8

GRAVITY

EXERCISES

Section 8.2 Universal Gravitation

11. **INTERPRET** This problem involves Newton's law of universal gravitation. We can use this law to find the radius of the planet given that we weigh twice the amount we do on Earth.

DEVELOP Newton's law of universal gravitation (Equation 8.1) is

$$F = \frac{GM_1M_2}{r^2}$$

On a spherical Earth, this gives $F_E = GM_E m/R_E^2$, where m is the mass of the explorer. On the spherical planet, this gives $F_P = GM_P m/R_P^2$. We are told that the planet has the same mass as the Earth, so $M_E = M_P$, and that the space explorers weigh twice as much on the new planet, so $2F_E = F_P$.

EVALUATE Taking the ratio of these expressions for force on each planet and solving for the radius R_P of the new planet gives

$$\frac{F_E}{F_P} = \frac{GM_E m}{R_E^2}\frac{R_P^2}{GM_P m}$$

$$\frac{1}{2} = \frac{R_P^2}{R_E^2}$$

$$R_P = \frac{R_E}{\sqrt{2}}$$

ASSESS Notice that the force due to gravity is not linear in the radius of the planet.

12. **INTERPRET** This problem involves Newton's law of universal gravitation and Newton's second law. We are to use astrophysical data to find the Moon's acceleration in its circular orbit about the Earth, and verify that it agrees with the acceleration we find from Newton's law of universal gravitation.

DEVELOP If we assume a circular orbit, the centripetal acceleration of the Moon is v^2/r, where r is the Earth-Moon separation and $v = 2\pi r/T$ is the velocity, with T being the orbital period of the Moon. The gravitational force between two masses m_1 and m_2 is given by Equation 8.1, $F = Gm_1m_2/r^2$, where r is their separation. The acceleration of the Moon in its orbit can be computed by considering the gravitational force between the Moon and the Earth and using Newton's second law (for constant mass, $F = ma$).

EVALUATE From the astrophysical data, we find that the Moon's centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2\left(3.85\times10^8\text{ m}\right)}{(27.3\text{ d})^2}\left(\frac{1\text{ d}}{24\times3600\text{ s}}\right)^2 = 2.73\times10^{-3}\text{ m/s}^2$$

Using Equation 8.1, the gravitational force between the Earth (mass M_E) and the Moon (mass M_M) is

$$F = \frac{GM_E M_M}{r^2}$$

where r is the distance between the Moon and the Earth. Using the data from Appendix E, Newton's second law gives the acceleration of the Moon as

$$a = \frac{F}{M_{\rm M}} = \frac{GM_{\rm E}}{r^2} = \frac{\left(6.67\times10^{-11}\ {\rm N\cdot m^2/kg^2}\right)\left(5.97\times10^{24}\ {\rm kg}\right)}{\left(3.85\times10^{8}\ {\rm m}\right)^2} = 2.69\times10^{-3}\ {\rm m/s^2}$$

This result is within 2% of the result from centripetal acceleration.

ASSESS What causes the difference in the results? For starters, we have approximated the Moon's orbit as circular, which is not the case. Can you think of other approximations?

13. **INTERPRET** This problem involves Newton's law of universal gravitation. We are to find the radius of the Earth that would result in gravity tripling at the surface of the Earth.

DEVELOP We shrink the Earth to a radius R such that the force due to gravity at its surface is three times the actual value. For this situation, Newton's law of universal gravitation gives

$$F = \frac{GM_{\rm E}m}{R^2} = \frac{3GM_{\rm E}m}{R_{\rm E}^2}$$

where $R_{\rm E}$ is the normal radius of the Earth.

EVALUATE Solving for the ratio of R to $R_{\rm E}$, we find $R/R_E = 1/\sqrt{3} = 57.7\%$.

ASSESS Thus, the reduced Earth would have about half the diameter of the actual Earth.

14. **INTERPRET** For this problem we need to use Newton's second law and Newton's law of universal gravitation and the astrophysical data of Appendix E to find the gravitational acceleration near the surface of **(a)** Mercury and **(b)** Saturn's moon Titan.

DEVELOP The gravitational force between two masses M and m is given by Equation 8.1, $F = GMm_2/r^2$, where r is their separation. By Newton's second law, $F = ma$, the gravitational acceleration near the surface of the gravitating body is

$$a = \frac{GM}{R^2}$$

EVALUATE With reference to the first two columns in Appendix E, we find

(a)
$$g_{\rm Mercury} = \frac{GM_{\rm Mercury}}{R_{\rm Mercury}^2} = \frac{\left(6.67\times10^{-11}\ {\rm N\cdot m^2/kg^2}\right)\left(0.330\times10^{24}\ {\rm kg}\right)}{\left(2.44\times10^{6}\ {\rm m}\right)^2} = 3.70\ {\rm m/s^2}$$

(b)
$$g_{\rm Titan} = \frac{GM_{\rm Titan}}{R_{\rm Titan}^2} = \frac{\left(6.67\times10^{-11}\ {\rm N\cdot m^2/kg^2}\right)0\left(.135\times10^{24}\ {\rm kg}\right)}{\left(2.58\times10^{6}\ {\rm m}\right)^2} = 1.35\ {\rm m/s^2}$$

ASSESS The measured values are $g_{\rm Mercury} = 3.70\ {\rm m/s^2}$ and $g_{\rm Titan} = 1.4\ {\rm m/s^2}$, so our results are in reasonable agreement with the data.

15. **INTERPRET** This involves using the gravitational force between two identical spheres to calculate their mass.

DEVELOP According to Newton's law of universal gravitation (Equation 8.1), the identical spheres $(m_1 = m_2 = m)$ generate a force between them of $F = Gm^2/r^2$.

EVALUATE Rearranging the gravitational force equation, the mass of each sphere is

$$m = r\sqrt{\frac{F}{G}} = (0.14\ {\rm m})\sqrt{\frac{0.25\times10^{-6}\,{\rm N}}{6.67\times10^{-11}\,{\rm N\cdot m^2/kg^2}}} = 8.6\ {\rm kg}$$

ASSESS Does this mass make sense given the small separation between the spheres? The density of lead is 11.34 g/cm^3, so the radius of each sphere is:

$$r = \sqrt[3]{\frac{3m}{4\pi\rho}} = \sqrt[3]{\frac{3(8600\ \text{g})}{4\pi(11.34\ \text{g/cm}^3)}} = 5.7\ \text{cm}$$

So yes, this is consistent with the fact that the centers of the two spheres are 14 cm apart, since there's about 3 cm of space between the closest edges of the spheres.

16. **INTERPRET** This problem involves using Newton's law of universal gravitation to find the gravitational attraction between an astronaut and his spaceship, given their mass and separation.

DEVELOP The gravitational force between two masses m_1 and m_2 separated by a distance r is given by Equation 8.1, $F = Gm_1m_2/r^2$.

EVALUATE Inserting the values given in the problem statement into Equation 8.1 gives

$$F = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(67\ \text{kg})(73{,}000\ \text{kg})}{(84\ \text{m})^2} = 4.62\times10^{-8}\ \text{N}$$

ASSESS The gravitational force between two masses is always attractive and decreases as $1/r^2$. Thus, when two masses are separated by a very large distance, the force between them becomes hardly noticeable. How long would it take an astronaut to return to the shuttle were he to be separated by 10 m? Assuming the shuttle to be massive enough that we can ignore its motion, we can apply Newton's second law to the astronaut to find his acceleration and use the kinematic Equation 2.10, $x = x_0 + v_0t + at^2/2$, to find the time. The result is

$$x - x_0 = \overbrace{v_0}^{=0} t + at^2/2 = at^2/2$$

$$t = \pm\sqrt{\frac{2(x-x_0)}{a}} = \sqrt{\frac{2m(x-x_0)}{F}} = \sqrt{\frac{2(67\ \text{kg})(10\ \text{m})}{4.62\times10^{-8}\ \text{N}}} \approx 2\ \text{days}$$

Thus, the astronaut would have a chance of regaining his ship.

17. **INTERPRET** We're asked to find the height of the building by using the difference in the gravitational acceleration at the top and bottom of the building. The change in the acceleration is due to the change in the distance to the center of the Earth.

DEVELOP In general, the acceleration due to gravity is given in Equation 8.2: $a = GM/r^2$. The acceleration is measured at street level, where $r = R_E$, and compared to reading at the top of the Willis Tower, where $r = R_E + h$. Here, R_E is the radius of the Earth, and h is the height of the building. The difference in the acceleration measurements should equal:

$$\Delta a = \frac{GM_E}{R_E^2} - \frac{GM_E}{(R_E+h)^2} = \frac{GM_E}{R_E^2}\left[1 - \frac{1}{(1+h/R_E)^2}\right]$$

Since $h \ll R_E$, we can use the binomial approximation from Appendix A: $(1+h/R_E)^{-2} \approx 1 - 2h/R_E$. The above expression reduces to: $\Delta a \approx 2gh/R_E$, where we have used $g = GM_E/R_E^2$ for the average value of the gravitational acceleration on the Earth's surface.

EVALUATE Using the above expression for the acceleration difference, we solve for the height of the tower:

$$h \approx R_E\frac{\Delta a}{2g} = (6.37\times10^6\,\text{m})\frac{(1.36\ \text{mm/s}^2)}{2(9.8\ \text{m/s}^2)} = 442\ \text{m}$$

ASSESS The 108-story Willis Tower is indeed 442 m tall. Note that present gravimeters can measure differences in the gravitational acceleration as small as a few tenths of a milligal, where $1\ \text{milligal} = 10^{-5}\,\text{m/s}^2$ is the unit used to measure gravity anomalies by geologists.

Section 8.3 Orbital Motion

18. **INTERPRET** The object of interest in this problem is the satellite. This problem involves Newton's second law and Newton's law of universal gravitation and explores the connection between the satellite's altitude and the period of its circular orbit.

DEVELOP By Newton's second law we have $F = ma_c$, where $a_c = v^2/r$ is the centripetal acceleration of the satellite with r being the radius of its orbit and v being the orbital speed. The orbital speed can be expressed as the circumference divided by the period T, or $v = (2\pi r)/T$. The gravitational force between the Earth and the satellite provides the centripetal force to keep the orbit circular. Thus,

$$\frac{GM_E m_s}{r^2} = \frac{m_s v^2}{r} = \frac{m_s}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 m_s r}{T^2}$$

The altitude is $h = r - R_E$, where R_E is the radius of the Earth.

EVALUATE Solving the above equation for r with $T = 2$ h = 7200 s, we obtain

$$r = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3} = \left(\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\ \text{kg}\right)\left(7200\ \text{s}\right)^2}{4\pi^2}\right)^{1/3} = 8.06\times10^6\ \text{m}$$

The altitude h is

$$h = r - R_E = 8.06\times10^6\ \text{m} - 6.37\times10^6\ \text{m} \approx 1690\ \text{km}$$

ASSESS The radius of the circular orbit is proportional to $T^{2/3}$ (Kepler's third law). This means that if the period T is to be doubled, then the radius has to increase by a factor of $2^{2/3} \approx 1.6$.

19. **INTERPRET** This problem involves using Newton's second law and Newton's universal law of gravitation to find the speed of a satellite in geosynchronous orbit (which means that the satellite completes one orbit in 24 hours, so it stays above the same place on the Earth).

DEVELOP By Newton's second law we have $F = ma_c$, where $a_c = v^2/r$ is the centripetal acceleration of the satellite with r being the radius of its orbit and v being the orbital speed. The gravitational force between the Earth and the satellite provides the centripetal force to keep the orbit circular. Thus,

$$\frac{GM_E m_s}{r^2} = \frac{m_s v^2}{r}$$

The orbital speed can be expressed as the circumference divided by the period T, or $v = (2\pi r)/T$, which we can use to eliminate the radius of the orbit so we can solve for the velocity.

EVALUATE Solving the above equation for v with $T = 24$ h = 86,400 s, we obtain

$$v = \sqrt[3]{\frac{2\pi GM_E}{T}} = \sqrt[3]{\frac{2\pi\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\ \text{kg}\right)}{86{,}400\ \text{s}}} = 3070\ \text{m/s}$$

ASSESS Dividing the circumference by this velocity and solving for the orbital radius gives

$$r = \frac{vT}{2\pi} = \frac{(3070\ \text{m/s})(86{,}400\ \text{s})}{2\pi} = 4.22\times10^7\ \text{m}$$

which agrees with the result of Example 8.3.

20. **INTERPRET** This problem involves Kepler's third law. We are asked to find the orbital period of Mars, given its orbital radius.

DEVELOP Kepler's third law (Equation 8.4) states that

$$T^2 = \frac{4\pi^2 r^3}{GM} \quad\rightarrow\quad \frac{T^2}{r^3} = \frac{4\pi^2}{GM} = \text{constant}$$

Note that this result is independent of the mass m of the orbiting object. Thus, for two celestial bodies whose semi-major axes are r_1 and r_2, the ratio of their periods would be

$$\frac{T_1^2}{T_2^2}=\frac{r_1^3}{r_2^3} \;\Rightarrow\; T_1=\left(\frac{r_1}{r_2}\right)^{3/2} T_2$$

EVALUATE Using the relation derived above, the period of Mars is

$$T_{\text{Mars}}=\left(\frac{r_{\text{Mars}}}{r_{\text{E}}}\right)^{3/2} T_{\text{E}}=(1.52)^{3/2}(1\text{ y})=1.87\text{ y}$$

ASSESS Because Mars has a larger orbit than Earth, we expect it to take longer to complete one revolution. Our result compares well with the measured value of 1.88 years given in Appendix E.

21. **INTERPRET** The problem involves finding the orbital period of one of Jupiter's moons.
DEVELOP We'll assume the orbit is circular, in which case Equation 8.4 gives the period: $T^2=4\pi^2r^3/GM$. We're told the radius of the orbit, and we can find the mass of Jupiter from Appendix E: $M=1.90\times10^{27}\,\text{kg}$.
EVALUATE Io's orbital period is

$$T=\sqrt{\frac{4\pi^2\left(4.22\times10^{8}\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.90\times10^{27}\,\text{kg}\right)}}=1.53\times10^5\,\text{s}=1.77\text{ d}$$

ASSESS The answer agrees with the rotation period given in Appendix E for the moon Io.

22. **INTERPRET** This problem involves Kepler's third law, which we can use to find the period of a golf ball orbiting the Moon.
DEVELOP Kepler's third law (Equation 8.4) states that

$$T^2=\frac{4\pi^2r^3}{GM} \;\Rightarrow\; T=2\pi\sqrt{\frac{r^3}{GM}}$$

Knowing the mass and the radius of the Moon allows us to determine the orbital period of the golf ball.
EVALUATE Using the equation obtained above, we find the period to be

$$T=2\pi\sqrt{\frac{R^3}{GM}}=2\pi\sqrt{\frac{\left(1.74\times10^6\ \text{m}\right)^3}{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.35\times10^{22}\ \text{kg}\right))}}=6.51\times10^3\ \text{s}=109\text{ min}$$

ASSESS This result is independent of the mass *m* of the golf ball. Any mass thrown into this orbit would have the same period *T*.

23. **INTERPRET** We're asked to find the altitude of a spacecraft orbiting Mars given its orbital period.
DEVELOP Equation 8.4 relates the period and radius of an orbiting body to the mass of the object it is orbiting around: $T^2=4\pi^2r^3/GM$. The mass of Mars from Appendix E is: $M_{\text{M}}=6.42\times10^{23}\,\text{kg}$. Once we solve for the orbital radius, we will have to subtract the radius of Mars $\left(R_{\text{M}}=3.38\times10^6\,\text{m}\right)$ to find the altitude: $h=r-R_{\text{M}}$.

EVALUATE The distance between the Mars Renaissance Orbiter and the center of the planet Mars is

$$r=\sqrt[3]{\tfrac{1}{4\pi^2}GMT^2}=\sqrt[3]{\tfrac{1}{4\pi^2}\left(6.67\times10^{-11}\,\tfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(6.42\times10^{23}\,\text{kg}\right)\left(112\cdot60\text{ s}\right)^2}=3.659\times10^6\,\text{m}$$

This implies that the altitude of the spacecraft is

$$h=r-R_{\text{M}}=3.659\times10^6\,\text{m}-3.38\times10^6\,\text{m}=0.28\times10^6\,\text{m}$$

ASSESS This is 280 km. Compare this to Example 8.2, where it was shown that a low Earth orbit with an altitude of 380 km has a period of about 90 min. Since Mars has less mass, spacecrafts must orbit at a smaller radius in order to have roughly the same orbital period.

Section 8.4 Gravitational Energy

24. **INTERPRET** The problem asks about the change in potential energy as Earth goes from perihelion to aphelion.

DEVELOP The potential energy difference between two points at distances r_1 and r_2 from the center of a gravitating mass *M* is, according to Equation 8.5:

$$\Delta U_{12} = -GMm\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

In this case, the gravitating mass is the sun $(M = M_S)$, acting on the Earth $(m = M_E)$. These mass values can be found in Appendix E. We want the change in potential energy as the Earth moves from its perihelion $(r_1 = r_p)$, which is the point of closest approach to the Sun, to its aphelion $(r_2 = r_a)$, which is the most distant point from the Sun.

EVALUATE Using the equation above and the fact that the prefix G stands for 10^9, the change in potential energy as Earth goes from perihelion to aphelion is

$$\Delta U = -GM_S m_E\left(\tfrac{1}{r_a} - \tfrac{1}{r_p}\right)$$

$$= -\frac{\left(6.67\times10^{-11}\,\tfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(1.99\times10^{30}\,\text{kg}\right)\left(5.97\times10^{24}\,\text{kg}\right)}{10^9\,\text{m}}\left(\frac{1}{152} - \frac{1}{147}\right) = 1.77\times10^{32}\,\text{J}$$

ASSESS Most planetary orbits are elliptical. As the Earth moves further away from the Sun $(r_p \to r_a)$, the change in potential energy is positive. It's as if the Earth is moving out of a gravitational potential well centered at the Sun. In order to conserve mechanical energy $(K + U = \text{constant})$, the Earth's kinetic energy will correspondingly decrease slightly.

25. **INTERPRET** This problem deals with the gravitational potential energy of an object. We are asked to find the energy required to raise an object to a given height in the Earth's gravitational field.

DEVELOP If we neglect any kinetic energy differences associated with the orbital or rotational motion of the Earth or package, the required energy is just the difference in gravitational potential energy given by Equation 8.5, $\Delta U = GM_E m\left[R_E^{-1} - (R_E + h)^{-1}\right]$, where h = 1800 km = 1.8 × 10^6 m.

EVALUATE Evaluating the expression above with the data from Appendix E gives

$$\Delta U = \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\ \text{kg}\right)\left(230\ \text{kg}\right)\left[\frac{1}{6.37\times10^6\ \text{m}} - \frac{1}{6.37\times10^6\ \text{m} + 1.8\times10^6\ \text{m}}\right] = 3.17\ \text{GJ}$$

ASSESS In terms of the more convenient combination of constants $GM_E = gR_E^2$, $\Delta U = mgR_E h/(R_E + h) = 3.17$ GJ

26. **INTERPRET** The problem asks for the maximum altitude the rocket can reach with an initial launch speed v_0. The problem sounds similar to what we encountered in Chapter 2, but here the acceleration is not constant. Instead, we will consider conservation of total mechanical energy.

DEVELOP If we consider the Earth at rest as approximately an inertial system, then a vertically launched rocket would have zero kinetic energy (instantaneously) at its maximum altitude, and the situation is the same as Example 8.5. Conservation of mechanical energy, $U_0 + K_0 = U + K$, can be used to solve for the maximum altitude.

EVALUATE The conservation equation gives

$$\frac{1}{2}mv_0^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{R_E + h}$$

or

$$h = \left(\frac{1}{R_E} - \frac{v_0^2}{2GM_E}\right)^{-1} - R_E$$

$$= \left(\frac{1}{6.37\times10^6\ \text{m}} - \frac{(5100\ \text{m/s})^2}{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}\right)^{-1} - 6.37\times10^6\ \text{m}$$

$$= 1.67\times10^6\ \text{m}$$

ASSESS If we assume a potential energy change of $\Delta U = mgh$, where $g = 9.8\ \text{m/s}^2$, then the result would have been

$$h = \frac{v_0^2}{2g} = \frac{(5100\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = 1.33\times10^6\ \text{m}$$

The decreasing gravitational acceleration $g(r)$ allows the rocket to go higher!

27. **INTERPRET** This problem involves conservation of total mechanical energy, as in Example 8.5, except that for this problem, we are given the final altitude and need to find the launch speed.

DEVELOP Apply conservation of total mechanical energy (Equation 7.7), $U_0 + K_0 = U + K$, where

$$U_0 + K_0 = -\frac{GM_\text{E}m}{R_\text{E}} + \frac{1}{2}mv^2$$

and

$$U + K = -\frac{GM_\text{E}m}{R_\text{E} + h} + 0$$

EVALUATE Solving the expression derived from conservation of total mechanical energy for the initial velocity v gives

$$v = \pm\sqrt{2GM_E\left(\frac{1}{R_E} - \frac{1}{R_E + h}\right)}$$

$$= \sqrt{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})\left(\frac{1}{(6.37\times10^6\ \text{m})} - \frac{1}{(6.37+1.1)\times10^6\ \text{m}}\right)}$$

= 4.29 km/s

ASSESS The positive square root was chosen because we are interested in the magnitude of the speed. Notice that our result is larger than the initial speed of 3.1 km/s for Example 8.5, which makes sense because the altitude attained (1100 km) is higher than that attained (530 km) in Example 8.5.

28. **INTERPRET** The problem asks about the energy needed to put a mass into the Earth's geosynchronous orbit. We can apply conservation of total mechanical energy to solve this problem.

DEVELOP The total mechanical energy of an object at rest ($K_0 = 0$) on the Earth's surface is

$$E_0 = U_0 + K_0 = -\frac{GM_\text{E}m}{R_\text{E}} + 0$$

(neglecting diurnal rotational energy, etc.), and its total mechanical energy in a circular orbit is given by Equation 8.8b:

$$E = -\frac{GM_E m}{2r}$$

The difference is the energy required to put the mass into the orbit.

EVALUATE The distance, measured from the Earth's center, that corresponds to the geosynchronous orbit is (see Example 8.3) $r = 4.22\times10^7\ \text{m}$. Thus, the energy necessary to put a mass of $m = 1$ kg into a circular geosynchronous orbit is

$$\Delta E = E - E_0$$
$$= \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\ \text{kg}\right)(1\ \text{kg})\left(\frac{1}{6.37\times10^6\ \text{m}} - \frac{1}{2(4.22\times10^7\ \text{m})}\right)$$
$$= 6\times10^7\ \text{J}$$

ASSESS We report the answer to a single significant figure because we are given the mass of the object to a single significant figure. The positive change in energy reflects the need to increase both the gravitational potential energy (making it less negative) and the kinetic energy of the mass.

29. **INTERPRET** This problem involves finding the total mechanical energy associated with the Earth's orbit about the Sun. We will assume the orbit is circular.

DEVELOP Apply Equation 8.8b, using data from Appendix E, to find the total mechanical energy. The two masses involved are the mass M_E of the Earth and the mass M_S of the Sun.

EVALUATE The total mechanical energy associated with the Earth's orbital motion is

$$E_{\text{Tot}} = \frac{1}{2}U = -\frac{GM_S M_E}{2r}$$
$$= -\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)\left(5.97\times10^{24}\ \text{kg}\right)}{2\left(1.50\times10^{11}\ \text{m}\right)} = -2.64\times10^{33}\ \text{J}$$

ASSESS Alternatively, from Equation 8.8a, $E_{\text{tot}} = -K = -\frac{1}{2}M_E\left(2\pi\, r/T\right)^2 = -2.66\times10^{33}\ \text{J}$, consistent with the accuracy of the data used in Appendix E.

30. **INTERPRET** This problem involves finding a planet's mass given the planet's escape speed, which is the speed needed to escape from the planet's gravitational field.

DEVELOP Solve Equation 8.7, $v_{\text{esc}} = \sqrt{2GM/r}$, for the radius r of the planet.

EVALUATE From the equation above, the radius of the planet is

$$r = \frac{2GM}{v_{\text{esc}}^2} = \frac{2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.9\times10^{24}\ \text{kg}\right)}{\left(7.1\times10^3\ \text{m/s}\right)^2} = 7.67\times10^6\ \text{m}$$

ASSESS The more massive the gravitating body, the greater is the speed required to escape from its gravitational field.

31. **INTERPRET** This problem involves calculating the escape speed from two celestial bodies with different characteristics.

DEVELOP Solve Equation 8.7, $v_{\text{esc}} = \sqrt{2GM/r}$, for the escape speed v_{esc} for each body. Use data from Appendix E as needed.

EVALUATE **(a)** For Jupiter's moon Callisto, the escape speed is

$$v_{\text{esc}} = \sqrt{2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.07\times10^{23}\ \text{kg}\right)/\left(2.40\times10^6\ \text{m}\right)} = 2.44\ \text{km/s}.$$

(b) For a neutron star, $v_{\text{esc}} = \sqrt{2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)/\left(6\times10^3\ \text{m}\right)} = 2.10\times10^8\ \text{m/s}$.

ASSESS The escape speed from a neutron star is about 70% of the speed of light.

32. **INTERPRET** This problem is about finding the radius of the Earth necessary to give an escape speed of 30 km/s.

DEVELOP Solve Equation 8.7, $v_{\text{esc}} = \sqrt{2GM/r}$, for the radius r of the Earth using Earth's mass from Appendix E and the given escape speed.

EVALUATE Given that $v_{\text{esc}} = 30\ \text{km/s} = 30{,}000\ \text{m/s}$, the would-be radius of the Earth is

$$r = \frac{2GM_E}{v_{\text{esc}}^2} = \frac{2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\ \text{kg}\right)}{(3.0\times10^4\ \text{m/s})^2} = 8.85\times10^5\ \text{m}$$

ASSESS Expressed in terms of the real value $R_E = 6.37\times10^6\ \text{m}$, we have $R \approx 0.139R_E$. The relationship between R and v_{esc} with M kept fixed is

$$\frac{v_{\text{esc},1}}{v_{\text{esc},2}} = \sqrt{\frac{R_2}{R_1}}$$

Thus, if we use the fact that the escape speed from the Earth is $v_{\text{ecs},0} = 11.2$ km/s , we can apply the above equation to find the would-be radius if the escape speed were $v_{\text{esc}} = 30$ km/s The result is

$$\frac{R}{R_E} = \left(\frac{v_{\text{esc},0}}{v_{\text{esc}}}\right)^2 = \left(\frac{11.2 \text{ km/s}}{30 \text{ km/s}}\right)^2 \approx 0.139$$

which is what we found above.

PROBLEMS

33. **INTERPRET** This problem involves Newton's second law (F = ma) and Newton's law of universal gravitation. We are to find the acceleration at an altitude equal to half the planet's radius, given the gravitational acceleration on the planet's surface.

DEVELOP By Newton's law of universal gravitation, Equation 8.1, we see that the force due to gravity on an object is $F = GMm/r^2$. Newtons' second law $F = ma$ relates force to acceleration, so we can express the acceleration on the surface of the planet and at an altitude above the surface equal to one-half of the planet's radius.

EVALUATE On the planet's surface, the acceleration due to gravity is

$$F_{\text{net}} = \frac{GMm}{r^2} = mg$$
$$g = \frac{GM}{r^2}$$

At the given altitude h, we substitute $3r/2$ for r, then solve the system of equations for g_h. This gives

$$g_{\text{h}} = \frac{GM}{(3r/2)^2} = \left(\frac{2}{3}\right)^2 \frac{GM}{r^2} = \frac{4}{9} g = \frac{4}{9}\left(22.5 \text{ m/s}^2\right) = 10.0 \text{ m/s}^2$$

ASSESS The acceleration scales with the inverse of the radial distance squared, to increasing the radius by a factor 3/2 results in a decrease in the acceleration by a factor $(2/3)^2$.

34. **INTERPRET** We can use the conservation of energy to find what height the high-jumper, Javier Sotomayor, might jump on a different planetary body.

DEVELOP Whether he's on Earth, Mars or the Moon, we assume that Sotomayor takes off with the same kinetic energy, K_0, and that his kinetic energy is zero at the top of his jump. By conservation of energy (Equation 7.6), this change in kinetic energy is equal to the negative of the change in potential energy: $\Delta K = 0 - K_0 = -\Delta U_{12}$. On a generic planetary body, let's call it X, the change in potential energy between the ground $(r_1 = R_\text{X})$ and the height Sotomayor reaches $(r_2 = R_\text{X} + h_\text{X})$ can be found from Equation 8.5:

$$\Delta U_{12} = GM_\text{X} m\left(\frac{1}{R_\text{X}} - \frac{1}{R_\text{X} + h_\text{X}}\right) \approx \frac{GM_\text{X} m}{R_\text{X}}\left[1 - \left(1 - h_\text{X}/R_\text{X}\right)\right] = mg_\text{X} h_\text{X}$$

Notice that we have used the binomial approximation from Appendix A, seeing as $h_\text{X} \ll R_\text{X}$. The term $g_\text{X} = GM_\text{X}/R_\text{X}^2$ is the surface gravity on whichever planetary body (see Appendix E). On Earth, Sotomayor reached a height h_E with gravity g_E, so with the same kinetic energy on planet X he would reach $h_\text{X} = \left(g_\text{E}/g_\text{X}\right) h_\text{E}$.

EVALUATE (a) The surface gravity on Mars is 3.74 m/s^2, so Sotomayor would jump:

$$h_{\text{Mars}} = \frac{9.81 \text{ m/s}^2}{3.74 \text{ m/s}^2}\left(2.45 \text{ m}\right) = 6.43 \text{ m}$$

(b) The surface gravity on the Moon is 1.62 m/s^2, so Sotomayor would jump:

$$h_{\text{Moon}} = \frac{9.81\ \text{m/s}^2}{1.62\ \text{m/s}^2}(2.45\ \text{m}) = 14.8\ \text{m}$$

ASSESS It makes sense that one could jump higher on a planet or moon with lower gravity than Earth. Notice that we could have gotten the same result by using Equation 2.11 and assuming that Sotomayor jumps with the same initial vertical velocity but under different gravitational accelerations.

35. **INTERPRET** This problem explores the gravitational acceleration of a gravitating body as a function of altitude h.

DEVELOP Using Equation 8.1, the gravitational force between a mass m and a planet of mass M_p is $F = GM^p m/r^2$ where r is their separation, measured from the center of the planet. From Newton's second law (for constant mass), $F = ma$, the acceleration of gravity at any altitude $h = r - R_p$ above the surface of a spherical planet of radius R_p, is

$$g(h) = \frac{GM_p}{(R_p + h)^2} = \frac{GM_p}{R_p^2}\left(\frac{R_p}{R_p + h}\right)^2 = g(0)\left(\frac{R_p}{R_p + h}\right)^2$$

where $g(0)$ is the value at the surface. Once the ratio $g(h)/g(0)$ is known, we can find the altitude h in terms of R_p.

EVALUATE Solving for h, we find

$$\frac{h}{R_p} = \sqrt{\frac{g(0)}{g(h)}} - 1$$

Therefore, for $g(h)/g(0) = 1/2$, we have $h/R_p = \sqrt{2} - 1 = 0.414$.

ASSESS To see if the result makes sense, we take the limit $h = 0$, where the object rests on the surface of the planet. In this limit, we recover $g(0)$ as the gravitational acceleration. The equation also shows that $g(h)$ decreases as the altitude h is increased, and $g(h)$ approaches zero as $h \to \infty$.

36. **INTERPRET** This problem involves Newton's law of universal gravitation. We are asked to find the fraction by which our weight on the surface of the Earth is reduced due to a spherical mass positioned directly over our head.

DEVELOP Apply Newton's law of universal gravitation to find the force exerted on you in the upward direction due to the spherical mass of water. Because the mass is spherical, we can treat it as if the entire mass were concentrated at the geometric center of the sphere. Divide the result by your weight $w = mg$ by this force to find the fraction by which your weight is reduced.

EVALUATE The force due to the water mass is

$$F_w = \frac{GM_w m}{r^2}$$

where $M_w = 4 \times 10^6$ kg and $r = 15$ m. Dividing this result by your weight gives a ratio of

$$\gamma = \frac{mgr^2}{GM_w m} = \frac{(9.8\ \text{m/s}^2)(15\ \text{m})^2}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(4\times10^6\ \text{kg})} = 1\times10^{-7}$$

to a single significant figure.

ASSESS We retain only a single significant figure because this is the precision to which we know the mass of the water. If we assume a 70-kg person that is 85% H_20, this fraction corresponds to the person's mass being reduced by the following estimated number n of water molecules:

$$n \approx \frac{(0.85)(1\times10^{-7})(70\times10^3\ \text{g})(6.02\times10^{23}\ \text{H}_2\text{O/mol})}{18\ \text{g/mo l}} = 2\times10^{20}\ \text{H}_2\text{O}$$

where the constant 6.02×10^{23} is Avogadro's constant (see Chapter 17 and the discussion preceding Equation 17.2).

37. **INTERPRET** In this problem we want to find the Moon's acceleration in its circular orbit about the Earth. In addition, we want to use the result to confirm the inverse-square law for the gravitational force.

DEVELOP Using Newton's second law ($F_{net} = ma$), and the Equation 5.1 for centripetal acceleration, we find that the centripetal force that keeps the Moon's orbit circular is:

$$F_{\text{net}} = M_{\text{M}} a_{\text{c}} = \frac{M_{\text{M}} v^2}{r_{\text{ME}}} = \frac{4\pi^2 M_{\text{M}} r_{\text{ME}}}{T^2}$$

The equation allows us to compute the acceleration of the Moon in its circular orbit.

EVALUATE Substituting the values given in the problem statement, we find

$$a_{\text{c}} = \frac{v^2}{r_{mE}} = \frac{4\pi^2 r_{mE}}{T^2} = \frac{4\pi^2 \left(3.84 \times 10^8 \text{ m}\right)}{\left(27.3 \text{ d} \times 86,400 \text{ s/d}\right)^2} = 2.73 \times 10^{-3} \text{ m/s}^2$$

As a fraction of the acceleration due to gravity on the surface of the Earth, this is

$$\frac{a_{\text{c}}}{g} = \frac{2.73 \times 10^{-3} \text{ m/s}^2}{9.81 \text{ m/s}^2} = 2.78 \times 10^{-4}$$

Comparing this result with the ratio of the radius of the Earth to the radius of the Moon's orbit gives

$$\left(\frac{R_E}{r_{ME}}\right)^2 = \left(\frac{6.37 \times 10^6 \text{ m}}{3.85 \times 10^8 \text{ m}}\right)^2 = 2.74 \times 10^{-4}$$

which suggests that Newton's inverse-square law for gravity is valid.

ASSESS Why do the answers differ by 0.04/2.78 = 1.4%? The problem is not the data used, nor its precision (all data is accurate to 3 significant figures). However, we assumed that the Moon's orbit was circular, which is not exactly true. At its nearest, the Moon is some 364×10^3 km from the Earth, whereas at its farthest, it is some 407×10^3 km from the Earth. This assumption is responsible for the difference in the results obtained above.

38. **INTERPRET** We're asked to derive Newton's law of gravitation from the function for the gravitational potential energy.

DEVELOP Equation 8.6 gives the gravitational potential energy at a distance r from a point mass: $U(r) = -GMm/r.$ Differentiating with respect to distance r and applying a minus sign should recover the force law (Equation 8.1).

EVALUATE From Equation 7.8, the force is equal to

$$F(r) = -\frac{dU}{dr} = -\frac{d}{dr}\left[\frac{-GMm}{r}\right] = \frac{-GMm}{r^2}$$

This has the same magnitude as Equation 8.1. The minus sign reminds us that the gravitational force is attractive, i.e., it points in a direction opposite to that of the radial vector, $\vec{r}$.

ASSESS Recall that we are free to define the zero of the gravitational potential, but that will only change the function by a constant: $U(r) = -GMm/r + C.$ When we take the derivative to find the force, this constant will disappear. The force is independent of our choice for the zero of the potential.

39. **INTERPRET** This problem involves Newton's law of universal gravitation, which is used to find the period of a circular orbit (Equation 8.4). We are asked to find the half-period of a circular orbit 130 m above the surface of the Moon.

DEVELOP Equation 8.4 gives the period of a circular orbit to be $T^2 = 4\pi^2 r^3/(GM)$, where M is the mass of the Moon and r is the radius of the orbit. For an orbit at a height h =130 m above the surface of the Moon, $r = R_{\text{M}} + h$. Use the data available in Appendix E to evaluate the half period $T/2$.

EVALUATE The half period of the astronaut's orbit was

$$\frac{T}{2} = \pi\sqrt{\frac{\left(R_{\text{M}} + h\right)^3}{GM}} = \pi\sqrt{\frac{\left(1.74 \times 10^6 \text{ m} + 0.13 \times 10^6 \text{ m}\right)^3}{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(7.35 \times 10^{22} \text{ kg}\right)}} = 3.63 \times 10^3 \text{ s} = 60.5 \text{ min}$$

or about an hour.

ASSESS During this hour, the astronaut could not communicate with the Earth.

40. **INTERPRET** In this problem we are asked to find the speed and period of an object orbiting about a gravitating body—a white dwarf.

DEVELOP Newton's law of universal gravitation describes the force between the spaceship and the white dwarf that provides the centripetal force for the spaceship to move in a circular path about the white dwarf:

$$F = \frac{GMm}{r^2} = ma_c = \frac{mv^2}{r}$$

where we used Equation 5.1, $a_c = v^2/r$, for centripetal acceleration. Solving for the orbital speed gives $v = \sqrt{GMm/r}$ (Equation 8.3). The period may be found by dividing the orbital circumference by the orbital velocity. This gives $T^2 = 4\pi^2 r^3/(GM)$ (Equation 8.4). Use the data from Appendix E to evaluate these formulas.

EVALUATE **(a)** The radius of a low orbit is approximately the radius of the white dwarf, or R_E, so Equation 8.3 gives

$$v = \sqrt{\frac{GM}{R_E}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)}{6.37\times10^{6}\ \text{m}}} = 4.56\times10^{6}\ \text{m/s}$$

or about 1.5% of the speed of light.

(b) The orbital period is

$$T = \frac{2\pi R_E}{v} = \frac{2\pi\left(6.37\times10^{6}\ \text{m}\right)}{4.56\times10^{6}\ \text{m/s}} = 8.77\ \text{s}$$

which is very short.

ASSESS According to Kepler's third law, the relationship between T and M is given by

$$T^2 = \frac{4\pi^2 r^3}{GM} \quad \Rightarrow \quad MT^2 = \frac{4\pi^2 R^3}{G} = \text{constant}$$

Thus, we see that if the mass of the gravitating body M is increased while keeping its radius R constant, then its period T must decrease.

41. **INTERPRET** We will be estimating the mass of the galaxy by using the Sun's orbit around the galaxy. This is similar to measuring the mass of the Earth by the orbit of the moon.

DEVELOP Equation 8.4 relates mass of a central object to the period and radius of an orbiting object: $T^2 = 4\pi^2 r^3 / GM$. However, the central object in this case is a point or a sphere, so we will have to assume that the galaxy is spherical and that most of its mass is located interior to the orbit of the Sun.

EVALUATE Using the radius and period given for the Sun's orbit, the mass of the galaxy is approximately

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2\left(2.6\times10^{20}\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(2\times10^{8}\times\pi\times10^{7}\,\text{s}\right)^2} = 2.6\times10^{41}\ \text{kg}$$

ASSESS If we divide our result by the mass of the Sun $\left(1.99\times10^{30}\,\text{kg}\right)$, we find that it is equivalent to about 100 billion Suns, which is a reasonable estimate for the number of stars in the galaxy. Astronomers plot the orbital velocity of objects (such as stars, clusters of stars, or clouds of hydrogen atoms) versus their distance from the galactic center to obtain "the rotation curve" for our galaxy and others. What's surprising about these curves is that they are flat (i.e., nearly constant) out to distances far beyond the central bright region of most galaxies. One would have expected the velocity of orbiting objects to drop off at large radii, as indicated in Equation 8.3. The fact that it doesn't seems to imply some sort of "dark matter," which doesn't emit or scatter light and yet accounts for over 80% of the mass in a galaxy. Dark matter is currently a topic of great interest in astronomy.

42. **INTERPRET** You ask yourself how much further can a golf ball go on the Moon compared to the Earth.

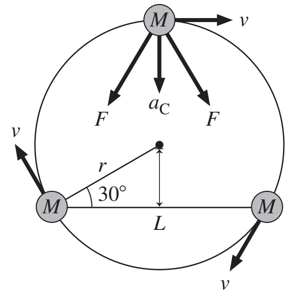
DEVELOP The horizontal range of a projectile was given in Equation 3.15: $x = v_0^2 \sin 2\theta_0 / g$, where v_0 and θ_0 are the initial speed and angle of the projectile. If we assume an astronaut can hit a golf ball on the Moon with the same speed and angle of the record one-arm hit on the Earth, then the range will be farther due to the lower surface gravity on the Moon.

EVALUATE Using the above range expression and the surface gravity values from Appendix E, the farthest an astronaut could hit a golf ball on the Moon is

$$x_M = x_E \frac{g_E}{g_M} = (257\text{ m})\frac{(9.81\text{ m/s}^2)}{(1.62\text{ m/s}^2)} = 1560\text{ m}$$

This is a little less than a mile, so Alan Shepard's claim of "miles and miles" was likely an exaggeration.
ASSESS We have not accounted for the fact that there's no wind resistance on the Moon, thus allowing lunar golf balls even greater range than what we have calculated on purely gravitational grounds. Even so, Shepard was wearing a bulky suit at the time, which didn't allow him to make a full swing. Although no measurements were made, the general consensus is that Shepard's ball couldn't have gone more than a couple hundred meters.

43. **INTERPRET** We're asked to solve a three-body problem, in which three identical stars are situated on the vertices of an equilateral triangle.
DEVELOP We're told that the system rotates. In order for the configuration to remain stable, each star must rotate with the same speed. Let's assume the rotational direction is clockwise, as shown in the figure below.



This is uniform circular motion about a radius $r = L/2\cos 30^\circ$. The centripetal acceleration $(a_c = v^2/r)$ is provided by gravity. Specifically, each star is pulled toward the two other stars. Taken separately, the magnitude of the force, F, between two stars is: $F = GM^2/L^2$ (Equation 8.1). Added together, the net force points toward the center of the triangle with a magnitude of

$$F_{net} = F\cos 30^\circ + F\cos 30^\circ = \frac{2GM^2\cos 30^\circ}{L^2}$$

It's this force that supplies the centripetal acceleration: $F_{net} = Ma_c$.
EVALUATE Pulling together all the information above, we can find an expression for the speed of the stars' rotation:

$$v = \sqrt{a_c r} = \sqrt{\left(\frac{F_{net}}{M}\right)\left(\frac{L}{2\cos 30^\circ}\right)} = \sqrt{\frac{GM}{L}}$$

Notice how this has a similar form to Equation 8.3: $v = \sqrt{GM/r}$, for the orbital speed of a two-body system. The period in the three-body system is:

$$T = \frac{2\pi r}{v} = \frac{\pi L}{\cos 30^\circ}\sqrt{\frac{L}{GM}}$$

To draw some comparison with Equation 8.4, we square the above equation and use $\cos^2 30^\circ = \frac{3}{4}$,

$$T^2 = \frac{4\pi^2 L^3}{3GM}$$

ASSESS This says the period becomes longer, the farther the stars are separated, which makes sense. The system rotates faster (shorter period) when the mass of the stars is larger, which also makes sense.

44. **INTERPRET** For this problem we compare, with the help of Kepler's third law, the orbital periods of two satellites located at different distances from the center of the Earth.
DEVELOP Kepler's third law (Equation 8.4) states that

$$T^2 = \frac{4\pi^2 r^3}{GM} \quad \Rightarrow \quad \frac{T^2}{r^3} = \frac{4\pi^2}{GM} = \text{constant}$$

Thus, for the two satellites A and B, the ratio of their period would be

$$\frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3} \quad \Rightarrow \quad \frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{3/2}$$

EVALUATE With $r_A = 2r_B$ the ratio of their period is

$$\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{3/2} = 2^{3/2} = 2.83$$

ASSESS By Kepler's third law, the orbital period is proportional to $r^{3/2}$. Therefore, the further away the satellite is from the Earth, the longer is its period.

45. **INTERPRET** This problem requires us to find the semimajor axis of an asteroid's orbit by using Kepler's third law (Equation 8.4). We are to express the results in terms of the Earth's orbital period and radius about the Sun.

DEVELOP Kepler's third law is

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

where r is the semimajor axis of the elliptical orbit. To find the semimajor axis of the asteroid's orbit, we take the ratio of Kepler's third law applied to both objects:

$$\left(\frac{T_{\text{ast}}}{T_{\text{E}}}\right)^2 = \left(\frac{r_{\text{ast}}}{r_{\text{E}}}\right)^3 \quad \Rightarrow \quad r_{\text{ast}} = r_{\text{E}}\left(\frac{T_{\text{ast}}}{T_{\text{E}}}\right)^{2/3}$$

EVALUATE Taking one unit of distance to be the Earth's orbit (≡ 1 AU) and one unit of time to be the Earth's period (= 1 y), we have

$$r_{\text{ast}} = (1\text{ AU})\left[\left(\frac{1417\text{ d}}{1\text{ y}}\right)\left(\frac{1\text{ y}}{363\text{ d}}\right)\right]^{2/3} = 2.47\text{ AU}$$

ASSESS Converting this distance to meters gives r_{ast} = (2.47 AU)(1.5 × 10^{11} m/AU) =3.71 × 10^8 m.

46. **INTERPRET** This problem involves conservation of total mechanical energy. We are to find the speed of an object as it hits the Sun, given that it starts from rest 1 AU (astronomical unit, see previous problem) from the Sun.

DEVELOP The canisters start at rest 1 AU from the Sun and free from the Earth's gravitational field. The change of potential energy as the waste canister travels into the Sun can be calculated by using conservation of total mechanical energy, $\Delta U + \Delta K = 0$ (Equation 7.6). The change in potential energy may be obtained from Equation 8.6:

$$\Delta U = U_{\text{final}} - U_{\text{initial}} = -\frac{GMm}{R_{\text{S}}} + \frac{GMm}{r_{\text{ES}}} = -GMm\left(\frac{1}{R_{\text{S}}} - \frac{1}{r_{\text{ES}}}\right)$$

where R_{S} is the radius of the Sun and r_{ES} is the radius of the Earth's orbit about the Sun. The gain in kinetic energy is $\Delta K = K_{\text{final}} - K_{\text{initial}} = mv^2/2 - 0 = mv^2/2$.

EVALUATE Solving Equation 7.6 for the speed v, we obtain

$$v = \sqrt{2GM\left(\frac{1}{R_{\text{S}}} - \frac{1}{r_{\text{ES}}}\right)}$$

$$= \sqrt{2\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\text{ kg}\right)\left(\frac{1}{6.96\times10^{8}\text{ m}} - \frac{1}{1.50\times10^{11}\text{ m}}\right)}$$

$$= 616\text{ km/s}$$

ASSESS How much energy per kg would be required to implement this solution? This may be found by using (again) conservation of total mechanical energy. The energy per kg to escape the Earth's gravitational field is (using Equation 8.6)

$$\Delta U = U_{\text{final}} - U_{\text{initial}} = -\frac{GM_{\text{E}}m}{\infty} + \frac{GM_{\text{E}}m}{R_{\text{E}}}$$

$$\frac{\Delta U}{m} = GM_{\text{E}}\left(\frac{1}{R_{\text{E}}}\right) = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\ \text{kg}\right)}{\left(6.37\times10^{6}\ \text{m}\right)} = 6.25\times10^{7}\ \text{J}$$

which is more energy than is contained in 1 kg of uranium!

47. **INTERPRET** This problem involves conservation of mechanical energy, which we can use to find the speed of Comet Halley when it reaches Neptune's orbit, given its speed at the perihelion.

DEVELOP At perihelion, the point of closest approach to the Sun, the comet's distance from the Sun is $r_{\text{i}} = 8.79\times10^{10}$ m and its speed is $v = 54.6\times10^{3}$ m/s. The distance of Neptune's orbit is at $r_f = 4.50\times10^{12}$ m. We use conservation of energy: $U_i + K_i = U_f + K_f$.

EVALUATE Inserting the given quantities into the formula above gives

$$U_i + K_i = U_f + K_f$$

$$-G\frac{Mm}{r_i} + \frac{1}{2}mv_i^2 = -G\frac{Mm}{r_f} + \frac{1}{2}mv_f^2$$

$$v_f = \pm\sqrt{2GM\left(\frac{1}{r_f} - \frac{1}{r_i}\right) + v_i^2}$$

$$= \pm\sqrt{2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)\left(\frac{1}{4.50\times10^{12}\ \text{m}} - \frac{1}{8.79\times10^{10}\ \text{m}}\right) + \left(54.6\times10^{3}\ \text{m/s}\right)^2}$$

$$= 4.48\ \text{km/s}$$

where we take the positive square root because we are interested in the speed, not in the direction at which the asteroid is orbiting.

ASSESS The speed changes by over 90%, but the energy does not change.

48. **INTERPRET** In this problem we compare the maximum height attained by a rocket when calculated with changing gravitational acceleration with that calculated under the assumption of a constant gravitational acceleration.

DEVELOP If the rocket has an initial vertical speed v_0, we can find the height h to which it can rise (where its kinetic energy is instantaneously zero) from conservation of total mechanical energy (Equation 7.7):

$$K_0 + U_0 = K + U \quad \Rightarrow \quad \frac{1}{2}mv_0^2 - \frac{GM_{\text{E}}m}{R_{\text{E}}} = -\frac{GM_{\text{E}}m}{R_{\text{E}} + h}$$

On the other hand, if we assume constant acceleration, then the height attained would be (from Equation 29)

$$h' = \frac{v_0^2}{2g} = \frac{v_0^2}{2GM_{\text{E}}/R_{\text{E}}^2} = \frac{v_0^2 R_{\text{E}}^2}{2GM_{\text{E}}}$$

where we have used $g = GM_{\text{E}}/R_{\text{E}}^2$.

EVALUATE Solving for h, we obtain

$$h = R_{\text{E}}\left(\frac{1}{1 - v_0^2 R_{\text{E}}/2GM_{\text{E}}} - 1\right) = R_{\text{E}}\left(\frac{1}{1 - h'/R_{\text{E}}} - 1\right) = h'\left(\frac{R_{\text{E}}}{R_{\text{E}} - h'}\right)$$

Since the factor multiplying h' is $R_E/\left(R_E - h'\right) > 1$, $h > h'$ and the equations of constant gravity underestimate the height. For h′ to differ from h by 1% [i.e., $(h - h')/h = 0.01$], we require that $h' = 0.99h$. Thus,

$$\frac{h}{h'} = \frac{1}{1 - h'/R_E}$$

$$\frac{1}{0.99} = \frac{1}{1 - h'/R_E}$$

or $h' = 0.01R_{\text{E}}$. This gives

$$h = \frac{h'}{0.99} = \frac{R_E}{99} = \frac{6.37\times10^6 \text{ m}}{99} = 6.43\times10^4 \text{ m} = 64.3 \text{ km}$$

Thus, the equations for constant acceleration would underestimate the height.

ASSESS We could have anticipated that $h > h'$ because the force of gravity decreases with increasing altitude.

49. **INTERPRET** We want to show that an object lands on Earth with essentially the escape speed when it starts from rest far away from the Earth.

DEVELOP Starting from rest means the initial kinetic energy is zero. The initial gravitational potential energy ($U_0 = -GMm/r_0$ from Equation 8.6) is nearly zero, given that $r_0 \gg R_E$. By conservation of energy, the object falls to the Earth's surface with kinetic and potential energy satisfying:

$$K + U = \tfrac{1}{2}mv^2 - \frac{GMm}{R_E} = K_0 + U_0 = -\frac{GMm}{r_0}$$

EVALUATE Solving for the velocity in the above equation gives:

$$v = \sqrt{2GM\left(\frac{1}{R_E} - \frac{1}{r_0}\right)} = \sqrt{\frac{2GM}{R_E}\left(1 - \frac{R_E}{r_0}\right)} \simeq v_{esc}\left(1 - \frac{R_E}{2r_0}\right) \approx v_{esc}$$

where we have used the definition of the escape velocity from Equation 8.7, as well as the binomial approximation from Appendix A, since $R_E/r_0 \ll 1$.

ASSESS Since gravity is a conservative force, the scenario where an object falls to Earth from a great distance is just the time-reversal of the scenario where the object leaves Earth with the essentially escape velocity. It's like one movie played either forwards or backwards. So the landing velocity in the falling scenario should be the same as the take-off velocity in the escaping scenario.

50. **INTERPRET** In this problem we are asked to compare the speed of an object in circular orbit with its escape speed.

DEVELOP The escape speed is the speed that makes the total energy zero:

$$E = U + K = -\frac{GMm}{r} + \frac{1}{2}mv^2 = 0$$

However, gravitational force is what provides the centripetal force for an object to move in a circular orbit:

$$F = \frac{GMm}{r^2} = \frac{mv^2}{r}$$

EVALUATE From the above equations we find

$$v_{esc} = \sqrt{\frac{2GM}{r}} \text{ and } v = \sqrt{\frac{GMm}{r}}$$

where we have taken the positive square root because we are only interested in the speed, not its direction. Thus, the ratio of the escape speed to the orbital speed is

$$\frac{v_{esc}}{v} = \sqrt{2}$$

ASSESS To escape from the orbit, the escape speed must be greater than its present orbital speed. Our calculation shows that the speed must increase by a factor of at least $\sqrt{2}$.

51. **INTERPRET** The question boils down to: is the comet's orbit open or closed? Will it orbit around the Sun multiple times (and therefore pass by the Earth again), or is it destined to escape our solar system?

DEVELOP The comet's orbit is open if the given velocity is greater than the escape velocity: $v = \sqrt{2GM/r}$. In this case, the mass is the Sun and the radius is the Earth's distance from the Sun.

EVALUATE The escape velocity from the Sun at Earth's orbital radius is

$$v_{esc} = \sqrt{\frac{2GM_S}{r_E}} = \sqrt{\frac{2\left(6.67\times10^{-11}\ \frac{N\cdot m^2}{kg^2}\right)\left(1.99\times10^{30}\ kg\right)}{\left(150\times10^9\ m\right)}} = 42.1\ km/s$$

The comet is going faster than this escape velocity, so it is on a open (hyperbola) orbit, see Figure 8.9. It will not return to Earth's vicinity.

ASSESS The escape velocity calculated here is much smaller than the 618 km/s escape velocity given in Appendix E for the Sun. But the larger value is the escape velocity from the Sun's surface. Farther away at the Earth's orbital radius the escape velocity doesn't need to be so high. Also note that the comet velocity is much greater than the Earth's escape velocity of 11.2 km/s, which just means that there's no danger of the comet getting captured in a closed orbit around Earth.

52. **INTERPRET** This problem involves conservation of total mechanical energy. We are to find the speeds of two meteoroids as they approach the Earth with different trajectories.

DEVELOP Conservation of total mechanical energy (Equation 7.7) applied to the meteoroids gives:

$$K_0 + U_0 = K + U \quad \Rightarrow \quad \frac{1}{2}mv_0^2 - \frac{GM_Em}{r_0} = \frac{1}{2}mv^2 - \frac{GM_Em}{r}$$

Solving for v, we obtain

$$v = \pm\sqrt{v_0^2 + 2GM_E\left(\frac{1}{r} - \frac{1}{r_0}\right)}$$

We will take the positive square root because we are interested in speed, not in direction.

EVALUATE **(a)** For the first meteoroid, its speed when it strikes the Earth ($r = R_E$) is

$$v_1 = \sqrt{(2100\ m/s)^2 + 2\left(6.67\times10^{-11}\ N\cdot m^2/kg^2\right)\left(5.97\times10^{24}\ kg\right)\left(\frac{1}{6.37\times10^6\ m} - \frac{1}{2.5\times10^8\ m}\right)}$$
$$= 1.12\times10^4\ m/s = 11.2\ km/s$$

(b) For the second meteoroid, its speed at the distance of closest approach ($r = 8.50 \times 10^6$ m) is

$$v_2 = \sqrt{(2100\ m/s)^2 + 2\left(6.67\times10^{-11}\ N\cdot m^2/kg^2\right)\left(5.97\times10^{24}\ kg\right)\left(\frac{1}{8.50\times10^6\ m} - \frac{1}{2.5\times10^8\ m}\right)}$$
$$= 9.74\times10^3\ m/s = 9.74\ km/s$$

(c) The escape velocity at a distance of $r = 8.50 \times 10^6$ m from the center of the Earth is

$$v_{esc} = \sqrt{\frac{2GM_E}{r}} = \sqrt{\frac{2\left(6.67\times10^{-11}\ N\cdot m^2/kg^2\right)\left(5.97\times10^{24}\ kg\right)}{8.5\times10^6\ m}} = 9.67\ km/s$$

Therefore, the second meteoroid will not return. Alternatively, v_{esc} at a distance of 250,000 km is 1.78 km/s < 2.1 km/s, which leads to the same conclusion.

ASSESS By conservation of total mechanical energy, the change in kinetic energy is equal to the negative of the change of potential energy: $\Delta K = -\Delta U$. Because $\Delta U_1 < \Delta U_2$ (the potential energy at the surface of the Earth is lower than that at $r = 8500$ km), we have $\Delta K_1 > \Delta K_2$, so the first meteoroid has a greater speed when it reaches the Earth, but less gravitational potential energy.

53. **INTERPRET** This problem involves the energy contained in orbital motion and position, which we will use to calculate the energy needed to put a satellite into circular orbit a height h above the surface of the Earth.

DEVELOP The total energy of a circular orbit is given by Equation 8.8b, $E_{orbit} = -GM_Em/2r$. In orbit, the radius $r = R_E + h$. On the ground, the total mechanical energy of the satellite is $E_{surface} = U_0 + K_0 = -GM_Em/R_E + mv^2/2$, where v is the velocity of the surface of the Earth as it rotates on its axis. However, we are instructed to neglect this ($mv^2/2$ ~ 0.34% of $|U_0|$), so we have $E_{surface} = -GM_Em/R_E$. Equate these two expressions for total mechanical energy to find the energy required to place a satellite in orbit.

EVALUATE The energy required to put a satellite into an orbit at a height h is therefore approximated by

$$E_{\text{orbit}} - E_{\text{surface}} = -\frac{GM_{\text{E}}m}{2(R_{\text{E}}+h)} + \frac{GM_{\text{E}}m}{R_{\text{E}}} = \left(\frac{GM_{\text{E}}m}{R_{\text{E}}}\right)\left(\frac{-1}{2(1+h/R_{\text{E}})} + \frac{2(1+h/R_{\text{E}})}{2(1+h/R_{\text{E}})}\right) = \left(\frac{GM_{\text{E}}m}{R_{\text{E}}}\right)\left[\frac{R_{\text{E}}+2h}{2(R_{\text{E}}+h)}\right]$$

which agrees with the formula in the problem statement.

ASSESS Notice that the second factor in the result is dimensionless, and the first factor has units of energy, so the units work out to units of energy, as required. If we let h $\to$ 0, we find

$$E_{\text{orbit}} - E_{\text{surface}} = \left(\frac{GM_{\text{E}}m}{2R_{\text{E}}}\right)$$

which is the energy of a satellite orbiting around the Earth at zero altitude (i.e., an object sitting on the surface of the Earth). This is as expected, and just represents the neglected kinetic energy of the satellite due to the Earth's rotation. Had we included this term, the difference would be zero.

54. **INTERPRET** In this problem we are asked about the speed of a projectile as a function of r given that its initial launch speed is twice the escape speed. Because all the forces (i.e. gravity) acting on the projectile are conservative forces, we can apply conservation of total mechanical energy.

DEVELOP Conservation of total mechanical energy applied to the projectile (initially at R_{E}) gives:

$$K_0 + U_0 = K + U \quad \Rightarrow \quad \frac{1}{2}mv_0^2 - \frac{GM_{\text{E}}m}{R_{\text{E}}} = \frac{1}{2}mv^2 - \frac{GM_{\text{E}}m}{r} \text{ or } v(r) = \pm\sqrt{v_0^2 + 2GM_{\text{E}}\left(\frac{1}{r} - \frac{1}{R_{\text{E}}}\right)}$$

We will use the positive square root because we are interested in speed, not direction. The escape speed is the speed that makes the total energy zero (Equation 8.7):

$$v_{\text{esc}} = \sqrt{\frac{2GM_{\text{E}}}{R_{\text{E}}}}$$

EVALUATE If $v_0 = 2v_{\text{esc}}$ then the speed as a function of r becomes

$$v(r) = \sqrt{v_0^2 + 2GM_{\text{E}}\left(\frac{1}{r} - \frac{1}{R_{\text{E}}}\right)} = \sqrt{\frac{8GM}{R} + 2GM_{\text{E}}\left(\frac{1}{r} - \frac{1}{R_{\text{E}}}\right)} = \sqrt{2GM_{\text{E}}\left(\frac{1}{r} + \frac{3}{R_{\text{E}}}\right)}$$

ASSESS When $r = R_{\text{E}}$, we find

$$v(R) = \sqrt{\frac{8GM_{\text{E}}}{R_{\text{E}}}} = 2\sqrt{\frac{2GM_{\text{E}}}{R_{\text{E}}}} = 2v_{\text{esc}}$$

as expected.

55. **INTERPRET** This problem involves using conservation of total mechanical energy to find the speed of a satellite for several different orbits. It also requires applying Kepler's third law to relate orbital radii to orbital periods.

DEVELOP The speed of a satellite in a circular orbit is given by Equation 8.3, $v^2 = GM/r$, where r is the distance to the center of the Earth. If the speed is to change to v' where $v' = 1.1v$, then the orbital radius will satisfy $v'^2 = (1.1)^2 v^2 = GM/r'$, which gives $r' = (1.1)^{-2} r$. From this, we can solve for the difference in orbital height $\Delta h = r - r'$. For part (b), take the ratio of Kepler's third law (Equation 8.4) applied to each orbit. This gives

$$\left(\frac{T}{T'}\right)^2 = \left(\frac{r}{r'}\right)^3$$

where the primed quantities are for the new orbit. Given that $T' = 0.9T$, we can again solve for $\Delta h = r - r'$

EVALUATE (**a**) For a 10% increase in orbital speed, the orbital height decreases by

$$\Delta h = r - r' = r\left[1-(1.1)^{-2}\right] = (R_{\text{E}}+h)\left[1-(1.1)^{-2}\right] = \left(6.37\times10^6 \text{ m} + 5.50\times10^6 \text{ m}\right)\left[1-(1.1)^{-2}\right] = 2.06\times10^6 \text{ m}$$

(**b**) For a 10% decrease in orbital period,

$$\left(\frac{T}{T'}\right)^2 = \left(\frac{1}{0.9}\right)^2 = \left(\frac{r}{r'}\right)^3$$

$$r' = (0.9)^{2/3} r$$

$$\Delta h = r - r' = r\left[1-(0.9)^{2/3}\right] = (R_E + h)\left[1-(0.9)^{2/3}\right] = \left(6.37\times10^6\ \text{m} + 5.50\times10^6\ \text{m}\right)\left[1-(0.9)^{2/3}\right] = 0.805\times10^6\ \text{m}$$

ASSESS We find that the orbital height is more sensitive to orbital speed than it is to orbital period.

56. **INTERPRET** In this problem we want to find the impact speeds of two meteoroids as they approach the Earth with different initial speeds. Because only conservative forces (i.e., gravity) act on the asteroids, we can apply conservation of total mechanical energy.

DEVELOP Conservation of energy applied to the meteoroids gives:

$$K_0 + U_0 = K + U \quad \Rightarrow \quad \frac{1}{2}mv_0^2 - \frac{GM_\text{E}m}{r_0} = \frac{1}{2}mv^2 - \frac{GM_\text{E}m}{r}$$

Solving for v, we obtain

$$v = \pm\sqrt{v_0^2 + 2GM_\text{E}\left(\frac{1}{r} - \frac{1}{r_0}\right)}$$

To find the impact speed, we set $r = R_\text{E}$.

EVALUATE For $v_{1,0} = 10\ \text{km/s} = 1.0\times10^4\ \text{m/s}$, the impact speed is

$$v_1 = \sqrt{(10{,}000\ \text{m/s})^2 + 2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\ \text{kg}\right)\left(\frac{1}{6.37\times10^6\ \text{m}} - \frac{1}{1.6\times10^8\ \text{m}}\right)}$$

$$= 1.48\times10^4\ \text{m/s} = 14.8\ \text{km/s}$$

where we have taken the positive square root because we are interested in speed, not in direction.

For $v_{2,0} = 20\ \text{km/s} = 2.0\times10^4\ \text{m/s}$, the speed is

$$v_2 = \sqrt{(20{,}000\ \text{m/s})^2 + 2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\ \text{kg}\right)\left(\frac{1}{6.37\times10^6\ \text{m}} - \frac{1}{1.6\times10^8\ \text{m}}\right)}$$

$$= 2.28\times10^4\ \text{m/s} = 22.8\ \text{km/s}$$

ASSESS The final impact speed depends on the initial speed and the change of gravitational potential energy. Because $v_{2,0} > v_{1,0}$, and the change in potential energy is the same for both, we have $v_2 > v_1$.

57. **INTERPRET** This problem again involves conservation of total mechanical energy, this time applied to two rockets so that we can find their speeds when they cross the Moon's orbit.

DEVELOP Conservation of energy applied to the rockets gives:

$$K_0 + U_0 = K + U \quad \Rightarrow \quad \frac{1}{2}mv_0^2 - \frac{GM_\text{E}m}{r_0} = \frac{1}{2}mv^2 - \frac{GM_\text{E}m}{r}$$

Solving for v, we obtain

$$v = \pm\sqrt{v_0^2 + 2GM_\text{E}\left(\frac{1}{r} - \frac{1}{r_0}\right)}$$

The initial position of the rockets is $r_0 = R_\text{E} =$, and the final position is $r = R_\text{M} = 3.85\times10^8$ m (Appendix E).

EVALUATE Evaluating the above expression for the final speed v gives

$$v_1 = \sqrt{(12{,}000\ \text{m/s})^2 + 2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\ \text{kg}\right)\left(\frac{1}{3.85\times10^8\ \text{m}} - \frac{1}{\left(6.37\times10^6\ \text{m}\right)}\right)} = 4.59\ \text{km/s}$$

and

$$v_2 = \sqrt{(18{,}000\ \text{m/s})^2 + 2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\ \text{kg}\right)\left(\frac{1}{3.85\times10^8\ \text{m}} - \frac{1}{\left(6.37\times10^6\ \text{m}\right)}\right)} = 14.2\ \text{km/s}$$

where we have taken the positive square root because we are interested in speed, not in direction.

Assess As $r \to \infty$, $v \to 0$ as expected because the second term under the radical becomes the escape speed, which would be equal to the first term for launching a rocket infinitely far from Earth.

58. **Interpret** This problem involves conservation of total mechanical energy, which we can use to find the speed of the satellite at the low point given its speed at the high point.

Develop Ignoring effects such as the gravitational influence of other bodies or atmospheric drag, we apply conservation of total mechanical energy (Equation 7.7, $U_0 + K_0 = U + K$) to the satellite in an elliptical Earth orbit. The speed and distance at perigee (the lowest point) are related to the same quantities at apogee (the highest point):

$$\frac{1}{2}mv_\text{a}^2 - \frac{GM_\text{E}m}{r_\text{a}} = \frac{1}{2}mv_\text{p}^2 - \frac{GM_\text{E}m}{r_\text{p}}$$

Solving for v_p, we obtain

$$v_\text{p} = \pm\sqrt{v_\text{a}^2 + GM_\text{E}\left(\frac{1}{r_\text{p}} - \frac{1}{r_\text{a}}\right)}$$

Evaluate The distances to the two points, as measured from the center of the Earth, are

$$r_\text{p} = h_\text{p} + R_\text{E} = 230\ \text{km} + 6370\ \text{km} = 6600\ \text{km} = 6.60\times10^6\ \text{m}$$

$$r_\text{a} = h_\text{a} + R_\text{E} = 890\ \text{km} + 6370\ \text{km} = 7260\ \text{km} = 7.27\times10^6\ \text{m}$$

Substituting the values given in the problem statement, the speed of the satellite at the perigee is

$$v_\text{p} = \sqrt{(7230\ \text{m/s})^2 + 2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\ \text{kg}\right)\left(\frac{1}{6.60\times10^6\ \text{m}} - \frac{1}{7.27\times10^6\ \text{m}}\right)} = 7.96\ \text{km/s}$$

where we have taken the positive square root because we are interested in speed, not in direction.

Assess In the limit where the orbit is circular, $r_\text{p} = r_\text{a}$, and we recover the expected result $v_\text{p} = v_\text{a}$. As we shall see in Chapter 11, the same result also follows from the conservation of angular momentum or Kepler's second law, which implies that $v_\text{a}r_\text{a} = v_\text{p}r_\text{p}$. Using this relation, we have

$$v_\text{p} = \left(\frac{r_\text{a}}{r_\text{p}}\right)v_\text{a} = \left(\frac{7.27\times10^6\ \text{m}}{6.60\times10^6\ \text{m}}\right)(7.23\ \text{km/s}) = 7.96\ \text{km/s}$$

The speed of the satellite at the low point (perigee) is greater than the speed at the high point (apogee).

59. **Interpret** This problem involves conservation of total mechanical energy, which we can use to find the speed of the missile at the apex of its trajectory. We ignore nonconservative forces such as air resistance, so that only gravity is considered to act on the missile.

Develop Applying conservation of total mechanical energy (Equation 7.7) gives

$$K_0 + U_0 = K + U \quad \Rightarrow \quad \frac{1}{2}mv_0^2 - \frac{GM_\text{E}m}{R_\text{E}} = \frac{1}{2}mv^2 - \frac{GM_\text{E}m}{r} \ \text{ or } \ v(r) = \pm\sqrt{v_0^2 + 2GM_\text{E}\left(\frac{1}{r} - \frac{1}{R_\text{E}}\right)}$$

We will take the positive square root because we are interested in the missile's speed, not its direction.

Evaluate Inserting the $r = R_\text{E}$ + 1200 km and v_0 = 6.1 km/s into the above expression, we find the speed at the apex of the trajectory is

$$v = \sqrt{(6100\ \text{m/s})^2 + 2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\ \text{kg}\right)\left(\frac{1}{\left(6.37\times10^6\ \text{m} + 1.20\times10^6\ \text{m}\right)} - \frac{1}{6.37\times10^6\ \text{m}}\right)}$$

$$= 4.17\ \text{km/s}$$

ASSESS If the missile was launched directly upward, it would reach a height of

$$-\frac{GM_{\rm E}m}{R_{\rm E}}+\frac{1}{2}mv_0^2=-\frac{GM_{\rm E}m}{R_{\rm E}+h}$$

$$h=\frac{GM_{\rm E}}{GM_{\rm E}/R_{\rm E}-v_0^2/2}-R_{\rm E}=2.70\times10^6\ \text{m}$$

which is just over twice the height given in the problem statement.

60. **INTERPRET** In this problem we are asked about the orbital radius, and the kinetic energy and speed of a spacecraft, given its mass and total energy.

DEVELOP The potential energy, kinetic energy and the total energy of an object in a circular orbit around the Sun are given by Equations 8.6, 8.8a and 8.8b:

$$U=-\frac{GM_{\rm S}m}{r}$$

$$K=\tfrac{1}{2}mv^2=\frac{GM_{\rm S}m}{2r}$$

$$E=U+K=-\frac{GM_{\rm S}m}{2r}$$

These equations can be used to solve for the physical quantities we are interested.

EVALUATE (a) Using the total energy equation and the fact that the prefix T stands for 10^{12}, the orbital radius of the spacecraft is

$$r=-\frac{GM_{\rm S}m}{2Er_{\rm E}}=-\frac{\left(6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(1.99\times10^{30}\,\text{kg}\right)(720\text{ kg})}{2\left(-5.3\times10^{11}\,\text{J}\right)}=9.0\times10^{10}\,\text{m}$$

(b) The kinetic energy of the spacecraft is

$$K=-E=5.3\times10^{11}\,\text{J}$$

(c) The speed of the spacecraft is

$$v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{-2E}{m}}=\sqrt{\frac{-2(-5.3\times10^{11}\,\text{J})}{720\text{ kg}}}=38\text{ km/s}$$

ASSESS The spacecraft's orbital radius (90 million km) puts it between the orbits of Mercury and Venus (57.6 and 108 million km, respectively). The spacecraft's speed is also reassuringly in between the orbital speeds of Mercury and Venus (48 and 35 km/s, respectively).

61. **INTERPRET** Only conservative forces (i.e., gravity) act on Mercury. Therefore, we can apply conservation of total mechanical energy to find Mercury's perihelion distance.

DEVELOP Conservation of total mechanical energy (Equation 7.7) gives

$$K_0+U_0=K+U$$

$$\frac{1}{2}mv_{\rm a}^2-\frac{GM_{\rm S}m}{r_{\rm a}}=\frac{1}{2}mv_{\rm p}^2-\frac{GM_{\rm S}m}{r_{\rm p}}$$

which we can solve for the perihelion distance $r_{\rm p}$.

EVALUATE Solving the expression above for $r_{\rm p}$ and inserting the given quantities gives

$$\begin{aligned}r_p&=\frac{2GM_{\rm S}}{v_{\rm p}^2-v_{\rm a}^2+2GM_{\rm S}/r_{\rm a}}\\&=\frac{2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)}{\left(59.0\times10^3\ \text{m/s}\right)-\left(38.8\times10^3\ \text{m/s}\right)+2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)/\left(6.99\times10^{10}\ \text{m}\right)}\\&=4.60\times10^{10}\ \text{m}\end{aligned}$$

ASSESS Kepler's second law provides a more direct solution:
$r_p = r_a(v_a/v_p) = (6.99\times10^{10}\ \text{m})(38.8/59.0) = 4.60\times10^{10}\ \text{m}$ (see the solution to Problem 58).

62. **INTERPRET** In this problem we are asked to show Equation 8.5 reduces to $\Delta U = mg\Delta r$ when $r_1 \approx r_2$.

DEVELOP Following the hint, we write $r_2 = r_1 + \Delta r$, so Equation 8.5 becomes

$$\Delta U = GMm\left(\frac{1}{r_1}-\frac{1}{r_2}\right) = GMm\left(\frac{r_2-r_2}{r_1r_2}\right) = \frac{GMm\,\Delta r}{r_1(r_1+\Delta r)} = \frac{GMm}{r_1^2(1+\Delta r/r_1)}\Delta r$$

EVALUATE Because $\Delta r \ll r$ we can neglect the second term in brackets in the denominator, leading to

$$\Delta U \approx \frac{GMm}{r_1^2}\Delta r$$

Because the gravitational acceleration at a distance r_1 from the center of the gravitating body of mass M is $g(r) = GM/r^2$, the above expression can be rewritten as

$$\Delta U \approx mg(r)\Delta r$$

Near the Earth's surface $g(R_E) = GM/R_E^2 = g = 9.8\ \text{m/s}^2$ is essentially constant, so $\Delta U \approx mg\,\Delta r$.

ASSESS When $r_1 \approx r_2$, the gravitational accelerations at these two distances are very close; $g(r_1) \approx g(r_2)$. In this limit, the change in gravitational potential energy only depends on $\Delta r = r_2 - r_1$. The equation $\Delta U = mg\Delta y$ is precisely what we used in Chapters 2 and 3 where the kinematics setting was always taken to be close to the Earth's surface, with g assuming a constant value of about 9.8 m/s^2.

63. **INTERPRET** This problem involves Kepler's third law, which we can apply to find the orbital periods of the satellites in their various orbits.

DEVELOP In a lower circular orbit (smaller r) the orbital speed is faster (see Equation 8.3). The time for 10 complete orbits of the faster satellite must equal the time for 9.5 geosynchronous orbits. Thus, $10T' = 9.5T$, where the prime indicates the faster, lower orbiting satellite. Thus, the ratio of the orbital periods is $T'/T = 0.95$. Knowing the ratio of the periods, we can find the radius of the lower orbit by applying Kepler's third law to both orbits and taking the ratio.

EVALUATE The ratio of Kepler's third law applied to both orbits gives

$$\left(\frac{T'}{T}\right)^2 = (0.95)^2 = \left(\frac{r'}{r}\right)^3$$

$$r' = r(0.95)^{2/3}$$

$$r - r' = r\left[1-(0.95)^{2/3}\right](42.2\times10^6\ \text{m}) = 1.42\times10^3\ \text{km}$$

where we have used the orbital radius r = 42,200 km from Example 8.3.

ASSESS To catch up with the other satellite in a single orbit, we would have to descend a distance

$$r - r' = r\left[1-(0.5)^{2/3}\right](42.2\times10^6\ \text{m}) = 15.6\times10^3\ \text{km}$$

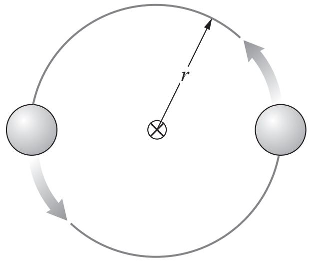
which would leave the satellite at a height of 42.2 Mm –15.6 Mm = 26.6 × 10^3 km.

64. **INTERPRET** In this problem we are asked to derive the period of a "binary system" that consists of two objects of equal mass M orbiting each other.

DEVELOP The gravitational force between two masses separated by a distance d is given by Equation 8.1:

$$F = -\frac{GM^2}{d^2}$$

The gravitational force is also the centripetal force that keeps the two masses orbiting about a common center, see figure below.



With radius $r = d/2$ and period *T*, the objects move around each other with a velocity of $v = 2\pi r/T$. Therefore, the centripetal force acting on one of the masses is

$$F = -\frac{Mv^2}{r} = -\frac{M(\pi d/T)^2}{(d/2)} = -\frac{2\pi^2 Md}{T^2}$$

The minus sign here indicates that the force is directed opposite to the radius vector, $\vec{r}$.

EVALUATE Equating the two force equations from above gives the following expression for the period:

$$T^2 = \frac{2\pi^2 d^3}{GM}$$

ASSESS Our result is in agreement with Kepler's third law, which states that the square of the period is proportional to the cube of the semi-major axis, which in this case translates into $T^2 \propto r^3$. If the diameter increases in this binary system, the period will correspondingly increase as well.

65. **INTERPRET** This problem involves Kepler's third law, which we will apply to convert the rate of change in the orbital period of the Moon to the rate of change in its orbital distance (i.e., its radial speed). We assume that the Moon's orbit is approximately circular for this calculation.

DEVELOP Kepler's third law relates the orbital period to the semimajor axis of an elliptical orbit (of which a circular orbit is a special case): $T^2 = 4\pi^2 r^3/(GM)$. We are told the rate of change is

$$\frac{dT}{dt} = \left(35\times10^{-3}\ \frac{\text{s}}{100\ \text{y}}\right)\left(\frac{1\ \text{y}}{365\ \text{d}}\right)\left(\frac{1\ \text{d}}{86,400\ \text{s}}\right) = 1.11\times10^{-11}$$

so we can differentiate Kepler's law to find the rate of change in the orbital radius *r*.

EVALUATE Differentiating Kepler's law gives

$$\frac{dT}{dt} = \pm\frac{d}{dT}\sqrt{\frac{4\pi^2 r^3}{GM}} = \frac{3}{2}\sqrt{\frac{4\pi^2 r}{GM}}\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{dT}{dt}\left(\frac{2}{3}\sqrt{\frac{GM}{4\pi^2 r}}\right) = \left(1.11\times10^{-11}\right)\left(\frac{2}{3}\sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.35\times10^{22}\ \text{kg}\right)}{4\pi^2\left(3.85\times10^{8}\ \text{m}\right)}}\right)$$

$$= \left(1.33\times10^{-10}\ \text{m/s}\right)\left(\frac{86,400\ \text{s}}{1\ \text{d}}\right)\left(\frac{36500\ \text{d}}{1\ \text{c}}\right)\left(\frac{100\ \text{cm}}{1\ \text{m}}\right) = 41.9\ \text{cm/c}$$

where we have taken the positive square root because the orbital radius is increasing, not decreasing.

ASSESS At this speed, we don't have to worry about the Moon leaving any time soon!

66. **INTERPRET** Your job is to avoid having a high jumper jump from an asteroid with enough kinetic energy to escape the asteroid. You will therefore have to find the minimum size asteroid that this won't happen.

DEVELOP From the derivation for the escape speed (Equation 8.7), we know that the jumper will not escape the asteroid if his/her total energy is less than zero: $K + U < 0$. The potential energy on the surface of the asteroid is given by Equation 8.6: $U_0 = -GMm/R$, where *M* is the mass of the asteroid, *m* is the mass of the jumper, and *R* is the radius of the asteroid. Assuming a spherical asteroid, the mass and radius are related through the given density: $M = \frac{4\pi}{3}\rho R^3$, and the potential energy at the surface becomes

$$U_0 = -\frac{G\left(\frac{4\pi}{3}\rho R^3\right)m}{R} = -\tfrac{4\pi}{3}G\rho R^2 m$$

EVALUATE For safety, you're told to be prepared for a jump that would reach 3 m on the Earth's surface. By conservation of energy arguments, this corresponds to a maximum kinetic energy of $K_{\max} = mgh$, where g is the Earth's gravitational acceleration and h is the height of the jump. In order to keep a jumper jumping with $K_{\max}$ from flying off into space, the asteroid's surface potential must satisfy $K_{\max} < -U_{0,}$ which translates into a lower limit on the asteroid radius:

$$R < \sqrt{\frac{3gh}{4\pi G\rho}} = \sqrt{\frac{3\left(9.8\text{ m/s}^2\right)\left(3\text{ m}\right)}{4\pi\left(6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(2500\text{ kg/m}^3\right)}} = 6500\text{ m}$$

The minimum asteroid diameter is therefore 13 km.

ASSESS Asteroid sizes vary over a wide range. The largest known asteroid is Ceres, with a diameter of 933 km. There should be plenty of asteroids big enough to meet the safety standard for the 2040 Olympics.

67. **INTERPRET** You need to determine if a hockey puck hit at the maximum known speed could somehow enter orbit around the Moon.

DEVELOP Let's assume the puck is hit in a direction more or less parallel to the Moon's surface. The easiest orbit to reach would be a circular orbit with radius just slightly greater than the Moon's surface radius: $r \simeq R_\text{M}$. From Equations 8.8a and b, this lowest energy orbit has kinetic energy, $K = GM_\text{M}m/2R_\text{M}$.

EVALUATE What speed would a hockey puck have to be hit at to reach the lowest energy orbit from above?

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{GM_\text{M}}{R_\text{M}}} = \frac{1}{\sqrt{2}}v_{\text{esc,M}} = \frac{1}{\sqrt{2}}\left(2.38\text{ km/s}\right) = 6100\text{ km/h}$$

Notice that we have used the escape velocity from the Moon, given in Appendix E. In any case, there's no danger that a 168 km/h puck will go into lunar orbit.

ASSESS Notice that the minimum orbital velocity is only 30% smaller than the escape velocity.

68. **INTERPRET** This problem involves Newton's law of universal gravitation. We will use it to estimate the ratio of tidal forces due to the Sun and the Moon, and compare that ratio with the ratio of gravitational forces due to the Sun and the Moon. Tidal forces are proportional to the change in the force due to gravity with respect to distance.

DEVELOP Differentiate Equation 8.1, $F = Gm_1m_2/r^2$, to find the ratio of the spatial derivative for the Moon (m_2 = 7.35×10^{22} kg, $r = 3.85 \times 10^8$ m) to the spatial derivative for the Sun ($m = 1.99 \times 10^{30}$ kg, $r = 1.5 \times 10^{11}$ m). We will also compare the ratio of the forces on the Earth due to the Moon and the Sun.

EVALUATE Inserting the known quantities, we find the ratio of the variation in force with distance to be

$$\frac{\frac{dF_\text{S}}{dr}}{\frac{dF_\text{M}}{dr}} = \frac{-2G\frac{M_\text{E}M_\text{S}}{r_\text{S}^3}}{-2G\frac{M_\text{E}M_\text{S}}{r_\text{M}^3}} = \frac{r_\text{M}^3M_\text{S}}{r_\text{S}^3M_\text{M}} = \frac{\left(3.85\times10^8\text{ m}\right)^3\left(1.99\times10^{30}\text{ kg}\right)}{\left(1.5\times10^{11}\text{ m}\right)^3\left(7.35\times10^{22}\text{ kg}\right)} = 0.45$$

The ratio of gravitational forces is

$$\frac{F_\text{S}}{F_\text{M}} = \frac{G\frac{M_\text{E}M_\text{S}}{r_\text{S}^2}}{G\frac{M_\text{E}M_\text{M}}{r_\text{M}^2}} = \frac{r_\text{M}^2M_\text{S}}{r_\text{S}^2M_\text{M}} = \frac{\left(3.85\times10^8\text{ m}\right)^2\left(1.99\times10^{30}\text{ kg}\right)}{\left(1.5\times10^{11}\text{ m}\right)^2\left(7.35\times10^{22}\text{ kg}\right)} = 177$$

ASSESS We see that the gravitational force due to the Sun is much higher than that due to the Moon, but the force due to the Moon changes more from one side of the Earth to the other. Thus, the Moon's gravity causes the majority of the tidal effects. Note that when the Moon and the Sun are both positioned on the same side of the Earth, their tidal forces add, so we have maximum tides. The opposite is true when the Moon and the Sun are on opposite sides of the Earth.

69. **INTERPRET** We're asked to calculate the position of the L1 Lagrange point, where the gravity of the Earth and the Sun combine to give a period of 1 year around the Sun. It's worth noting that when only the Sun's gravity is considered, the only place with a 1-year period would be at the Earth's orbital radius:

$$T_E = \sqrt{\frac{4\pi^2 r_E^3}{GM_S}} = 1 \text{ y}$$

where we have used Equation 8.4 with r_E the Earth's distance from the Sun, and M_S the mass of the Sun.

DEVELOP Let's assume that the point L1is at a distance r_{LE} from the Earth and a distance r_{LS} from the Sun. From the remarks in the text, we know that L1 is between the Earth and the Sun, so $r_{LE} + r_{LS} = r_E$. The sum of the gravitational attraction from both bodies is

$$F_{net} = \frac{GM_S m}{r_{LS}^2} - \frac{GM_E m}{r_{LE}^2}$$

This sum supplies a centripetal force that keeps any object there in uniform circular motion around the Sun: $F_{net} = mv^2 / r_{LS}$. The orbital speed results in a period of $T = 2\pi r_{LS} / v$, which by definition is equal to one year. Combining all this information, we have:

$$\frac{GM_S}{r_{LS}^2} - \frac{GM_E}{r_{LE}^2} = \frac{4\pi^2 r_{LS}}{T^2}$$

We will now substitute $r_{LS} = r_E - r_{LE}$, as well as introduce the variables $x = r_{LE} / r_E$ and $y = M_E / M_S$, in order to obtain:

$$\frac{1}{(1-x)^3} - \frac{y}{x^2(1-x)} = \frac{4\pi^2 r_E^3}{GM_S T^2} = \frac{T_E^2}{T^2} = 1$$

Notice how we were able to substitute the Earth's orbital period, T_E, into the right-hand side of the equation. Both the Earth's period and the period at L1 are equal to 1 year, so they cancel.

EVALUATE We are now faced with a rather difficult equation to solve:

$$\frac{1}{(1-x)^2} - \frac{y}{x^2} = 1 - x$$

But we can assume that the Lagrange point is much closer to Earth than to the Sun, so $x \ll 1$. In which case, the first term on the left can be reduced using the binomial approximation: $(1-x)^{-2} \approx 1 + 2x$ (see Appendix A). We then have

$$x \approx \sqrt[3]{\frac{y}{3}} = \sqrt[3]{\frac{M_E}{3M_S}} = \sqrt[3]{\frac{(5.97\times 10^{24}\text{ kg})}{3(1.99\times 10^{30}\text{ kg})}} = 0.01$$

This implies that L1 is 1% of the distance between the Earth and the Sun. Relative to the Earth, L1 is at a distance of

$$r_{LE} \approx 0.01 r_E = 0.01(150\times 10^6\text{ km}) = 1.5\times 10^6\text{ km}$$

ASSESS One can check in an outside reference that indeed the L1 Lagrange point is around 1.5 million km from Earth. There are 4 other Lagrange points, called L2, L3, L4 and L5. Like L1, they are all stationary points, meaning an object situated there will not move relative to the Earth and Sun.

70. **INTERPRET** We consider the characteristics of the Global Positioning System.

DEVELOP We're told that the GPS satellites are in orbit at an altitude of about 20,200 km. To find the period, we use Equation 8.4:

$$T^2 = \frac{4\pi^2 (R_E + h)^3}{GM_E}$$

where for the orbital radius we take into account the radius of the Earth, R_E, and the altitude, h.

Evaluate Plugging in the known values, the period of one of the satellites is

$$T = \sqrt{\frac{4\pi^2\left(6.37\times10^6\,\text{m} + 20.2\times10^6\,\text{m}\right)^3}{\left(6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(5.97\times10^{24}\,\text{kg}\right)}} = 4.31\times10^4\,\text{s} = 12\ \text{h}$$

The answer is (c).

Assess The GPS satellites circle the Earth twice a day, which means that a given satellite is in the sky over a particular point for a few hours. Optimally a GPS receiver will have 4 satellites in view in order to compare their different signals and provide an accurate reading of the receiver's position.

71. Interpret We consider the characteristics of the Global Positioning System.

Develop The satellite speed can be found from Equation 8.3: $v = \sqrt{GM_E/(R_E + h)}$, where as in the previous problem we write the orbital radius as the sum of the radius of the Earth, R_E, and the altitude, h.

Evaluate Plugging in the known values, the speed of one of the satellites is

$$v = \sqrt{\frac{GM_E}{(R_E + h)}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(5.97\times10^{24}\,\text{kg}\right)}{\left(6.37\times10^6\,\text{m} + 20.2\times10^6\,\text{m}\right)}} = 3.9\ \text{km/s}$$

The answer is (d).

Assess This is slower than the International Space Station, which orbits at 7.7 km/s (see Example 8.2). However, the station is much closer to Earth at an altitude of 380 km. It completes an orbit in 90 min, as compared to 12 hours.

72. Interpret We consider the characteristics of the Global Positioning System.

Develop The escape speed is given by Equation 8.7: $v = \sqrt{2GM_E/(R_E + h)}$.

Evaluate Notice that this is $\sqrt{2}$ times the orbital speed found in the previous problem:

$$v_{esc} = \sqrt{2}\cdot v = \sqrt{2}\left(3.9\ \text{km/s}\right) = 5.5\ \text{km/s}$$

The answer is (b).

Assess At the Earth's surface, the escape speed is roughly twice this: 11.2 km/s, but that appears to be just a coincidence.

73. Interpret We consider the characteristics of the Global Positioning System.

Develop The total energy of an object in a circular orbit is given in Equation 8.8b: $E = -GM_E m/2(R_E + h)$.

Evaluate Using the mass for the next generation of GPS satellites, the total energy is

$$E = -\frac{GM_E m}{2(R_E + h)} = -\frac{\left(6.67\times10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(5.97\times10^{24}\,\text{kg}\right)\left(844\ \text{kg}\right)}{2\left(6.37\times10^6\,\text{m} + 20.2\times10^6\,\text{m}\right)} = -6.3\ \text{GJ}$$

The answer is (d).

Assess The total energy is negative because the satellites are in bound orbits around the Earth.