W}\right)\left(\frac{10^6}{\cancel{\text{M}}}\right)=10^9\ \text{W}$$

where we have used 1000 = 10^3. Thus 1000 MW = 10^9 W. Notice how we have cancelled the prefix "M" in the intermediate step.

(**b**) Using the same strategy to express 1000 MW in terms of kW, we find

$$1000\text{ MW}=\left(1000\text{ M W}\right)\overbrace{\left(\frac{10^6}{\text{M}}\right)}^{=1}\overbrace{\left(\frac{\text{k}}{10^3}\right)}^{=1}=\left(\cancel{10^3}\ \cancel{\text{M}}\ \text{W}\right)\left(\frac{10^6}{\cancel{\text{M}}}\right)\left(\frac{\text{k}}{\cancel{10^3}}\right)=10^6\ \text{k W}$$

where we have used the algebraic relation that 1/1 = 1 (i.e., 10^3/k = k/10^3). Thus, 1000 MW = 10^6 kW.

(**c**) Repeating the same strategy to express 1000 MW in terms of GW, we find

$$1000\text{ MW}=\left(1000\text{ M W}\right)\overbrace{\left(\frac{10^6}{\text{M}}\right)}^{=1}\overbrace{\left(\frac{G}{10^9}\right)}^{=1}=\left(1\ \cancel{\text{M}}\ \text{W}\right)\left(\frac{\cancel{10^9}}{\cancel{\text{M}}}\right)\left(\frac{G}{\cancel{10^9}}\right)=1\,\text{G W}$$

Thus, 1000 MW = 1 GW.

ASSESS Notice that we find the factor 10^9 in going from W in part (a) to GW in part (c), which is consistent with the definition of giga (1 G = 10^9). You may be familiar with some of the prefixes from such things as computer memory, where a common amount of RAM for a laptop computer (as of the date at which this book is published!) is hundreds of Mb (512 Mb), whereas the hard drive can hold hundreds of Gb of data, and external storage devices can hold terabytes (Tb) of data.

11. INTERPRET This problem involves comparing the sizes of two objects (a hydrogen atom and a proton), where the size of each object is expressed in different multiples of the meter (i.e., a distance).

DEVELOP Before any comparison can be made, the quantity of interest must first be expressed in the same units. From Table 1.1, we see that a nanometer (nm) is 10^{-9} m, and a femtometer (fm) is 10^{-15} m. Expressed mathematically, these relations are 1 nm = 10^{-9} m and 1 fm = 10^{-15} m, or 1 = (10^{-9} m)/nm = (10^{-15} m)/fm.

EVALUATE Using these conversion factors, the diameter of a hydrogen atom is $d_{\text{H}} = (0.1\ \cancel{\text{nm}})(10^{-9}\ \text{m}/\cancel{\text{nm}}) = 10^{-10}$ m, and the diameter of a proton is $d_{\text{p}} = (1\ \cancel{\text{fm}})(10^{-15}\ \text{m}/\cancel{\text{fm}}) = 10^{-15}$ m. Therefore, the ratio of the diameters of a hydrogen atom to a proton (its nucleus) is

$$\frac{d_{\text{H}}}{d_{\text{p}}} = \frac{10^{-10}\ \text{m}}{10^{-15}\ \text{m}} = 10^5$$

ASSESS The hydrogen atom is about 100,000 times larger than its nucleus. To get a feel for this difference in size, consider the diameter of the Earth (~ 10^7 m). If a hydrogen atom were the size of the Earth, the proton would have a size of (10^7 m)/10^5 = 10^2 m, which is the size of a football field!

12. INTERPRET This problem involves using the definition of the meter to find the distance traveled by light (in a vacuum) in a given time t = 1 ns. Because the meter is defined using the unit s (seconds), we will need to convert between ns and s.

DEVELOP By definition, the speed of light $c \equiv 299{,}792{,}458$ m/s (where $\equiv$ means "is defined as"). From Table 1.1, we find that 1 ns = 10^{-9} s, so the time interval $t = 10^{-9}$ s.

EVALUATE The distance d traveled by light (in vacuum) is

$$d = ct = (299{,}792{,}458\ \text{m/s})(10^{-9}\ \text{s}) = 0.3\ \text{m}$$

ASSESS This distance is roughly a third of a meter, or about the size of your average textbook. Notice that the number of significant figures in the answer corresponds to that given for the time interval t. Had t been given as 1.00000000 ns, then we could have calculated d = 0.299792458 m.

13. INTERPRET This problem involves inverting the definition of the second to find the period of the given ^{133}Cs radiation. Note that the "period" in this context is a length of time, so our result should be in units of time.

DEVELOP By definition, $1\ \text{s} \equiv 9{,}192{,}631{,}770T$, where T is the period of the radiation corresponding to the transition between the two hyperfine levels of the ^{133}Cs ground state. Because this is a definition, the "second" is actually given to infinite precision, so we can write $1.000000000\ \text{s} \equiv 9{,}192{,}631{,}770T$ periods of ^{133}Cs radiation.

EVALUATE One period T of cesium radiation is thus

$$T = \frac{1.000000000\ \text{s}}{9{,}192{,}631{,}770} = 1.087827757 \times 10^{-10}\ \text{s} = 108.7827757\ \text{ps}$$

ASSESS Because one nanosecond corresponds to about 9 periods of the cesium radiation, each period is about $\frac{1}{9}$ of a nanosecond. Note that there exists an alternative definition based on the frequency of the cesium-133 hyperfine transition, which is the reciprocal of the period.

14. INTERPRET For this problem, we need to convert unit prefixes from exa (E) to kilo (k). Note that the "g" in Eg refers to grams.

DEVELOP From Table 1.1, we find that E = 10^{18}, and k = 10^3, so to convert from E to k we multiply the quantity given in units of Eg by 1 = (k/10^3)(10^{18}/E).

EVALUATE The mass m of water in the lake is

$$m = (14\ \cancel{\text{E}}\text{g})\overbrace{\left(\frac{\text{k}}{10^3}\right)}^{=1}\overbrace{\left(\frac{10^{18}}{\cancel{\text{E}}}\right)}^{=1} = 14 \times 10^{15}\ \text{kg}$$

ASSESS If you were to drink 1 liter of water (= 1.0 kg) per day from Lake Baikal, how long would it take to drink the lake dry?

$$t = \left(\frac{14 \times 10^{15}\ \cancel{\text{kg}}}{1.0\ \cancel{\text{kg}}/\cancel{\text{d}}}\right)\left(\frac{1.0\ \text{y}}{365\ \cancel{\text{d}}}\right) = 3.8 \times 10^{13}\ \text{y}$$

which is some 3000 times longer than the age of the universe!

15. **Interpret** For this problem, we need to divide a 1-cm length by the given diameter of the hydrogen atom to find how many hydrogen atoms we need to place side-by-side to span this length.

Develop We first express the quantities of interest (diameter of a hydrogen atom and 1-cm line) in the same units. Since a nanometer is 10^{-9} m (Table 1.1), we see that $d_{\rm H} = 0.1\ {\rm nm} = 10^{-10}\ {\rm m}.$ In addition, $1\ {\rm cm} = 10^{-2}\ {\rm m}.$

Evaluate The desired number of atoms *n* is the length of the line divided by the diameter of a single atom:

$$n = \frac{10^{-2}\ {\rm m}}{10^{-10}\ {\rm m}} = 10^{8}$$

Assess If 1 cm corresponds to 10^8 hydrogen atoms, then each atom would correspond to 10^{-8} cm = 10^{-10} m = 0.1 nm, which agrees with the diameter given for the hydrogen atom.

16. **Interpret** This problem asks us to calculate the length of the arc described by the given angle and radius.

Develop Figure 1.2 gives the relationship between angle θ in radians, arc *s*, and radius *r*, and shows that the angle in radians is simply the (dimensionless) ratio of arc to radius ($\theta = s/r$). Reformulating this equation to give s in terms of θ and *r*, we find $s = \theta r$.

Evaluate Inserting the given values into the equation for arc length *s* gives

$$s = \theta r = (1.4\ {\rm rad})(8.1\ {\rm cm}) = 11\ {\rm cm}$$

where we have used the fact that radians are a dimensionless quantity and so do not need to be retained in the solution.

Assess Because the given quantities are known only to two significant figures, we round the answer down to the same number of significant figures.

17. **Interpret** For this problem, we are looking for an angle subtended by a circular arc, so we will use the definition of angle in radians (see Fig. 1.2).

Develop From Fig. 1.2, we see that the angle in radians is the circular arc length *s* divided by the radius *r*, or $\theta = s/r$.

Evaluate Inserting the known quantities (s = 2.1 km, *r* = 3.4 km), we find the angle subtended is

$$\theta = \frac{s}{r} = \frac{2.1\ {\rm km}}{3.4\ {\rm km}} = 0.62\ {\rm rad}$$

Using the fact that $\pi\ {\rm rad} = 180°,$ the result can be expressed as

$$\theta = 0.62\ {\rm rad} = (0.62\ {\rm rad})\left(\frac{180°}{\pi\ {\rm rad}}\right) \approx 35°$$

Assess Because a complete circular revolution is $360°$, $35°$ is roughly 1/10 of a circle. The circumference of a circle of radius $r = 3.4$ km is $C = 2\pi(3.4\ {\rm km}) = 21.4\ {\rm km}.$ Therefore, we expect the jetliner to fly approximately 1/10 of *C*, or 2.1 km, which agrees with the problem statement.

18. **Interpret** We must convert from speed in m/h to speed in m/s and ft/s. Because speed contains two units (length and time) we must convert both to convert the speed.

Develop From Appendix C we find the following conversion equations:

$1\ {\rm mi} = 1609\ {\rm m} = 5280\ {\rm ft};\quad 1\ {\rm h} = 3600\ {\rm s}.$

Evaluate Using the conversion equations, we can convert mi/h into the desired units. (**a**) In m/s, the car is moving at

$$35.0\ {\rm mi/h} = \left(35.0\frac{\cancel{\rm mi}}{\cancel{\rm h}}\right)\overbrace{\left(\frac{1609\ {\rm m}}{1\ \cancel{\rm mi}}\right)}^{=1}\overbrace{\left(\frac{1\ \cancel{\rm h}}{3600\ {\rm s}}\right)}^{=1} = 15.6\ {\rm m/s}$$

(b) In ft/s, the car is moving at

$$35.0\ {\rm mi/h} = \left(35.0\frac{\cancel{\rm mi}}{\cancel{\rm h}}\right)\overbrace{\left(\frac{5280\ {\rm ft}}{1\ \cancel{\rm mi}}\right)}^{=1}\overbrace{\left(\frac{1\ \cancel{\rm h}}{3600\ {\rm s}}\right)}^{=1} = 51.3\ {\rm ft/s}$$

Assess Note that each conversion factor is simply another way to express unity. Also note that the speed is larger in ft/s than in m/s, which is reasonable because feet are shorter than meters.

19. **INTERPRET** For this problem, we must convert the weight of the letter in ounces to its mass in grams.

DEVELOP Two different units for mass appear in the problem—ounces and grams. The conversion from ounces to grams is given in Appendix C (1 oz = weight of 0.02835 kg).

EVALUATE The maximum weight of the letter is 1 oz. Using the conversion factor above, we see that this corresponds to a weight of 0.02835 kg, or 28.35 g. Because the weight in oz is given to a single significant figure, we must round our answer to a single significant figure, which gives 30 g (or 3×10^1 g).

ASSESS The conversion factor between oz and g may be obtained based on some easily remembered conversion factors between the metric and English systems (e.g., 1 lb = weight of 0.454 kg, and 1 lb = 16 oz).

20. **INTERPRET** This problem involves calculating the number of seconds in a year, and comparing the result to $\pi \times 10^7$ s.

DEVELOP From Appendix C we find that 1 y = 365.24 d, and that 1 d = 86,400 s.

EVALUATE Using the conversion equations above, we find that one year is

$$1\text{ y} = (1\ \cancel{\text{y}})\left(\frac{365.24\ \cancel{\text{d}}}{1\ \cancel{\text{y}}}\right)\left(\frac{86400\text{ s}}{1\ \cancel{\text{d}}}\right) = 3.16\times10^7\text{ s}$$

The percent difference e between this result and $\pi \times 10^7$ s is

$$e = \frac{\left(3.16\times10^7\text{ s} - 3.14\times10^7\text{ s}\right)}{3.16\times10^7\text{ s}}(100\%) = 0.446\%$$

where the result is given to 3 significant figures to match the 3 significant figures of the input (86,400 s/d).

ASSESS To calculate the percent error, we take the difference between the correct result and the erroneous result and divide this difference by the correct result.

21. **INTERPRET** For this problem, we must express a given volume (1 m^3) in units of cm^3.

DEVELOP Because volume has dimension of $(\text{length})^3$, the problem reduces to converting m to cm. The conversion equation is 1 m = 100 cm, so the conversion factor to convert m to cm is 1 = (100 cm)/(1 m).

EVALUATE Using this conversion factor, we obtain

$$1\text{ m}^3 = (1\ \cancel{\text{m}})^3 \overbrace{\underbrace{\left(\frac{100\text{ cm}}{1\ \cancel{\text{m}}}\right)}_{=1}^{3}}^{=1} = 10^6\text{ cm}^3$$

ASSESS Another way to remember this relationship is to note that 1 m^3 = 1000 liters, and 1 liter = 1000 cm^3, so 1 m^3 = 1000 × 1000 cm^3 = 10^6 cm^3.

22. **INTERPRET** We are asked to convert carbon emission in exagrams to the more familiar tons, where $1\text{ t} = 1000\text{ kg}$.

DEVELOP The emission is given as roughly 0.5 Eg, which from Table 1.1 is $0.5\times10^{18}\text{g}$.

EVALUATE The mass of carbon emissions in tons is

$$m = 0.5\times10^{18}\text{g}\left(1\text{ kg}/1000\text{ g}\right)\left(1\text{ t}/1000\text{ kg}\right) = 0.5\times10^{12}\text{t}$$

ASSESS A half trillion tons of carbon dioxide is hard to fathom.

23. **INTERPRET** For this problem, we have to convert units of volume and units of area. We are told the coverage of the paint (in English units) is 350 ft^2/gal.

DEVELOP From Appendix C, we find the following conversion equations:

$$1\text{ gal} = 3.785\times10^{-3}\text{ m}^3 = 3.785\text{ L}$$
$$1\text{ ft}^2 = 9.290\times10^{-2}\text{ m}^2$$

Thus, the conversion factors are 1 = 3.785 L/gal and 1 = 0.09290 m^2/ft^2.

EVALUATE Combining the two conversion factors, we have

$$350\ \text{ft}^2/\text{gal} = \left(350\frac{\text{ft}^2}{\text{gal}}\right)\overbrace{\left(\frac{1\ \text{gal}}{3.785\ \text{L}}\right)}^{=1}\overbrace{\left(\frac{9.290\times10^{-2}\ \text{m}^2}{1\ \text{ft}^2}\right)}^{=1} = 8.6\ \text{m}^2/\text{L}$$

ASSESS Dividing this result by 350 gives $1\ \text{ft}^2/\text{gal} = 0.025\ \text{m}^2/\text{L}$.

24. **INTERPRET** For this problem, we will need to convert miles to kilometers.

DEVELOP From Appendix C, we know 1 mi = 1.609 km, so 1 = 1 mi/(1.609 km).

EVALUATE Using the conversion factor above, we find

$$100\frac{\text{km}}{\text{h}} = \left(100\frac{\text{km}}{\text{h}}\right)\overbrace{\left(\frac{1\ \text{mi}}{1.609\ \text{km}}\right)}^{=1} = 62.2\frac{\text{mi}}{\text{h}}$$

ASSESS Thus the speed limit in Canada is about 2.8 mi/h less than in the United States.

25. **INTERPRET** This problem will require us to convert units of both length and time. Specifically, we have to convert km to m and h to s.

DEVELOP From Appendix C, we find the following conversion equations, which we convert into conversion factors by dividing through by km and h, respectively:

$$1\ \text{km} \equiv 1000\ \text{m} \Rightarrow 1 = 1\ \text{km}/(1000\ \text{m})$$
$$1\ \text{h} \equiv 3600\ \text{s} \Rightarrow 1 = 3600\ \text{s}/\text{h}$$

EVALUATE Combining the two conversion factors, we have

$$1\ \text{m/s} = \left(1\frac{\text{m}}{\text{s}}\right)\overbrace{\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)}^{=1}\overbrace{\left(\frac{1\ \text{km}}{1000\ \text{m}}\right)}^{=1} = 3.6\ \text{km/h}$$

ASSESS The units of the results are distance per unit time, as expected for a speed. Because one km is *defined* as 1000 m, and 1 h is *defined* as 3600 s, the conversion factor is exact. Thus, if you walk 1 meter in one second, you can walk a distance of 3.6 km in one hour (assuming you don't get tired and slow down).

26. **INTERPRET** For this problem, we will convert lbs to kg and ft to m.

DEVELOP From Appendix C, we find the following conversion equations, which we convert to conversion factors by dividing by kg and ft, respectively:

$$1\ \text{kg} = 2.205\ \text{lbs} \Rightarrow 1 = 2.205\ \text{lbs} / \text{kg}$$
$$1\ \text{ft} = 3.281\ \text{m} \Rightarrow 1 = 3.281\ \text{m} / \text{ft}$$

EVALUATE Using these conversion factors, we find

$$\frac{3.0}{2100}\left(\frac{\text{ft}^2}{\text{lbs}}\right) = \left(\frac{3.0\ \text{ft}^2}{2100\ \text{lbs}}\right)\overbrace{\left(\frac{2.205\ \text{lbs}}{\text{kg}}\right)}^{=1}\overbrace{\left(\frac{3.281\ \text{m}}{\text{ft}}\right)^2}^{=1} = 3.4\times10^{-2}\ \text{m}^2/\text{kg}$$

ASSESS The conversion factor for length is squared because we are dealing with an area. Thus, the result has units of area per unit mass, as expected. Also, note that the result is reported to the same number of significant figures (i.e., 2) as given by the data (3.0 and 2100 have 2 significant figures).

27. **INTERPRET** This problem involves converting radians to degrees.

DEVELOP An angle in radians is the circular arc length s divided by the radius r, or $\theta = s/r$. Because a complete revolution (360°) is *defined* as 2π radians, we have $360^\circ \equiv 2\pi$ rad, or $1 = 360^\circ/(2\pi)$.

EVALUATE Using the conversion factor above, we find that

$$1\ \text{rad} = (1\ \text{rad})\overbrace{\left(\frac{360^\circ}{2\pi\ \text{rad}}\right)}^{=1} = 57.3^\circ$$

ASSESS Because this conversion involves a definition, we know the quantities involved to infinite precision, so we could report the result to as many significant figures as desired. The result is that 1 rad is approximatley one-sixth of a complete revolution, so 6 rad corresponds approximately to one revolution.

Section 1.3 Working with Numbers

28. **INTERPRET** We are asked to add two distances together, paying attention to significant figures.

DEVELOP We first must put the two distances in the same units. We are free to choose, so let's convert the second quantity from km to m: $2.13103\text{ km}(1000\text{ m}/1\text{ km}) = 2131.03\text{ m}$.

EVALUATE In Section 1.3, we're told that when adding two numbers, the answer should have the same number of digits to the right of the decimal point as the term in the sum that has the smallest number of digits to the right of the decimal point. In this case, the second number has the smallest number of digits (2), so

$$3.63105\text{ m} + 2131.03\text{ m} = 2134.66\text{ m}$$

ASSESS One could do the problem in terms of km, but that means dealing with a lot more digits to the right of the decimal point.

29. **INTERPRET** We interpret this as a problem involving the conversion of time to different units.

DEVELOP With reference to Table 1.1 for SI prefixes, we have $1\text{ ms} = 10^{-3}\text{s}$.

EVALUATE Using the above conversion factor, we obtain

$$\frac{4.23103\text{ m/s}}{0.57\text{ ms}}\left(\frac{1\text{ ms}}{10^{-3}\text{ s}}\right) = 7.4\text{ m/s}^2$$

ASSESS Acceleration is the physical quantity with such units. An average acceleration of 7.4 m/s^2 changes the speed of an object by 4.23103 m/s in 0.57 ms. Note that we only kept two significant figures in the answer, since that was the number of significant figures in the time quantity (see Section 1.3).

30. **INTERPRET** This problem requires both adding and multiplying physical quantities.

DEVELOP To add the two distances, we need to convert them to the same units and the same exponent. For this, let's choose 10^{-3} meters: $5.131022\text{ cm} = 53.1022\times10^{-3}\text{m}$ and $6.83103\text{ mm} = 6.83103\times10^{-3}\text{m}$.

EVALUATE Adding first the two distances

$$(53.1022 + 6.83103)\times10^{-3}\text{m} = 59.93323\times10^{-3}\text{m}$$

Note that by the rules for significant figures (see Section 1.3), we only are supposed to keep 4 digits to the right of the decimal point, since that is the smallest number of digits in the two terms of the sum. But this is an intermediate result, so we keep one extra digit. Now we multiply by the force term

$$(59.93323\times10^{-3}\text{m})(1.83104\text{ N}) = 0.109740\text{ N}\cdot\text{m}$$

Here we keep 6 significant figures, since that is the number of significant figures in the least precise quantity entering the calculation (the force term).

ASSESS The unit $\text{N}\cdot\text{m}$, as we'll discover later, is used for torque.

31. **INTERPRET** This problem asks that we take the cube root of a number in scientific notation without a calculator.

DEVELOP As shown in the Tactics 1.1 box in the text, we can take the cube root by multiplying the exponent by 1/3.

EVALUATE We know that the cube root of 64 is 4. So let's rewrite the given number as 64×10^{18}. We now can calculate the cube root more easily:

$$\sqrt[3]{6.4\times10^{19}} = \left(64\times10^{18}\right)^{1/3} = (64)^{1/3}\times10^{18/3} = 4\times10^6$$

ASSESS We can check our work by cubing 4×10^6 and verifying that it indeed equals 6.4×10^{19}.

32. **INTERPRET** This problem involves adding two distances that are given in different units. Therefore, before performing the sum, we must express both distances in the same units. We will choose to express them in m, so we must convert cm into m, then sum the result with 1.46 m.

DEVELOP From Table 1.1, we find that 1 cm = 10^{-2} m, or 1 = 10^{-2} m/cm. Therefore, 2.3 cm = (2.3 cm)(10^{-2} m/cm) = 0.023 m.

EVALUATE Summing the distances gives 1.46 m + 0.023 m = 1.483 m. Rounding down to two significant figures gives the final result of 1.48 m.

ASSESS We can check our result by converting the distances to miles first, performing the sum, then converting back to km to compare with the original result. We find

$$3.63105\text{ mi}+\left(2.13103\ \cancel{\text{km}}\right)\left(\frac{1\text{ mi}}{1.609\ \cancel{\text{km}}}\right)=4.95549\text{ mi}$$

$$\left(4.95549\ \cancel{\text{mi}}\right)\left(\frac{1.609\text{ km}}{1\ \cancel{\text{mi}}}\right)=7.97339\text{ km}$$

which agrees with the original result.

33. **INTERPRET** This problem involves adding two distances that are given in different units. Therefore, before performing the sum, we must express both distances in the same units. We will choose to convert cm to m, then sum the result with 41 m.

DEVELOP From Table 1.1, we see that 1 cm = 10^{-2} m, or 1 = 10^{-2} m/cm.

EVALUATE An airplane of initial length $L_0 = 41\text{ m}$ is increased by $\Delta L = 3.6$ cm, so the final length L is

$$L = L_0 + \Delta L = 41\text{ m} + \left(3.6\ \cancel{\text{cm}}\right)\overbrace{\left(\frac{10^{-2}\text{ m}}{\cancel{\text{cm}}}\right)}^{=1} = 41\text{ m} + 0.036\text{ m} = 41.036\text{ m}$$

However, because the data are given to two significant figures, we must round the result to two significant figures, so L = 41 m is the final result.

ASSESS To two significant figures, the result remains unchanged. In this context, 41 m means a length greater than or equal to 40.5 m, but less than 41.5 m, and 41 m + 0.036 m = 41.036 m satisfies this condition.

34. **INTERPRET** This problem is the same as the preceding problem, except that the data are given to four significant figures.

DEVELOP From Table 1.1, we see that 1 cm = 10^{-2} m, or 1 = 10^{-2} m/cm.

EVALUATE An airplane of initial length $L_0 = 41.05\text{ m}$ is increased by $\Delta L = 3.6$ cm, so the final length L is

$$L = L_0 + \Delta L = 41.05\text{ m} + \left(3.6\ \cancel{\text{cm}}\right)\overbrace{\left(\frac{10^{-2}\text{ m}}{\cancel{\text{cm}}}\right)}^{=1} = 41.05\text{ m} + 0.036\text{ m} = 41.086\text{ m}$$

Rounding the result to four significant figures gives L = 41.09 m as the final result.

ASSESS In this context, 41.09 m means a length greater than or equal to 41.085 m, but less than 41.095 m, and 41.086 m satisfies this condition.

PROBLEMS

35. **INTERPRET** This problem involves exploring the numerical precision of results by evaluating $\left(\sqrt{3}\right)^3$ using two different approaches: the first involves retaining only 3 significant figures in the intermediate step, and the second involves retaining 4 significant figures in the intermediate step.

DEVELOP For part (a), retain 3 significant figures in calculating $\sqrt{3}$ and in part (b) retain 4 significant figures in the same calculation. Cube the results and round to 3 significant figures for the final answer and compare the results of (a) and (b).

EVALUATE (**a**) $\sqrt{3} \approx 1.73$ (to 3 significant figures), so $(1.73)^3 = 5.177 \approx 5.18$, to 3 significant figures.
(**b**) To 4 significant figures, $\sqrt{3} \approx 1.732$ so $(1.732)^3 \approx 5.1957$, or 5.20 to 3 significant figures.

ASSESS With a calculator, we find $(\sqrt{3})^3 = 3^{3/2} = 5.196\ldots$ This example shows that it is important to carry intermediate calculations to more digits than the desired accuracy for the final answer. Rounding of intermediate results could affect the final answer.

36. **INTERPRET** We are being asked to estimate how many trees it takes to print a day's worth of newspapers.

DEVELOP Let's assume that this big city daily has a circulation of about 500,000 newspapers. Each of these has a mass of somewhere around 500 grams (or about the weight of a pound). So the total mass of paper is

$$M_{\text{p}} = (500{,}000)(500\text{ g}) = 2\times10^8\text{ g}$$

There's no point in keeping more than one significant figure, since all these are rough estimates. Note that this mass is equal to 200 tons, which is maybe 20 or so truck loads. That sounds about right.

Next, we need to estimate how much tree wood is required. We're told that half of the paper is recycled, and the other half comes from new wood pulp. We'll assume that it takes one gram of tree wood to make one gram of newspaper (neglecting the mass of paper additives and ink). We can describe this as a conversion efficiency of one. Therefore, the total mass of wood in one day's printing is

$$M_{\rm w} = \left(\tfrac{1}{2}\right)(1)\,M_{\rm p} = 10^8\,{\rm g}$$

The last thing we need to estimate is the amount of wood per tree. We'll assume each tree is a 10-meter-high cone with a radius of 10 cm at the bottom. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. To convert this volume to mass, we need the density of wood. Since wood floats, it must have a density slightly less than water $(\rho = 1\,{\rm g/cm^3})$, so for simplicity we'll assume the two are equal. The mass of a single tree is then

$$m_{\rm w} = \rho V = \left(1\ {\rm g/cm^3}\right)\left(\tfrac{1}{3}\pi(10\ {\rm cm})^2(10\ {\rm m})\right) = 10^5\,{\rm g}$$

EVALUATE The total number of trees needed to print one day's worth of a big city newspaper is the total mass of wood divided by the mass of one tree

$$N = \frac{M_{\rm w}}{m_{\rm w}} = \frac{10^8\,{\rm g}}{10^5\,{\rm g}} = 1000$$

ASSESS Does this sound right? Some references say that a cord of wood produces 2700 copies of newspaper. A cord is about 2 tons. How many trees to a cord depends on their size, but if we assume 30 trees to a cord, then this would imply just under 3000 trees to print out a day's worth of a big city newspaper.

37. **INTERPRET** This problem requires us to make a rough, "order-of-magnitude" estimate, instead of a finding a precise numerical answer. Thus, although answers may vary, they should be within an order of magnitude of the given answer.

DEVELOP For problems that involve rough estimates, various assumptions typically need to be made. Such assumptions must be physically motivated with reasonable order-of-magnitude estimates. We shall assume that there are 300 million people in the United States, and that each person drinks 250 ml = 0.250 kg of milk per day.

EVALUATE Based on the assumption above, the amount consumed per year would be approximately

$$(300\times10^6)(365\ \not{d}/{\rm y})(0.250\ {\rm kg}/\not{d}) \approx 3\times10^{10}\ {\rm kg/y}$$

Dividing this by the average annual production of one cow, we estimate the number of cows needed to be

$$N = \frac{3\times10^{10}\ {\rm kg/y}}{10^4\ {\rm kg/y}} = 3\times10^6$$

ASSESS There are currently approximately 9 million milk cows in the United States. Based on our estimate, we do not expect any shortage of milk in the near future.

38. **INTERPRET** This problem asks us to compare the size of the Earth to that of the Sun by finding how many Earth volumes would fit into a single Sun volume.

DEVELOP From Appendix E, we find that the radius of the Earth is 6.37×10^6 m, and the radius of the Sun is 696×10^6 m. Recall that the volume V of a sphere is $V = (4/3)\pi r^3$, where r is the radius of the sphere.

EVALUATE The number N of Earths that would fit into the Sun is

$$N = \frac{V_{\rm Sun}}{V_{\rm Earth}} = \frac{(4/3)\pi r_{\rm Sun}^3}{(4/3)\pi r_{\rm Earth}^3} = \frac{r_{\rm Sun}^3}{r_{\rm Earth}^3} = \frac{\left(696\times\not{10^6}\ \not{\rm m}\right)^3}{\left(6.37\times\not{10^6}\ \not{\rm m}\right)^3} = 1.30\times10^6$$

ASSESS Thus, a little over a million Earths could fit into the Sun.

39. **INTERPRET** This problem calls for a rough estimate. The quantities of interest here are the total rate of energy consumption (i.e., power consumption) and the area needed for solar cells. The electrical power consumed by the entire population of the United States, divided by the power converted by one square meter of solar cells, is the area required by this question.

DEVELOP For problems that involve rough estimates, various assumptions typically need to be made. Such assumptions must be physically motivated with reasonable order-of-magnitude estimates. Note that we will be required to convert all data regarding power to kW, and that we must express length in common units.

EVALUATE Assume there are 300 million people in the United States, and that the power consumption for each person is 3 kW (a per capita average over 24 h periods of all types of weather), then the total power consumption P_{tot} is

$$P_{\text{tot}} = 300\times10^6 \times 3\text{ kW} = 9\times10^8\text{ kW}$$

For a solar cell with 20% efficiency in converting sunlight to electrical power, the power-yield P_{sc} per unit area A is $P_{\text{sc}}/A = (0.20)(300\ \cancel{\text{W}}/\text{m}^2)(10^{-3}\ \text{kW}/\cancel{\text{W}}) = 0.060\ \text{kW/m}^2$. Therefore, the total area A_{tot} needed is

$$A_{\text{tot}} = \frac{P_{\text{tot}}}{P_{\text{sc}}/A} = \frac{9\times10^8\text{ kW}}{0.060\text{ kW/m}^2} = 1.5\times10^{10}\text{ m}^2$$

The land area of the continental United States A_{US} can be approximated as the area of a rectangle the size of the distance from New York to Los Angeles by the distance from New York to Miami, or $A_{\text{US}} \approx (5000\text{ km})\times(2000\text{ km}) = 10^7\text{ km}^2$. From Table 1.1, we know that 1 km = 10^3 m, or 1 = 10^{-3} km/m. Then the fraction of area to be covered by solar cells would be

$$\frac{A_{\text{tot}}}{A_{\text{US}}} \approx \left(\frac{1.5\times10^{10}\ \cancel{\text{m}^2}}{10^7\ \cancel{\text{km}^2}}\right)\overbrace{\left(\frac{10^{-3}\ \cancel{\text{km}}}{1\ \cancel{\text{m}}}\right)^2}^{=1} = 1.5\times10^{-3}$$

or approximately 0.15%.

ASSESS This represents only a small fraction of land to be used for solar cells. The area A_{tot} is comparable to the fraction of land now covered by airports.

40. **INTERPRET** This problem calls for a rough estimate instead of a precise numerical answer. Answers will differ depending on the assumptions used, so each assumption should be explained. We are asked to estimate the number of piano tuners in Chicago using publicly available information (population, tuning frequency, etc.).

DEVELOP Assume that each piano tuner takes 2 hours to tune a piano, and works an 8-h day for 50 weeks per year. In one year, each piano tuner could then tune a number N_{p} of pianos given by

$$N_{\text{p}} = \frac{(8\text{ h})(5\text{ d/w})(50\text{ w/y})}{2\text{ h}} = 1000\text{ y}^{-1}$$

We can furthermore assume that the percent of people that play the piano in Chicago mirrors the figure for the entire United States, which is 20 million (out of 300 million), and that each piano-playing person owns a piano and has it tuned each year. Given that the population of Chicago is about 3 million, the number of pianos N_{t} that need tuning each year is

$$N_{\text{t}} = 3\times10^6\frac{20}{300}\text{ y}^{-1} = 2\times10^5\text{ y}^{-1}$$

EVALUATE Based on the above assumptions, the number N of piano tuners in Chicago is

$$N = \frac{N_{\text{t}}}{N_{\text{p}}} = \frac{2\times10^5\text{ y}^{-1}}{1000\text{ y}^{-1}} = 200$$

ASSESS Thus, we can be fairly confident that Chicago counts several hundred piano tuners among its citizens.

41. **INTERPRET** This problem calls for a rough estimate instead of a precise numerical answer. Answers will differ depending on the assumptions used, so each assumption should be explained. We are asked to estimate the rate (volume per unit time) at which water flows over the Niagara Falls, and the time it would take for Lake Eerie to rise 1 m if the falls were shut off.

DEVELOP For problems that involve rough estimates, various assumptions typically need to be made. Such assumptions must be physically motivated and explained.

(**a**) The discharge of Niagara Falls (i.e., the volume V flowing per unit time over the falls) is the current speed multiplied by the cross-sectional area of the channel, $D = vA$ (see Section 15.4). A road map of Niagara Falls shows that the river is about 1-km wide at the falls. The cross-sectional area A at the falls is the river width multiplied by the average depth, which is probably on the order of 1 m. The current speed may be estimated to be approximatley 3 m/s.

(**b**) To estimate the time it takes for Lake Erie to rise by 1 m, we need to know its surface area. From a road map of the area, we find that Lake Erie is about 375 km long by 75 km wide.

EVALUATE (**a**) Based on the these assumptions, the volume V of water going over Niagara Falls each second is

$$V = D(1\text{ s}) = vA(1\text{ s}) = (3\text{ m/}\cancel{\text{s}})(1\text{ }\cancel{\text{km}})\overbrace{\left(\frac{10^3\text{ m}}{\cancel{\text{km}}}\right)}^{=1}(1\text{ m})(1\text{ }\cancel{\text{s}}) = 3\times10^3\text{ m}^3$$

(b) To find the time t it would take Lake Erie to rise 1 m if the falls were blocked, we divide the discharge obtained for part (a) into the volume V_L of a 1-m layer of water covering Lake Erie. This gives

$$\begin{aligned} t = \frac{V_L}{D} &= \frac{(375\text{ }\cancel{\text{km}})(75\text{ }\cancel{\text{km}})(1\text{ }\cancel{\text{m}})}{3\times10^3\text{ }\cancel{\text{m}^3}\text{/s}}\overbrace{\left(\frac{10^3\text{ }\cancel{\text{m}}}{\cancel{\text{km}}}\right)^2}^{=1} \\ &= (9\times10^6\text{ }\cancel{\text{s}})\left(\frac{1\text{ }\cancel{\text{h}}}{3600\text{ }\cancel{\text{s}}}\right)\left(\frac{1\text{ d}}{24\text{ }\cancel{\text{h}}}\right) \\ &= 100\text{ days} \end{aligned}$$

ASSESS Checking our estimate of the discharge against public information, we find that the average discharge of the Niagara Falls is roughly 2×10^5 ft^3/s. Converting this to m^3/s gives

$$2\times10^5\text{ ft}^3\text{/s} = (2\times10^5\text{ }\cancel{\text{ft}^3}\text{/s})\overbrace{\left(\frac{0.3048\text{ m}}{\cancel{\text{ft}}}\right)^3}^{=1} = 6\times10^3\text{ m}^3\text{/s}$$

which is within a factor of 2 or our very rough estimation. Not bad!

42. **INTERPRET** This problem calls for a rough estimate instead of a precise numerical answer. We are asked to estimate the total number of air molecules in a typical university dormitory room.

DEVELOP For problems that involve rough estimates, various assumptions typically need to be made. Such assumptions must be physically motivated and explained. A typical dormitory room might have dimensions of 3 m × 4 m ×3 m, which makes a volume of $V = 36$ m^3. In addition, we shall regard the air in the room as an ideal gas at standard temperature and pressure, so one "mole of air" contains Avogadro's number of molecules, which is about 6.02×10^{23}, and occupies a volume of 22.4 liters.

EVALUATE Based on these assumptions, the number of molecules in the dormitory room is approximately

$$N = \left(\frac{6.02\times10^{23}\text{ }\cancel{\text{mol}^{-1}}}{22.4\text{ }\cancel{\text{L}}/\cancel{\text{mol}}}\right)\overbrace{\left(\frac{10^3\text{ }\cancel{\text{L}}}{\cancel{\text{m}^3}}\right)}^{=1}(36\text{ }\cancel{\text{m}^3}) = 9.7\times10^{26}$$

ASSESS This is a fairly large number. But the result is reasonable; each cubic meter contains on the order of 10^{25} molecules.

43. **INTERPRET** We can estimate the number of hairs in a braid by dividing the cross-sectional area of a braid by the cross-sectional area of a hair.

DEVELOP We are not given the area of a braid, but we can assume it has a diameter of about an inch or so, which in centimeters is $d_B \approx 3\,\text{cm}$. We're told that the hair has $d_h \approx 100\,\mu\text{m}$. The cross-sectional area of each is given by the formula $A = \pi r^2 = \pi(d/2)^2$.

EVALUATE The number of hairs in a braid is roughly

$$N \approx \frac{A_b}{A_h} = \left(\frac{d_b}{d_h}\right)^2 \approx \left(\frac{3\times10^{-2}\text{m}}{100\times10^{-6}\text{m}}\right)^2 = 9\times10^4 \sim 10^5$$

ASSESS We can compare this to the number of hairs on the typical person's head, which is often quoted as being around 100,000. Assuming the braid is holding all of the hair from a person's head, our answer is pretty good.

44. **INTERPRET** The problem asks us how lucky you would have to be to randomly guess a credit card number. This probability is the number of valid credit card numbers divided by the number of possibilities. The pool of 16-digit numbers contains 10^{16} numbers. All we have to do then is estimate how many of these are valid.

DEVELOP The U.S. population is around 300 million. Not every American has a credit card, but some of them have more than one. Let's simply assume that there are 300 million valid credit card numbers out there.

EVALUATE The probability of randomly picking a valid credit card number is

$$P = \frac{N_{\text{valid}}}{N_{\text{total}}} \approx \frac{300\times 10^6}{10^{16}} \sim 10^{-8}$$

ASSESS One out of 100 million is not very likely, so credit card thieves use something more than just luck.

45. **INTERPRET** This problem calls for a rough estimate instead of a precise numerical answer. The quantity of interest here is the thickness of the bubble made from the bubble gum.

DEVELOP For problems that involve rough estimates, various assumptions typically need to be made. Such assumptions must be physically motivated and explained. The volume V of the *bubble* (a thin spherical shell) is $V = 4\pi r^2 d$ where r is the radius and $d \ll r$ is the thickness. The mass m of such a bubble is the volume multiplied by the density, and this mass must equate with the mass (i.e., 8 g) of the wad of bubble gum. Thus, we can equate these two expressions for the mass of the bubble gum and solve for the thickness d of the bubble. Recall that the radius $r = D/2 = 5$ cm, where D is the diameter ($D = 10$ cm).

EVALUATE With these assumptions, the thickness of the bubble is

$$m = V\rho = 4\pi r^2 d\rho$$

$$d = \frac{m}{4\pi r^2 \rho} = \frac{8\text{ g}}{4\pi(5\text{ cm})^2\left(1\text{ g}/\text{cm}^3\right)} = 0.025\text{ cm}$$

ASSESS The thickness of the bubble is very small. But our estimate is reasonable. Four layers of bubble-gum bubble would be about 1-mm thick. Note that this solution assumes that the bubble gum density does not change in going from the wad in the package to the chewed gum in the mouth. Is this assumption reasonable?

46. **INTERPRET** After giving us several astronomical parameters, this problem asks us to calculate the size of the Sun relative to the Moon and the absolute size of the Sun.

DEVELOP Because the Moon just covers the Sun during a solar eclipse, the Moon and the Sun must subtend the same angle when viewed from the Earth. From Fig. 1.2 we know that $\theta = s/r$, where s is the subtended arc and r is the radius. For $r \gg s$, the arc subtended may be approximated by a straight line, and so may be taken as the diameter of the Moon and the Sun for this problem. Thus, we have $\theta = d_S/r_S = d_M/r_M$, where the subscripts S and M refer to the Sun and the Moon, respectively.

EVALUATE Inserting the given distances to the Sun and the Moon, the ratio of their diameters is

$$\frac{d_S}{r_S} = \frac{d_M}{r_M}$$

$$\frac{d_S}{d_M} = \frac{r_S}{r_M} = \frac{1.5\times 10^8\text{ km}}{4\times 10^5\text{ km}} = 375$$

Rounding this result to a single significant figure gives 400 as the final result. Thus, the Sun has about 400 times the diameter of the Moon. If the Moon's radius is 1800 km, then the Sun's radius is

$$d_S = d_M\frac{r_S}{r_M} = (1800\text{ km})(375) = 6.75\times 10^5\text{ km}$$

Rounding this result to a single significant figure gives 7×10^5 km as the final radius of the Sun.

ASSESS Notice that we retained more significant figures than warranted when we entered the result of the intermediate calculation into the final calculation. However, at the end of each calculation, we rounded the result to the correct number of significant figures. The angle subtended by each object is (using data for the Moon)

$$\theta = d_M/r_M = (3600\text{ km})/(4\times 10^5\text{ km}) = (4.5\times 10^{-3}\text{ rad})(180°/\pi\text{ rad}) = 0.5°$$

47. **INTERPRET** This is a problem that calls for a rough estimate, instead of a precise numerical answer. The quantities of interest here are the size of the electronic components on a PC chip, and the number of calculations that can be performed each second.

DEVELOP The area of each component is the area of the chip divided by the number of components. We'll take the square root of that to get the component size $d_{\text{comp}} = \sqrt{A_{\text{comp}}}$. For part (b), to estimate the number of calculations performed per second, we take the inverse of how long it takes to do one calculation. We are told that one calculation requires that an electric impulse traverse 10^4 components each one million times. The time it takes to traverse one component is the distance across one component divided by the velocity, $t_{\text{comp}} = d_{\text{comp}}/v$. We assume the velocity is approximately the speed of light.

EVALUATE (a) The area of each component is

$$A_{\text{comp}} = \frac{A_{\text{chip}}}{N_{\text{comp}}} = \frac{(4\text{ mm})^2}{10^9} = 1.6\times10^{-14}\,\text{m}^2$$

Assuming the component is square

$$d_{\text{comp}} = \sqrt{A_{\text{comp}}} = \sqrt{1.6\times10^{-14}\,\text{m}^2} = 1.3\times10^{-7}\,\text{m} \sim 0.1\ \mu\text{m}$$

(b) The time to do one calculation is the number of components to be traversed, multiplied by the number of traversals, and then multiplied by the time to traverse one component

$$t_{\text{calc}} = \left(10^4\right)\left(10^6\right)t_{\text{comp}} = 10^{10}\frac{d_{\text{comp}}}{v} = 10^{10}\frac{1.3\times10^{-7}\,\text{m}}{3\times10^8\,\text{m/s}} = 4.2\times10^{-6}\,\text{s}$$

This means the chip can do 2×10^5 calculations per second.

ASSESS The number of calculations per second is often referred to as FLOPS (Floating Point Operations Per Second). The performance of the above chip is 200,000 FLOPS. Modern supercomputers use parallel computing to perform at the level of more than a trillion FLOPS (or TFLOPS).

48. **INTERPRET** This problem calls for a rough estimate, instead of a precise numerical answer. We are asked to estimate the number of atoms and cells in the human body.

DEVELOP Human tissue is mostly water, so for a rough estimate we can consider the human body to contain about as many atoms as an equivalent mass of water. One mole of water (H_2O) is 18 g and contains Avogadro's number of molecules ($N_A = 6 \times 10^{23}$). Because H_2O has 3 atoms per molecule, the number n of atoms per kg water is about

$$n = \frac{3\times6\times10^{23}}{18\ \cancel{\text{g}}}\overbrace{\left(\frac{10^3\ \cancel{\text{g}}}{\text{kg}}\right)}^{=1} = 1\times10^{26}\ \text{kg}^{-1}$$

To estimate the number of cells in the body, take the size of a red blood cell (= 10 μm, see Table 1.2) as a typical cell. Assuming again that the human body is mostly water, the volume V_H of a human body can be estimated by dividing its mass m = 60 kg by the density ρ = 1 gm/cm^3 of water, or

$$V_H = \frac{m}{\rho} = \left(\frac{60\ \cancel{\text{kg}}}{1\ \cancel{\text{g}}/\text{cm}^3}\right)\overbrace{\left(\frac{10^3\ \cancel{\text{g}}}{\cancel{\text{kg}}}\right)}^{=1} = 6\times10^4\ \text{cm}^3$$

EVALUATE **(a)** Assuming that the mass of the average human is m = 60 kg, we find that the number N_a of atoms in the human body is approximately

$$N_a = mn = (60\text{ kg})\left(1\times10^{26}\ \text{kg}^{-1}\right) = 6\times10^{27}$$

(b) To estimate the number N_c of cells in the human body we divide the volume V_H by the volume $V_b = (10\ \mu\text{m})^3$ of a red blood cell. This gives

$$N_c = \frac{V_H}{V_b} = \frac{6\times10^4\ \cancel{\text{cm}^3}}{(10\ \cancel{\mu\text{m}})^3}\overbrace{\left(\frac{10^4\ \cancel{\mu\text{m}}}{\cancel{\text{cm}}}\right)^3}^{=1} = 6\times10^{13}$$

ASSESS Conventional wisdom claims that the human body has about 10^{13} cells, so our estimate for part (**b**) seems reasonable.

49. **INTERPRET** This problem involves estimating uncertainty given the number of significant figures.

DEVELOP Because the value 3.6 can be used to represent any number between 3.55 and 3.65, rounding to two significant figures, we see that the uncertainty in the first decimal place is ±0.05 Therefore, the percent uncertainty Δ in a one-decimal-place number *N* is

$$\Delta = 100\left(\frac{\pm 0.05}{N}\right)\%$$

which decreases as *N* increases.

EVALUATE For the numbers given, the percent uncertainty is **(a)** $\Delta = 100(\pm 0.05/1.1)\% \approx \pm 5\%$; **(b)** $\Delta = 100(\pm 0.05/5.0)\% \approx \pm 1\%$; and **(c)** $\Delta = 100(\pm 0.05/9.9)\% \approx \pm 0.5\%$.

ASSESS Our result indicates that, for a one-decimal place number *N*, whereas the uncertainty in the first decimal place is ±0.05 independent of *N*, the percent uncertainty Δ becomes smaller for larger *N*.

50. **INTERPRET** This problem calls for a rough estimate, instead of a precise numerical answer. We are asked to estimate the age of the Atlantic Ocean, given that it has been widening at the rate at which fingernails grow.

DEVELOP For problems that involve rough estimates, various assumptions typically need to be made. Such assumptions must be physically motivated and explained. Assume that you cut about 1 mm of new growth from your fingernails every week which is a rate *s* = 50 mm/y. Take the width *w* of the Atlantic Ocean to be *w* = 5000 km.

EVALUATE To find the age *A* of the Atlantic Ocean, divide its width by the rate at which it is growing. This gives

$$A = \frac{w}{s} = \left(\frac{5000\ \cancel{\text{km}}}{50\ \cancel{\text{mm}}/\text{y}}\right)\overbrace{\left(\frac{10^6\ \cancel{\text{mm}}}{\cancel{\text{km}}}\right)}^{=1} = 1\times 10^8\ \text{y}$$

or 100 My.

ASSESS Geological evidence indicates that the age of the Atlantic Ocean is in fact very close to this value, although actual spreading rates for mid-oceanic ridges vary between 12 and 160 mm per year, and are believed to be steady for periods of only a few million years.

51. **INTERPRET** This problem involves comparing the cost of gasoline in the U.S. and Canada. To do this we need to have both prices in the same currency and for the same volume of gas.

DEVELOP Let's change the Canadian pricing into its equivalent American pricing. Therefore we take the given cost in Canadian dollars per liter, and multiply by 3.785 liters per gallon (from Appendix C) and also by 87¢ per Canadian dollar.

EVALUATE To differentiate the two currencies, we will denote the Canadian dollar with a "hat": $\hat{\$}$. Converting the Canadian cost gives

$$\hat{\$}0.94/\text{L}\left(\frac{3.785\ \text{L}}{1\ \text{gal}}\right)\left(\frac{\$0.87}{\hat{\$}1.00}\right) = \$3.09/\text{gal}$$

This is 12¢ more than the cost in the U.S., so it would be better to buy the gas before crossing the border.

ASSESS It certainly pays to know how to do a unit conversion!

52. **INTERPRET** This problem involves converting units. We are asked to convert a distance given in miles and yards to kilometers.

DEVELOP We will convert the two distances, 26 miles and 385 yards, into meters, and then add the two for our final answer. From Appendix C, 1 mi = 1609 m (or 1 = 1609 km/mi) and 1 yard = 0.9144 m (or 1 = 0.9144 m/yard). Note that we will also need to use 1 = 10^{-3} km/m to convert m to km.

EVALUATE Multiplying the distances by their conversion factors, then summing, we find a total distance *d* of

$$d = \left(26\ \cancel{\text{mi}}\right)\overbrace{\left(\frac{1.609\ \text{km}}{\cancel{\text{mi}}}\right)}^{=1} + \left(385\ \cancel{\text{yard}}\right)\overbrace{\left(\frac{0.9144\ \cancel{\text{m}}}{\cancel{\text{yard}}}\right)}^{=1}\overbrace{\left(\frac{10^{-3}\ \text{km}}{\cancel{\text{m}}}\right)}^{=1} = 41.83\ \text{km} + 0.35\ \text{km} = 42\ \text{km}$$

where we have rounded the final answer to 2 significant figures.

ASSESS A mile is about 1.6 km, so this answer seems reasonable.

53. **INTERPRET** This problem involves converting between different units and retaining the correct number of significant figures.

DEVELOP First convert the values given in mm to m using the conversion1 mm = 10^{-3} m, or 1 = 10^{-3} m/mm, then apply the rules for significant figures given in the chapter: When multiplying or dividing, report your answer with the number of significant figures in the least-precise factor. When adding or subtracting, report your answer to the least-precise decimal place.

EVALUATE **(a)** To the least-precise decimal place, we have

$$1.0\text{ m}+\left(1\ \cancel{\text{mm}}\right)\left(\frac{10^{-3}\text{ m}}{\cancel{\text{mm}}}\right)=1\text{ m}+0.001\text{ m}=1.001\text{ m}=1.0\text{ m}$$

(b) The least-precise factor, 1 m, has one significant figure, so we find

$$\left(1.0\text{ m}\right)\left(1\ \cancel{\text{mm}}\right)\left(\frac{10^{-3}\text{ m}}{\cancel{\text{mm}}}\right)=0.001\text{ m}^2$$

(c) The second decimal place in 1.0 m is the least precise so

$$1.0\text{ m}-\left(999\ \cancel{\text{mm}}\right)\overbrace{\left(\frac{10^{-3}\text{ m}}{\cancel{\text{mm}}}\right)}^{=1}=1.0\text{ m}-0.999\text{ m}=0.001\text{ m}=0.0\text{ m}$$

(d) Again, 1.0 m has two significant figures, so we find

$$\left(\frac{1.0\ \cancel{\text{m}}}{999\ \cancel{\text{mm}}}\right)\left(\frac{10^{3}\ \cancel{\text{mm}}}{\cancel{\text{m}}}\right)=\frac{1.0}{0.999}=1.001001\ldots\ =1.0\ .$$

ASSESS It is important to note that sometimes the answer you get, to the right precision, is unchanged! It might help to think of this in terms of money: If you're a millionaire, and you drop a nickel down a storm drain, you're still a millionaire.

54. **INTERPRET** This problem involves dividing the area of the chip by the area of a single electronic component in order to verify the number of components on the chip.

DEVELOP The areas of the chip and a single component are obtained by squaring the given lengths: 5 mm and 32 nm. Dividing these areas gives an estimate of the number of components.

EVALUATE The number of components is approximately

$$N=\frac{A_{\text{chip}}}{A_{\text{comp}}}=\frac{\left(\ell_{\text{chip}}\right)^2}{\left(\ell_{\text{comp}}\right)^2}=\left(\frac{5\times10^{-3}\text{ m}}{32\times10^{-9}\text{ m}}\right)^2=2\times10^{10}$$

This is 20 billion, so the salesperson apparently was not exaggerating when he/she said there were 10 billion components.

ASSESS Our estimate is likely too high, since there may be areas on the chip that don't contain electronic components. Regardless, 10 billion appears to be the right order of magnitude.

55. **INTERPRET** We have to adjust the cost per bag of coffee to include the shipping costs.

DEVELOP To find the final cost per bag, calculate the total cost for the 6 bags of coffee and then divide by 6.

EVALUATE The total purchase is 6 bags plus shipping

$$6\left(\$8.95\right)+\$6.90=\$60.60$$

Dividing by the number of bags gives \$10.10 per bag.

ASSESS If the same shipping costs applied to one bag of coffee, then the price per bag would be \$15.85. So sometimes it pays to buy in bulk.

56. **INTERPRET** This problem involves converting worldwide energy use into the average individual's power consumption.

DEVELOP The energy used in a year is a quantity with the dimensions of power (energy/time). So all we have to do is convert this to the more common power unit of the watt ($1\text{ W} = 1\text{ J/s}$). To get the average per capita energy consumption, we will divide by the world population, which is currently around 6.8 billion.

EVALUATE The worldwide energy consumption in watts is

$$450\text{ EJ/y} = \frac{\left(450\times 10^{18}\text{ J}\right)}{\left(\pi\times 10^{7}\text{ s}\right)} = 1.4\times 10^{13}\text{ W}$$

We have used the approximation for converting years to seconds. Dividing this by the number of people gives

$$\frac{1.4\times 10^{13}\text{ W}}{6.8\times 10^{9}} \approx 2\text{ kW}$$

ASSESS The United States is said to account for 25% of the world's energy consumption (105 EJ), while accounting for about 5% of the world's population (0.3 billion). The U.S. per capita consumption is 11 kW, which is the highest in the world.

57. **INTERPRET** We can use the given cell size and atom size to estimate the number of atoms that fit in a cell and then compare that to the number of cells in a body.

DEVELOP By dividing the diameter of a cell by the diameter of an atom, we can say about how many atoms span the length of a cell. But since cells and atoms are three dimensional, we will cube this number to get the number of atoms in a cell.

EVALUATE The number of atoms in a cell is just the cube of the ratio of diameters

$$N \approx \left(\frac{d_{\text{cell}}}{d_{\text{atom}}}\right)^3 = \left(\frac{10\ \mu\text{m}}{0.1\text{ nm}}\right)^3 = \left(10^5\right)^3 = 10^{15}$$

We're told the body has about 10^{14} cells, which is technically less than our calculation for the number of atoms in a cell, but these are rough estimates. It's safer to say that the two quantities are about the same.

The answer is (c).

ASSESS Does this make sense? A human being is roughly 2 meters tall. This is roughly 100,000 times bigger than the diameter of a cell. And we find out here that a cell is roughly 100,000 times bigger than an atom. So yes, it's reasonable that the number of cells in the body is about the same as the number of atoms in the cell.

58. **INTERPRET** We're being asked to use the given cell diameter to calculate the volume.

DEVELOP Assuming that a cell is spherical, then its volume can be gotten from the diameter, $V = \frac{4\pi}{3}\left(d/2\right)^3$.

EVALUATE The cell volume is

$$V = \frac{4\pi}{3}\left(\frac{d}{2}\right)^3 = \frac{4\pi}{3}\left(\frac{10\times 10^{-6}\text{ m}}{2}\right)^3 = 5\times 10^{-16}\text{ m}^3$$

The closest answer is (b).

ASSESS If we assume a human being is a cylinder that is 30 centimeters across and 175 centimeters tall, then the volume is roughly 0.1 m^3. Dividing this by the cell volume tells us that there are roughly 10^{14} cells in the body, which agrees with what is written in the text.

59. **INTERPRET** The problem asks us to estimate the mass of a cell. We can use the previous answer for the volume and make some approximation for the cell's density.

DEVELOP Since living things are mostly water, we can assume that the cell has a density roughly equal to that of water ($\rho = 1\text{ g/cm}^3$).

EVALUATE Using the volume from Problem 1.58 and the density of water, the cell mass is

$$m = \rho V = \left(\frac{10^{-3}\text{ kg}}{(10^{-2}\text{ m})^3}\right)\left(5\times 10^{-16}\text{ m}^3\right) = 5\times 10^{-13}\text{ kg}$$

The closest answer is (c).

ASSESS If we multiply this by the number of cells in the body (10^{14}), we get 50 kg, which is roughly what a human weighs.

60. **INTERPRET** The problem asks us to estimate the number of atoms in the body, but we already have done most of the work in the previous problems.

DEVELOP In Problem 1.57, we found the number of atoms in a cell to be 10^{15}. Multiply that by the given number of cells in the body (10^{14}) to obtain the number of atoms in the body.

EVALUATE Multiplying the two estimates gives 10^{29} atoms in the body.

The closest answer is (c).

ASSESS This estimate is probably a bit too high. The maximum number atoms we could have is if all the atoms in the body were hydrogen. The hydrogen atom has a mass roughly equal to that of a single proton, 1.67×10^{-27} kg (from Appendix D). If we divide the typical human body mass by this hydrogen mass, we get 4×10^{28}. Therefore, 10^{29} is too high.