USING NEWTON'S LAWS

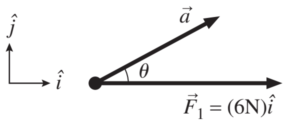
5

EXERCISES

Section 5.1 Using Newton's Second Law

12. INTERPRET This problem involves applying Newton's second law in a two-dimensional situation. The object of interest is the 1.5-kg mass, and we are asked to find the second of two forces acting on the mass that would give it the given acceleration.

DEVELOP Draw a free-body diagram of the situation (see figure below). Include the acceleration of the mass, and write the acceleration in component form. Now apply Newton's second law (for constant mass) in vector form to find the second force.



EVALUATE The acceleration in component form is $\vec{a} = a\cos(\theta)\hat{i} + a\sin(\theta)\hat{j}$, with a = 7.3 m/s^2 and θ = 30°. Thus, Newton's law gives

$$\vec{F}_{\text{net}} = m\vec{a}$$
$$\vec{F}_2 + \vec{F}_1 = ma\left[\cos(\theta)\hat{i} + \sin(\theta)\hat{j}\right]$$
$$\vec{F}_2 = (1.5\text{ kg})(7.3\text{ m/s}^2)\left[\cos(30^\circ)\hat{i} + \sin(30^\circ)\hat{j}\right] - (6.8\text{ N})\hat{i} = (2.7\text{ N})\hat{i} + (5.5\text{ N})\hat{j}$$

ASSESS This force has magnitude $F_2 = \sqrt{(2.68\text{ N})^2 + (5.48\text{ N})^2} = 6.1\text{ N}$ and points in the direction $\theta = \text{atan}(5.48\text{ N}/2.68\text{ N}) = 64^\circ$ counterclockwise from the $\hat{i}$ direction.

13. INTERPRET This problem requires an application on Newton's second law in two dimensions. Two forces are exerted on the object of interest (i.e., the 3.1-kg mass) and produce an acceleration. With the mass of the object and one force given, we are asked to find the other force.

DEVELOP Newton's second law for this mass says $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = m\vec{a}$, where we assume no other significant forces are acting. Thus, the second force is given by $\vec{F}_2 = m\vec{a} - \vec{F}_1$.

EVALUATE Inserting the expressions given in the problem statement for $\vec{F}_1$ and $\vec{a}$, we obtain

$$\vec{F}_2 = m\vec{a} - \vec{F}_1 = (3.1\text{ kg})\left[(0.91\text{ m/s}^2)\hat{i} - (0.27\text{ m/s}^2)\hat{j}\right] - \left[(-1.2\text{ N})\hat{i} - (2.5\text{ N})\hat{j}\right] = (4.0\text{ N})\hat{i} + (1.7\text{ N})\hat{j}$$

ASSESS This force has magnitude $F_2 = \sqrt{(4.02\text{ N})^2 + (1.66\text{ N})^2} = 4.3\text{ N}$ and points in the direction $\theta = \text{atan}(1.66\text{ N}/4.02\text{ N}) = 22^\circ$ counterclockwise from the x axis.

14. INTERPRET In this problem, we are asked to find the tilt angle of an air table such that the acceleration of an object sliding on the surface of the table is the same as the gravitational acceleration near the surface of the Moon.

DEVELOP Example 5.1 shows that the acceleration down an incline is $a = g\sin(\theta)$. By setting the acceleration equal to the acceleration due to gravity on the surface of the Moon ($a = g_{\text{M}}$ = 1.6 m/s^2), we can solve for the tilt angle θ.

EVALUATE The angle of tilt should be

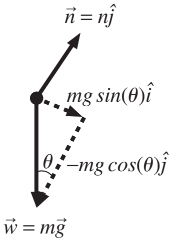
$$\theta = \text{asin}\left(\frac{a}{g}\right) = \text{asin}\left(\frac{1.6\ \text{m/s}^2}{9.8\ \text{m/s}^2}\right) = 9.4°$$

above the horizontal.

ASSESS Notice that the tilt angle does not depend on the mass of the object.

15. **INTERPRET** This problem involves Newton's second law applied to a two-dimensional situation to find the acceleration of the skier, then kinematics to find the time it takes him to reach the bottom of the slope.

DEVELOP Draw a free-body diagram of the situation (see figure below) and apply Newton's second law to find the acceleration. The angle is $\theta = 24°$ and weight of the skier is $w = mg$. Given the acceleration (which is constant), we can use Equation 2.10, $x = x_0 + v_0 t + at^2/2$, with $x - x_0 = 1.3$ km = 1300 m, to find how long it takes him to reach the bottom.



EVALUATE Applying Newton's second law to the skier gives two equations (one for the x direction and one for the y direction):

$$\vec{F}_{\text{net}} = m\vec{a}$$

$$n\hat{j} + mg\left[\sin(\theta)\hat{i} - \cos(\theta)\hat{j}\right] = m\vec{a} \quad \Rightarrow \quad \begin{cases} n - mg\cos(\theta) = 0 \\ g\sin(\theta) = a \end{cases}$$

Solving the second scalar equation for acceleration, we find $a = (9.8\ \text{m/s}^2)\sin(24°) = 3.986\ \text{m/s}^2$, and inserting this result into Equation 2.10 gives

$$x = x_0 + \overbrace{v_0}^{=0} t + at^2/2$$

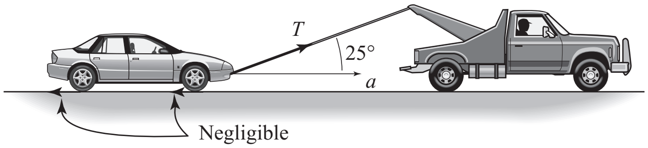
$$t = \pm\sqrt{\frac{2(x - x_0)}{a}} = \sqrt{\frac{2(1300\ \text{m})}{3.986\ \text{m/s}^2}} = 26\ \text{s}$$

where we have chosen the positive square root and we have used $v_0 = 0$ because the skier starts at rest.

ASSESS If the slope becomes vertical, $\theta \to 90°$ and $a = g$, which is what we expect because the skier would be free-falling.

16. **INTERPRET** In this problem, the physical quantity of interest is the tension force in the cable. To compute the tension, we apply Newton's second law to the car + cable system.

DEVELOP Make a diagram of the situation showing the car and tow rope, and label the tension, acceleration, and angle (see figure below). Write the tension in component form: $\vec{T} = T\cos(\theta)\hat{i} + T\sin(\theta)\hat{j}$, where $\hat{i}$ is in the direction of the truck's acceleration. From this, we see that the net horizontal force acting on the car is $T\sin(\theta)$. By Newton's second law, the net horizontal force must be equal to the horizontal acceleration times the mass of the car. If the cable does not stretch, we know the acceleration of the car must be the same as that of the tow truck, which allows us to solve Newton's second law for the tension T.



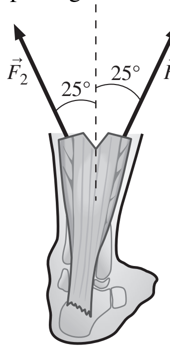
EVALUATE Applying Newton's second law to the motion in the horizontal direction, we obtain $T\cos\theta = ma$ or

$$T = \frac{ma}{\cos(\theta)} = \frac{(1400\text{ kg})(0.57\text{ m/s}^2)}{\cos(25°)} = 880\text{ N}$$

ASSESS This force is oriented at 25° above horizontal. In component form, the tension is $\vec{T} = (880\text{ N})\left[\cos(25°)\hat{i} + \sin(25°)\hat{j}\right] = (800\text{ N})\hat{i} + (370\text{ N})\hat{j}$. To see that our expression for T makes sense, let's consider the limiting case where the cable is completely horizontal with $\theta = 0$. The tension in this case would simply be $T = ma$, which is the force required to accelerate the car.

17. INTERPRET This is a static problem in which we are looking for the force exerted on the tendon by the two muscles.

DEVELOP In this case, there are two forces pulling on the tendon as shown below in the figure:



We are told that the horizontal pulls are opposite each other (meaning that the two forces are in the *x*-*y* plane), and we assume that the net horizontal force is zero: $F_1\sin 25° - F_2\sin 25° = 0$, in which case $F_1 = F_2$. The net vertical force pulls up on the tendon with a force equivalent to ten times the gymnast's weight:

$$F_1\sin 25° + F_2\sin 25° = 10mg$$

EVALUATE Solving for the force in each muscle gives

$$F_1 = F_2 = \frac{10(55\text{ kg})(9.8\text{ m/s}^2)}{2\sin 25°} = 6.4\text{ kN}$$

ASSESS The Achilles tendon is the thickest and strongest tendon in the body. In simply walking, it has to withstand strains of as much as 4 times the body weight.

Section 5.2 Multiple Objects

18. INTERPRET This problem is similar to Example 5.4. We assume that your baby sister and the turkey are tied together through the tablecloth, so their accelerations should be equal.

DEVELOP As the baby sister is pulling with all her weight, we'll assume that her feet are off the ground and that she is hanging from the tablecloth. In which case, the relevant forces acting on her are the tension (upwards) from the tablecloth and her own weight (downwards), which result in a downward acceleration:

$$\vec{T}_{\text{sis}} - \vec{w}_{\text{sis}} = m_{\text{sis}}\vec{a}_{\text{sis}} \quad \rightarrow \quad T = m_{\text{sis}}\left(g - a_y\right)$$

We'll assume the tablecloth is massless, so it applies the same tension to the turkey (albeit in a different direction). Although the turkey experiences vertical forces (its weight and normal force), the only relevant force in this case is the tension from the tablecloth in the horizontal direction (we assume the table is frictionless):

$$\vec{T}_{\text{tky}} = m_{\text{tky}}\vec{a}_{\text{tky}} \quad \rightarrow \quad T = m_{\text{tky}}a_x$$

As we mentioned above, the horizontal acceleration of the turkey is equal to the vertical acceleration of the sister: $a_x = a_y = a$.

EVALUATE (a) Equating the tension on the two objects, we can solve for the acceleration:

$$a = g\frac{m_{\text{sis}}}{m_{\text{sis}} + m_{\text{tky}}} = \left(9.8 \text{ m/s}^2\right)\frac{12 \text{ kg}}{12 \text{ kg} + 6.8 \text{ kg}} = 6.26 \text{ m/s}^2 \approx 6.3 \text{ m/s}^2$$

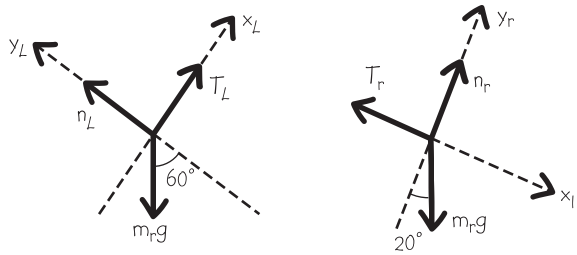
(b) Assuming the turkey started at rest, it will reach the end of the table in a time of

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(0.60 \text{ m})}{\left(6.26 \text{ m/s}^2\right)}} = 0.43 \text{ s}$$

ASSESS Less than half a second is not a lot of time to save the turkey, but it's likely that there is some friction between the tablecloth and the table, so you probably have more time than this.

19. **INTERPRET** In this problem, two masses that rest on slopes at unequal angles are connected by a rope that passes over a pulley. We are asked to find the ratio of the two masses if they both remain at rest.

DEVELOP Let the masses on the right and on the left be denoted as m_R and m_L, respectively. The free-body diagrams for m_R and m_L are shown in the sketch below, where the forces acing on the masses are the normal force from the slope, the weight, and the tension force. Note also that we use different coordinate systems for the left and right mass. If the masses don't slide, the net force on each must be zero (by Newton's second law), from which we can find the ratio of the masses.



EVALUATE Applying Newton's second law to the force component that is parallel to each surface gives

$$T_L - m_L g \sin(\theta_L) = 0$$
$$m_R g \sin(\theta_R) - T_R = 0$$

Now, if the masses of the string and pulley are negligible and there is no friction, then the tension must be the same throughout the entire string, so $T_L = T_R$. By adding the two equations, we find

$$m_R g \sin(\theta_R) - m_L g \sin(\theta_L) = 0$$
$$\frac{m_R}{m_L} = \frac{\sin(\theta_L)}{\sin(\theta_R)} = \frac{\sin(60^\circ)}{\sin(20^\circ)} = 2.5$$

ASSESS We conclude that when $\theta_L > \theta_R$, $m_R > m_L$ for the system to remain at rest. This makes sense because when the angle θ_L on the left-hand side increases, there is a greater tendency for m_L to slide down. To counter this, the mass m_R on the right-hand-side must go up. We may also consider the extreme case where $\theta_R = 0$. This situation corresponds to having a mass m_R on a frictionless horizontal surface, with m_L hanging over the edge and connected to m_R by the string. For this system to remain stationary, m_R must be infinitely massive compared to m_L.

20. **INTERPRET** This problem is similar to Problem 5.19. We are again dealing with two masses that are resting on slopes at unequal inclines, but this time we are given one of the masses and we must calculate the other mass needed so that the masses accelerate at the given rate.

DEVELOP Refer to the sketch for Problem 5.19. Let the masses on the right and on the left again be denoted as m_R and m_L, respectively. Apply Newton's second law to each mass to find the right-hand mass required so that each mass accelerates at the given rate.

EVALUATE (**a**) Applying Newton's second law in the *x* directions gives $T_L - m_L g \sin(60°) = m_L a_L$ and $m_R g \sin(20°) - T_R = m_R a_R$. If the string is massless, the tensions are the same, so $T_R = T_L$. If the string doesn't stretch, then the acceleration of each mass is the same, so $a_L = a_R \equiv a$. Thus, we can sum the two equations given by Newton's second law to find

$$m_R g\sin(\theta_R) - m_L g\sin(\theta_L) = (m_L + m_R)a$$
$$m_R = m_L \frac{g\sin(\theta_L)+a}{g\sin(\theta_R)-a}$$

For the coordinate system we are using, a downward acceleration of the right-hand mass is positive, so $a = 0.64$ m/s^2, which gives

$$m_R = (2.1\text{ kg})\frac{(9.8\text{ m/s}^2)\sin(60^\circ)+0.64\text{ m/s}^2}{(9.8\text{ m/s}^2)\sin(20^\circ)-0.64\text{ m/s}^2} = 7.1\text{ kg}$$

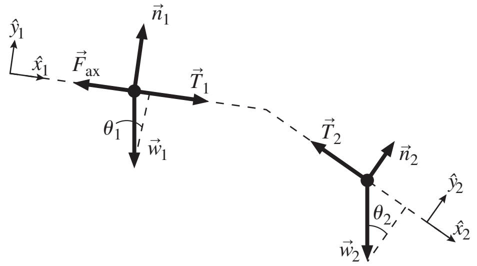
(**b**) If the right-hand mass accelerates up the slope, $a = -0.76$ m/s^2. This gives

$$m_R = (2.1\text{ kg})\frac{(9.8\text{ m/s}^2)\sin(60^\circ)-0.76\text{ m/s}^2}{(9.8\text{ m/s}^2)\sin(20^\circ)+0.76\text{ m/s}^2} = 3.9\text{ kg}$$

ASSESS When the right-hand mass is smaller, the masses accelerate toward the left (and vice versa), which is what we would expect.

21. **INTERPRET** This is a two-dimensional problem that involves applying Newton's second law to two climbers, tied together by a rope and sliding down an icy mountainside. The physical quantities of interest are their acceleration and the force required to bring them to a complete stop.

DEVELOP We choose two coordinate systems where the x axes are parallel to the slopes and y axes are perpendicular to the slopes (see figure below). Assume that the icy surface is frictionless and that the climbers move together as a unit with the same magnitude of down-slope acceleration a. If the rope is not stretching, the tension forces are equal in magnitude, so $T_1 = T_2 \equiv T$. To find the acceleration of the climbers, apply Newton's second law in the direction of the slope. To find the force F_{ax} exerted by the ax, again apply Newton's second law, but this time include F_{ax} and set the acceleration to zero; $a = 0$.



EVALUATE (**b**) For this part, we neglect the force due to the ax. Because we are now working in one-dimension (the x dimension), we forego vector notation, and insert the sign (±) according to the direction of the force. Of course, at the end we must interpret the sign of the resulting force as indicating its direction (positive or negative x direction). The magnitude of the net force in the $\hat{x}_1$ and $\hat{x}_2$ directions (downward positive) is thus

$$F_{net} = m_1 g\sin(\theta_1) + \overbrace{T_1 - T_2}^{=0} + m_2 g\sin(\theta_2)$$
$$= (75\text{ kg})(9.8\text{ m/s}^2)\sin(12°) + (63\text{ kg})(9.8\text{ m/s}^2)\sin(38°) = 533\text{ N}$$

Thus, the magnitude of the acceleration of the pair is

$$a = \frac{F_{net}}{m_1+m_2} = \frac{533\text{ N}}{75\text{ kg}+63\text{ kg}} = 3.9\text{ m/s}^2$$

so the pair accelerate down the slope at 3.9 m/s^2.

(b) After they have stopped, we include the force of the ax. Thus, the magnitude of the force due to the ax is

$$F_{net} = -F_{ax} + m_1 g\sin(\theta_1) + \overbrace{T_1 - T_2}^{=0} + m_2 g\sin(\theta_2) = m\overbrace{a}^{=0} = 0$$
$$F_{ax} = m_1 g\sin(\theta_1) + m_2 g\sin(\theta_2) = 530\text{ N}$$

so two significant figures. That is, the force exerted by the ax must be 530 N up the slope.

ASSESS If the two climbers were not roped together, then their acceleration would have been

$$a_1 = \frac{F_1}{m_1} = g\sin(\theta_1) = (9.8\text{ m/s}^2)\sin(12°) = 2.04\text{ m/s}^2$$

$$a_2 = \frac{F_2}{m_2} = g\sin(\theta_2) = (9.8\text{ m/s}^2)\sin(38°) = 6.03\text{ m/s}^2$$

The acceleration of the pair is the mass-weighted average of the individual accelerations:

$$a = \frac{F_{\text{net}}}{m_1+m_2} = \frac{m_1 g\sin(\theta_1) + m_2 g\sin(\theta_2)}{m_1+m_2} = \left(\frac{m_1}{m_1+m_2}\right) g\sin(\theta_1) + \left(\frac{m_2}{m_1+m_2}\right) g\sin(\theta_2)$$

$$= \left(\frac{m_1}{m_1+m_2}\right) a_1 + \left(\frac{m_2}{m_1+m_2}\right) a_2 = 3.9\text{ m/s}^2$$

Section 5.3 Circular Motion

22. **INTERPRET** This problem involves circular motion and the force required to centripetally accelerate the Moon towards the Earth.

DEVELOP We can calculate the force required to centripetally acceleration of the Moon from the data in Appendix E and Equation 5.1.

EVALUATE From Equation 5.1 the force required is $F_c = ma_c = mv^2/r$. The orbital speed of the Moon is given in Appendix E as v = 1.0 km/s = 1000 m/s, its orbital radius is $r = 0.385 \times 10^6$ km $= 3.85 \times 10^8$ m, and its mass is $m = 7.35 \times 10^{22}$ kg. Thus the cable tension required is

$$T = \frac{mv^2}{r} = \frac{(7.35\times10^{22}\text{ kg})(1000\text{ m/s})^2}{3.85\times10^8\text{ m}} = 1.91\times10^{20}\text{ N}$$

ASSESS We could also calculate the velocity by dividing the circumference of the Moon's orbit by its period. This gives $v = 2\pi r/T = 2\pi(3.85\times10^8\text{ m})/(27.3\text{ d}) = 1.03\text{ m/s}$ Inserting this speed into Equation 5.1 gives a tension of 2.01×10^{20} N. Which answer (2.01 or 1.91×10^{20} N) is likely to more accurate (i.e., closer to the real answer)? For the latter calculation, we made the assumption that the Moon's orbit is circular in calculating its orbital speed, which is an approximation. In reality the orbit is slightly elliptical. The average speed reported in Appendix E is therefore a more reliable datum, so that the calculation using the value v = 1000 m/s is probably more accurate.

23. **INTERPRET** In this problem we are asked to show that the force required to keep a mass m in a circular path of radius r with period T is $4\pi^2 mr/T^2$.

DEVELOP To derive the formula, we first note that for an object of mass m in uniform circular motion, the magnitude of the net force is given by Equation 5.1: $F = ma = mv^2/r$. Next, we make use of the fact that the period of the motion (i.e., time for one revolution) is the circumference $C = 2\pi r$ divided by the speed v. Thus, $T = 2\pi r/v$.

EVALUATE Combining the two expressions, the force can be rewritten as

$$F = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2}$$

ASSESS Our result indicates that for a fixed radius r, the centripetal force is inversely proportional to T^2. For example, if T is very large (i.e., it takes a very long time for the mass to complete one revolution), then the speed v is very small and the centripetal force F is also very small.

24. **INTERPRET** For the rock to whirl around in a circle, the string has to supply the centripetal force through its tension. To keep this tension below the limit, the string makes an angle with the horizontal, as shown in Figure 5.11 in the text.

DEVELOP The situation is the same as described in Example 5.5. The net vertical force is zero, so the weight is balanced by the vertical component of the tension $T\sin\theta = mg$. As for the horizontal component of the tension, it is providing the needed centripetal force, $T\cos\theta = mv^2/r$. The radius of the rock's trajectory is $r = L\cos\theta$.

EVALUATE (a) We are first asked to find the minimum angle that keeps the tension under the string's breaking limit:

$$\theta_{\min} = \sin^{-1}\left(\frac{mg}{T_{\max}}\right) = \sin^{-1}\left(\frac{(0.940\text{ kg})(9.8\text{ m/s}^2)}{(120\text{ N})}\right) = 4.40^\circ$$

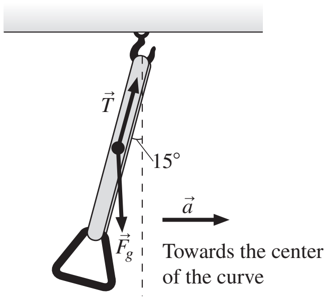
(b) At this angle, the speed of the rock is

$$v = \sqrt{\frac{T_{\max}L}{m}}\cos\theta_{\min} = \sqrt{\frac{(120\text{ N})(1.30\text{ m})}{(0.940\text{ kg})}}\cos 4.40^\circ = 12.8\text{ m/s}$$

ASSESS The stronger the string is, the closer to the horizontal it can whirl the rock around (i.e. $\theta_{\min} \to 0$ as $T_{\max}$ increases). But to maintain such a trajectory, the velocity has to increase, so that the centripetal acceleration is sufficient.

25. **INTERPRET** You're asked to find the velocity of the subway while rounding the curve. We'll assume uniform circular motion, in which case all the objects in the subway are accelerating according to $a = v^2/r$.

DEVELOP The only information you have is that a strap is dangled at 15° to the vertical during the turn. You can assume that the strap hung straight down before the turn and that its displacement was outward with respect to the center of the curve. We can think of the strap as a mass hanging from an attachment (perhaps a chain or belt) that provides a tension, as shown in the figure below.



As we said above, the strap and everything else in the subway experience an acceleration, which obeys Newton's 2nd law: $\vec{F}_{\text{net}} = \vec{F}_g + \vec{T} = m\vec{a}$. We'll choose our coordinate system such that the acceleration is in the +x-direction. Therefore, in component form, the second law is

$$T_x = T\sin 15^\circ = ma = \frac{mv^2}{r}$$
$$T_y - F_g = T\cos 15^\circ - mg = 0$$

We'll evaluate these equations to find the velocity of the subway.

EVALUATE We see from above that $T = mg/\cos 15^\circ$, so $a = g\tan 15^\circ$ and

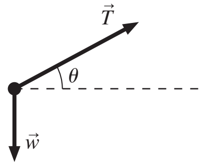
$$v = \sqrt{ar} = \sqrt{(9.8\text{ m/s}^2)(\tan 15^\circ)(132\text{ m})} = 18.6\text{ m/s} = 67.0\text{ km/h}$$

The train did exceed the 45 km/h speed limit on this curve by 22 km/h, thus provoking the derailment.

ASSESS As part of the derivation, we found that the acceleration obeys: $a = g\tan\theta$. Does that make sense? If the strap were hanging straight down $\theta = 0$, the acceleration would be zero, as we would expect for a train that is moving straight at a constant speed.

26. **INTERPRET** This problem involves circular motion and Newton's second law. We are asked to find the speed at which a tetherball circles given the angle its cord makes with the horizontal.

DEVELOP Draw a free-body diagram of the ball (see figure below). Because the vertical component of the ball's acceleration is zero, we know from Newton's second law that the vertical component of the tension force pulling the ball up must cancel the gravitation force pulling the ball down, so $w = T\sin(\theta)$. Applying Newton's second law in the horizontal direction, and using Equation 5.1 for the acceleration of the ball, gives $F_{net} = T\cos(\theta) = mv^2/r$. Note that the radius involved is the horizontal radius, so $r = (1.7\text{ m})\cos(\theta)$.



EVALUATE Combining these two formulas, we can solve for the speed v. This gives

$$v = \pm\sqrt{\frac{Tr\cos(\theta)}{m}} = \pm\sqrt{\frac{mgr\cos(\theta)}{m\sin(\theta)}} = \pm\sqrt{\frac{gr}{\tan(\theta)}} = \pm\sqrt{\frac{(9.8\text{ m/s}^2)(1.7\text{ m})\cos(15^\circ)}{\tan(15^\circ)}} = \pm 7.7\text{ m/s}$$

ASSESS This expression is the same as found in Example 5.5, step 6. The ± sign indicates that the result is the same whether the ball travels clockwise or counterclockwise.

27. **INTERPRET** This problem involves circular motion and Newton's second law. The object of interest is the plane. By analyzing the force acting on the plane while it travels a circular path, its speed can be determined.
DEVELOP We follow the strategy outlined in Example 5.6. There are two forces acting on the plane: the gravitational force $\vec{w}$ and the normal force $\vec{n}$. Applying Newton's second law gives $\vec{F}_{net} = \vec{w} + \vec{n} = m\vec{a}$. Breaking this vector equation up into its components gives

$$x:\quad n\sin(\theta) = \frac{mv^2}{r}$$
$$y:\quad n\cos(\theta) = mg$$

EVALUATE Dividing the two equations allows us to eliminate n and obtain

$$\frac{n\sin(\theta)}{n\cos(\theta)} = \frac{mv^2/r}{mg} \quad\Rightarrow\quad \tan(\theta) = \frac{v^2}{rg}$$

With $\theta = 28^\circ$ and $r = 3.6$ km = 3600 m, the speed of the plane is

$$v = \pm\sqrt{gr\tan(\theta)} = \pm\sqrt{(9.8\text{ m/s}^2)(3600\text{ m})\tan(28^\circ)} = \pm 140\text{ m/s} = \pm 490\text{ km/h}$$

ASSESS The result shows that the speed of the plane is proportional to $\sqrt{\tan\theta}$. If we want to increase the speed v while the radius r is kept fixed, then the banking angle would also need to be increased. This is due to the fact that the horizontal component of the normal force $n\sin(\theta)$ is what keeps the plane in circular motion. The ± sign indicates that the result is the same whether the plane travels clockwise or counterclockwise.

Section 5.4 Friction

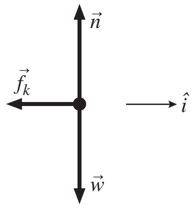
28. **INTERPRET** This problem involves calculating the force due to friction that is exerted upon an object of a given mass with a given coefficient of kinetic friction. It also involves Newton's second law to find the normal force.
DEVELOP If the floor is level then, by Newton's second law, the normal force must exactly cancel the weight of the cabinet. Thus $n = mg$. Use this result in Equation 5.3 for kinetic friction to solve this problem, with $m = 73$ kg and $\mu_k = 0.81$.
EVALUATE From Equation 5.3, the force exerted on the cabinet due to friction is
$f_k = \mu_k mg = (0.81)(73\text{ kg})(9.8\text{ m/s}^2) = 580\text{ N}$. The direction of the friction force is opposite to the motion of the cabinet.
ASSESS Note that the coefficient of friction is dimensionless, so the units work out to be N.

29. **INTERPRET** This problem involves Newton's second law, friction, and kinematics. The object of interest is the hockey puck. Three forces are involved: gravity, the normal force, and friction, and friction is what causes the acceleration of the puck (in the direction opposing the motion of the puck). The physical quantity to be computed is the coefficient of kinetic friction.

DEVELOP Draw a free-body diagram of the situation (see figure below). We define the positive-x direction to be the direction of the puck's initial velocity $v_0 = 14$ m/s. Given that it travels a distance of $x - x_0 = 56$ m, we can find the distance traveled by using the kinematic Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$. The result is

$$a = -\frac{v_0^2}{2(x - x_0)}$$

where we have used the fact that the final velocity is $v = 0$. Notice that the acceleration is negative, meaning that the puck decelerates. By applying Newton's second law in the vertical direction, we know that the normal force must have the same magnitude as the weight, because the puck does not accelerate vertically. Thus, $n = w$. The force of friction is the only horizontal force acting on the puck, so Newton's second law and Equation 5.3 tell us that $F_{\text{net}} = -f_k = -\mu_k mg = ma$, where we have inserted a minus sign because the friction force acts to oppose the motion, which we take to be in the positive-x direction.



EVALUATE Inserting the known quantities into the Newton's second law gives

$$-\mu_k mg = ma$$
$$\mu_k = -\frac{a}{g} = -\frac{-v_0^2}{2g(x - x_0)} = \frac{(14 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)(56 \text{ m})} = 0.18$$

ASSESS The result $a = -\mu_k g$ shows that increasing the coefficient of friction would result in a greater acceleration. This makes sense because friction is what causes the acceleration.

30. **INTERPRET** This problem involves Newton's second law, the force due to kinetic friction, and kinematics. The object of interest is the skier, and we are asked to find how much longer it would take him to descend a slope with a non-zero coefficient of kinetic friction as compared to if there were no friction.

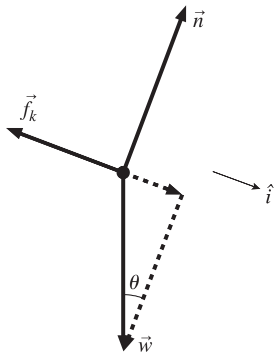
DEVELOP Start with a free-body diagram, and choose a coordinate system in which the positive-x direction is down the slope (see figure below). The forces acting on the skier are the force of gravity, $w = mg$, the normal force n exerted by the slope, and the force f_k due to kinetic friction. To find the skier's acceleration, apply Newton's second law in the $\hat{i}$ direction This gives

$$F_{\text{net}} = ma$$
$$-f_k + w\sin(\theta) = ma$$

where $w = mg$ and we have made $f_k < 0$ because it always acts to oppose the motion, so in this case it acts in the negative-x direction. The force due to kinetic friction may be found from Equation 5.3, $f_k = \mu_k n = \mu_k mg\cos(\theta)$, where we have used Newton's second law in the $\hat{j}$ direction in the final equality:

$$F_{\text{net}} = n - mg\cos(\theta) = ma = 0 \quad \Rightarrow \quad n = mg\cos(\theta)$$

We can now calculate the acceleration a, from which we can find the time to descend the slope using kinematic Equation 2.10 for constant acceleration, $x = x_0 + v_0 t + at^2/2$, with $x - x_0 = 100$ m and $v_0 = 0$ m/s.



EVALUATE If there is no kinetic friction (μ_k = 0 so fk = 0), then, from the first equation above, the acceleration is $a_1 = g\sin(\theta)$. From Equation 2.10, the time to descend the slope is

$$x - x_0 = \overbrace{v_0}^{0} t_1 + \frac{a_1 t_1^2}{2}$$
$$t_1 = \sqrt{\frac{2(x-x_0)}{a_1}} = \sqrt{\frac{2(x-x_0)}{g\sin(\theta)}} = \sqrt{\frac{2(100\text{ m})}{(9.8\text{ m/s}^2)\sin(28^\circ)}} = 6.59\text{ s}$$

where we have taken the positive square root. If μ_k = 0.17, then the acceleration is $a = g\sin(\theta) - g\mu_k\cos(\theta)$, so the time to descend the slope is

$$t_2 = \sqrt{\frac{2(x-x_0)}{a_2}} = \sqrt{\frac{2(x-x_0)}{g\sin(\theta) - g\mu_k\cos(\theta)}} = \sqrt{\frac{2(100\text{ m})}{(9.8\text{ m/s}^2)\left[\sin(28^\circ) - 0.17\cos(28^\circ)\right]}} = 7.99\text{ s}$$

The difference in the time to descend the slopes is $t_2 - t_1$ = 7.99 s – 6.59 s = 1.4 s.

ASSESS Notice that the units for the formulas giving the time are seconds. Considering that the fastest runners can cover 100 m in slightly less than 10 s, we see that our skier travels considerably faster than an extremely fast runner.

31. **INTERPRET** In this problem, the car is moving in uniform circular motion thanks to the friction provided by the road.

DEVELOP To maintain uniform circular motion through the unbanked turn, the car must be able to accelerate at $a = v^2/r$. This requires a force of friction: $f_s = ma = mv^2/r$, directed inward along the curve's radius. As explained in Example 5.9, this friction is static because the car's motion is perpendicular to this force. Using Equation 5.2: $f_s \le \mu_s n$, we will find the minimum value for the coefficient of friction.

EVALUATE Because the curve is unbanked, the normal is just equal to the weight of the car (see Figure 5.24). Solving for the coefficient of friction

$$\mu_s \ge \frac{v^2}{rg} = \frac{(90\text{ km/h})^2}{(120\text{ m})(9.8\text{ m/s}^2)}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)^2 = 0.53$$

ASSESS This seems reasonable for the minimum friction coefficient. Rubber tires on a dry concrete road will typically have $\mu_s \approx 1$, but notice that if the concrete is wet, the coefficient drops to about 0.4.

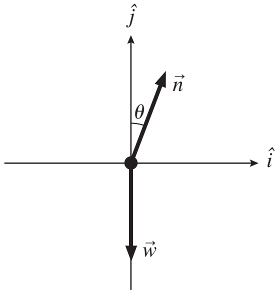
PROBLEMS

32. **INTERPRET** We wish to find the acceleration of a skier on a frictionless slope, which we can do using Newton's second law (for constant mass). This problem was done for us in Example 5.1, but this time we do it with a coordinate system that has horizontal and vertical axes.

DEVELOP Start with a free-body diagram, as shown in the figure below. To find the acceleration, apply Newton's second law, which gives

$$\vec{F}_{\text{net}} = m\vec{a}$$
$$m\vec{a} = n\sin(\theta)\hat{i} + \left[n\cos(\theta) - w\right]\hat{j}$$

We know that the skier does not accelerate in the direction of the normal force, so the forces in this direction must cancel. This gives $n = w\cos(\theta)$, with $w = mg$. This allows us to find the acceleration and the normal force, which is the force the snow exerts on the skier.



EVALUATE (**a**) Inserting the expression for the normal force into Newton's second law gives an acceleration of

$$\begin{aligned}\vec{a} &= g\sin(\theta)\cos(\theta)\hat{i} + \left[g\cos^2(\theta) - g\right]\hat{j} \\ &= \left(9.8\ \text{m/s}^2\right)\left\{\sin\left(32^\circ\right)\cos\left(32^\circ\right)\hat{i} + \left[\cos^2\left(32^\circ\right) - 1\right]\hat{j}\right\} = \left(4.40\ \text{m/s}^2\right)\hat{i} - \left(2.75\ \text{m/s}^2\right)\hat{j}\end{aligned}$$

The magnitude of this acceleration is $a = \sqrt{\left(4.40\ \text{m/s}^2\right)^2 + \left(2.75\ \text{m/s}^2\right)^2} = 5.2\ \text{m/s}^2$ and the direction is $\theta =$ atan(2.75/4.40) = −32°, which is the same result as found in Example 5.1.

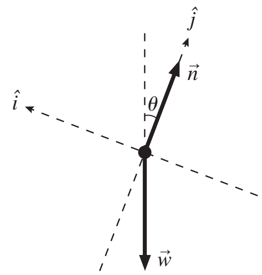
(b) The force exerted by the snow on the skier is

$$n = mg\cos(\theta) = (65\ \text{kg})\left(9.8\ \text{m/s}^2\right)\cos\left(32^\circ\right) = 540\ \text{N}$$

ASSESS The answers we get are independent of the coordinate system used. However, choosing the best coordinate system can make the problem easier to understand and to solve.

33. **INTERPRET** This problem involves a block with an initial velocity sliding up a frictionless ramp. The quantity of interest is the distance it travels before coming to a complete stop. We will apply Newton's second law and kinematics to solve this problem.

DEVELOP Draw a free-body diagram of the situation (see figure below). Applying Newton's second law in the $\hat{i}$ direction gives $-w\sin(\theta) = ma$, or $a = -g\sin(\theta)$. The stopping distance can be calculated by solving the kinematic Equation 2.11: $v^2 = v_0^2 + 2a\Delta x$.



EVALUATE Inserting the acceleration into Equation 2.11, the distance traveled by the block is

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{v^2 - v_0^2}{2\left[-g\sin(\theta)\right]} = \frac{(0\ \text{m/s})^2 - (2.2\ \text{m/s})^2}{-2\left(9.8\ \text{m/s}^2\right)\sin(35°)} = 0.43\ \text{m}$$

ASSESS The result shows that the distance traveled is inversely proportional to sin(θ). To see that this makes sense, consider the limit where $\theta \to 0$. This situation would correspond to a frictionless horizontal surface. In this case, we expect the block to travel indefinitely, in agreement with our expression for Δx.

34. **INTERPRET** We are asked to find the sum of the forces provided by two motor proteins.

DEVELOP The net force on the spindle pole is $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2$. We're only asked to find the magnitude of this sum, so let's choose a coordinate system that makes our life easy. If the $+x$ axis splits the middle between the two forces, then $\theta = \pm\frac{1}{2}65^\circ = \pm 32.5^\circ$ and the y-components of the forces ($F_y = F\sin\theta$) cancel each other out. Therefore, the net force points completely in the x-direction.

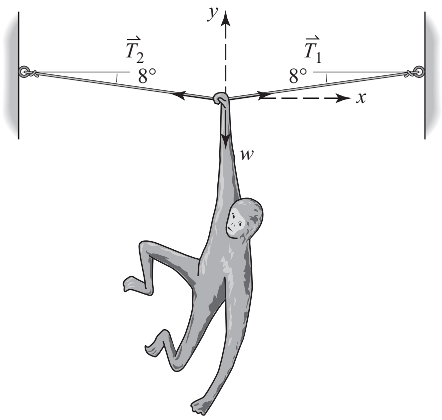
EVALUATE The magnitude is just the sum of the x-components:

$$F_{\text{net}} = F_{1x} + F_{2x} = (7.3\text{ pN})\left[\cos(+32.5^{\circ}) + \cos(-32.5^{\circ})\right] = 12\text{ pN}$$

ASSESS The answer is within the limits of what the sum could be. If the forces were aligned ($\theta = 0$), then the net force would be $2F = 14.6$ N . Whereas if the forces were completely opposite ($\theta = 180^{\circ}$), then the net force would be zero.

35. **INTERPRET** This problem involves Newton's second law. We are asked to find the tension in a rope needed to support an object of a given mass.

DEVELOP Draw a diagram of the situation (see figure below). Apply Newton's second law in the y direction and solve for the tension of the rope. Note that the tension of the rope is everywhere the same (for a massless rope), so $T_1 = T_2 = T$,



EVALUATE Applied in the y direction, Newton's second law gives

$$T_1 \sin(\theta) + T_2 \sin(\theta) = w$$

$$T = \frac{mg}{2\sin(\theta)} = \frac{(15\text{ kg})(9.8\text{ m/s}^2)}{2\sin(8^{\circ})} = 530\text{ N}$$

The monkey's weight is w = mg = (15 kg)(9.8 m/s2) = 150 N (to two significant figures). This is over three times less than the tension force in the rope.

ASSESS Notice that the $T \to \infty$ as $\theta \to 0$ because there is a vanishingly small component of the tension acting in the vertical direction. The majority of the tension simply serves to pull the two support points together.

36. **INTERPRET** This problem is similar to Problem 35, except that the angles at each end of the rope are different. We will apply Newton's second law to solve this problem.

DEVELOP Draw a free-body diagram for the pack (see below). The angles are $\theta_1 = 28^{\circ}$ and $\theta_2 = 71^{\circ}$, and the weight of the pack is $w = mg$. Because the pack does not move, its acceleration in both the horizontal and vertical direction is zero. Thus, applying Newton's second law in the vertical direction gives

$$F_{\text{net}} = ma = 0$$

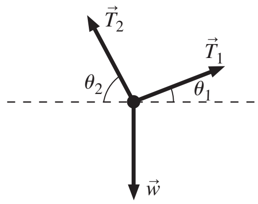
$$T_2 \sin(\theta_2) + T_1 \sin(\theta_1) - w = 0$$

and in the horizontal direction it gives

$$F_{\text{net}} = ma = 0$$

$$-T_2 \cos(\theta_2) + T_1 \cos(\theta_1) = 0$$

Using these two equations, we can solve for T_1 and T_2.



EVALUATE Using the second equation above to express T_2 in terms of T_1 gives $T_1 = T_2\cos(\theta_2)/\cos(\theta_1)$, which when inserted in the first equation gives

$$T_2\sin(\theta_2) + T_2\cos(\theta_2)\tan(\theta_1) = mg$$

$$T_2 = \frac{mg}{\sin(\theta_2)+\cos(\theta_2)\tan(\theta_1)} = \frac{(26\text{ kg})(9.8\text{ m/s}^2)}{\sin(71^\circ)+\cos(71^\circ)\tan(28^\circ)}$$
$$= 230\text{ N}$$

Inserting this value into the expression for T_1 gives

$$T_1 = T_2\cos(\theta_2)/\cos(\theta_1) = (228\text{ N})\cos(71^\circ)/\cos(28^\circ) = 84\text{ N}$$

ASSESS Notice that the tension in the rope on each side of the pack is unequal because the angle made by the rope is different.

37. **INTERPRET** The key concepts in this problem are circular motion and Newton's second law. The object of interest is the mass m_1 that travels in a circular path. By analyzing the force acting on m_1 and m_2, the tension in the string and the period of the circular motion of m_1 can be determined.

DEVELOP Apply Newton's second law to each mass (see Figure 5.31). Because the table is frictionless, the only force acting on m_1 in the horizontal plane is the tension. Assuming the massless rope does not encounter any friction when it goes through the hole in the table, the tension T acting on each mass is of the same magnitude. By Newton's second law, and because m_2 does not accelerate, this tension must cancel the force due to gravity acting on m_2. Thus, we have

$$m_1:\quad F_{\text{net}} = T = m_1a_1 = \frac{m_1v^2}{R} \quad\Rightarrow\quad T = \frac{m_1v^2}{R}$$
$$m_2:\quad F_{\text{net}} = T - m_2g = m_2a_2 = 0 \quad\Rightarrow\quad T = m_2g$$

EVALUATE **(a)** Newton's second law applied to the stationary mass m_2 yields $T = m_2g$.

(b) The tension in the string also provides the centripetal force for m_1. Let $\tau = 2\pi R/v$ be the period of the circular motion. The above equation for m_1 then gives

$$T = \frac{m_1v^2}{R} = \frac{m_1}{R}\left(\frac{2\pi R}{\tau}\right)^2 = \frac{4\pi^2 m_1R}{\tau^2}$$

But from **(a)**, we also have $T = m_2g$. By combining the two equations, we obtain

$$\frac{4\pi^2 m_1R}{\tau^2} = m_2g$$
$$\tau = 2\pi\sqrt{\frac{m_1R}{m_2g}}$$

ASSESS From the expression for τ, we can draw the following conclusions: **(i)** Because $\tau \propto \sqrt{R}$, the larger the radius; the longer the period. **(ii)** With the tension (or m_2) kept fixed, increasing m_1 also leads to a longer period. Note that the derivation above only applies for uniform circular motion. If m_1 is pulled in toward the hole (i.e., R is reduced), then the motion is no longer circular (although the acceleration is still centripetal), and the formula $a = v^2/R$ does not apply.

38. **INTERPRET** We're asked to calculate the horizontal traction force supplied by a mass and a set of massless/frictionless pulleys.

DEVELOP Because the pulleys are massless and frictionless, the tension T in the cord will be the same throughout the system. This tension has to support the mass from falling, so $T = mg$. The horizontal force on the leg is the sum from the cord above and below the pulley attached to the foot:

$$F_y = T\cos\theta_1 + T\cos\theta_2$$

EVALUATE Using the values given, the traction force on the leg is:

$$F_y = (4.8\ \text{kg})(9.8\ \text{m/s}^2)(\cos 70^\circ + \cos 20^\circ) = 60\ \text{N}$$

ASSESS This force is about 10% of the weight of a 60 kg person, so this seems reasonable for the amount of force needed to counter some of the forces exerted by muscles in the leg.

39. **INTERPRET** This problem involves Newton's second and third laws and uniform circular motion. The objects of interest are the roller-coaster seat, the seat belt, and the rider. We are asked to find the force exerted on a rider at the top of the turn by the roller-coaster seat and by the seatbelt, and to determine what would happen should the rider unbuckle his seatbelt.

DEVELOP The free-body diagram for the rider at the top of the track will be the same as for the roller coaster (see Figure. 5.16). The rider thus has two forces acting on him, the normal force due to the seat and the force due to gravity, both of which are pushing him down. By Newton's second law, the sum of these forces must be proportional to the acceleration. Expressed mathematically, we have

$$F_{\text{net}} = n + mg = ma = \frac{mv^2}{R}$$

where $m = 60$ kg, $v = 9.7$ m/s, and $R = 6.3$ m. Solve this equation for n, which is the force exerted by the roller-coaster seat on the rider. To find the force exerted by the seatbelt on the rider, make a free-body diagram of the seatbelt to find the force exerted by the rider on the seatbelt, then use Newton's third law to find the force exerted by the seatbelt on the rider.

EVALUATE (**a**) From Newton's second law, the seat exerts a force

$$n = \frac{mv^2}{R} - mg = (60\ \text{kg})\left[\frac{(9.7\ \text{m/s})^2}{6.3\ \text{m}} - (9.8\ \text{m/s}^2)\right] = 310\ \text{N}$$

on the rider. As indicated in Figure 5.16, the normal force is oriented downward, accelerating the rider toward the Earth.

(b) A free-body diagram of the seatbelt is exactly like that for the rider (see Figure 5.16), but the mass of the seatbelt is different. Therefore, Newton's second law applied to the seatbelt gives

$$F_{\text{net}} = n_{\text{R-SB}} + m_{\text{SB}}g = m_{\text{SB}}a = \frac{m_{\text{SB}}v^2}{R}$$

where $n_{\text{R-SB}}$ is now the force exerted by the rider on the seatbelt and m_{SB} is the (unknown) mass of the seatbelt. From Newton's third law, we know that the force exerted by the seatbelt on the rider has the same magnitude as $n_{\text{R-SB}}$, but is oriented in the opposite direction. Therefore,

$$n_{\text{SB-R}} = -\frac{m_{\text{SB}}v^2}{R}$$

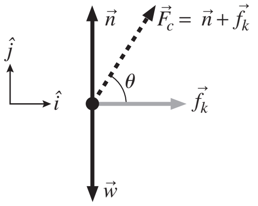
The negative sign means that this force is oriented upward, as expected. Without knowledge of the mass of the seatbelt, we cannot find the force $n_{\text{SB-R}}$, but we can see that this force is simply the force needed to give the seatbelt the same centripetal acceleration $a = v^2/R$ as the rider.

(c) If the rider unbuckles at this point, nothing would happen because the seatbelt does not enter into Newton's second law in part (a). Thus, the rider would remain in his seat.

ASSESS Because the seat pushes down on the rider with the normal force, the ride's centripetal acceleration a toward the Earth is greater than that due to gravity; $a > g$. This can be verified by calculating the centripetal acceleration, which gives $a = v^2/R = (9.7\ \text{m/s})^2/(6.3\ \text{m}) = 15\ \text{m/s}^2 > g$. If the normal force were to go to zero, the rider's acceleration due to gravity g would be greater than the centripetal acceleration of the roller coaster, and the rider would fall out of his seat (unless the seat belt were there to restrain him).

40. **INTERPRET** This problem involves uniform circular motion and Newton's second law. The object of interest is the skater, and the three forces involved are gravity $w = mg$, the normal force n, and static friction f_s. The physical quantities of interest are the vertical and horizontal components of the force exerted on the blade as the skater makes a circular turn.

DEVELOP Draw a free-body diagram for the skater (see figure below). By applying Newton's second law $\vec{F}_{\text{net}} = m\vec{a}$, we find the (vector) equation of motion for the skater: $\vec{w}+\vec{n}+\vec{f}_k = m\vec{a}$, which can be decomposed into one equation from the vertical direction and one from the horizontal direction. Assuming the skater executes uniform circular motion, her acceleration will be given by Equation 5.1, $a = mv^2/r$ (Equation 5.1) By analyzing the force acting on the skater, we can find the maximum angle at which she can lean without falling over.



EVALUATE **(a)** By equating the x and y components of the vector equation of motion, we have

$$x: \quad f_s = ma_x = \frac{mv^2}{r}$$
$$y: \quad n - mg = ma_y = 0$$

Thus, the vertical force exerted on her skate is upward and is equal to her weight,

$$\vec{n} = (mg)\,\hat{j} = (45\text{ kg})\left(9.8\text{ m/s}^2\right)\hat{j} = (440\text{ N})\,\hat{j}$$

The horizontal force on her skate, which causes the centripetal acceleration, is

$$\vec{f}_s = \frac{mv^2}{r}\hat{i} = \frac{(45\text{ kg})(6.3\text{ m/s})^2}{5.0\text{ m}}\hat{i} = (360\text{ N})\,\hat{i}$$

(b) Stability requires that the center of gravity of the skater be along the line of action of the contact force $\vec{F}_c$. Therefore, she should lean at an angle

$$\theta = \text{atan}\left(\frac{n}{f_k}\right) = \text{atan}\left(\frac{441\text{ N}}{357\text{ N}}\right) = 51°$$

above the horizontal.

ASSESS We see that friction is what provides the centripetal force for circular motion. Without the friction, the skater would never be able to turn. Note that the skate experiences significant frication perpendicular to the skate, and very little friction parallel to the skate.

41. **INTERPRET** This problem involves applying Newton's second law to an object that is executing uniform circular motion. The object of interest is the plane, and we are to find the minimum turning radius it can make given the maximum angle at which it can bank.

DEVELOP Apply Newton's second law $\vec{F}_{\text{net}} = m\vec{a}$ to the plane as sketched in Figure 5.33. In component form, this gives

$$F_w\cos(\theta) - mg = ma_y = 0$$
$$F_w\sin(\theta) = ma_x = \frac{mv^2}{r}$$

where the subscripts x and y indicate the horizontal (to the left) and upward directions, respectively. Note that the acceleration in the horizontal direction is centripetal acceleration given by Equation 5.1, because the plane is executing uniform circular motion. Given the maximum bank angle is $\theta = 40°$ and the speed is $v = 950$ km/h = 263.89 m/s, solve for the turning radius r.

EVALUATE The minimum turning radius r is

$$F_w \cos(\theta) = mg \quad \Rightarrow \quad F_w = \frac{mg}{\cos\theta}$$

$$F_w \sin(\theta) = \frac{mv^2}{r}$$

$$r = \frac{mv^2 \cos(\theta)}{mg \sin(\theta)} = \frac{v^2}{g \tan(\theta)} = \frac{(263.89 \text{ m/s})^2}{(9.8 \text{ m/s}^2)\tan(40^\circ)} = 8500 \text{ m=8.5 km}$$

where we report the result to two significant figures.

ASSESS This may seem like a rather wide turn—but the speed of the plane is high and the bank angle is low. These are actually reasonable values for a passenger jet. Will the passengers' coffee spill from their cups during such a turn?

42. **INTERPRET** This problem involves uniform circular motion and Newton's second law. The object of interest is the water in the bucket. The forces acting on the water are gravity w and the normal force n of the bucket. We are asked to find the minimum speed at which the water will remain in the bucket.

DEVELOP We will consider the bucket at the top of its trajectory, because this is the point at which the normal force has zero upward component to counteract the downward pull of gravity, so it is the point at which the water would be most susceptible to falling out. Draw a free-body diagram of the bucket at the top of the trajectory (see Figure 5.16 for a suitable diagram). Applying Newton's second law (for constant mass) to the water gives $n + F_g = ma = mv^2/r$. Solve this equation for the speed, and then consider that if the bucket loses contact with the water, it cannot exert a normal force on the water, so $n = 0$ (see Problem 39 for a discussion of a similar question).

EVALUATE Solving the above equation for speed, we obtain $v = \pm\sqrt{gr + nr/m}$. The positive and negative speeds correspond to the two directions in which the bucket may turn. Without loss of generality, we choose to work with the positive speed. The minimum possible speed corresponds to the situation where the water just loses contact with the bucket, so $n = 0$. Inserting $n = 0$ into the expression for the speed gives

$$v_{\text{min}} = \sqrt{gr} = \sqrt{(9.8 \text{ m/s}^2)(0.85 \text{ m})} = 2.9 \text{ m/s}$$

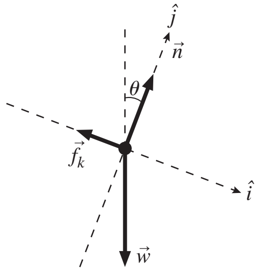
ASSESS With this minimum speed, the normal force vanishes at the top of the trajectory, and gravity alone provides the centripetal force that keeps the water moving in its circular path. If the bucket's speed is too slow (less than v_{min}), then the acceleration of the water due to gravity will exceed the centripetal acceleration of the bucket, and the water will accelerate downward faster than the bucket (i.e., spill out).

43. **INTERPRET** This problem involves Newton's second law. The object of interest is the child and sled, and we are to find the frictional force needed for them to have zero acceleration.

DEVELOP Draw a free-body diagram of the situation (see figure below). Note that the frictional force is drawn so that opposes the motion (although this is not necessary—were we to draw it in the opposite direction we would find a negative friction force). If the child does not accelerate, Newton's second law gives $\vec{F}_{\text{net}} = 0$. In component form, this is

$$n - mg\cos(\theta) = 0$$
$$mg\sin(\theta) - f_k = 0$$

where we have used $w = mg$. From Equation 5.3, we know that $f_k = \mu_k n$, so we can solve this system of equations for the coefficient of kinetic friction μ_k.



EVALUATE Inserting the expression for f_k and using the first equation to eliminate the normal force gives

$$mg\sin(\theta) - \mu_k mg\cos(\theta) = 0$$
$$\mu_k = \tan(\theta) = \tan(12^\circ) = 0.21$$

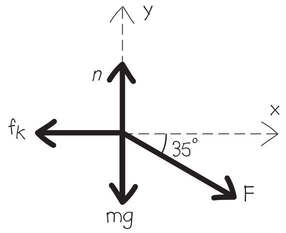
ASSESS If the coefficient of kinetic friction does not depend on the speed of the sled, then the child will slide down the hill at her initial speed. Thus, if she takes a running start, she will continue at the speed at which she runs. If she sits on her sled with no running start, she will remain stationary. If both situations, she experiences no acceleration.

44. **INTERPRET** This problem involves Newton's second law in a situation with zero acceleration. The object of interest is the mower, and the forces involved are the weight w due to gravity, the normal force n, the force f_k due to friction, and the force F applied by the gardener at an angle of 35° with the horizontal. We are asked to find the force F required to produce a (nonzero) constant velocity.

DEVELOP Draw a free-body diagram of the mower (see figure below), assuming the ground is horizontal. Newton's second law applied to the mower gives $\vec{F}_{\rm net} = \vec{F} + m\vec{g} + \vec{n} + \vec{f}_{\rm k} = m\vec{a} = 0$, because the acceleration is zero. In component form, this vector equation yields two scalar equations, one for the vertical direction and one for the horizontal direction. These are

$$x: \quad F\cos(\theta) - f_{\rm k} = 0$$
$$y: \quad n - F\sin(\theta) - mg = 0$$

which we can solve for F given that $f_{\rm k} = \mu_{\rm k} n$ (Equation 5.3).



EVALUATE Substituting the expression for $f_{\rm k}$ into the x equation and solving for F, gives

$$F = \frac{\mu_{\rm k} mg}{\cos(\theta) - \mu_{\rm k}\sin(\theta)} = \frac{(0.68)(22\text{ kg})(9.8\text{ m/s}^2)}{\cos(35°) - (0.68)\sin(35°)} = 342\text{ N} \approx 340\text{ N}$$

to two significant figures. Compared with the weight of the mower, we find

$$\tfrac{F}{mg} = \frac{\mu_k}{\cos(\theta) - \mu_k\sin(\theta)} = 1.58.$$
$$\frac{F}{mg} = \frac{\mu_k}{\cos(\theta) - \mu_k\sin(\theta)} = \frac{0.68}{\cos(35^\circ) - (0.68)\sin(35^\circ)} = 1.6$$

so the force applied is 60% more than the weight of the mower.

ASSESS To show that our result makes sense, let's check some limiting cases. First, if $\mu_{\rm k} = 0,$ our expression gives $F = 0$. This is reasonable because without friction, no force needs to be applied to keep the mower moving at constant speed. However, if $\theta = 0$, we get $F = \mu_{\rm k} mg.$ In this limit, the normal force is simply $n = mg$ and the friction force is $f_{\rm k} = \mu_{\rm k} n = \mu_{\rm k} mg.$ Thus, the force applied has the same magnitude as friction, but points in the opposite direction: $\vec{F} = -\vec{f}_{\rm k}$.

45. **INTERPRET** The interpretation of the problem is the same as given in Example 5.4, except that the rock now has a fourth force acting on it, which is the force due to kinetic friction.

DEVELOP Because friction always acts to oppose motion, the force f_k due to kinetic friction acting on the rock will be oriented in the negative-x direction. With the addition of the friction force, Newton's second law applied to the rock gives $\vec{T}_r + \vec{F}_{gr} + \vec{n} + \vec{f}_k = m\vec{a}_r.$ Writing this vector equation in component form gives

$$x: \quad T_r - \mu_k n = m_r a$$
$$y: \quad n - m_r g = 0$$

where we have inserted a negative sign for the friction force and have used the fact that the rock does not accelerate in the y direction, and that the force due to gravity on the rock is $F_{gr} = m_r g$. Applying Newton's law to the climber gives the same result as in Example 5.4:

$$T - m_c g = -ma$$

Note that the acceleration of the climber and the rock has the same magnitude, although they act in different directions. This is so because the rope is considered to not stretch, so both objects must move at the same rate.

EVALUATE Solve the 3 equations above for the acceleration, which gives

$$\mu_k m_r g - m_r a - m_c g = m_c a$$

$$a = g\left(\frac{m_c - \mu_k m_r}{m_c + m_r}\right) = \left(9.8 \text{ m/s}^2\right)\left[\frac{70 \text{ kg} - (0.057)(940 \text{ kg})}{70 \text{ kg} + 940 \text{ kg}}\right] = 0.16 \text{ m/s}^2$$

Inserting this result into the kinematic Equation 2.10 gives $t = \sqrt{2(51 \text{ m})/(0.159 \text{ m/s}^2)} = 25$ s.

ASSESS Notice that our expression for the acceleration reverts to that found in Example 5.4 if we let $m_k \to 0$.

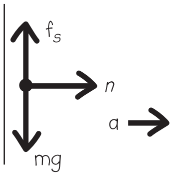
46. **INTERPRET** This problem involves Newton's second law. The object of interest is the bat, and the forces involved are gravity $\vec{F}_g$, the normal force $\vec{n}$, and the force due to static friction $\vec{f}_s$. We are asked to find the minimum acceleration of the train that prevents the bat from sliding down the window.

DEVELOP Draw a free-body diagram for the situation (see figure below). Using Newton's second law, the force equation for the bat is $\vec{F}_{\text{net}} = \vec{F}_g + \vec{n} + \vec{f}_\text{s} = m\vec{a}$. This can be decomposed into two equations, one for the y direction and one for the x direction:

$$x: \quad n = ma$$

$$y: \quad f_\text{s} - mg = 0$$

The force due to static friction is $f_\text{s} \le \mu_\text{s} n$, which we can use in the equations above to find the minimum accelration.



EVALUATE Inserting the inequality for static friction into Newton's second law gives

$$mg = f_\text{s} \le \mu_\text{s} n$$

$$mg \le \mu_\text{s} ma$$

$$a \ge \frac{g}{\mu_\text{s}} = \frac{9.8 \text{ m/s}^2}{0.86} = 11 \text{ m/s}$$

ASSESS The minimum acceleration is inversely proportional to μ_s,which is the coefficient of static friction between the bat and the train. The smaller the value of μ_s the greater the acceleration is needed to keep the bat in place. In the limit $\mu_\text{s} \to 0$ (frictionless surface), the acceleration would have to be infinitely large. However, for $\mu_\text{s} \to \infty$ (infinitely sticky surface), we get $a \to 0$.

47. **INTERPRET** This problem involves Newton's second law and kinematics. The object of interest is the train, and we are asked to find if the train can stop within 150 m if the wheels maintain static contact with the rails (i.e., the wheels do not skid on the rails).

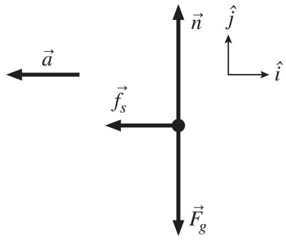
DEVELOP Considering all the wheels as one point of contact, make a free-body diagram for the train (see figure below). Applying Newton's law to the train wheels gives $\vec{F}_{\text{net}} = \vec{f}_s + \vec{n} + \vec{F}_g = m\vec{a}$, and writing this in component form gives

$$x: \quad F_g + n = 0$$

$$y: \quad -f_s = -ma$$

where we have used the fact that there is zero acceleration in the x direction and we have explicitly noted the sign of the friction force and the acceleration to emphasize that they are in the same direction (negative-x direction).

The force due to static friction is $f_s \le \mu_s n$ and the force due to gravity is $F_g = -mg$ (because gravity acts in the downward direction). Insert these values into the above equations to find the maximum acceleration possible without having the wheels slip on the rails, then use the kinematic Equation 2.11 $v^2 = v_0^2 + 2a(x - x_0)$ to find the stopping distance.



EVALUATE Newton's second law thus gives

$$ma = f_s \le \mu_s n = \mu_s mg$$
$$a \le \mu_s g$$

so the maximum acceleration possible is $\mu_s g$. Inserting this result for the acceleration into Equation 2.11 gives a stopping distance of

$$\overbrace{v^2}^{=0} = v_0^2 + 2a(x - x_0)$$
$$x - x_0 = \frac{v_0^2}{2a} = \frac{v_0^2}{2\mu_s g} = \frac{(140\text{ km/h})^2}{2(0.58)(9.8\text{ m/s}^2)}\left(\frac{10^3\text{ m}}{\text{km}}\right)^2\left(\frac{\text{h}}{3600\text{ s}}\right)^2 = 130\text{ m}$$

so the train will stop before hitting the car.

ASSESS The stopping time for the train is

$$x - x_0 = (v_0 + v)t/2$$
$$t = \frac{2(x - x_0)}{v_0} = \frac{2(133\text{ m})}{38.9\text{ m/s}} = 6.8\text{ s}$$

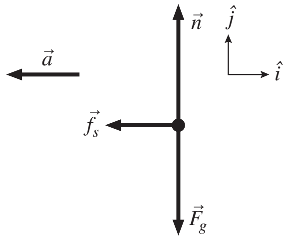
which should be just enough time for the passengers to get out of the car.

48. **INTERPRET** This problem involves uniform circular motion (assuming the bug walks at a speed that is much, much less then the tangential speed of the CD at the bug's position). and Newton's second law. The object of interest is the bug, and the forces acting on it are gravity $\vec{F}_g$ normal force $\vec{n}$ and static friction $\vec{f}_s$. We want to find out how far the bug gets from the center before it begins to slip.

DEVELOP Draw a free-body diagram of the bug as seen from the side (see figure below). We have chosen the *x* direction to be away from the center of the CD, so the centripetal acceleration *a* is oriented in the negative-*x* direction. Using Newton's second law, the force equation for the bug is $\vec{F}_{\text{net}} = \vec{F}_g + \vec{n} + \vec{f}_s = m\vec{a}$. Assume that the disc is level. Assuming uniform circular motion (see comment in Interpret), we can use Equation 5.1 $a = v^2/r$. Writing Newton's second law in component form thus gives :

$$x:\quad -f_s = -\frac{mv(r)^2}{r}$$
$$y:\quad n = mg$$

The force due to static friction is $f_s \le \mu_s n$, and the speed of the disc as a function of the bug's radial position and the frequency of revolution ($\phi = 200\text{ s}^{-1}$) is $v(r) = 2\pi r\phi$, so we can find the distance *r* at which static friction can no longer supply the necessary acceleration (i.e., the bug starts to slip).



EVALUATE Inserting the given quantities into Newton's second law gives

$$\frac{mv^2}{r} = f_s \le \mu_s n$$

$$\frac{m(2\pi r\phi)^2}{r} \le \mu_s mg$$

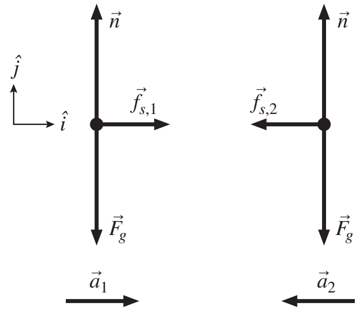
$$r \le \frac{\mu_s g}{4\pi^2\phi^2} = \frac{1.2(9.8\ \text{m/s}^2)}{4\pi^2(200\ \text{min}^{-1})^2}\left(\frac{60\ \text{s}}{1\ \text{min}}\right)^2 = 0.027\ \text{m}$$

Thus, the radial distance traveled by the bug before slipping occurs is 2.7 cm.

ASSESS Our result indicates that the larger the friction coefficient, the greater the distance the bug can travel. This makes sense because friction is what produces the centripetal force. In the limit $\mu_s \to 0$ (frictionless surface), the bug would not be able to move at all $(r \to 0)$. On the other hand, if the surface of the disc is very sticky (large μ_s) we would then expect the distance traveled by the bug to be very large.

49. INTERPRET This problem involves kinematics, Newton's second law, and frictional forces. We are given information to find the acceleration of the textbook, and are asked to calculate what coefficient of static friction is necessary to keep the paperback book stuck to the textbook during the acceleration, and the maximum coefficient of static friction possible that would still let the paperback book slide off the textbook during deceleration.

DEVELOP Draw a free-body diagram for the paperback book for both the case of acceleration a_1 and deceleration a_2 (see figure below). The accelerations can be calculated using the kinematic Equation 2.7 $v = v_0 + at$. This gives $\vec{a}_1 = \vec{v}/t = (0.96\ \text{m/s})/(0.42\ \text{s})\hat{i} = (2.286\ \text{m/s}^2)\hat{i}$ and $\vec{a}_2 = -\vec{v}_0/t = -(0.96\ \text{m/s})/(0.33\ \text{s}) = -(2.909\ \text{m/s}^2)\hat{i}$. Apply Newton's second law to the paperback book in both situations to find the coeffecient of static friction.



EVALUATE When the paperback book accelerates, Newton's second law (in component form) gives

$$\left.\begin{aligned} ma_1 &= f_{s,1} \le \mu_{s,1} n \\ n - mg &= 0 \end{aligned}\right\} \mu_{s,1} \ge \frac{a_1}{g} = \frac{2.286\ \text{m/s}^2}{9.8\ \text{m/s}^2} = 0.23$$

When the paperback book decelerates, we have

$$\left.\begin{aligned} ma_2 &> f_{s,2}^{\max} = \mu_{s,2} n \\ n - mg &= 0 \end{aligned}\right\} \mu_{s,2} = \frac{a_2}{g} = \frac{0.33\ \text{m/s}^2}{9.8\ \text{m/s}^2} = 0.30$$

Thus, the actual coefficient of static friction must lie in the range $0.23 \le \mu_s \le 0.30$.

ASSESS Notice that the force due to friction acts to oppose any potential velocity of the paperback book with respect to the reference book.

50. **INTERPRET** This problem involves Newton's second law. The object of interest is a child on a sled. Three forces act on this object: gravity $\vec{F}_g$, the normal force $\vec{n}$, and the force $\vec{f}_\text{k}$ from kinetic friction By analyzing the forces acting on the child while he travels down the hill and using kinematics, we can determine the child's acceleration a and his speed v when he reaches level ground. Subsequently, we can solve the one-dimensional kinematics problem to determine the distance he slides before coming to a stop.

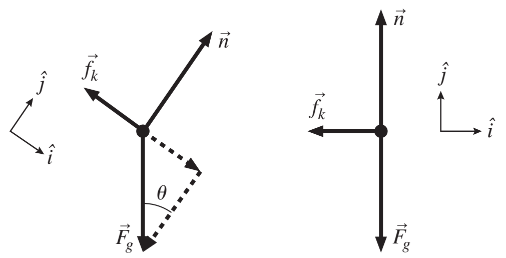
DEVELOP Draw a free-body diagram for the sled on the slope and on level ground (see figure below), and apply Newton's second law in both cases to find the acceleration of the sled. Using Equation 5.3 $f_\text{k} = \mu_\text{k} n$, this gives

$$\left.\begin{aligned} x:&\quad mg\sin(\theta) - f_\text{k} = ma_\text{slope} \\ y:&\quad n - mg\cos(\theta) = 0 \end{aligned}\right\} a_\text{slope} = g\left[\sin(\theta) - \mu_k\cos(\theta)\right]$$

for the acceleration on the slope. Repeating the calculation for level ground gives

$$\left.\begin{aligned} x:&\quad f_\text{k} = ma_\text{level} \\ y:&\quad n - mg = 0 \end{aligned}\right\} a_\text{level} = \mu_k g$$

Use these results for acceleration in the kinematic Equation 2.11 $v^2 = v_0^2 + 2a(x - x_0)$ to find the speed at the bottom of the hill, then insert this result into the same equation (but as the initial speed this time) to find the distance traveled on level ground.



EVALUATE Using Equation 2.11, the speed of the child at the bottom of the hill is

$$\begin{aligned} v &= \pm\sqrt{v_0^2 + 2a(x - x_0)} = \pm\sqrt{0 + 2g\left[\sin(\theta) - \mu_k\cos(\theta)\right](x - x_0)} \\ &= \pm\sqrt{2\left(9.8\text{ m/s}^2\right)\left[\sin\left(25^\circ\right) - 0.12\cos\left(25^\circ\right)\right](41\text{ m})} = 15.88\text{ m/s} \end{aligned}$$

where we have chosen the positive square root arbitrarily with no loss of generality (the negative answer simply corresponds to motion in the opposite direction). Now, on level ground the distance traveled before stopping is

$$x - x_0 = \frac{v^2 - v_0^2}{2a_\text{level}} = \frac{-v_0^2}{2(-\mu_\text{k} g)} = \frac{-(15.9\text{ m/s})^2}{-2(0.12)\left(9.8\text{ m/s}^2\right)} = 110\text{ m}$$

where we have used the final speed calculated for the slope as the initial speed for the level ground, and we have used $a_\text{level} = -\mu_\text{k} g$ because the acceleration is in the negative-x direction.

ASSESS After traveling 41 m down the hill, the speed of the child at the bottom of the hill is 15.9 m/s, or roughly 36 mi/h! So sliding 110 m on the level ground before coming to a complete stop sounds reasonable.

51. **INTERPRET** This problem involves Newton's second law. We are asked to find the maximum acceleration of a front-wheel-drive car, given that 70% of its mass rests on the front wheels.

DEVELOP Draw a free-body diagram for the combined front wheels of the car, which will be the same diagram as for the sled on level ground in the preceding problem. Note that the mass resting on the front wheels is $m_\text{f} = 0.7m$, so $F_g = 0.7mg$. The mass entering into Newton's second law, however, is m, because the entire car is being accelerated. Applying Newton's second law to the combined front-wheel system gives

$$\begin{aligned} x:&\quad f_\text{s} = ma \\ y:&\quad n - 0.7mg = 0 \end{aligned}$$

EVALUATE Using Equation 5.2 $f_s \le \mu_\text{s} n$ in Newton's second law gives

$$a = \frac{f_s}{m} \le \frac{\mu_s n}{m} = \frac{\mu_s(0.7mg)}{m}$$

$$a \le 0.7\mu_s g = 0.7(0.61)(9.8 \text{ m/s}^2) = 4.2 \text{ m/s}^2$$

Thus, the car's maximum acceleration is 4.2 m/s^2.

ASSESS Note that as the car accelerates, the proportion of weight carried by the back wheels increases. To take advantage of this effect, some people drive backwards in front-wheel-drive cars when the road is very slick.

52. **INTERPRET** This problem involves Newton's second law, the kinetic friction force, and kinematics The object of interest is the car, and the three forces involved are gravity $\vec{F}_g$, the normal force $\vec{n}$, and the force due to kinetic friction $\vec{f}_k$ By analyzing the force acting on the car, we can solve the one-dimensional kinematics problem to determine its initial speed.

DEVELOP On a level road, the acceleration of a skidding car is $a = -\mu_k g$ (see Example 5.8), where the negative sign indicates that the acceleration is opposite to the velocity. Given that the final speed of the car (i.e., just before hitting the stationary car) is v = 25 km/h = 6.94 m/s, we can solve Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$ to find the car's initial speed.

EVALUATE Using $a = -\mu_k g$ in Equation 2.11, we have

$$v_0 = \sqrt{v^2 - 2a(x - x_0)} = \sqrt{(6.94 \text{ m/s})^2 + 2(0.71)(9.8 \text{ m/s}^2)^2 (47 \text{ m})} = 26.5 \text{ m/s} = 95.4 \text{ km/h} = 59.3 \text{ mi/h}$$

ASSESS Note that friction is what causes the acceleration.

53. **INTERPRET** This problem involves Newton's second law, kinematics, and the force due to kinetic friction. The object of interest is a swimmer that slides down the slide, and we are asked to find the coefficient of kinetic friction given the relative time it takes a swimmer to slide down the slide when it's wet and when it's dry.

DEVELOP The free-body diagram for this problem is the same as that for the sled on the slope in Problem 5.50. Apply Newton's second law to a swimmer sliding down the dry slide ($\mu_k \ne 0$) to find the acceleration, then let $\mu_k \to 0$ to find the acceleration for the wet slide. This gives

$$\left.\begin{array}{ll} x: & mg\sin(\theta) - f_k = ma \\ y: & n - mg\cos(\theta) = 0 \end{array}\right\} a = g\left[\sin(\theta) - \mu_k \cos(\theta)\right]$$

To find the coefficient of kinetic friction, use the kinematic Equation 2.10 $x = x_0 + v_0 t + at^2/2$ to relate the acceleration to the time it takes to travel down the slide with $v_0 = 0$.

EVALUATE The time it takes to slide down the dry slide is

$$x = x_0 + \overbrace{v_0}^{=0} t + at^2/2$$

$$t_{\text{dry}} = \pm\sqrt{\frac{2(x - x_0)}{a}} = \pm\sqrt{\frac{2(x - x_0)}{g\left[\sin(\theta) - \mu_k \cos(\theta)\right]}}$$

Letting $\mu_k \to 0$, we get the time it takes to descend the wet slide:

$$t_{\text{wet}} = \pm\sqrt{\frac{2(x - x_0)}{g\sin(\theta)}} = t_{\text{dry}}\sqrt{1 - \mu_k \cot(\theta)}$$

We are told that $t_{\text{wet}}/t_{\text{dry}} = 1/3$, so

$$\frac{t_{\text{wet}}}{t_{\text{dry}}} = \frac{1}{3} = \sqrt{1 - \mu_k \cot(\theta)}$$

$$\mu_k = \frac{8\tan(35^\circ)}{9} = 0.62$$

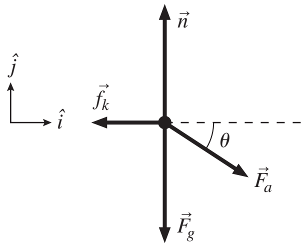
ASSESS Notice that we did not need to evaluate the acceleration, we simply needed to find that the ratio of the accelerations is $1 - \mu_k \cot(\theta)$.

54. **INTERPRET** This is a problem involving Newton's second law with zero acceleration. The object of interest is the trunk, and the forces involved are gravity $\vec{F}_g$, the normal force $\vec{n}$, friction $\vec{f}_s$, and the applied force $\vec{F}_a$. We need to show that if the coefficient of static friction exceeds the given value, the trunk won't move no matter how hard you push.

DEVELOP Draw a free-body diagram of the situation (see figure below). Applying Newton's second law to the trunk gives $\vec{F}_{net} = \vec{F}_a + \vec{F}_g + \vec{n} + \vec{f}_s = m\vec{a}$, which in component form gives

$$x:\quad F_a\cos(\theta) - f_s = 0$$
$$y:\quad n - mg - F_a\sin(\theta) = 0$$

From Equation 5.2, we know $f_s \le \mu_s n = \mu_s\left[mg - F_a\sin(\theta)\right]$, which allows us to find an expression for the appliced force F_a.



EVALUATE Substituting the expression for f_s back to the x component of Newton's second law and solving for F_a we obtain $F_a\cos(\theta) - \mu_s\left[F_a\sin(\theta) + mg\right] = 0$ or

$$F_a = \frac{\mu_s mg}{\cos(\theta) - \mu_s\sin(\theta)}$$

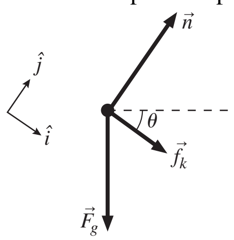
If $\cos(\theta) - \mu_s\sin(\theta) < 0$ then the applied force F_a will become negative, which means that the trunk will not budge and remain in equilibrium. Thus, the equilibrium condition will always be satisfied if

$$\mu_s > \cot(\theta) = \cot(50°) = 0.84$$

ASSESS As the coefficient of friction increases, we must apply a greater force in order to move the trunk. However, because the applied force has a downward component, this adds to the normal force and results in a greater friction force. Obviously when the force is applied vertically downward, it's impossible to move the trunk. On the other hand, if $\theta = 0$, then we get $F_a = \mu_s mg$. In this limit the normal force is simply $n = mg$, and the friction force is $f_s = \mu_s n = \mu_s$mg. Thus, the applied force has the same magnitude as the friction, but points in the opposite direction: $F_a = -f_s$.

55. **INTERPRET** This problem involves Newton's second law and kinematics. The object of interest is the box, and the forces involved are gravity $\vec{F}_g$, the normal force $\vec{n}$, static and kinetic friction $\vec{f}_k$ and $\vec{f}_s$. We are asked to find how far up a slope the box will travel given its initial speed and coefficient of kinetic friction.

DEVELOP Draw a free-body diagram for the situation (see figure below). Use Newton's second law to find the acceleration of the block as it travels up the slope, then insert the result into the kinematic Equation 2.11 $v^2 = v_0^2 + 2a(x - x_0)$ to find the distance the box travels up the slope.



EVALUATE (**a**) In component form, Newton's second law gives

$$\left.\begin{aligned} x:&\quad f_k + mg\sin(\theta) = ma \\ y:&\quad n - mg\cos(\theta) = 0 \end{aligned}\right\} a = g\sin(\theta) + \mu_k g\cos(\theta)$$

where we have used $F_g = mg$ and Equation 5.3 $f_k = \mu_k n$ for the force due to kinetic friction. Inserting this result into Equation 2.11 and solving for the distance $x - x_0$ gives

$$\overbrace{v^2}^{=0} = v_0^2 + 2a(x - x_0)$$

$$x_0 - x = \frac{v_0^2}{2a} = \frac{v_0^2}{2[g\sin(\theta) + \mu_k g\cos(\theta)]} = \frac{(1.4\text{ m/s})^2}{2(9.8\text{ m/s}^2)[\sin(22^\circ) - (0.70)\cos(22^\circ)]} = 0.10\text{ m}$$

Thus, the block travels 10 cm up the slope.

(b) When the block has stopped, Newton's second law still applies, but with the force due to static friction, which will be oriented up the incline to resist motion down the incline. Thus, Newton's second law gives us

$$\left.\begin{aligned} x:&\quad -f_s + mg\sin(\theta) = ma \\ y:&\quad n - mg\cos(\theta) = 0 \end{aligned}\right\} a \ge g\sin(\theta) - \mu_k g\cos(\theta)$$

where we have used Equation 5.2 $f_s \le \mu_s n$. The acceleration will be positive if $\mu_s < \tan(\theta) = \tan(22^\circ) = 0.404$. However, this value is less than that for kinetic friction, so it is likely that μ_s exceeds this value, because $\mu_s > \mu_k$ is generally true. Thus, we conclude that the block does not slide back down the slope.

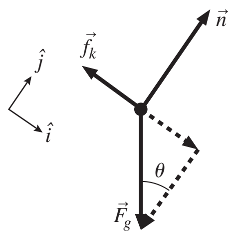
ASSESS Notice how the direction of the frictional force depends on the circumstances: to resist the motion it was oriented down the incline in part (**a**) and up the incline in part (**b**).

56. **INTERPRET** This problem involves Newton's second law, kinematics, and frictional forces. The object of interest is the box, and the three forces involved are gravity $\vec{F}_g = m\vec{g}$, the normal force $\vec{n}$, and the force due to kinetic friction $\vec{f}_k$. We are asked to find the maximum coefficient of kinetic friction that would still allow the boxes to slide down the 5.4-m ramp in 3.3 s or less.

DEVELOP Draw a free-body diagram of a box (see figure below). Applying Newton's second law gives $\vec{F}_g + \vec{n} + \vec{f}_k = m\vec{a}$, which in component form is

$$\begin{aligned} x:&\quad mg\sin\theta - f_k = ma \\ y:&\quad n - mg\cos\theta = 0 \end{aligned}$$

Using Equation 5.3 for the frictional force, $f_k = \mu_k n$, gives an acceleration of $a = g\sin(\theta) - \mu_k g\cos(\theta)$. Using this result in the kinematic Equation 2.10 $x = x_0 + v_0 t + at^2/2$, with $v_0 = 0$, to find the maximum coefficient of kinetic friction.



EVALUATE The time required for the box to travel a distance $x - x_0$ down the ramp is

$$x - x_0 = \frac{1}{2}at^2 \quad \Rightarrow \quad a = \frac{2(x - x_0)}{t^2}$$

Equating this expression with the expression for acceleration from Newton's second law leads to

$$g[\sin(\theta) - \mu_k\cos(\theta)] = \frac{2(x - x_0)}{t^2}$$

$$\mu_k = \tan(\theta) - \frac{2(x - x_0)}{gt^2\cos(\theta)} = \tan(30^\circ) - \frac{2(5.4\text{ m})}{(9.8\text{ m/s}^2)(3.3\text{ s})^2\cos(30^\circ)} = 0.46$$

This is the maximum coefficient that can be tolerated.

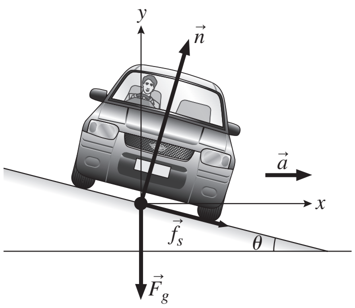
Assess The above expression shows that increasing t gives a greater value of μ_k. What this means is that if the coefficient of friction were greater than 0.46, the slide would take longer than 3.3 s. The value $\mu_k = \tan(\theta) = \tan(30°) = 0.577$ corresponds to $t = \infty$, which means that the box would stay indefinitely on the ramp without sliding when the coefficient of kinetic friction exceeds 0.577.

57. **Interpret** Your speed can be determined by assuming uniform circular motion through the turn. Because the curve is banked, the centripetal force is provided by the normal force, but there may also be a contribution from the friction between your tires and the road.

Develop Example 5.6 describes a curve designed for a certain speed must be banked at an angle given by:

$$\tan\theta = \frac{v_d^2}{gr} = \frac{(80 \text{ km/h})^2}{(9.8 \text{ m/s}^2)(210 \text{ m})} = 0.240$$

In this case, however, we investigate what would have happened if you went faster than the designed speed. To stay in your lane (at the same radius), you would need to turn the steering wheel slightly, causing some friction parallel to the road and perpendicular to the car's motion. The three forces acting on the car are represented in the figure below.



Notice that we have chosen the coordinate axes so that the centripetal acceleration, $\vec{a} = (v^2/r)\hat{r}$, points in the +x direction. The sum of the forces for the two components are

$$x:\quad n\sin\theta + f_s\cos\theta = mv^2/r$$
$$y:\quad n\cos\theta - f_s\sin\theta - F_g = 0$$

The friction has a maximum limit: $f_s \le \mu_s n$. We will use this to find the maximum limit on your speed.

Evaluate We don't know the mass of the car, but we can combine the two component equations to eliminate the mass. With some algebra, we arrive at:

$$\frac{f_s}{n} = \frac{\frac{v^2}{gr}\cos\theta - \sin\theta}{\cos\theta + \frac{v^2}{gr}\sin\theta} \le \mu_s$$

Dividing the numerator and denominator by $\cos\theta$, and using $\tan\theta = v_d^2/gr$, we get

$$\frac{v^2}{gr} - \frac{v_d^2}{gr} \le \mu_s\left(1 + \frac{v^2 v_d^2}{(gr)^2}\right)$$

With a little more algebra and using the value of v_d^2/gr from above, we obtain

$$v \le v_d\sqrt{\frac{1+\mu_s\frac{gr}{v_d^2}}{1-\mu_s\frac{v_d^2}{gr}}} = (80 \text{ km/h})\sqrt{\frac{1+0.15/0.240}{1-0.15\cdot 0.240}} = (80 \text{ km/h})(1.30) = 100 \text{ km/h}$$

So you could have been speeding around the curve by as much as 20 km/h over the posted limit.

Assess We can check whether our final expression for the maximum speed makes sense. If $\mu_s = 0.9$ (characteristic of a dry road), we get $v \le 200$ km/h. If $\mu_s = 0$, our expression becomes to $v \le v_d$, which just says that we must drive at the posted speed if the road is frictionless. If the road is not banked, then $v_d \to 0$, and our expression reduces to $v \le \sqrt{\mu_s gr}$, which agrees with our derivation for the unbanked curve in Problem 3.31.

58. **INTERPRET** This problem involves uniform circular motion. The object of interest is the astronaut, and we are to find the frequency of revolution that would lead to a centripetal acceleration equal in magnitude to the acceleration due to gravity at the surface of the Earth.

DEVELOP Because this is uniform circular motion (i.e., the space station rotates at a constant rate), we can apply Equation 5.1 $a = v^2/r$ to find the centripetal acceleration, with r = (450 m)/2 = 225 m. To relate the tangential velocity v to the parameters of rotation, note that the velocity is the frequency of rotation ϕ multiplied by the circumference $C = 2\pi r$, which gives $v = 2\pi r\phi$.

EVALUATE Equating the centripetal acceleration to the Earth's gravity gives

$$g = \frac{v^2}{r} = \frac{(2\pi r\phi)^2}{r} = 4\pi^2 r\phi^2$$

$$\phi = \pm\frac{1}{2\pi}\sqrt{\frac{g}{r}} = \pm\frac{1}{2\pi}\sqrt{\frac{(9.8\text{ m/s}^2)}{225\text{ m}}} = (0.0332\text{ s}^{-1})\left(\frac{60\text{ s}}{1\text{ min}}\right) = 2.0\text{ min}^{-1}$$

so the satellite would have to rotate twice per minute.

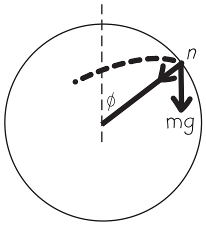
ASSESS The units work out to inverse time, as expected for a frequency. Notice that the larger the satellite radius, the lower the frequency of rotation.

59. **INTERPRET** This problem involves Newton's second law and centripetal acceleration. The forces involved are gravity $\vec{F}_g = m\vec{g}$ and the normal force $\vec{n}$ that is everywhere perpendicular to the track. The aim of this problem is to establish the condition under which a car moving too slowly as it goes around a loop-the-loop roller coaster would leave the track.

DEVELOP Assume the roller coaster travels counter-clockwise around the loop-the-loop. Draw a free-body diagram for the roller coaster at an arbitrary point on the track (see figure below), with the angle ϕ of departure from the track indicated. Applying Newton's second law to the roller coaster gives $\vec{F}_g + \vec{n} = m\vec{a}$. For the roller coaster to stay on the track, the radial component of the net force (toward the center of the track) must equate to the mass times the centripetal acceleration,

$$n + mg\cos(\phi) = \frac{mv^2}{r}$$

The tangential component of the net force is not of interest for this problem.



EVALUATE The car leaves the track when the normal force becomes zero (no more contact):

$$mg\cos\phi = \frac{mv^2}{r} \quad\Rightarrow\quad \cos(\phi) = \frac{v^2}{gr}$$

which is the expression given in the problem statement.

ASSESS The result implies that the car leaves the track when the speed is too small: $v < \sqrt{gr}$. Otherwise, the car never leaves the track, as in Example 5.7.

60. **INTERPRET** This problem looks at friction on a banked curve.

DEVELOP The curve has a radius R and was engineered for a speed of v_0, which from Example 5.6 means that its banking angle satisfies the equation: $\tan\theta = v_0^2/gR$.

Let's assume that the car is going faster than the designed speed. To maintain the same circular trajectory, there must be friction between the tires and the road. See the figure in Problem 5.57 to have an idea of the forces acting on the car. As in this previous problem, the forces in the x and y direction obey Newton's 2nd law:

$$x: \quad n\sin\theta + f_s\cos\theta = mv^2/R$$
$$y: \quad n\cos\theta - f_s\sin\theta - F_g = 0$$

We will use these two equations, and the above expression for the banking angle to derive an expression for the friction coefficient.

EVALUATE We first divide the equation of motion in the x direction by the equation of motion in the y direction:

$$\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = \frac{v^2}{gR}$$

where we have used $f_s = \mu_s n$ (Equation 5.2). Dividing the numerator and denominator on the left-hand-side by $\cos\theta$ and rearranging terms gives:

$$gR(\tan\theta + \mu_s) = v^2(1 - \mu_s\tan\theta)$$

Finally, using the expression for the banking angle with the designed speed,

$$\mu_s = \frac{v^2 - v_0^2}{gR + (v^2v_0^2/gR)}$$

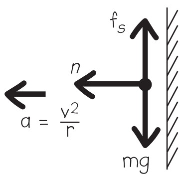
ASSESS Notice that if the car moves at the designed speed ($v = v_0$), the coefficient of friction is zero. This makes sense, since no friction is supposed to be necessary in a banked turn when you drive at the designed speed. Also note that if $v < v_0$, the coefficient is negative, which simply reflects the fact that driving below the designed speed requires that the friction point outward with respect to the center of the curve.

61. **INTERPRET** This problem involves Newton's second law, uniform circular motion, and frictional forces. The forces involved are the force due to gravity $\vec{F}_g = m\vec{g}$, the normal force $\vec{n}$ that acts perpendicular to the wall, and the force due to static friction $\vec{f}_s$. We are to find the frequency of revolution needed to prevent the book from slipping down the wall.

DEVELOP Draw a free-body diagram of the situation (see figure below). Because this is uniform circular motion (i.e. the centrifuge rotates at a constant rate), Equation 5.1 $a = v^2/r$ applies for the centripetal acceleration. The direction of this acceleration is toward the center of the circle of rotation, as indicated in the drawing. Applying Newton's second law in the vertical and horizontal directions gives

$$x: \quad n = ma = mv^2/r$$
$$y: \quad f_s - mg = 0$$

Using Equation 5.2 $f_s = \mu_s n$ in Newton's second law leads to $g/\mu_s = v^2/r$. From this expression, we can find the frequency needed to maintain the book in place.



EVALUATE The frequency of revolution ϕ is related to the tangential velocity by $v = 2\pi r\phi$. Inserting this into the expression above and solving for ϕ gives

$$\frac{g}{\mu_s} = \frac{(2\pi r\phi)^2}{r}$$

$$\phi \geq \left|\pm\sqrt{\frac{g}{4\pi^2 r\mu_s}}\right| = \sqrt{\frac{(9.8\text{ m/s}^2)}{4\pi^2(5.1\text{ m})(0.62)}} = (0.28\text{ s}^{-1})\left(\frac{60\text{ s}}{1\text{ min}}\right) = 17\text{ min}^{-1}$$

Thus, the centrifuge must rotate at a frequency of 17 min^{-1} or greater.

ASSESS The two possible signs for the frequency correspond to rotations clockwise and counter clockwise.

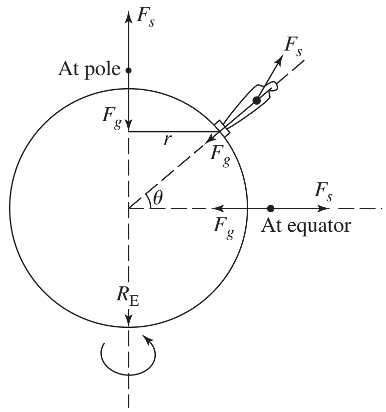
62. **INTERPRET** In this problem we would like to find out the difference in a person's weights measured at the north pole and at the equator.

DEVELOP When standing on the Earth's surface, you are rotating with the Earth about its axis through the poles, with a period of $T = 1\text{ d}$. The radius of your circle of rotation (your perpendicular distance to the axis) is $r = R_E \cos\theta$, where R_E is the radius of the Earth (constant if geographical variations are neglected) and θ is your latitude.

Your rotational speed depends on your latitude: $v = 2\pi r/T$. Consequently, your centripetal acceleration has magnitude of

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 R_E \cos\theta}{T^2}$$

and is directed toward the axis of rotation. We assume there are only two forces acting on you, gravity, $\vec{F}_g$ (with magnitude *mg* approximately constant, directed towards the center of the Earth), and the force exerted by the scale, $\vec{F}_s$. Regard the figure below:



From Newton's 2nd law, $\vec{F}_g + \vec{F}_s = m\vec{a}_c$.

EVALUATE At the north pole, $\theta = 90^\circ$ and there's no centripetal acceleration, $a_c = 0$. The scale and gravity forces are balanced, so the scale reads your actual weight: $F_{s,p} = mg$. On the other hand, at the equator the centripetal acceleration has a maximum magnitude and points directly opposite to gravity. So the scale reads

$$F_{s,e} = mg - ma_c = mg\left(1 - \frac{4\pi^2 R_E}{gT^2}\right)$$

Therefore, the equator reading will be less than the north pole reading by a percentage of

$$\frac{F_{s,p} - F_{s,e}}{F_{s,p}} = \frac{4\pi^2 R_E}{gT^2} = \frac{4\pi^2\left(6.37\times 10^6\text{ m}\right)}{\left(9.81\text{ m/s}^2\right)\left(86{,}400\text{ s}\right)^2} = 0.343\%$$

ASSESS Our result shows that you weigh more at the north pole because the centripetal acceleration is zero there. But the difference is hardly noticeable.

63. **INTERPRET** This problem involves Newton's second law, uniform circular motion, and frictional forces. The object of interest is the car, and we are to find whether braking in a straight line will stop the car before it hits the truck, or whether it's better to swerve in as tight a circular turn as possible. The forces acting on the car are the force due to gravity $\vec{F}_g = m\vec{g}$ and the force due to kinetic friction $\vec{f}_k$ for the former option and the force due to static friction $\vec{f}_s$ for the latter option.

DEVELOP For the braking option, Newton's second law applied to the car in the *x* and *y* directions gives

$$\left.\begin{aligned} x:&\quad f_k = ma \\ y:&\quad n - mg = 0 \end{aligned}\right\} \mu_s g = a,$$

where we have used Equation 5.3 $f_s = \mu_s n$. For the swerve option, Newton's second law applied in the x and y directions gives

$$\left.\begin{array}{ll} x: & f_s = ma = mv^2/r \\ y: & n - mg = 0 \end{array}\right\} \mu_s g = v^2/r$$

Use the kinematic Equation 2.11 $v^2 = v_0^2 + 2a(x - x_0)$ to find the stopping distance in the braking option, and calculate the turning radius r for the swerve option. Compare these results to decide which option to take.

EVALUATE For the braking option, the stopping distance is

$$\overbrace{v^2}^{=0} = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = -\frac{v_0^2}{2a} = -\frac{v_0^2}{2(-\mu_s g)} = \frac{v_0^2}{2\mu_s g}$$

where the acceleration has a negative sign because it is oriented opposite to the velocity. For the swerving option, the turning radius is $r = v^2/\mu_s g = (x - x_0)$. Thus the turning radius is greater then the stopping distance, so you should chose to brake in a straight line rather than swerve.

ASSESS Note that if the coefficient of static friction decreases from its maximum value of μ_s, the turning radius will get larger, and the linear acceleration will decrease, as expected.

64. **INTERPRET** This problem involves Newton's second law and frictional forces. The object of interest is the block that slides up an incline and back down again. We want to show that when the coefficient of kinetic friction is $\mu_k = \frac{3}{5}\tan\theta$, the final speed of the block half its initial speed.

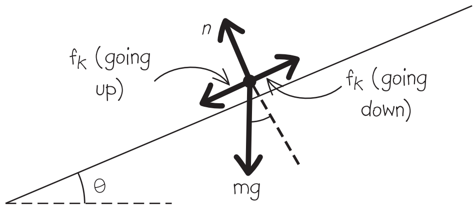
DEVELOP Draw a free-body diagram of the block going up and going down the incline (see figure below). Using Equation 5.3 $f_k = \mu_k n$, and applying Newton's law to the block going up the incline gives

$$\left.\begin{array}{ll} x: & mg\sin\theta + f_k = ma_{up} \\ y: & n - mg\cos\theta = 0 \end{array}\right\} a_{up} = g\sin(\theta) + \mu_k g\cos(\theta)$$

When going down, the acceleration is

$$\left.\begin{array}{ll} x: & mg\sin\theta - f_k = ma_{down} \\ y: & n - mg\cos\theta = 0 \end{array}\right\} a_{down} = g\sin(\theta) + \mu_k g\cos(\theta)$$

Knowing the acceleration, use Equation 2.11, $v^2 = v_0^2 + 2a(x - x_0)$ to express the initial upward speed and the final downward speed, then use the fact that $v_{down}/v_{up} = 1/2$ to find an expression for the coefficient of kinetic friction.



EVALUATE Suppose the block slides up a distance L. From Equation 2.11, its initial speed upward is

$$v_{up} = \sqrt{-2a_{up}L}$$

Similarly, as the block slides down the same distance, it returns to the bottom with speed

$$v_{down} = \sqrt{2a_{down}L}$$

The condition $v_{down}/v_{up} = 1/2$ implies

$$\frac{1}{4} = \left(\frac{v_{down}}{v_{up}}\right)^2 = \frac{2a_{down}L}{-2a_{up}L} = \frac{g\left[\sin(\theta) - \mu_k\cos(\theta)\right]}{g\left[\sin(\theta) + \mu_k\cos(\theta)\right]} = \frac{\tan(\theta) - \mu_k}{\tan(\theta) + \mu_k}$$

which gives $\mu_k = \frac{3}{5}\tan(\theta)$.

ASSESS To see that our result makes sense, let's check some limiting cases. If the incline were frictionless with $\mu_k = 0$ then we would expect $v_{down} = v_{up}$, which is satisfied. However, if the coefficient of kinetic friction becomes too large [$\mu_k > \tan(\theta)$, see Example 5.10], the block will not slide down at all $v_{down} = 0$.

65. **INTERPRET** This problem involves Newton's second law, Hooke's law (see Equation 4.9), and uniform circular motion. The object of interest is the m = 2.1-kg mass, and we are to find the radius of its circular trajectory given the spring constant and the tangential speed at which it travels. Because the table is frictionless, we only need consider the forces acting horizontally, so the only force of interest is the radial force due to the spring.

DEVELOP In the horizontal plane, Newton's second law gives $k(r - r_0) = ma = mv^2/r$, where r_0 = 18 cm is the unstretched spring length and r is the stretched spring length that we are to find.

EVALUATE Inserting the given quantities into the above expression gives

$$k(r - r_0) = mv^2/r$$
$$kr^2 - kr_0 r - mv^2 = 0$$
$$r = \frac{kr_0 \pm \sqrt{k^2 r_0^2 + 4kmv^2}}{2k} = \frac{1}{2}\left[0.18\text{ m} \pm \sqrt{(0.18\text{ m})^2 + 4(2.1\text{ kg})(1.4\text{ m/s})^2/(150\text{ N/m})}\right] = 0.28\text{ m}$$

ASSESS Can you convince yourself that the units under the radical work out to be meters?

66. **INTERPRET** We are asked to graph the tension force from Example 5.11 as a function of angle. From the graph, we can determine the minimum force necessary to move the trunk and the angle for this minimum force.

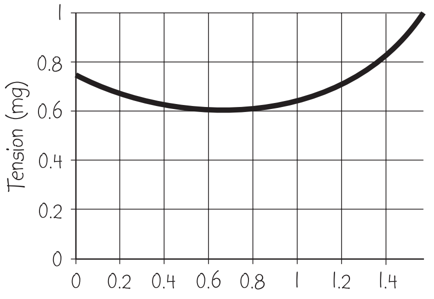
DEVELOP The equation given in Example 5.11 is

$$T = \frac{\mu_k mg}{\cos(\theta) + \mu_k \sin(\theta)}$$

If we plot T/mg accurately versus θ we can read the minimum force ratio and the optimal angle directly from the plot.

EVALUATE **(a)** From the figure below, we see that the minimum force ratio is 0.6, so the minimum force required is 60% of the weight of the trunk.

(b) The angle at which the minimum occurs is about 0.65 radians, or $\theta \approx (0.65\text{ radian}) \times 180/\pi = 37°$.



ASSESS The shape of the graph makes sense, intuitively. As we increase the angle, we decrease the frictional force, up to a point. At too large of an angle, we spend too much effort trying to lift the trunk instead of moving it. At an angle of $\pi/2$ (90°), the force is equal to the trunk's weight, because we are lifting it straight up.

The exact solution (see Problem 5.64) is 60% of the trunk's weight, at an angle of 36.9°. This graphical method of solution is not bad!

67. **INTERPRET** We need to find the minimum force necessary to move the trunk in Example 5.11, for an arbitrary value of μ_k. We also need to find the angle at which this minimum occurs. We can do this using calculus to find the minimum of the function.

DEVELOP For simplicity, we define the reduced tension

$$T' \equiv \frac{T}{mg} = \frac{\mu_k}{\cos(\theta) + \mu_k \sin(\theta)}$$

Then we can find the minimum value of T' by setting $dT'/d\theta = 0$ and solving for the optimal angle θ.

EVALUATE

$$\frac{dT'}{d\theta} = 0 = \mu_k \frac{d}{d\theta}\left[\cos(\theta) + \mu_k \sin(\theta)\right]^{-1} = -\mu_k \frac{-\sin(\theta) + \mu_k \cos(\theta)}{\left[\cos(\theta) + \mu_k \sin(\theta)\right]^2}$$

The numerator must be zero for this equation to be satisfied, so

$$\sin\theta = \mu_k \cos\theta_k$$
$$\theta = \text{atan}(\mu_k)$$

We substitute this value of θ into the expression for T' to obtain

$$\begin{aligned} T' &= \frac{\mu_k}{\cos\left[\text{atan}(\mu_k)\right] + \mu_k \sin\left[\text{atan}(\mu_k)\right]} \\ &= \frac{\mu_k}{\left(\sqrt{1+\mu_k^2}\right)^{-1} + \mu_k^2\left(\sqrt{1+\mu_k^2}\right)^{-1}} \\ &= \frac{u_k\sqrt{1+\mu_k^2}}{1+\mu_k^2} \\ &= \frac{u_k}{\sqrt{1+\mu_k^2}} \end{aligned}$$

ASSESS We can check our answer by substituting $\mu_k = 0.75$ in our answer and comparing the result with the graphical solution from Problem 5.66. We find $\theta = \text{atan}(0.75) = 36.9°$ and $T' = 0.75\Big/\sqrt{1+(0.75)^2} = 0.6.$ These answers match previously with what we obtained.

68. **INTERPRET** We need to find an equation for the speed of an object as a function of time, where instead of just a constant acceleration due to gravity there is also a drag force that depends on velocity. We also need to find the terminal velocity of the object.

DEVELOP We choose the upward direction to be positive. Instead of $F = -mg = ma$ we have $F = -mg - bv = ma$. Since the acceleration is $a = dv/dt$, we have a differential equation: $\frac{dv}{dt} + \frac{b}{m}v + g = 0$. The initial velocity of the object is zero. We find terminal velocity by finding the value of $v(t)$ in the limit as $t \to \infty$.

EVALUATE (a) We assume that $v(t) = Ae^{-bt/m} + B$. Substituting this into the differential equation, we find $B = -mg/b$. Using the fact that $v(0) = 0$, we find $A = mg/b$, so the object's speed as a function of time is

$$v(t) = \frac{mg}{b}\left(e^{-bt/m} - 1\right)$$

(b) The terminal velocity is the speed of the object at large time:

$$v_{\text{term}} = \lim_{t\to\infty} v(t) = \frac{-mg}{b}$$

The minus sign is here because the velocity is downwards in the negative direction.

ASSESS Our units check out for the equation in part (a), and the equation has a form consistent with what we would expect. A force of $-bv$ is sometimes called Stokes' drag. It applies for viscous fluids in which objects move relatively slowly.

69. **INTERPRET** We need to develop an equation for the trajectory of an object with an initial horizontal velocity. The object has a velocity-dependent drag force like that of Problem 68, and the initial x velocity is equal to the terminal velocity in Problem 68. We also need to find the maximum horizontal distance the object can move.

DEVELOP We find the horizontal and vertical motions separately. We can do this because the force is linear with velocity, as are the individual components. The equation for the vertical component of velocity has been solved in Problem 68: $v_y(t) = \frac{mg}{b}\left(e^{-bt/m} - 1\right)$; we will need to integrate this once more to find vertical position $y(t)$. As for the horizontal motion, the only force is the drag ($F = -bv_x$), so by Newton's second law we have differential equation: $-bv_x = m\frac{dv_x}{dt}$. We find the solution to this, then integrate it to get the equation for horizontal motion as a function of time, $x(t)$, and then take the limit as $t \to \infty$ to find the maximum range. We then invert the equation for horizontal motion to get an equation for $t(x)$, and plug this into our equation for $y(t)$ to find $y(x)$.

EVALUATE First, we find $y(t)$ by separating the y variable from the t variable in the velocity equation:

$$v_y(t) = \frac{dy}{dt} = \frac{mg}{b}\left(e^{-bt/m} - 1\right) \quad \to \quad dy = \frac{mg}{b}\left(e^{-bt/m} - 1\right)dt$$

Integrating both sides of the equation gives

$$\int dy = \int \frac{mg}{b}\left(e^{-bt/m} - 1\right)dt \quad \to \quad y(t) = \frac{mg}{b}\left[-\frac{m}{b}e^{-bt/m} - t + C_1\right]$$

We'll assume the object starts at the origin, so $y(0) = 0 = \frac{mg}{b}[-\frac{m}{b} + C_1]$, which implies $C_1 = \frac{m}{b}$. In final form, the vertical position as a function of time is

$$y(t) = \frac{mg}{b}\left[\frac{m}{b}\left(1 - e^{-bt/m}\right) - t\right]$$

Now, turning to the horizontal direction, the differential equation for the velocity can be solved with a function of the form $v_x(t) = Ae^{-bt/m}$. The initial horizontal velocity is $v_{x0} = \frac{mg}{b}$ so $A = \frac{mg}{b}$ and $v_x(t) = \frac{mg}{b}e^{-bt/m}$. We integrate this like we did $v_y(t)$ to find the horizontal position:

$$v_x(t) = \frac{dx}{dt} = \frac{mg}{b}e^{-bt/m} \quad \to \quad x(t) = \int \frac{mg}{b}e^{-bt/m}dt = -\frac{m^2g}{b^2}e^{-bt/m} + C_2$$

We have assumed $x(0) = 0$, so it follows that $C_2 = \frac{m^2g}{b^2}$ and

$$x(t) = \frac{m^2g}{b^2}\left(1 - e^{-bt/m}\right)$$

Since x gets larger as t gets larger, the maximum value of x is

$$x_{\max} = \lim_{t\to\infty}[x(t)] = \frac{m^2g}{b^2} = \frac{mv_{x0}}{b}$$

We now invert the equation $x(t)$ by rearranging the terms and taking the natural log of both sides:

$$t(x) = -\frac{m}{b}\ln\left(1 - \frac{x}{x_{\max}}\right)$$

Plugging this into the $y(t)$ equation

$$\begin{aligned} y(t) &= \frac{mg}{b}\left[\frac{m}{b}\left(1 - \exp\left(\frac{-b}{m}\left\{\frac{-m}{b}\ln\left(1 - \frac{x}{x_{\max}}\right)\right\}\right)\right) - \left\{\frac{-m}{b}\ln\left(1 - \frac{x}{x_{\max}}\right)\right\}\right] \\ &= x_{\max}\left[\left(1 - \left(1 - \frac{x}{x_{\max}}\right)\right) + \ln\left(1 - \frac{x}{x_{\max}}\right)\right] \\ &= x + x_{\max}\ln\left(1 - \frac{x}{x_{\max}}\right) \end{aligned}$$

ASSESS Does our answer make sense? Plugging in $x = 0$, $y = 0 + x_{\max}\ln(1) = 0$, which is exactly what we assumed for the starting position. As $x \to x_{\max}$, $y \to -\infty$, which is what we would expect as the object continues to fall straight down through the fluid.

70. **INTERPRET** We are asked to derive an expression for how far a block can be slid up an incline. We then use this equation to calculate the angle of the incline that would minimize this sliding distance.

DEVELOP The three forces acting on the block are gravity, the normal force and kinetic friction. Choose a coordinate system in which the x-axis points up the slope, so that the block is initially moving in the $+x$-direction. The normal force and the y-component of the gravity balance each other out: $n = mg\cos\theta$. However, the forces in the x-direction do not balance out:

$$F_{\text{net},x} = -mg\sin\theta - f_{\text{k}} = ma_x$$

where the kinetic friction is $f_{\text{k}} = \mu_{\text{k}} n$ (Equation 5.3). Notice that here the frictional force points down the slope, in opposition to the direction of motion.

EVALUATE (a) We are asked to derive an expression for the distance the block moves before stopping. Using Equation 2.11,

$$d = \frac{-v_0^2}{2a_x} = \frac{v_0^2}{g\left(\sin\theta + \mu_{\text{k}}\cos\theta\right)}$$

(b) To find the angle that minimizes the distance, we first take the derivative of the above equation:

$$\frac{\text{d}}{\text{d}\theta}[d] = \frac{-v_0^2}{g\left(\sin\theta + \mu_{\text{k}}\cos\theta\right)^2}\left(\cos\theta - \mu_{\text{k}}\sin\theta\right)$$

This derivative equals zero when $\cos\theta = \mu_{\text{k}}\sin\theta$, or $\theta = \cot^{-1}\left(\mu_{\text{k}}\right)$.

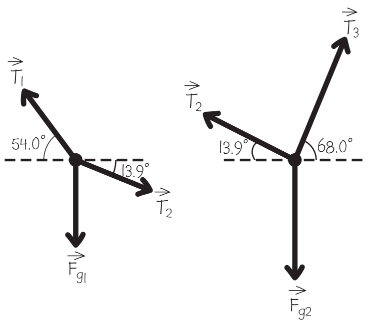
ASSESS One can plug the angle in part (b) into the equation of part (a) and find that the minimum distance is

$$d = \frac{v_0^2}{g\sqrt{1+\mu_{\text{k}}^2}}$$

Furthermore, one can verify that this is a minimum (as opposed to a maximum or an inflection point) by showing that the second derivative of the function $d(\theta)$ is positive at the determined value.

71. **INTERPRET** We need to find the tension in the three strings holding up the flower arrangement. If the tension in any one segment is more than 100 N, then we need heavier string. The flowers are in equilibrium, so we use Newton's first law: The components of the forces on each pot will sum to zero.

DEVELOP We start with a free-body diagram, as shown in the figure below. After resolving each tension into the horizontal and vertical components, we set the sum of components along each axis, for each pot, equal to zero and solve for the tensions.



EVALUATE For the left pot,

$$\left.\begin{aligned} T_{x1} &= -T_1\cos(54.0°) \\ T_{y1} &= T_1\sin(54.0°) \\ T_{x2} &= T_2\cos(13.9°) \\ T_{y2} &= -T_2\sin(13.9°) \\ F_{g1} &= -m_1 g \end{aligned}\right\} \begin{aligned} -T_1\cos(54.0°) + T_2\cos(13.9°) = 0 \\ T_1\sin(54.0°) - T_2\sin(13.9°) - m_1 g = 0 \end{aligned}$$

For the right pot,

$$\left.\begin{aligned} T_{x3} &= T_3\cos(68.0°) \\ T_{y3} &= T_3\sin(68.0°) \\ T_{x2} &= -T_2\cos(13.9°) \\ T_{y2} &= T_2\sin(13.9°) \\ F_{g2} &= -m_2 g \end{aligned}\right\} \begin{aligned} T_3\cos(68.0°) - T_2\cos(13.9°) = 0 \\ T_3\sin(68.0°) + T_2\sin(13.9°) - m_3 g = 0 \end{aligned}$$

This problem is over-specified: There are three unknowns and four equations. However, we can still estimate if any of the forces are greater than 100 N.

Start with the left pot and solve the top equation for T_1 then substitute that value into the second equation to find T_2:

$$T_1 = T_2 \frac{\cos(13.9°)}{\cos(54.0°)}$$

$$T_2 \frac{\cos(13.9°)}{\cos(54.0°)} \sin(54.0°) - T_2 \sin(13.9°) = m_1 g$$

$$T_2\ (1.336 - 0.240) = (3.85\ \text{kg})(9.8\ \text{m/s}^2)$$

$$T_2 = 34.4\ \text{N}$$

Now substitute this back into the equation for T_1 to find that tension: $T_1 = T_2 \cos(13.9°)/\cos(54°) = 56.9$ N.

Do the same thing for the right pot.

$$T_3 = T_2 \frac{\cos(13.9°)}{\cos(68.0°)}$$

$$T_2 \frac{\cos(13.9°)}{\cos(68.0°)} \sin(68.0°) + T_2 \sin(13.9°) = m_3 g$$

$$T_2 (2.403 + 0.240) = (9.28\ \text{kg})(9.8\ \text{m/s}^2)$$

$$T_2 = 34.4\ \text{N}$$

Now substitute this back into the equation for T_3 to find $T_3 = T_2 \cos(13.9°)/\cos(68.0°) = 89.2$ N . Thus, the 100 N string will suffice.

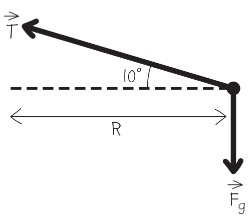
ASSESS Although 100 N string will work, we would be better off with a bigger margin of error. As it is, the far-right string will break if anyone waters the plants!

72. **INTERPRET** This problem involves Newton's second law and uniform circular motion. We need to compare the tangential speed of the hammer as it goes around the circle with that of a "speeding bullet." The forces acting on the hammer are the force of gravity $\vec{F}_g = m\vec{g}$, and the tension force from the cable.

DEVELOP Draw a free-body diagram of the hammer as seen from the side (see figure below). Applying Newton's second law in the horizontal and vertical directions gives

$$\left.\begin{array}{ll} x: & T\cos(\theta) = ma = mv^2/r \\ y: & T\sin(\theta) = mg \end{array}\right\} \frac{v^2}{r} = \frac{g}{\tan(\theta)}$$

where we have used Equation 5.1 $a = mv^2/r$ for the centripetal acceleration that the hammer experiences. We can now solve for the speed.



EVALUATE Solving the above equation for the tangential speed v gives

$$v^2 =$$

$$v = \pm\sqrt{\frac{rg}{\tan(\theta)}} = \pm\sqrt{\frac{(2.4\ \text{m})(9.8\ \text{m/s}^2)}{\tan(10°)}} = \pm 11.5\ \text{m/s}$$

which is an order of magnitude slower than a speeding bullet.

ASSESS Notice that the units under the radical are m^2/s^2. The positive and negative answers correspond to the hammer turned clockwise and counter clockwise around the circle.

73. **INTERPRET** This problem involves uniform circular motion. We need to find the minimum radius, at a given speed for a vertical circle, if the acceleration is not to exceed six times that of gravity. We find the centripetal acceleration of the plane, and remember that gravity is also a factor.

DEVELOP Converting the speed and the acceleration to SI units gives $1.8\times(340\text{ m/s}) = 612\text{ m/s}$. Note that gravity provides 1g of acceleration no matter how fast the pilot is flying, so the centripetal acceleration has to provide the remaining 5g. Because we assume uniform circular motion, we can use Equation 5.1 a = v^2/r to calculate the centripetal acceleration.

EVALUATE Inserting $a = 5g = 49\text{ m/s}^2$ into Equation 5.1 and solving for the radius r gives

$$r = \frac{v^2}{a} = \frac{(612\text{ m/s})^2}{5(9.8\text{ m/s}^2)} = 7.6\text{ km}$$

ASSESS If he wants to make a smaller loop, he will have to slow down! But notice that the acceleration is a quadratic function of velocity ($a \propto v^2$), so a 50% reduction in speed (while maintaining the same radius) would reduce the total acceleration to $5g/4 + 1g = 2.25g$.

74. **INTERPRET** We are asked to analyze the movement of a figure skater.

DEVELOP The skater is leaning to her left.

EVALUATE We can imagine that her leaning will lead to a turn to her left.

The answer is (a).

ASSESS Although not many of us can perform figure skating maneuvers such as this, we probably can relate to leaning into a turn while running or roller skating or riding a bicycle.

75. **INTERPRET** We are asked to analyze the movement of a figure skater.

DEVELOP The forces acting on the skater are gravity, the normal force, and friction. The friction is the only force pointing in the horizontal direction, and it must account for the centripetal acceleration that points to her left.

EVALUATE To turn left, the net force will have to point left.

The answer is (a).

ASSESS If there were no friction force pointing to the left, the leaning skater's foot would slip out from underneath her and she would fall. In Chapter 12, we'll learn how to analyze a problem such as this by considering the net torque on the skater.

76. **INTERPRET** We are asked to analyze the movement of a figure skater.

DEVELOP At higher speed but the same radius, the skater will need a higher centripetal acceleration ($a_c = v^2/r$). This can happen only if she leans more into the turn.

EVALUATE At higher speed, the tilt would need to be greater.

The answer is (b).

ASSESS Again, from our own experience we know that the faster we go through a turn, the harder we have to lean into it.

77. **INTERPRET** We are asked to analyze the movement of a figure skater.

DEVELOP The three forces do not all act on the same part of the skater's body. The weight is applied at the center of mass, while the normal and friction forces are applied at the skate. If we imagine a line from the skate to the center of mass, there cannot be any net force perpendicular to this line, otherwise the skater will start to rotate around her center of mass (see Problem 5.40). The sum of the vertical normal force and the horizontal friction force is a force pointing in the direction $\theta = \tan^{-1}(f_s/n)$ with respect to the vertical. The skater should be leaning at this angle to avoid having a rotating force (or torque).

EVALUATE The normal balances the downward weight, $n = mg$, and the friction is providing the centripetal force: $f_s = ma_c$. Therefore, if the skater tilt is $\theta = \tan^{-1}(0.5)$, the centripetal acceleration must be $a_c = 0.5g \approx 5\text{ m/s}^2$.

The answer is (c).

ASSESS As mentioned in Problem 5.75, we will later learn specific techniques for how to deal with forces applied away from the center of mass.