WORK, ENERGY, AND POWER

6

EXERCISES

Section 6.1 Work

11. **INTERPRET** This problem involves the concept of work. You are doing work on the shopping cart by pushing it around.

DEVELOP Assume the force is constant and is applied in the horizontal direction, in which case this is a one-dimensional problem and Equation 6.1 applies.

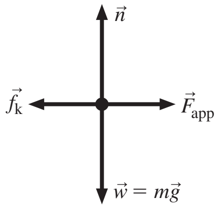
EVALUATE Inserting the given quantities into Equation 6.1 gives the work done as

$$W = F\Delta x = (75\ \text{N})(12\ \text{m}) = 900\ \text{J}$$

ASSESS If it takes you 30 seconds to cover this distance, the power expended would be $P = W/\Delta t = (900\ \text{J})/(30\ \text{s}) = 30\ \text{W}$. This gives you some appreciation for the energy needed to power a 60-W light bulb.

12. **INTERPRET** This problem involves work and forces due to friction (see Chapter 5). The relevant physical quantity here is the work done by the person on the box.

DEVELOP Draw a free-body diagram for the box (see figure below). Because the box moves at a constant speed, we know from Newton's second law $F_{\text{net}} = ma$ that the net force is constant at zero, so the force applied $\vec{F}_{\text{app}}$ must be constant. Being a one-dimensional problem, we can apply Equation 6.1. From the free-body diagram, we see that $F_{\text{app}} = f_{\text{k}}$, and $n = mg$. From Equation 5.3 $\vec{f}_{\text{k}} = \mu_k n$, we find that the force applied must be $F_{\text{app}} = \mu_{\text{k}} mg$. Insert this into Equation 6.1 to find the work done.



EVALUATE The work done by pushing the box a distance $\Delta x = 4.8$ m is

$$W = F_{\text{app}}\Delta x = \mu_{\text{k}} mg\Delta x = (0.21)(50\ \text{kg})(9.8\ \text{m/s}^2)(4.8\ \text{m}) = 490\ \text{J}$$

to two significant figures.

ASSESS The units are correct, $1\ \text{J} = 1\ \text{N}\cdot\text{m} = 1\ \text{kg}\cdot\text{m/s}^2\cdot\text{m}$. If the floor were frictionless ($\mu_{\text{k}} = 0$), then the work done would be zero, as expected.

13. **INTERPRET** The problem involves work, which is done by the crane on the beam. We are to find the work done to lift the box vertically 23 m.

DEVELOP From the definition of work as the scalar product of force and distance (see Equation 6.3), we see that no work is done when the force applied is perpendicular to the displacement. This is the case for the crane when it swings the beam eastward by 18 m. The crane applies a vertical force (to counter gravity) and the displacement is horizontal (eastward). Thus, we need only concern ourselves with the vertical displacement of the beam. Furthermore, if the beam moves with constant speed, we know that the vertical force applied must be constant and be equal to the weight of the box ($F_{app} = mg$, see previous problem). Thus, we can apply Equation 6.1 to the vertical displacement to find the work done.

EVALUATE Inserting the given quantities into Equation 6.1 gives the work done:

$$W = F_{app}\Delta y = mg\Delta y = (650 \text{ kg})(9.8 \text{ m/s}^2)(23 \text{ m}) = 150 \text{ k J}$$

to two significant figures.

ASSESS We could have used the more general Equation 6.5 to find the work. This gives

$$W = \vec{F}\cdot\Delta\vec{r} = F_{app}\hat{j}\cdot(18 \text{ m}\hat{i} + 23 \text{ m}\hat{j}) = F_{app}(23 \text{ m}) = 150 \text{ k J}$$

which agrees with our previous result.

14. **INTERPRET** This problem is about the work done by gravity (i.e., by the Earth) on the water that passess over the lip of Cherun-Meru. Thus, it is a one-dimensional problem involving work done by a constant force.

DEVELOP Since the density of water is 1000 kg/m^3, the mass of a cubic meter of water is 1000 kg, and the force of gravity at the Earth's surface on a cubic meter of water is constant at

$$F_g = mg = (1000 \text{ kg})(9.8 \text{ m/s}^2) = 9800 \text{ N}$$

vertically downward. We can then use Equation 6.1, $W = F\Delta x$, to find the work done.

EVALUATE The work done by gravity on the water is

$$W = F_g\Delta y = (9800 \text{ N})(980 \text{ m}) = 9.6\times 10^6 \text{ J}$$

ASSESS The units are correct; 1 J = 1 N·m. The greater the distance the water falls, the larger the amount of work done by gravity. Would the work be different if the water followed a different path from the top of the falls to the bottom?

15. **INTERPRET** This problem involves the average force exerted by the meteorite on the Earth. It is a one-dimensional problem because all forces and displacements are in the same direction (i.e., vertical).

DEVELOP Because we are interested in the average force, which is constant during the meteorite's deceleration period, we can use Equation 6.1 $W = F\Delta x$ to find the average force. We are given the W = 140 MJ and Dx = 75 cm = 0.75 m.

EVALUATE Solving Equation 6.1 for the force and inserting the given quantities gives an average force of

$$W = F\Delta x$$
$$F = \frac{W}{\Delta x} = \frac{140 \text{ MJ}}{0.75 \text{ m}} = 190 \text{ MN}$$

to two significant figures.

ASSESS Notice that we did not need to convert from MJ to J, we simply retained the prefactor M (= 10^6) in our calculation. Thus, the units of MN are units of force. Using the fact that dynamite carries 7.5 MJ/kg of explosive energy, this meteorite impact delivered the equivalent of (140 MJ)/(7.5 MJ/kg) ≈ 19 kg of dynamite (about 41 lbs).

16. **INTERPRET** This problem is about the work done by the elevator cable on the elevator as it accelerates upward. It is a one dimensional problem and also involves Newton's second law. We are asked to find an expression for the work done to lift the elevator the given height.

DEVELOP Applying Newton's second law to the elevator gives

$$T - mg = ma_y \quad \Rightarrow \quad T = m(g + a_y)$$

where T is the cable tension force and $a_y = 0.1g$ is the upward acceleration of the elevator. Because the elevator is displaced parallel to the force, we can insert this result for the tension force into Equation 6.1, $W = F\Delta y$, to find an expression for the work done by the cable.

EVALUATE The work done by the cable on the elevator is

$$W = T\Delta y = m(g + a_y)\Delta y = m(g + 0.1g)h$$
$$T = 1.1mgh$$

ASSESS The units are correct, 1 J = 1 N·m. The greater the upward acceleration a_y, the more work must be done by the cable. Of course, if the elevator undergoes free fall, $a_y = -g$ and the tension in the cable is zero, so no work is done on the elevator.

17. **INTERPRET** This problem is an exercise in vector properties. We are asked to show that the scalar product (or dot product) of two vectors is distributive.

DEVELOP Use the definition of the scalar product (Equation 6.4) to demonstrate the distributive property of the vector scalar product.

EVALUATE Using the definition of the vector scalar product, we see that

$$\begin{aligned}\vec{A}\cdot\left(\vec{B}+\vec{C}\right) &= A_x\left(B_x + C_x\right) + A_y\left(B_y + C_y\right) + A_z\left(B_z + C_z\right)\\ &= A_xB_x + A_yB_y + A_zB_z + A_xC_x + A_yC_y + A_zC_z\\ &= \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}\end{aligned}$$

ASSESS We could also use Equation 6.3 to demonstrate the distributive property. This gives

$$\vec{A}\cdot\left(\vec{B}+\vec{C}\right) = AB\cos\left(\theta_{AB}\right) + AC\cos\left(\theta_{AC}\right) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$$

18. **INTERPRET** This problem involves finding the work done by a force moving an object through a given displacement.

DEVELOP Because this is a two dimensional problem, we will use Equation 6.5, $W = \vec{F}\cdot\Delta\vec{r}$, to find the work. We are given that $\vec{F} = 1.8\hat{i} + 2.2\hat{j}$ N and $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \left(56\hat{i} + 31\hat{j}\text{ m}\right) - \left(0\hat{i} + 0\hat{j}\text{ m}\right) = 56\hat{i} + 31\hat{j}$ m .

EVALUATE Inserting the given quantities into Equation 6.5, we find that the work done is

$$W = \left(1.8\hat{i} + 2.2\hat{j}\text{ N}\right)\cdot\left(56\hat{i} + 31\hat{j}\text{ m}\right) = (1.8\text{ N})(56\text{ m}) + (2.2\text{ N})(31\text{ m}) = 170\text{ J}$$

to two significant figures.

ASSESS We find that the units of the scalar product come out to N·m, just as when we multiply scalars quantities.

19. **INTERPRET** This problem involves the concept of work. We are asked to find the distance a stalled car can be moved by a given amount of work.

DEVELOP Because the force is directed at 17° to the car's displacement vector, we must use Equation 6.2, W = $F\cos(\theta)\Delta r$.

EVALUATE Solving Equation 6.2 for Dr, and inserting the given quantitites, we find that the distance the car is moved is

$$\Delta r = \frac{W}{F\cos\theta} = \frac{860\text{ J}}{(470\text{ N})\cos(17°)} = 1.9\text{ m}$$

ASSESS Only the horizontal component of the force, $F_x = F\cos\theta$, does the work. The vertical part of the force simply modifies the normal force experienced by the car.

Section 6.2 Forces that Vary

20. **INTERPRET** This problem involves calculating the work done by a varying force to cover two distances. This is a one-dimensional problem.

DEVELOP We will apply Equation 6.8,

$$W = \int_{x_1}^{x_2} F(x)\,dx$$

to find the work done. The force is $F(x) = \mu_k(x)n = \left(\mu_0 + ax^2\right)mg$, and we are to find the work done from $x_1 = 0$ km to $x_2 = 3$ km and from $x_1 = 3$ km to $x_2 = 4$ km.

EVALUATE (**a**) Inserting $F(x)$ into the integral and evaluating it from $x_1 = 0$ km to $x_2 = 3$ km gives

$$W = \int_{x_1}^{x_2} mg\left(\mu_0 + ax^2\right)dx = mg\left(\mu_0 x + \frac{1}{3}ax^3\right)\Bigg|_{x_1}^{x_2}$$

$$= mg\left[\mu_0\left(x_2 - x_1\right) + \frac{a}{3}\left(x_2^3 - x_1^3\right)\right] = (180\text{ kg})(9.8\text{ m/s}^2)\left\{(0.17)(3\text{ m} - 0\text{ m}) + \frac{0.0062\text{ m}^{-2}}{3}\left[(3\text{ m})^3 - (0\text{ m})^3\right]\right\}$$

$$= 1900\text{ J}$$

to two significant figures.

(**b**) Repeating the exercise for x_1 = 3 km to x_2 = 4 km gives

$$W = (180\text{ kg})(9.8\text{ m/s}^2)\left\{(0.17)(4\text{ m} - 3\text{ m}) + \frac{0.0062\text{ m}^{-2}}{3}\left[(4\text{ m})^3 - (3\text{ m})^3\right]\right\}$$

$$= 1600\text{ J}$$

to two significant figures.

ASSESS Notice that the constant has dimensions of m^{-2}, so that the units work out to be N·m = J.

21. **INTERPRET** This problem involves the work done to stretch a spring from equilibrium to a given distance, and from that distance to a further distance.

DEVELOP The problem can be solved by using Equation 6.8, from which Equation 6.10 is derived. [Notice that Equation 6.10 applies to the special case where one of the endpoints is the equilibrium position of the spring, which is not the case for part (b) of the problem.] The force applied to the spring is $F(x) = kx$, so Equation 6.8 gives

$$W = \int_{x_1}^{x_2} F(x)\,dx = \int_{x_1}^{x_2} (-kx)\,dx = \frac{k}{2}\left(x_1^2 - x_2^2\right)$$

where x_1 and x_2 are the initial and final displacements from equilibrium, respectively.

EVALUATE **(a)** The amount of work done in stretching from x_1 = 0 m to x_2 = 0.1 m is

$$W = \frac{200\text{ N/m}}{2}\left[(0.1\text{ m})^2 - (0\text{ m})^2\right] = 1\text{ J}$$

(**b**) Similarly, to stretch from x_1 = 0.1 m to x_2 = 0.2 m from equilibrium requires

$$W = \frac{200\text{ N/m}}{2}\left[(0.2\text{ m})^2 - (0.1\text{ m})^2\right] = 3\text{ J}$$

ASSESS Another way to solve part **(b)** is to note that the work to stretch the spring from 0 to 20 cm is four times the work from part **(a)**, or 4.0 J, so the work in part **(b)** is 4.0 J − 1.0 J = 3.0 J.

22. **INTERPRET** We must find the work necessary to compress a spring a given distance, given the spring constant. We will use the most general equation for work in one dimension.

DEVELOP The general equation for work in one dimension is $W = \int_{x_1}^{x_2} \vec{F}\cdot d\vec{x}$. By Newton's third law, the force applied by the mechanic is equal and opposite to the spring force, $\vec{F} = -k\vec{x}$, so we can substitute $\vec{F} = k\vec{x}$ into the equation for *W*. The initial displacement with respect to the equilibrium position is x_1 = 0 m and the final displacement is x_2 = 0.45 m – 0.32 m = 0.13 m.

EVALUATE Inserting the given quantities into the expression for work gives

$$W = \int_{x_1}^{x_2} \vec{F}(x)\cdot d\vec{x} = \int_{x_1}^{x_2} kx\,dx = \frac{k}{2}\left(x_2^2 - x_1^2\right) = \frac{\left(3.8\times10^3\text{ N/m}\right)(0.13\text{ m})^2}{2} = 32\text{ J}$$

ASSESS Note that we don't use the initial and final lengths of the spring! The *x* in the spring force equation is the displacement from the *equilibrium* position, which in this case is 45 cm – 32 cm = 13 cm.

23. **INTERPRET** The problem is about work done to stretch a spring. We want to find out how much the spring can be stretched with a given amount of work.

DEVELOP Because the spring is stretched starting from its equilibrium position, the result of Equation 6.10, $W = kx^2/2$ can be applied. In this expression, x represents the distance from equilibrium that the spring is stretched (or compressed).

Evaluate Solve Equation 6.10 for x and insert the given quantities. This gives

$$x = \pm\sqrt{\frac{2W}{k}} = \sqrt{\frac{2(8.5\text{ J})}{190\text{ N/m}}} = 0.299\text{ m} = 30\text{ cm}$$

to two significant figures. We have chosen the positive square root to reflect the fact that the spring is stretched, not compressed.

Assess Notice that x is inversely proportional to $\sqrt{k}$. This means that the stiffer the spring (greater k), the less it will be stretched, and vice versa. Also note that the work needed to stretch a spring an amount x is the same as is needed to compress it by this same amount.

24. **Interpret** We're asked how much work does a fly impart on a spider silk strand, assuming the strand acts like a simple spring.

Develop As calculated for Equation 6.10, the work done on a spring when stretching it is: $W = \frac{1}{2}kx^2$.

Evaluate Using the spring constant of the strand and the distance it stretches when a fly hits it, the work done is

$$W = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(70\text{ mN/m})(0.096\text{ m})^2 = 0.32\text{ mJ}$$

Assess When the fly hits the strand, it transfers some significant fraction of its kinetic energy into the work used to stretch the strand. We can estimate the fly's kinetic energy before the impact. Let's assume the fly has a mass of roughly 1 g and that its speed is around 1 m/s. Then, its kinetic energy $\left(K = \frac{1}{2}mv^2\right)$ is 0.5 mJ. So the work we calculated above seems reasonable.

Section 6.3 Kinetic Energy

25. **Interpret** This problem involves kinetic energy. The object of interest is the airplane, and we are to find its kinetic energy given its mass and velocity.

Develop This a straight-forward application of Equation 6.13, $K = mv^2/2$, where K is the kinetic energy, $m = 2.4 \times 10^5$ kg is the mass, and $v = 900$ km/h is the speed.

Evaluate The kinetic energy of the airplane is thus

$$K = \frac{1}{2}mv^2 = \frac{(2.4\times10^5\text{ kg})(900\text{ km/h})^2}{2}\left(\frac{10^3\text{ m}}{\text{km}}\right)^2\left(\frac{\text{h}}{3600\text{ s}}\right)^2 = 7.5\times10^9\text{ J} = 7.5\text{ G J}$$

Assess The units work out to be

$$\frac{\text{kg}\cdot \cancel{\text{km}}^2\cdot\text{m}^2\cdot\cancel{\text{h}}^2}{\cancel{\text{h}}^2\cdot\cancel{\text{km}}^2\cdot\text{s}^2} = \text{N}\cdot\text{m} = \text{J}$$

as expected.

26. **Interpret** How much work is done in accelerating a particle from rest to some final speed? We use the work-energy theorem.

Develop The relationship between work and kinetic energy is $W = \Delta K$ (Equation 6.14). $K \equiv \frac{1}{2}mv^2$, so we can use the mass of a proton (1.67×10^{-27} kg from the physical constants table on the front inside cover) and the given final velocity ($21\text{ Mm/s} = 2.1\times10^7\text{ m/s}$) to find the change in K and thus the work.

Evaluate Using the fact that the initial velocity is zero, the work is

$$W = \tfrac{1}{2}mv_\text{f}^2 - \tfrac{1}{2}mv_\text{i}^2 = \tfrac{1}{2}mv_\text{f}^2 = \tfrac{1}{2}(1.67\times10^{-27}\text{ kg})(2.1\times10^7\text{ m/s})^2 = 3.7\times10^{-13}\text{ J}$$

Assess In particle physics problems such as this one, energies are often given in the more conveniently sized unit of *electron volts*. $1\text{ eV} = 1.6\times10^{-19}$ J, so the amount of work done in this case is 2.3 MeV.

27. **Interpret** This problem involves kinetic energy. We are to find the speed at which the small car must travel so that it has the same kinetic energy as the large truck.

Develop We will use Equation 6.13, K =mv2/2, to find the kinetic energy of each vehicle. By setting their kinetic energies equal, we can solve for the speed of the car.

EVALUATE Let the car's variables carry the subscript c, and the truck's variables carry the subscript T. The kinetic energy of each is $K_c = m_c v_c^2/2$ for the car and $K_T = m_T v_T^2/2$ for the truck. Setting these equal and solving for v_c gives

$$\frac{1}{2}m_c v_c^2 = \frac{1}{2}m_T v_T^2$$

$$v_c = \pm v\sqrt{\frac{m_T}{m_c}} = \pm(20\text{ km/h})\sqrt{\frac{3.2\times10^4\text{ kg}}{950\text{ kg}}} = \pm120\text{ km/h}$$

ASSESS The plus/minus sign indicates that the car can either travel in the same direction as the truck, or in the opposite direction. Notice that we did not need to convert km/h to m/s for this problem, because the units of kg under the radical cancel.

28. **INTERPRET** The object of interest is the skateboarder. We are asked to find the total (i.e., net) work done on the skateboarder between the top and bottom of the hill.

DEVELOP The work-energy theorem states that the net work done on an object equates to its change in kinetic energy, which we can calculate for the skateboarder from the information given. The relevant equations here are Equations 6.12 $K = mv^2/2$ that gives the kinetic energy and Equation 6.14 (work-energy theorem), $\Delta K = W_{\text{net}}$, which relates the net work done to the change in kinetic energy. Given the initial velocity v_1 and the final velocity v_2, the net work done on the skateboarder can be calculated.

EVALUATE From the work-energy theorem we find the net work done by gravity (i.e., the Earth) on the skateboarder is

$$W_{\text{net}} = \Delta K = \frac{1}{2}m\left(v_2^2 - v_1^2\right) = \frac{1}{2}(60\text{ kg})\left[(10\text{ m/s})^2 - (5.0\text{ m/s})^2\right] = 2.3\text{ kJ}$$

ASSESS The work-energy theorem states that the change in kinetic energy of an object is equal to the net work done on the object. Therefore, the greater the difference in kinetic energy, ΔK, the more the work required.

29. **INTERPRET** This problem involves work and the work-energy theorem. Given a force acting on an object and the distance over which the force acts, we are asked to find the initial velocity of the object.

DEVELOP The work-energy theorem, Equation 6.14 ($W_{\text{net}} = \Delta K$) tells us that the net work done on the straw is its change in kinetic energy, which involves the straw's initial speed. Because the stopping force acts in the same direction as the straw's displacement in the tree (i.e., it's a one-dimensional problem), and assuming the stopping force is constant, we can use Equation 6.1, $W = F_x \Delta x$ to find the net work done on the straw by the tree. Because the force of the tree acts to oppose the displacement of the straw, the work is negative : $W = -F_x x$, where $x = 4.5$ cm. Equating this to the change in kinetic energy by the work-energy theorem allows us to find the initial velocity of the straw.

EVALUATE Equating the work done by the tree to the change in the straw's kinetic energy, then solving for the initial speed of the straw gives

$$W_{\text{net}} = -Fx = \frac{m}{2}\left(\overbrace{v_2^2}^{=0} - v_1^2\right)$$

$$v_1 = \pm\sqrt{\frac{2Fx}{m}} = \sqrt{\frac{2(70\text{ N})(0.045)}{0.5\times10^{-3}\text{ kg}}} = 110\text{ m/s}$$

to two significant figures. Because the plus/minus sign simply indicates an initial velocity to the left or to the right, we have arbitrarily chosen the positive sign.

ASSESS This speed is reasonable for tornados, which usually have wind speeds between 18 and 140 m/s.

30. **INTERPRET** The object of interest in this problem is the car, and we are asked to find the height from which to drop the car so that it has the same energy upon impact as for a 20 mi/h collision with a stationary object. This problem involves work, kinetic energy, and the work-energy theorem.

DEVELOP The force acting on the car as it falls is the force due to gravity, $F = mg$. Because the car's displacement is in the same direction (i.e., downward) as the force, we can use Equation 6.1 to find the work done by gravity on the car as a function of the height y from which we drop the car: $W = Fy$. From the work-energy theorem $W_{net} = \Delta K$ (Equation 6.14), we can equate this work to the work done on the car in going from 20 mi/h to 0 mi/h in a collision and solve for the height y. Converting mi/h to m/s with the aid of Appendix C, we find that $(20 \text{ mi/h})(1609 \text{ m/mi})(1 \text{ h}/3600 \text{ s}) = 8.94 \text{ m/s}$.

EVALUATE By the work-energy theorem, we have

$$W_{net} = Fy = \frac{m}{2}\left(v_2^2 - v_1^2\right)$$

$$y = \frac{m}{2mg}\left(v_2^2 - v_1^2\right) = \frac{(0 \text{ m/s})^2 - (8.94 \text{ m/s})^2}{2\left(9.8 \text{ m/s}^2\right)} = -4.1 \text{ m}$$

ASSESS The net work done by the stopping force on the car is negative because it acts to oppose the car's displacement, so $F(x_2 - x_1) < 0$, so it reduces the car's kinetic energy instead of increasing it.

Section 6.4 Power

31. **INTERPRET** This problem is an exercise in converting power from kcal/day to Watts.

DEVELOP From Appendix C, we find that 1 cal = 4.184 J, and we know that 1 day = (24 h)(3600 s/h) = 86,400 s.

EVALUATE Performing the conversion gives

$$\frac{2000 \text{ kcal}}{1 \text{ d}}\overbrace{\left(\frac{1 \text{ d}}{86{,}400 \text{ s}}\right)}^{=1}\overbrace{\left(\frac{1000 \text{ cal}}{1 \text{ kcal}}\right)}^{=1}\overbrace{\left(\frac{1 \text{ J}}{4.184 \text{ cal}}\right)}^{=1} = 5.53 \text{ J/s} = 5.53 \text{ W}$$

ASSESS This is an *average* power. Human power output is higher during exercise.

32. **INTERPRET** This problem involves calculating an average power, and converting that power from J to horsepower.

DEVELOP Because the horse pulls in the same direction as the displacement of the plow, and if we assume the horse pulls with a constant force ($F_x = 750$ N), then Equation 6.1, $W = F_x x$ gives the work done by the horse. The average power supplied by the horse is simply the work divided by the time it takes to do the work (Equation 6.15), or $\bar{P} = \Delta W/\Delta t$ .With SI units, the result will be in Watts, so to convert to horsepower use the conversion equation in Appendix C, 1 hp = 746 W.

EVALUATE The work done by the horse is

$$\bar{P} = \frac{\Delta W}{\Delta t} = \frac{F_x x}{\Delta t} = \frac{(750 \text{ N})(200 \text{ m})}{(5 \text{ min})(60 \text{ s/min})} = 500 \text{ W}$$

Converting this result to horsepower gives (500 W)(1 hp/746 W) = 0.67 hp.

ASSESS Note that modern car engines deliver hundreds of horsepower, so the equivalent of several hundred horses pulling!

33. **INTERPRET** This problem involves calculating the power output of a car battery, or the rate at which energy is drained from the battery.

DEVELOP According to Equation 6.15, if work ΔW is done in time Δt, then the average power is $\bar{P} = \Delta W/\Delta t$.

EVALUATE Using Equation 6.15, the power output for each of the three cases is

(a) $$\bar{P} = \frac{\Delta W}{\Delta t} = \frac{(1 \text{ kW}\cdot\text{h})}{(1/60) \text{ h}} = 60 \text{ kW}$$

(b) $$\bar{P} = \frac{\Delta W}{\Delta t} = \frac{(1 \text{ kW}\cdot\text{h})}{1 \text{ h}} = 1 \text{ kW}$$

(c) $$\bar{P} = \frac{\Delta W}{\Delta t} = \frac{(1\text{ kW}\cdot\text{h})}{24\text{ h}}\left(\frac{1000\text{ W}}{\text{kW}}\right) = 41.7\text{ W}$$

ASSESS From Equation 6.15, we see that when the amount of work done is fixed, the average power is inversely proportional to Δt. Thus, the average power output is the greatest in case **(a)** and smallest in case **(c)**.

34. **INTERPRET** This problem involves calculating the average power output of a sprinter, given the work she does and the time in which she does it.

DEVELOP The average power is simply the work done divided by the time it takes to do the work, $\bar{P} = \Delta W/\Delta t$ (Equation 6.15).

EVALUATE Inserting the given quantities into Equation 6.15 gives the average power as

$$\bar{P} = \frac{22.4\text{ k J}}{10.6\text{ s}} = 2.1\text{ k W}$$

ASSESS Notice that the length of the sprint is not relevant to the problem. Also note that we did not need to convert kJ to J, provided we retained the factor k in the result.

35. **INTERPRET** This problem involves calculating the total work done, given average power and time.

DEVELOP From Equation 6.15, if the average power is $\bar{P}$, then the amount of work done over a period Δt is $\Delta W = \bar{P}\Delta t$. Note that we need to convert hp to SI units, which we can do with the help of Appendix C, where we find $1\text{ hp} = 746\text{ W}$.

EVALUATE The work done by the lawnmower is

$$\Delta W = \bar{P}\Delta t = (3.5\text{ hp})(746\text{ W/hp})(3600\text{ s}) = 9.4\times 10^{6}\text{ J}$$

ASSESS Given a constant average power, the work done is proportional to the time interval Δt. Note that the work done is positive, which means that the lawnmower is doing the work on the grass.

36. **INTERPRET** This problem involves the work-energy theorem and average power. We are asked to find the power output of a long-jumper during his prejump run.

DEVELOP The work-energy theorem (Equation 6.14) states that $\Delta K = W_{\text{net}}$, and from the net work we can calculate the power output using Equation 6.15, $\bar{P} = \Delta W/\Delta t$.

EVALUATE The energy expended in the prejump run is

$$W_{\text{net}} = \frac{m}{2}\left(v_2^2 - \overbrace{v_1^2}^{=0}\right) = \frac{mv_2^2}{2}$$

Therefore, the average power is

$$\bar{P} = \frac{\Delta W}{\Delta t} = \frac{mv_2^2}{2\Delta t} = \frac{(75\text{ kg})(10\text{ m/s})^2}{2(3.1\text{ s})} = 1.2\text{ k W}$$

to two significant figures.

ASSESS Note that the power output is proportional to the final speed squared.

37. **INTERPRET** In this problem we are asked to estimate the power output or rate of work, while doing deep knee bends at a given rate.

DEVELOP For a single deep knee bend, our final position is the same as the initial position, so our net displacement is zero. Considering that this is a one-dimensional problem, we can use Equation 6.8 to find the total work done in a single deep knee bend, then divide this by the time required for a single deep knee bend to find the power (Equation 6.15, $\bar{P} = \Delta W/\Delta t$).

EVALUATE Because the final position is the same as the initial position, we have $x_1 = x_2 \equiv x$ in the limits of the integral in Equation 6.8. Thus the work done for a single deep knee bend is

$$W = \int_x^x F(x)\,dx = 0\text{ W}$$

Thus, no work is done, so (in theory) no power is expended!

ASSESS We work up a sweat doing deep-knee bends because our bodies are working against a host of frictional forces. Thus, we are not working against gravity, because gravity gives us as much energy on the way down as it takes on the way up. Instead, we get our exercise from working against friction.

38. **INTERPRET** This problem is an exercise in converting from power to energy. We are to find the area needed to collect a given amount of energy given the parameters of solar radiation reaching the Earth's surface.

DEVELOP If we multiply the power density hitting the surface of the Earth (1 kW/m^2) by the surface area (m^2) of our perfectly efficient solar collector, we get power (kW). This can be seen by dimensional analysis:

$$\bar{P} = \left(\frac{\text{kW}}{\cancel{\text{m}}^2}\right)\cancel{\text{m}}^2 = \text{kW}$$

The relationship between average power and time is given by Equation 6.14, $\bar{P} = \Delta W/\Delta t$, which we can use to solve this problem, given that the energy desired is $\Delta W = 40$ kW·h.

EVALUATE The time it takes to collect $\Delta W = 40$ kW·h is thus

$$40\ \text{kW}\cdot\text{h} = \bar{P}\Delta t = \left(1\ \text{kW/m}^2\right)\left(15\ \text{m}^2\right)\Delta t$$

$$\Delta t = \frac{40\ \cancel{\text{kW}}\cdot\text{h}}{\left(1\ \cancel{\text{kW}}/\cancel{\text{m}}^2\right)\left(15\ \cancel{\text{m}}^2\right)} = 2.7\ \text{h}$$

ASSESS This problem was simplified by dimensional analysis, which allowed us to combine the power collected per unit area (1 kW/m^2) with the collection area (m^2) to get power.

39. **INTERPRET** This problem involves the concept of average power. We are asked to find the time it takes to melt an ice cube given the energy needed for the task and the average power supplied.

DEVELOP Use the definition of average power (Equation 6.15), $\bar{P} = \Delta W/\Delta t$, to solve the problem, given that W = 20 kJ and $\bar{P} = 900$ W .

EVALUATE The time required to melt the ice cube is

$$\Delta t = \frac{\Delta W}{P} = \frac{20\times 10^3\ \text{J}}{900\ \text{W}} = 22\ \text{s}$$

ASSESS This result seems reasonable given common experience with microwave ovens. Note that the result will depend on the mass of the ice cube (can you deduce the relationship?).

40. **INTERPRET** This problem is an exercise in converting between power and energy. We are given two objects that require a different amount of power to operate, and we are to determine which one consumes the most energy if left on for the given periods of time.

DEVELOP Use Equation 6.15, $\bar{P} = \Delta W/\Delta t$, to calculate the energy consumed (ΔW) for each object given power and time.

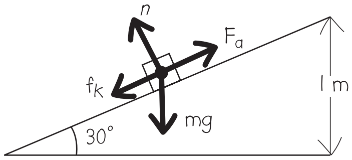
EVALUATE The hair dryer will consume an energy of $\Delta W = \bar{P}\Delta t = (1.2\ \text{kW})(10\ \text{min})(60\ \text{s/min}) = 720\ \text{kJ}$, whereas the night light will consume an energy of $\Delta W = \bar{P}\Delta t = (7\ \text{W})(24\ \text{h})(3600\ \text{s/h}) = 605\ \text{kJ}$. Thus, the hair dryer consumes more energy (but not much!).

ASSESS Notice that we had to report both answers in the same units to facilitate comparing the results.

PROBLEMS

41. **INTERPRET** The problem is about calculating work, given force and displacement. The object of interest is the box, which is being pushed up a ramp. For part (b) of the problem, we consider the work-energy theorem.

DEVELOP Make a free-body diagram of the box (see figure below). Use Equation 6.5, $W = \vec{F}\cdot\Delta\vec{r}$, to calculate the work done in pushing the box up the ramp.



EVALUATE **(a)** The box rises $\Delta y = 1$ m vertically. This means that the displacement up the ramp (parallel to the applied force) is

$$\Delta r = \frac{\Delta y}{\sin(\theta)} = \frac{1\text{ m}}{\sin(30^\circ)} = 2\text{ m}$$

Therefore, the work done during this process is

$$W_{\text{app}} = \vec{F}_{\text{app}} \cdot \Delta\vec{r} = (200\text{ N})(2\text{ m})\cos(0^\circ) = 400\text{ J}$$

because the angle between the applied force and the displacement vector is 0°.

(b) To find the mass, we first note that the work done by gravity is

$$W_g = \vec{F}_g \cdot \Delta\vec{r} = (-mg\,\hat{j})\cdot(\Delta x\hat{i} + \Delta y\hat{j}) = -mg\Delta y = -mg\Delta r\sin\theta$$

The work done by friction is

$$W_f = \vec{f}_k \cdot \Delta\vec{r} = -f_k\Delta r = -\mu_k n\ \Delta r = -\mu_k(mg\cos\theta)\Delta r$$

where in the last step we have used $n - mg\cos(\theta) = 0$, which results from applying Newton's second law to the box in the direction perpendicular to the incline. Because the speed of the box remains unchanged, the work-energy theorem $W = \Delta K$, says the total work must be zero:

$$W_{\text{Tot}} = W_{\text{app}} + W_g + W_f = 0$$

This implies

$$W_{\text{app}} = -W_g - W_f = mg\Delta r\sin\theta + \mu_k(mg\cos\theta)\Delta r = mg\Delta r(\sin\theta + \mu_k\cos\theta)$$

from which the mass is found to be

$$m = \frac{W_a}{g\Delta r(\sin\theta + \mu_k\cos\theta)} = \frac{F_a}{g(\sin\theta + \mu_k\cos\theta)} = \frac{200\text{ N}}{(9.8\text{ m/s}^2)[\sin(30^\circ) + (0.18)\cos(30^\circ)]}$$
$$= 31\text{ kg}$$

ASSESS The mass could also be found by solving Newton's second law, with zero acceleration:

$$F_{\text{net}} = F_{\text{app}} - mg(\sin\theta + \mu_k\cos\theta) = ma = 0$$
$$m = \frac{F_a}{g(\sin\theta + \mu_k\cos\theta)}$$

42. **INTERPRET** This problem involves the concept of work. The object of interest is the car, and we are to calculate the work done in pushing it with the given force a distance of 5.6 m.

DEVELOP Because the forces applied to the car are not in the same direction as its displacement, we will use Equation 6.2, $W = F\Delta r\cos(\theta)$, to calculate the work done. The angle θ is the angle between the force vectors and the displacement vector ($\theta = 25^\circ$ for the two forces in this case).

EVALUATE Each person applies a force $F = 280$ N at $\theta = 25^\circ$ to the car, and they push the car $\Delta r = 5.6$ m. Inserting these quantities into Equation 6.2 gives the work per person as

$$W = F\Delta r\cos(\theta) = (280\text{ N})(5.6\text{ m})\cos(25^\circ) = 1400\text{ J}$$

to two significant figures.

ASSESS Because the people push at 25° to the displacement, they each supply $F\sin(\theta) = (280\text{ N})\sin(25^\circ) = 118$ N of force perpendicular to the displacement, which does no work at all.

43. **INTERPRET** You want to find out how much work you do during a particular exercise.

DEVELOP You only do work when lifting the weight (gravity does the work to bring the weight back down). The work required to lift the weight the given distance is $W = w\Delta y$ (it's irrelevant at what angle the force from your arms is applied – the net result is that the weight moves up by Δy). We'll need to convert the work to kcal using $1\ \text{kcal} = 4184\ \text{J}$ from Appendix C.

EVALUATE (a) Each repetition requires you to exert

$$W = w\Delta y = (20\ \text{N})(0.55\ \text{m}) = 11\ \text{J}\left(\frac{1\ \text{kcal}}{4184\ \text{J}}\right) = 2.63\times10^{-3}\,\text{kcal}$$

To get a 200 kcal workout, the number of reps you'd have to do is

$$N = \frac{200\ \text{kcal}}{2.63\times10^{-3}\,\text{kcal}} = 76{,}000$$

(b) If your workout takes 1.0 min, then the power output is just the work divided by the time:

$$P = \frac{W}{\Delta t} = \frac{200\ \text{kcal}}{1.0\ \text{min}}\left(\frac{4184\ \text{J}}{1\ \text{kcal}}\right) = 14\ \text{kW}$$

ASSESS The answers seem unreasonably large. Typically, lifting weights burns around 300 kcal per hour.

44. **INTERPRET** The problem involves calculating the average force given work and displacement. The object of interest is the locomotive pulling a train.

DEVELOP Use Equation 6.5, $W = \vec{F}\cdot\Delta\vec{r}$, to solve for the average force in the coupling between the locomotive and the rest of the train. Because the force is always in the direction of the displacement, the dot product reduces to the scalar product because cos(0) = 1. Thus, Equation 6.5 reduces to $W = F\Delta r$.

EVALUATE The distance pulled by the locomotive is $\Delta r = 180\ \text{km} = 1.80\times10^5\ \text{m}$ and the work done is $W = 7.9 \times 10^{11}$ J, so the average force is

$$F_{\text{av}} = \frac{W}{\Delta r} = \frac{7.9\times10^{11}\ \text{J}}{1.80\times10^5\ \text{m}} = 4.4\times10^6\ \text{N}$$

ASSESS The average force depends only on the total work done and the displacement. The train's mass is not required to answer this question.

45. **INTERPRET** This problem involves calculating the work done as a result of a force acting at a nonzero angle with respect to the displacement. We are asked to find the angle that the rope makes with the horizontal, given the work, force, and distance over which the force acts.

DEVELOP Because the force is not parallel to the displacement, we must use the more general equation for work; Equation 6.5, $W = \vec{F}\cdot\Delta\vec{r}$. In scalar form, dot product gives $W = F\Delta r\cos(\theta)$, where θ is the angle between the rope and the displacement direction (i.e., horizontal).

EVALUATE We are given that $W = 2500$ J, $F = 120$ N, $\Delta r = 23$ m, so the angle θ is

$$\theta = \text{acos}\left(\frac{W}{F\Delta r}\right) = \text{acos}\left(\frac{2500\ \text{J}}{(120\ \text{N})(23\ \text{m})}\right) = 0.44\ \text{rad} = 25^\circ$$

ASSESS Notice that the argument of the acos function is dimensionless, as it should be. The angle 25° is a physically reasonable result.

46. **INTERPRET** This problem is an exercise in vector multiplication. We are asked to evaluate the scalar products between different pairs of the unit vectors $\hat{i}, \hat{j}$, and $\hat{k}$.

DEVELOP As shown in Equation 6.3, the scalar product of two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

where A and B are the magnitudes of the vectors and θ is the angle between them. With this definition, the scalar products between different pairs of unit vectors can be computed.

EVALUATE **(a)** The dot product of any vector with itself equals its magnitude squared, $\vec{A}\cdot\vec{A} = A^2\cos 0^\circ = A^2$, and the magnitude of any unit vector is unity: $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$. Therefore

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1^2\cos(0) = 1$$

(**b**) If two vectors $\vec{A}$ and $\vec{B}$ are perpendicular, then their dot product is zero, $\vec{A}\cdot\vec{B} = AB\cos(90°) = 0$. Because the unit vectors $\hat{i}, \hat{j}$, and $\hat{k}$ are mutually perpendicular, the angle between any pair of them is 90°. Therefore

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 1^2\cos(90^\circ) = 0$$

(**c**) Using the distributive law, we have

$$\begin{aligned}\vec{A}\cdot\vec{B} &= \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\cdot\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)\\ &= A_xB_x\hat{i}\cdot\hat{i} + A_xB_y\hat{i}\cdot\hat{j} + A_xB_z\hat{i}\cdot\hat{k} + A_yB_x\hat{j}\cdot\hat{i} + A_yB_y\hat{j}\cdot\hat{j} + A_yB_z\hat{j}\cdot\hat{k} + A_zB_x\hat{k}\cdot\hat{i} + A_zB_y\hat{k}\cdot\hat{j} + A_zB_z\hat{k}\cdot\hat{k}\\ &= A_xB_x + A_yB_y + A_zB_z\end{aligned}$$

where we have used the results from **(a)** and **(b)**. The final expression indeed agrees with Equation 6.4.

ASSESS The quantity $\vec{A}\cdot\vec{B}$ is a scalar formed by two vectors. The scalar product is commutative ($\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$) and distributive [$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$].

47. **INTERPRET** This problem is an exercise in vector multiplication. We are asked to find the scalar product between two vectors of the form $a\hat{i} + b\hat{j}$ and $b\hat{i} - a\hat{j}$, and to find the angle between them, for arbitrary *a* and *b*.

DEVELOP Use Equations 6.3 and 6.4 ($\vec{A}\cdot\vec{B} = AB\cos(\theta)$ and $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$, respectively).

EVALUATE (**a**) The scalar product of $a\hat{i} + b\hat{j}$ and $b\hat{i} - a\hat{j}$ is $\left(a\hat{i} + b\hat{j}\right)\cdot\left(b\hat{i} - a\hat{j}\right) = ab - ab = 0$.

(**b**) The angle between the two vectors is θ = acos(0) = 90°.

ASSESS Thus, for arbitrary *a* and *b*, the vectors $a\hat{i} + b\hat{j}$ and $b\hat{i} - a\hat{j}$ are perpendicular.

48. **INTERPRET** Your job is to determine whether the force applied by the tractor agrees with the limit given on the amount of work done per distance.

DEVELOP To maximize the work done by the tractor, we'll assume it pushes in the direction parallel to the airplane's motion. Using Equation 6.1, $W = F\Delta x$, we'll check to see whether it uses more or less energy than it's claimed to.

EVALUATE The tractor exerts a force of 0.42 MN over a distance of 25 m, then the work it does is

$$W = F\Delta x = (0.4\text{ MN})(25\text{ m}) = 10.5\text{ MJ}$$

So, no, the tractor does not meet its specifications. It requires more energy than 10 MJ to move an airplane 25 m.

ASSESS It doesn't seem like the tractor is too far off. The work it does is over its specifications by half a MJ, which is the energy in roughly 0.3% of a gallon of gasoline (see Appendix C). But since the airline company has to presumably move hundreds of airplanes every day across the country, this inefficiency may add up to something significant.

49. **INTERPRET** This problem involves finding the work done by the given force vector that acts through the given displacement.

DEVELOP Use the general form of the expression for work, Equation 6.5: $W = \vec{F}\cdot\Delta\vec{r}$, with $\vec{F} = 67\hat{i} + 23\hat{j} + 55\hat{k}$ N and

$$\begin{aligned}\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 &= (21-16)\hat{i} + (10-31)\hat{j} + (14-0)\hat{k}\text{ m}\\ &= 5\hat{i} - 21\hat{j} + 14\hat{k}\end{aligned}$$

EVALUATE Inserting the given force and displacement into Equation 6.5 gives

$$W = \left(67\hat{i} + 23\hat{j} + 55\hat{k}\text{ N}\right)\cdot\left(5\hat{i} - 21\hat{j} + 14\hat{k}\text{ m}\right)\vec{F}\cdot\Delta\vec{r} = (335 - 483 + 770)\text{ Nm} = 622\text{ J}$$

ASSESS Notice that we must keep track of the signs of the individual terms in doing the dot product to be sure to get the correct result.

50. **INTERPRET** In this problem we are asked to find the work done by a non-constant force that varies with position.

DEVELOP We are dealing with a one-dimensional varying force, $F(x)$, so to evaluate the work done, we need to integrate using Equation 6.8:

$$W = \int_{x_1}^{x_2} F\,dx$$

EVALUATE The work done on the particle moving from $x = 0$ to $x = 6$ m is

$$W = \int_{x_1}^{x_2} F\,dx = \int_0^{6\text{m}} ax^2 dx = \frac{1}{3}ax^3\Big|_0^{6\text{m}} = \frac{1}{3}\left(5\ \text{N/m}^2\right)\left(6\ \text{m}\right)^3 = 360\ \text{J}$$

ASSESS Notice that the units of $F(x)$ are in N when x is in m, but once we integrate we get an extra factor of x, which means the integral has units of J, as expected.

51. **INTERPRET** This problem involves calculating spring constants given the work it takes to deform the springs.
DEVELOP Use Equation 6.10, $W = kx^2/2$, to express the work W done in terms of the deformation x for each spring. We are given that $2W_A = W_B$ and $x_A = 2x_B$.
EVALUATE For spring A, $W_A = k_A x_A^2/2$, and for spring B $W_B = k_B x_B^2/2$. Taking the ration of these two equations and using the given relations between springs A and B gives

$$\frac{W_A}{W_B} = \frac{k_A}{k_B}\frac{x_A^2}{x_B^2}$$
$$\frac{1}{2} = 4\frac{k_A}{k_B}$$
$$k_B = 8k_A$$

ASSESS Note that the spring constant is linear in work, but quadratic in spring deformation.

52. **INTERPRET** This is a one-dimensional problem in which we are asked to find the work done by a non-constant force that varies with position.
DEVELOP Because we are dealing with a force $F(x)$ that varies with position, we need to use the more general expression for work in one dimension, which is Equation 6.8:

$$W = \int_{x_1}^{x_2} F(x)dx$$

With $F(x) = a\sqrt{x}$, we obtain $W_{x_1\to x_2} = \int_{x_1}^{x_2} ax^{1/2}dx = \frac{2}{3}ax^{3/2}\Big|_{x_1}^{x_2} = \left(\frac{2a}{3}\right)\left(x_2^{3/2} - x_1^{3/2}\right)$.

EVALUATE Evaluating the above expression for the work for each case, we find

(a) $W_{0\to 3} = \frac{2}{3}(9.5\ \text{N/m}^{1/2})(3\ \text{m})^{3/2} = 33\ \text{J}$

(b) $W_{3\to 6} = \frac{2}{3}(9.5\ \text{N/m}^{1/2})[(6\ \text{m})^{3/2} - (3\ \text{m})^{3/2}] = 60\ \text{J}$

(c) $W_{6\to 9} = \frac{2}{3}(9.5\ \text{N/m}^{1/2})[(9\ \text{m})^{3/2} - (6\ \text{m})^{3/2}] = 78\ \text{J}$

ASSESS Because the force increases with x (as $\sqrt{x}$) more work is done as the object is displaced further in the x direction.

53. **INTERPRET** This is a one-dimensional problem that involves calculating the work done given a non-constant force.
DEVELOP The force given varies with position, so we need to use the more general expression for work in one dimension; Equation 6.10:

$$W = \int_{x_1}^{x_2} F(x)dx$$

with $F(x)$ given in the problem statement. The limit of the integration are from $x_1 = 0$ to $x_2 = x$.
EVALUATE Evaluating the integral gives

$$W = \int_0^x F_0 \left[\frac{L_0 - x'}{L_0} - \frac{L_0^2}{(L_0 + x')^2} \right] dx'$$
$$= F_0 \left| \frac{1}{L_0} \left(L_0 x' - \frac{x'^2}{2} \right) + \frac{L_0^2}{L_0 + x'} \right|_0^x$$
$$= F_0 \left(x - \frac{x^2}{2L_0} + \frac{L_0^2}{L_0 + x} - L_0 \right)$$

ASSESS Note that we changed the integration variable from x to x' simply to avoid confusing it with the upper limit x of the integration.

54. **INTERPRET** As you push the swing, you are doing work against gravity. While gravitational force is constant, the path is a circular arc so the force required varies. Therefore, this is a two-dimensional problem in which we need to calculate the work done by a varying force.

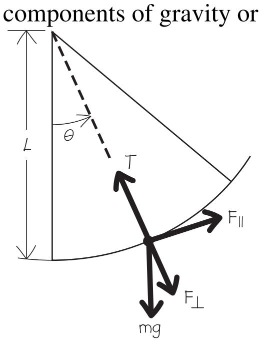
DEVELOP Draw a free-body diagram of the situation (see figure below). Because this is a two-dimensional problem in which the orientation of the force varies with respect to the displacement, we need to use the most general form of the expression for work,

$$W = \int_{r_1}^{r_2} \vec{F}(r) \cdot d\vec{r}$$

Because the path of the swing is a circular arc (radius L and differential arc length $|d\vec{r}| = L d\theta$), only the components of force acting tangent to the circle (labeled $F_\parallel$ in the figure below) do work on the swing. From the free-body diagram, we see that the force acting in the tangential direction is $F_\parallel = mg \sin(\theta)$. To pull the swing up to an angle f at constant speed means that we supply a force equal in magnitude to this force but in the opposite direction, so the work we do is

$$W = \int_0^\phi \vec{F} \cdot d\vec{r} = \int_0^\phi F_\parallel |dr| = \int_0^\phi mg \sin\theta \cdot L d\theta$$

The radial (or perpendicular) components do no work because the scalar product with the path element is zero. Thus, the tension in the chains and the radial components of gravity or the applied force do no work.



EVALUATE Evaluating the integral just derived for the work gives

$$W = mgL \left| -\cos\theta \right|_0^\phi = mgL(1 - \cos\phi)$$

which is the expression given in the problem statement.

ASSESS The result can also be derived using $W = mgh$, where $h = L[1 - \cos(\phi)]$ is the vertical distance measured from the bottom of the swing. Thus, the work is the energy required to lift the child on the swing by a vertical distance h.

55. **INTERPRET** This problem involves calculating the (relative) speed of two particles, given their relative kinetic energy and mass.

DEVELOP Use Equation 6.13, $K = mv^2/2$ to express the kinetic energy of each particle. Thus, the kinetic energy of particle 1 is $K_1 = mv_1^2/2$ and $K_2 = mv_2^2/2$. The problem states that $K_1 = K_2$, and $m_1 = 4m_2$, so we can find the ratio of the speeds by taking the ratio of the equations.

EVALUATE Taking ratio K_1/K_2 gives

$$\frac{K_1}{K_2} = \frac{m_1 v_1^2}{m_2 v_2^2}$$
$$1 = 4\frac{v_1^2}{v_2^2}$$
$$v_2 = \pm 2v_1$$

ASSESS The positive/negative sign indicates that the orientation of the speeds does not matter, only the magnitude matters. In other words, it does not matter if both particles move in the same direction, or if they move in opposite directions.

56. **INTERPRET** This problem is about the distance the plane can be towed with a given amount of work done by the tractor.
DEVELOP Equation 6.2, $W = F\Delta r\cos\theta$, applies here. The displacement is horizontal but the applied force (tension in the link) is at an angle $\theta = 22°$ with the horizontal.
EVALUATE Using Equation 6.2, the distance the plane moves is

$$\Delta x = \frac{W}{F_x} = \frac{W}{F\cos\theta} = \frac{8.7\times 10^6\ \text{J}}{(4.1\times 10^5\ \text{N})\cos 22°} = 22.9\ \text{m}$$

ASSESS Only the horizontal component of the force, $F_x = F\cos\theta$, does the work.

57. **INTERPRET** This is a one-dimensional problem that involves calculating the work done given a non-constant force.
DEVELOP The force given varies with position, so we need to use the more general expression for work in one dimension; Equation 6.10:

$$W = \int_{x_1}^{x_2} F(x)dx$$

with $F(x)$ given in the problem statement. The limits of the integration are from $x_1 = 0$ to $x_2 = x$.
EVALUATE Evaluating the integral gives

$$W = \int_0^{x_0}\left(\frac{F_0}{x_0}\right)xdx = \left(\frac{F_0}{x_0}\right)\frac{x^2}{2}\Bigg|_0^{x_0} = \left(\frac{F_0}{x_0}\right)\frac{x_0^2}{2} = \frac{1}{2}F_0 x_0$$

ASSESS Thus, if the force varies linearly with position, the work varies quadratically.

58. **INTERPRET** In this one-dimensional problem, we are asked to find the work done by a non-constant force that varies with position.
DEVELOP Because we are dealing with a force $F(x)$ that varies with position, we need to evaluate the work using Equation 6.8:

$$W = \int_{x_1}^{x_2} F\,dx$$

With $F = ax^{3/2}$ we obtain $W_{x_1\to x_2} = \int_{x_1}^{x_2} ax^{3/2}dx = \frac{2}{5}ax^{5/2}\Big|_{x_1}^{x_2} = \left(\frac{2a}{5}\right)\left(x_2^{5/2} - x_1^{5/2}\right)$.
EVALUATE The work required in moving the object from $x = 0$ to $x = 14$ m is

$$W_{0\to 14\ \text{m}} = \left(\frac{2}{5}\right)\left(0.75\ \text{N/m}^{3/2}\right)\left(14\ \text{m}\right)^{5/2} = 220\ \text{N}\cdot\text{m} = 220\ \text{J}$$

ASSESS Because the force increases with x (as $x^{3/2}$), more work is done as the object moves further in the x direction.

59. **INTERPRET** This problem is an exercise in vector multiplication. We are given two vectors of equal magnitude and the relationship between their scalar product. With this information, we are to find the angle between the vectors.
DEVELOP We are told that $A = B$ and that $\vec{A}\cdot\vec{B} = A^2/3$. Use Equation 6.3 to find the angle θ between the vectors.
EVALUATE Evaluating the scalar product using Equation 6.3 gives

$$\vec{A}\cdot\vec{B} = AB\cos(\theta) = A^2\cos(\theta) = A^2/3$$
$$\theta = \text{acos}(1/3) = 70.5^\circ$$

ASSESS Note that an equivalent condition is $\vec{A}\cdot\vec{B} = B^2/3$ because $A = B$.

60. **INTERPRET** In this problem the pump (with a given power) is doing work against gravity to deliver water to a tank above the ground. The quantity of interest is the amount of water that the pump can deliver during a given time interval.

DEVELOP According to Equation 6.15, if the average power is $\bar{P}$, then the amount of work done over a period Δt is $\Delta W = \bar{P}\Delta t$. Because the work required to lift an object of mass m to a vertical height h is $W = mgh$, the rate at which the mass can be delivered is

$$\bar{P} = \frac{\Delta W}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)gh \quad \Rightarrow \quad \frac{\Delta m}{\Delta t} = \frac{\bar{P}}{gh}$$

In SI units, 1 hp = 746 W.

EVALUATE Using the expression above, we find the rate at which water is delivered to the tank to be

$$\frac{\Delta m}{\Delta t} = \frac{\bar{P}}{gh} = \frac{(0.5\text{ hp})(746\text{ W/hp})}{(9.8\text{ m/s}^2)(60\text{ m})} = 0.63\text{ kg/s}$$

to two significant figures. Because the mass of 1 gallon ($1\text{ gal} = 3.786\times10^{-3}\text{ m}^3$) of water is $(1000\text{ kg/m}^3)(3.786\times10^{-3}\text{ m}^3) = 3.786\text{ kg}$, the rate can also be written as

$$\frac{\Delta m}{\Delta t} = \left(0.634\frac{\text{kg}}{\text{s}}\right)\left(60\frac{\text{s}}{\text{min}}\right)\left(3.786\frac{\text{kg}}{\text{gal}}\right) = 10\text{ gal/min}$$

to two significant figures.

ASSESS Given a constant average power, the rate of delivery $\Delta m/\Delta t$ is inversely proportional to the height h. The greater the height h, the slower is the rate, as expected.

61. **INTERPRET** This problem involves converting power from W to gallons per day.

DEVELOP From Appendix C we find that the energy content of oil is 39 kW·h/gal. Let the units guide you in converting from GW to gallons/day.

EVALUATE The import rate is

$$(800\ \cancel{G}\cancel{W})\overbrace{\left(\frac{1\text{ gal}}{39\ \cancel{k}\cancel{W}\cdot\cancel{h}}\right)}^{=1}\overbrace{\left(\frac{10^6\ \cancel{k}}{\cancel{G}}\right)}^{=1}\overbrace{\left(\frac{24\ \cancel{h}}{\text{day}}\right)}^{=1} = 490\times10^{12}\text{ gal/day}$$

ASSESS This may also be express as 490 Tgal/day.

62. **INTERPRET** This problem involves the total work done, given the average power and time. The object of interest is the runner, and we are to calculate the work done given a formula for the runner's power output in terms of mass and speed.

DEVELOP According to Equation 6.15, if the average power is $\bar{P}$, then the amount of work done over a period Δt is $\Delta W = \bar{P}\Delta t$. In this problem, $\bar{P}$ is a function of the runner's speed, which is $v = 5.2$ m/s. The time for the runner to complete the race is Δt = (10,000 m)/(5.2 m/s) = 1923 s.

EVALUATE Over the entire race time, the runner's work output is

$$\begin{aligned}\Delta W &= \bar{P}\Delta t = m(bv - c)\Delta t \\ &= (54\text{ kg})\left[(4.27\text{ J/kg}\cdot\text{m})(5.2\text{ m/s}) - 1.83\text{ W/kg}\right](1923\text{ s}) \\ &= 2.12\times10^6\text{ J} = 0.588\text{ kW}\cdot\text{h}\end{aligned}$$

ASSESS Note that the units work out to be units of power, as expected. Also, were the power a function of time (which it undoubtedly is in reality—runner's have less power at the end of the race than at the beginning), we would have to use Equation 6.18 to find the work done.

63. **INTERPRET** You have the mass and power of a car, and need to find the highest rate at which it can climb a given slope. You'll need to use work and energy techniques.

DEVELOP Assume the car is moving at constant speed, such that the net force on the car is zero. That means the force from the engine propelling the car forward along the road, F_c, must balance the component of the gravitational force that is parallel to the ground and points back down the slope. In other words, $F_c = F_g \sin\theta$. This force is related to the car's power through Equation 6.19: $P = \vec{F}_c \cdot \vec{v}$. As we have defined it, the force is in the same direction as the velocity of the car, so $P = F_c v$.

EVALUATE Using all its available power, the car can climb the slope at a speed of

$$v = \frac{P}{F_c} = \frac{P}{mg\sin\theta} = \frac{35\text{ kW}}{(1750\text{ kg})(9.8\text{ m/s}^2)\sin 4.5°} = 26\text{ m/s}$$

ASSESS This speed (58 mph) seems reasonable for the grade involved. The actual maximum speed will be lower due to air resistance, which is not negligible at this speed. Note, as well, that you can derive the same result by arguing that the car must work against gravity to climb the slope. Therefore, the component of its force pointing straight up must equal mg. The angle between this upward force and the velocity of the car is $90° - 4.5° = 85.5°$, so the power provided by the car is $P = mgv\cos(85.5°)$, which gives the same answer as the above equation.

64. **INTERPRET** In this problem a constant average power is supplied to the car as it climbs a slope against the air resistance. We want to know the angle of the slope if the car is moving at a steady speed.

DEVELOP At constant velocity, there is no change in kinetic energy, so the net work done on the car is zero. Therefore, the power supplied by the engine equals the power expended against gravity and air resistance. The power can be found from Equation 6.19, $P = \vec{F} \cdot \vec{v}$.

EVALUATE Because gravity $m\vec{g}$ makes an angle of $\theta + 90°$ with the velocity $\vec{v}$ (where θ is the angle of the slope with respect to the horizontal), the power expended against gravity is

$$P_g = m\vec{g}\cdot\vec{v} = mgv\cos(\theta + 90°) = -mgv\sin(\theta)$$

Similarly, the air resistance makes an angle of 180° to the velocity, so

$$P_{\text{air}} = \vec{F}_{\text{air}}\cdot\vec{v} = F_{\text{air}}v\cos(180°) = -F_{\text{air}}v$$

In SI units, v = 60 km/h = 16.7 m/s. Because the car moves at a constant speed, $P_{\text{Tot}} = P_{\text{car}} + P_g + P_{\text{air}} = 0$, or

$$P_{\text{car}} = -P_g - P_{\text{air}} = mgv\sin\theta + F_{\text{air}}v$$

Solving the equation gives

$$\theta = \text{asin}\left(\frac{P_{\text{car}} - F_{\text{air}}v}{mgv}\right) = \text{asin}\left[\frac{38000\text{ W} - (450\text{ N})(16.7\text{ m/s})}{(1400\text{ kg})(9.8\text{ m/s}^2)(16.7\text{ m/s})}\right] = 7.7°$$

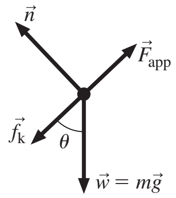
ASSESS To see that the result makes sense, we first note that increasing P_{car}, (i.e., increasing the power output of the car's engine), will allow the car to climb a steeper slope. On the other hand, when $P_{\text{car}} < P_{\text{air}}$, we get a negative value for θ, which means that the car's power is not large enough to overcome the air resistance, and the car will not be able to climb the slope at all.

65. **INTERPRET** This problem involves the concept of work and Newton's second law (for constant mass), $\vec{F}_{\text{net}} = m\vec{a}$. The object of interest is the box, and we are asked to find the work done to push it up an inclined slope a given distance.

DEVELOP Draw a free-body diagram of the situation (see figure below). To express the forces in terms of known quantities, apply Newton's second law to the box. This gives

$$\left.\begin{aligned} -f_k + F_{\text{app}} - mg\sin(\theta) &= 0 \\ n - mg\cos(\theta) &= 0 \end{aligned}\right\} -\mu_k mg\cos(\theta) + F_{\text{app}} - mg\sin(\theta) = 0$$

which we can solve for μ_k given that we know the work done by you pushing the box up the slope is $F_{\text{app}}d$ = 2.2 kJ (see Equation 6.1) because the force you apply is in the same direction as the displacement of the box.



EVALUATE Inserting the known quantities into the expression above and solving for μ_k gives

$$\mu_k = \frac{F_{app} - mg\sin(\theta)}{mg\cos(\theta)} = \frac{(2200\text{ J})/(3.1\text{ m}) - (78\text{ kg})(9.8\text{ m/s}^2)\sin(22^\circ)}{(78\text{ kg})(9.8\text{ m/s}^2)\cos(22^\circ)} = 0.60$$

66. **INTERPRET** We're asked to estimate the power output of the human heart.

DEVELOP Imagine that blood circulates through the body through one "tube" that goes from the heart down to the feet, then up to the head, and finally back to the heart where it starts over again. In this simplified model, the heart only has to do work when pushing blood upwards from the feet to the head (gravity will do the work when blood is falling downwards). We will determine the work required to pump 1L of blood up from the feet to the head. We will use this to approximate the power output of the heart, assuming 5L of blood is pumped through the body per minute.

EVALUATE (a) Since the heart has to work against gravity, the work required to move 1L (1 kg) of blood the distance between feet and head is just

$$W_{1\text{L}} = mgh = (1\text{ kg})(9.8\text{ m/s}^2)(1.7\text{ m}) = 17\text{ J}$$

(b) If the heart pumps blood at 5L/min, the power output is

$$P = \frac{5W_{1\text{L}}}{\Delta t} = \frac{5 \cdot 17\text{ J}}{60\text{ s}} = 1.4\text{ W}$$

ASSESS A typical male expends about 1800 kcal/day during rest (this is called the basal metabolic rate). In SI units, that's about 87 W. About 20% of this energy expenditure is used by the heart, so the heart power output is somewhere around 17 W.

ASSESS The units all cancel to give μ_k as a dimensionless quantity, as expected.

67. **INTERPRET** The object of interest here is the chest. The physical quantity we are asked to find is the power needed to push the chest against friction. This problem involves the concept of work and power, and we will have to use Newton's second law.

DEVELOP If you push parallel to a level floor, the applied force equals the frictional force (from Newton's second law, $F_{\text{net}} = ma$, where the acceleration is zero), so $F_a = f_{\text{k}}$. Because (again by Newton's second law) the normal force equals the weight of the box ($n = mg$) the applied force is

$$F_a = \mu_{\text{k}} n = \mu_{\text{k}} mg$$

Use Equation 6.19, $P = \vec{F} \cdot \vec{v}$, to find the power needed. Because we are applying a force in the same direction as the displacement of the box, we can use Equation 6.1, $W = F\Delta x$, to find the work done.

EVALUATE **(a)** The power required is

$$P_a = F_a v = \mu_{\text{k}} mgv = (0.78)(95\text{ kg})(9.8\text{ m/s}^2)(0.62\text{ m/s}) = 450\text{ W}$$

which is about 0.6 hp.

(b) The work done by the applied force acting over a displacement $\Delta x = 11$ m is

$$W_a = F_a \Delta x = \mu_{\text{k}} mg\Delta x = (0.78)(95\text{ kg})(9.8\text{ m/s}^2)(11\text{ m}) = 8.0\text{ kJ}$$

ASSESS An alternative way to calculate the power is to note that the time required to push the chest 11 m is $\Delta t = \Delta x/v = (11\text{ m})/(0.62\text{ m/s}) = 17.74\text{ s}$. Using Equation 6.17, we have

$$W_a = P_a \Delta t = (450\text{ W})(17.74\text{ s}) = 8.0\text{ kJ}$$

68. **INTERPRET** This problem involves calculating the power supplied by you to the spoon, and the work you do if you supply this power for 1 min.

DEVELOP Apply Equation 6.19, $P = \vec{F} \cdot \vec{v}$, to find the power expended. Because the force you supply is always in the direction of the spoon's velocity, the angle in the scalar product between the force and the velocity is zero, so Equation 6.19 reduces to $P = Fv$. To find the work done, apply Equation 6.17, $W = P\Delta t$.

EVALUATE (**a**) Inserting the given quantities into the expression for power derived above gives

$$P = Fv = (45\text{ N})(0.29\text{ m/s}) = 13\text{ W}$$

(b) The work done in applying this power for 1.0 min is $W = P\Delta t = (13.05\text{ W})(60\text{ s}) = 780\text{ J}$ to two significant figures.

ASSESS Notice that in part (b), we used the result of part (a), but retained extra significant figures because the result for part (a) was an intermediate result in this case.

69. **INTERPRET** This problem is about the total work done, given the power and time. The object of interest is the machine whose power output is given, and we are to find the total work is done over the given time interval.

DEVELOP The power given in this problem is time-varying. Therefore, use Equation 6.18: $W = \int_{t_1}^{t_2} P(t)dt$ to find the total work done, with $t_1 = 10$ s, $t_2 = 20$ s, and $P = ct^2$.

EVALUATE Inserting the given quantities into Equation 6.18, we obtain

$$W_{t_1 \to t_2} = \int_{t_1}^{t_2} ct^2 dt = \frac{1}{3}ct^3\bigg|_{t_1}^{t_2} = \frac{c}{3}\left(t_2^3 - t_1^3\right) = \frac{1}{3}\left(18\text{ W/s}^2\right)\left[(20\text{ s})^3 - (10\text{ s})^3\right] = 42\text{ kJ}$$

ASSESS Because the power increases quadratically with t (i.e., as t^2), as time progresses, more work is done by the machine over the same interval of time. For example, the work done in a 10-s interval from $t_1 = 20$ s to $t_2 = 30$ s ($W_{20\,s \to 30\text{ s}} = 114\text{ kJ}$) is greater than the work in a 10-s interval from $t_1 = 10$ s to $t_2 = 20$ s ($W_{10\,s \to 20\text{ s}} = 42\text{ kJ}$).

70. **INTERPRET** In this problem we consider the power output of a bumblebee.

DEVELOP As the bee's wings beat, they complete a circle: first flapping down and then flapping back up to where they began. Let's assume for simplicity that the upstroke takes negligible time, so that the wing is essentially always in the downstroke. During a downstroke, the wings push down on the air, and by Newton's third law the air pushes back up on the bee. Therefore, in order to hover, the average downward force supplied by the wings has to equal the bee's weight, $\bar{F} = mg$, otherwise the air wouldn't push back up on the bee enough to keep it at a constant height above the ground. To find the average power exerted by the bee, we'll need to multiply this average downwards force by the average downward velocity, $\bar{P} = \bar{F}\bar{v}$ (from Equation 6.19, where by definition the vectors point in the same direction). We can estimate the downward velocity by taking the average wing displacement, $\Delta\bar{r} = 1.5$ mm, and dividing by the time of one wingbeat, $\Delta t = \frac{1}{100}$ s.

EVALUATE Using the arguments above, the average power is

$$\bar{P} = \frac{mg\Delta\bar{r}}{\Delta t} = \frac{\left(0.25 \times 10^{-6}\text{ kg}\right)\left(9.8\text{ m/s}^2\right)\left(1.5 \times 10^{-3}\text{ m}\right)}{\left(\frac{1}{100}\text{ s}\right)} = 0.37\ \mu\text{W}$$

ASSESS We've treated the wing movement in a very simplistic way, but the answer seems reasonable. However, one could assume that the upstroke takes roughly the same amount of time as the downstroke. This modifies the answer slightly, since the wings essentially exert no force during the upstroke. Therefore, the force during the downstroke would have to be twice the bee's weight, $2mg$, to keep the average force equal to the bee's weight. The downward velocity would also be double the value we calculated, since the wing would be moving down for only half the wingbeat. The power during a downstroke would be 4 times what we calculated above. But averaged over an entire wingbeat (both downstroke and upstroke) the power would be twice what we calculated above.

71. **INTERPRET** This problem involves the concepts of power and work (or energy). Over a given period of time, the refrigerators will consume different amounts of energy, which we can calculate given their power consumption. We are to find when the cost difference for the energy consumed equals the difference in the price of the refrigerators.

DEVELOP To find the energy consumed, use Equation 6.17, $W = P\Delta t$. Thus, the work done (i.e., energy consumed) by the standard refrigerator is $W_s = P_s\Delta t_s$, where $P_s = 425$ W and $\Delta t_s = 0.20\Delta t$. The work done by the energy-efficient refrigerator in the same time interval is $W_{ee} = P_{ee}\Delta t_{ee}$, where $P_{ee} = 225$ W and $\Delta t_{ee} = 0.11\Delta t$. The cost difference Δc for the energy consumed is $\Delta c = p(W_s - W_{ee})$, where $p = 9.5$ ¢/kW·h is the price. We need to find the time interval for which the cost difference is equal to the difference in the price of the refrigerators.

EVALUATE The difference in the original price of the refrigerators is $\Delta p = \$1150 - \$850 = \$300$. The time interval to recuperate this difference is

$$\Delta p = \Delta c = p\left(P_s\Delta t_s - P_{ee}\Delta t_{ee}\right) = p\Delta t\left[(0.20)P_s - (0.11)P_{ee}\right]$$

$$\Delta t = \frac{\Delta p}{p\left[(0.20)P_s - (0.11)P_{ee}\right]} = \frac{\cancel{\$}300}{\left(\cancel{\$}0.095\ \cancel{\text{kW}}^{-1}\cdot\text{h}^{-1}\right)\left[(0.20)(0.425\ \cancel{\text{kW}}) - (0.11)(0.225\ \cancel{\text{kW}})\right]} = 5.24\times10^4\ \text{h} = 6.0\ \text{y}$$

ASSESS Notice that we converted the units so that all quantities were expressed in the same units. The answer is expressed to two significant figures because that is the least number of significant figures in the data.

72. **INTERPRET** Your friend is lifting weights and you want to verify how much energy she is using in her workout. This is similar to Problem 6.43.

DEVELOP Your friend is doing work against gravity in lifting the weight the given height: $W = mgh$ (see Figure 6.13 in the text). We're going to want to compare this to the energy content of a candy bar, so we'll use the conversion $1\ \text{kcal} = 4.184\ \text{kJ}$ from Appendix C.

EVALUATE Your friend does 5 repetitions, which requires the work of

$$W = 5mgh = 5(45\ \text{kg})(9.8\ \text{m/s}^2)(0.50\ \text{m}) = 1.10\ \text{kJ}\left(\frac{1\ \text{kcal}}{4.184\ \text{J}}\right) = 0.264\ \text{kcal}$$

This is not enough to burn off the 230 kcal candy bar. To do that, your friend would need to do significantly more reps:

$$N = \frac{230\ \text{kcal}}{\frac{1}{5}(0.264\ \text{kcal})} = 4400$$

ASSESS The work calculated above underestimates the amount of energy used while exercising, since the body burns calories just to keep the heart, lungs and other organs working. About 30 minutes of moderate weight training should burn off the candy bar.

73. **INTERPRET** This problem is about the total work done, given the power and time. The object of interest is the machine, and we are to show that the total work done is finite, even though the machine runs forever.

DEVELOP The power given in this problem is time-varying. Therefore, to find the work done in a given time interval, we need to use Equation 6.18, $W = \int_{t_1}^{t_2} P(t)\,dt$.

EVALUATE With $P(t) = P_0t_0^2/(t+t_0)^2$, we obtain

$$W = \int_0^\infty \frac{P_0t_0^2}{(t+t_0)^2}dt = P_0t_0^2\int_0^\infty \frac{dt}{(t+t_0)^2} = \frac{P_0t_0^2}{(t+t_0)}\Bigg|_0^\infty = P_0t_0$$

ASSESS The result shows that even though the machine operates forever, the total amount of work done is finite. This is not surprising because the power output decreases quadratically with time.

74. **INTERPRET** This problem involves power, work, and kinetic energy. We will also need to use the kinematic Equation 3.4, $\vec{v} = d\vec{r}/dt$, to find an expression for the distance covered by the train.

DEVELOP Because the power is constant in time, we apply Equation 6.17, $W = P\Delta t$, to find the work done. The work done is related to the speed of the train by Equation 6.14, $\Delta K = W_{net}$, and because the train starts from rest, $\Delta K = mv^2/2$, where v is the final speed of the train. The position of the train can be found by integrating Equation 3.4.

EVALUATE Equating the change in kinetic energy to the net work done, we find the following expression for the train's speed:

$$\Delta K = W_{\text{net}}$$

$$\frac{1}{2}mv^2 = Pt$$

$$v = \pm\sqrt{\frac{2Pt}{m}}$$

The position of the train is given by

$$x = \int_0^t v(t')dt' = \pm\int_0^t \sqrt{\frac{2Pt'}{m}}dt' = \pm\frac{2}{3}\sqrt{\frac{2Pt^3}{m}}$$

ASSESS The positive and negative signs correspond to the train moving to the right or to the left.

75. **INTERPRET** In this one-dimensional problem we are asked to find the work done by a non-constant force that varies with position. We want to show that although the force becomes arbitrarily large as x approaches zero, the work done remains finite.

DEVELOP Because we are dealing with a one-dimensional non-constant force $F(x)$ use Equation 6.8, $W = \int_{x_1}^{x_2} F(x)\,dx$, to find the work done. Let x_1 approach zero to find the limiting expression for the work.

EVALUATE With $F(x) = bx^{-1/2}$ we obtain

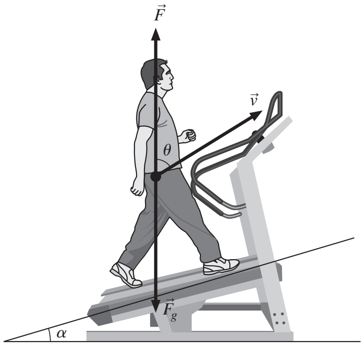
$$W_{x_1\to x_2} = \int_{x_1}^{x_2} bx^{-1/2}dx = 2bx^{1/2}\Big|_{x_1}^{x_2} = 2b\left(\sqrt{x_2} - \sqrt{x_1}\right)$$

Thus, we see that $W_{x_1\to x_2}$ is finite as $x_1 \to 0$. In fact, $W \to 2b\sqrt{x_2}$, for $x_1 \to 0$.

ASSESS The result demonstrates that even though a function $F(x)$ may diverge at some value $x = x_0$, the integral $\int F(x)dx$ can be finite at $x = x_0$.

76. **INTERPRET** You're asked to incline the treadmill so that the patient exerts energy at the desired rate.

DEVELOP The patient will have to work against gravity in walking up the inclined treadmill, $\vec{F} = -\vec{F}_g$, so the power output will be: $P = \vec{F}\cdot\vec{v} = mgv\cos\theta.$ The angle between the gravitational force and the patient's velocity is equal to $\theta = 90^\circ - \alpha$, where α is the inclination angle. See the figure below.



EVALUATE Solving for the inclination angle gives:

$$\alpha = 90^\circ - \cos^{-1}\left(\frac{P}{mgv}\right) = 90^\circ - \cos^{-1}\left(\frac{(350\text{ W})\left(3.6\frac{\text{km/h}}{\text{m/s}}\right)}{(75\text{ kg})(9.8\text{ m/s}^2)(8.0\text{ km/h})}\right) = 12^\circ$$

ASSESS The angle seems reasonable for a treadmill. Notice that you can arrive at the same answer by using the fact that $\cos\theta = \cos(90^\circ - \alpha) = \sin\alpha.$

77. **INTERPRET** Your task is to find the work needed to stretch a bungee-jump cord to double its unstretched length. The force exerted by the cord is similar to that of a spring, but with extra terms.

DEVELOP The applied force is equal and opposite to the cord's restorative force, applied to the cord , $\vec{F}_{\text{app}} = -\vec{F}.$ To find the work required to double the length of the cord, we integrate the applied force from $x = 0$ to $x = L_0$.

EVALUATE (a) Integrating the force equation gives

$$W=\int_0^{L_0} kx+bx^2+cx^3+dx^5dx=\tfrac{1}{2}kL_0^2+\tfrac{1}{3}bL_0^3+\tfrac{1}{4}cL_0^4+\tfrac{1}{5}dL_0^5$$

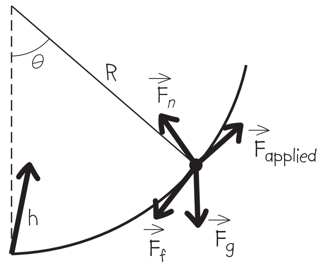
(b) With the given values the work becomes

$$W=\tfrac{1}{2}\left(420\,\tfrac{\text{N}}{\text{m}}\right)(10\text{m})^2+\tfrac{1}{3}\left(-86\,\tfrac{\text{N}}{\text{m}^2}\right)(10\text{m})^3+\tfrac{1}{4}\left(12\,\tfrac{\text{N}}{\text{m}^3}\right)(10\text{m})^4+\tfrac{1}{5}\left(-0.50\,\tfrac{\text{N}}{\text{m}^4}\right)(10\text{m})^5=12\text{ kJ}$$

ASSESS Unlike for a spring, the work formula for the cord is not symmetric around $x=0$. This is because the cord is easier to stretch than to squeeze. For example, the work needed to squeeze the cord to half its length $\left(x=-\tfrac{1}{2}L_0\right)$ is 11 kJ, which is practically the same as the work to double it.

78. **INTERPRET** We are to find the work done against friction while pushing an object up a circular ramp. The normal force (and thus the frictional force) varies, so we will need to use the integral equation for work. We want to show that the work done against friction is $\mu mg(2hR-h^2)^{1/2}$.

DEVELOP We begin by drawing a free-body diagram, as shown in the figure below. The normal force is $\vec{F}_g=mg\cos\theta$, so the frictional force is $\vec{F}_f=\mu\vec{F}_n=\mu mg\cos\theta$. From Equation 6.11, the work done against friction is $W=\int_{s_1}^{s_2}\vec{F}_f\cdot d\vec{s}$ (where we use *s* instead of *r* because we are dealing with an arc element).



EVALUATE Inserting the expression for force into Equation 6.11 gives $W=\int_{s_1}^{s_2}\mu mg\cos\theta\,ds=\mu mg\int_{\theta=0}^{\theta=\theta_f}\cos\theta(R\,d\theta)$. We need to relate the final angle θ_f to the height *h*:

$$h=R-R\cos\theta_f\quad\Rightarrow\quad\cos\theta_f=1-\frac{h}{R}\quad\Rightarrow\quad\theta_f=\text{acos}\left(1-\frac{h}{R}\right)$$

Inserting this result into the expression for work gives

$$W=\mu mgR\int_0^{\text{acos}\left(1-\frac{h}{R}\right)}\cos\theta\,d\theta=\mu mgR\left\{\sin\left[\cos^{-1}\left(1-\frac{h}{R}\right)\right]-\sin(0)\right\}$$

$$W=\mu mgR\sin\left[\cos^{-1}\left(1-\frac{h}{R}\right)\right]=\mu mgR\sqrt{2\frac{h}{R}-\frac{h^2}{R^2}}=\mu mg\sqrt{2hR-h^2}$$

which is the expression for work given in the problem statement.

ASSESS Note that this is only the work done against friction. It does not include the work done against gravity.

79. **INTERPRET** In this two-dimensional problem, we need to calculate the work done against a given vector force, along a vector path. We will use the most general integral equation for work to find the work done.

DEVELOP Calculate the work using Equation 6.11, $W=\int_{s_1}^{s_2}\vec{F}\cdot d\vec{r}$. The path taken follows $y=ax^2-bx$, where a = 2 m^{-1} and b = 4, so $\frac{dy}{dx}=2ax-b$ and $d\vec{r}=dx\hat{i}+(2ax-b)\,dx\hat{j}$. The force is $\vec{F}=cxy\hat{i}+d\hat{j}$, where c = 10 N/m^2 and d = 15 N. The position x goes from x= 0 to x = 3 m.

EVALUATE Inserting the expression for the force and the differential $d\vec{r}$ into

$$W=\int_{x=0}^{x=3\text{m}}\left(cxy\hat{i}+d\hat{j}\right)\cdot\left[\hat{i}+(2ax-b)\,\hat{j}\right])dx=\int_0^3\left[cxy+d(2ax-b)\right]dx$$

$$W=\int_0^3\left[cx\left(ax^2-bx\right)+d(2ax-b)\right]dx=\int_0^3\left(cax^3-cbx^2+2adx-bd\right)dx$$

$$W=\left[\tfrac{1}{4}cax^4-\tfrac{1}{3}cbx^3+adx^2-bdx\right]_0^3=405\text{ J}-360\text{ J}+270\text{ J}-180\text{ J}=135\text{ J}$$

ASSESS Because it is not obvious to what physical situation this problem relates, it's not possible to compare the result with an estimate or a limit gained from our understanding of physics. Notice, however, that the units work out as expected.

80. **INTERPRET** Repeat Problem 79, but instead of taking the path described therein, take "right-angle" paths. We still use the general integral equation for *W* to find the work in each case.

DEVELOP In part **(a)**, we first move along the *x* axis to the point (3 m, 0) and then parallel to the *y* axis to the point (3 m, 6 m). In part **(b)**, we move first along the *y* axis to (0, 6 m) and then parallel to the *x* axis to (3 m, 6 m). We use $W = \int_{s_1}^{s_2} \vec{F}\cdot d\vec{r}$ to find the work in each case, where $\vec{F} = cxy\hat{i} + d\hat{j}$, $c = 10\ \text{N/m}^2$ and $d = 15$ N.

EVALUATE

(a)

$$
\begin{aligned}
W &= \int_0^{3\text{m}} \left(cxy_0\hat{i} + d\hat{j}\right)\cdot d\vec{x} + \int_0^{6\text{m}} \left(cxy\hat{i} + d\hat{j}\right)\cdot d\vec{y} \\
&= \int_0^{3\text{m}} \left(cx\overbrace{y_0}^{=0}\right) dx + \int_0^{6\text{m}} (d)\,dy = (d)\,y\,\Big|_0^{6\text{m}} \\
&= (d)\,y\Big|_{y=0}^{y=6\text{ m}} = (15\text{ N})(6\text{ m} - 0\text{ m}) = 90\text{ J}
\end{aligned}
$$

(b)

$$
\begin{aligned}
W &= \int_0^{6\text{m}} \left(c\overbrace{x_0}^{=0} y\hat{i} + d\hat{j}\right)\cdot d\vec{y} + \int_0^{3\text{ m}} \left(cxy_f\hat{i} + d\hat{j}\right)\cdot d\vec{x} \\
&= \int_0^{6\text{m}} (d)\,dy + \int_0^{3\text{ m}} \left(cxy_f\right) dx \\
&= (d)\,y\Big|_0^{6\text{ m}} + \frac{cx^2 y_f}{2}\Bigg|_0^{3\text{ m}} = (15\text{ N})(6\text{ m} - 0\text{ m}) + \left(5\text{ N/m}^2\right)(6\text{ m})\left[(3\text{ m})^2 - (0\text{ m})^2\right] = 360\text{ J}
\end{aligned}
$$

ASSESS The answers vary because the force given is a *non-conservative* force. There will be more on these in the next chapter.

81. **INTERPRET** A mass falls a given distance, and we are asked to find the force necessary to stop the mass within a another given distance. From the work-energy theorem (Equation 6.14, $\Delta K = W_{\text{net}}$), we see that the work done by gravity on the way down is equal in magnitude to the work done by the stopping force, because there is no change in kinetic energy between the initial (leg on bed) and final (leg on floor) state.

DEVELOP The height dropped is $h = 0.7$ m and the stopping distance is $s = 0.02$ m. The mass of the leg is m = 8 kg. From the work-energy theorem, we know that $|W_{\text{down}}| = |W_{\text{stop}}|$. The work done by gravity is $W_{\text{down}} = mgh$, and the absolute value of the work done by the stopping force is $|W_{\text{stop}}| = F_s s$, where F_s is the stopping force.

EVALUATE From the work-energy theorem, we have

$$
\begin{aligned}
\left|W_{\text{down}}\right| &= \left|W_{\text{stop}}\right| \\
mgh &= F_s s \\
F_s &= mg\frac{h}{s}
\end{aligned}
$$

The value $h/s = (0.7\text{ m})/(0.02\text{ m}) = 35$, so the average stopping force is 35 times the weight of the leg.

ASSESS The shorter the distance over which something is stopped, the greater the force required. This is why cars are built to "crumple" on impact: The increased distance traveled by the passengers during the crash means a lower average force on their bodies.

82. **INTERPRET** We're asked to analyze a graph of the power a bat imparts on a ball as a function of time.

DEVELOP The power, by definition is the rate at which the bat supplies energy to the ball.

EVALUATE The peak in the power is where the bat is delivering energy to the ball at the greatest rate.

The answer is (c).

ASSESS We can check that the other answers are incorrect. The power from the bat does work on the ball according to Equation 6.18: $W = \int P dt$. This work increases the kinetic energy of the ball, and thereby increases its speed. After the peak, there is still more work being done on the ball. Therefore, the work, the kinetic energy and the speed do not reach their maxima at the peak – they will keep increasing until the power goes to zero.

83. **INTERPRET** We're asked to analyze a graph of the power a bat imparts on a ball as a function of time.

DEVELOP As argued in the previous problem, the speed continues to increase as long as the power is non-zero.

EVALUATE The speed will reach its maximum at the end of the hit, which occurs around 0.185 s on the graph. The answer is (c).

ASSESS If we neglect wind resistance during the hit, the only horizontal force on the ball is the force from the bat. Consequently, there is nothing to slow the ball down while the bat and ball are in contact. It would be illogical, therefore, for the maximum speed to occur before the bat's force was finished acting on the ball.

84. **INTERPRET** We're asked to analyze a graph of the power a bat imparts on a ball as a function of time.

DEVELOP The change in the kinetic energy is equal to the work done by the bat: $\Delta K = W = \int P dt.$ We can estimate this integral by roughly determining the area under the curve in the graph.

EVALUATE Each square in the grid has an area of

$$\Delta W = 1 \text{ kW} \cdot 0.01 \text{ s} = 10 \text{ J}$$

There are roughly 55 squares under the curve in the graph, so the total work done is $550 \text{ J},$ which is also the increase in the kinetic energy.

The answer is (a).

ASSESS Does this make sense? Suppose the batter hits a 90 mi/h fastball. Given that a baseball has a mass of around 140 g, the initial kinetic energy of the ball is about 110 J. The ball is initially moving in the opposite direction of the bat, so the bat will have to do 110 J of work to bring the ball to rest. That leaves 440 J to propel the ball from rest to a final velocity of

$$v_\text{f} = \sqrt{\frac{2(440 \text{ J})}{(0.14 \text{ kg})}} = 79 \text{ m/s} = 180 \text{ mi/h}$$

If we assume the ball leaves the bat at a 45° with the horizontal, then the range of the ball (Equation 3.15) is: $x = v_\text{f}^2 / g = 640 \text{ m}.$ This is unreasonably far (outfield fences in typical ballparks are around 400 ft, or 120 m, from home plate), but we have neglected wind resistance.

85. **INTERPRET** We're asked to analyze a graph of the power a bat imparts on a ball as a function of time.

DEVELOP We can assume that the force provided by the bat and the velocity of the ball are parallel. Therefore, the bat force is given by: $F = P / v.$ The power is maximum at the peak in the graph, P_pk, whereas the velocity constantly increases while the ball and bat are in contact (recall Problem 6.83).

EVALUATE We can rule out answer (a), since the power is zero there, which implies the force is too. Near the peak, the power is not changing much (the derivative with respect to time is zero at the maximum). Therefore, at a point slightly before the peak, the power is essentially the same, but the velocity is smaller by some amount we will call $\Delta v.$ The force at a point before the peak can be approximated as:

$$F_\text{before} = \frac{P_\text{before}}{v_\text{before}} \approx \frac{P_\text{pk}}{v_\text{pk} - \Delta v} \approx \frac{P_\text{pk}}{v_\text{pk}}\left(1 + \frac{\Delta v}{v_\text{pk}}\right) > F_\text{pk}$$

where we have used the binomial approximation from Appendix A: $(1-x)^{-1} \approx 1 + x$ for $x \ll 1.$ By a similar argument, $F_\text{after} < F_\text{pk},$ so the force is greatest just before the peak.

The answer is (c).

ASSESS One might question the reasoning above. If the velocity were changing more slowly than the power near the peak, then the force would be maximum at the peak, not before. However, we can show that this leads to a contradiction. The derivative of the force with respect to time is zero when the force is maximum:

$$\frac{dF}{dt} = \frac{d}{dt}\left[\frac{P}{v}\right] = \frac{1}{v}\frac{dP}{dt} - \frac{P}{v^2}\frac{dv}{dt} = 0$$

Assuming the maximum force occurs at the peak, then the derivative of the power would also be zero $\left(dP/dt = 0\right)$, since the peak is a maximum of the power as well. The equation above reduces to $dv/dt = 0$, which implies zero acceleration, zero force. But that contradicts the assumption that the peak is a maximum of the force. In conclusion, the maximum force has to occur before the peak.