10

ROTATIONAL MOTION

EXERCISES

Section 10.1 Angular Velocity and Acceleration

13. **INTERPRET** This problem involves calculating the angular speed of a variety of rotating objects.

DEVELOP Apply Equation 10.1, $\bar{\omega} = \Delta\theta/\Delta t$, where $\Delta\theta$ is the rotation and Δt is the time interval for the rotation.

EVALUATE (**a**) $\omega_E = (1\text{ rev})/(1\text{ d}) = 2\pi/(86{,}400\text{ s}) = 7.27\times10^{-5}\text{ s}^{-1}$

(**b**) $\omega_{min} = (1\text{ rev})/(1\text{ h}) = 2\pi/3600\text{ s} = 1.75\times10^{-3}\text{ s}^{-1}$

(**c**) $\omega_{hr} = (1\text{ rev})/(12\text{ h}) = \omega_{min}/12 = 1.45\times10^{-4}\text{ s}^{-1}$

(**d**) $\omega = (300\text{ rev})/\text{min} = 300\times2\pi/(60\text{ s}) = 31.4\text{ s}^{-1}$

ASSESS Note that radians are a dimensionless angular measure, i.e., pure numbers; therefore angular speed can be expressed in units of inverse seconds.

14. **INTERPRET** We are asked to compute the linear speed at the equator and at an arbitray other location on Earth. The problem involves the rotational motion of the Earth.

DEVELOP We first calculate the angular speed of the Earth using Equation 10.1:

$$\omega_E = \frac{\Delta\theta}{\Delta t} = \frac{1\text{ rev}}{1\text{ d}} = \frac{2\pi\text{ rad}}{86{,}400\text{ s}} = 7.27\times10^{-5}\text{ s}^{-1}$$

The linear speed can then be computed using Equation 10.3: $v = \omega r$.

EVALUATE **(a)** At the equator,

$$v = \omega_E R_E = \left(7.27\times10^{-5}\text{ s}^{-1}\right)\left(6.37\times10^{6}\text{ m}\right) = 463\text{ m/s}$$

(b) At latitude θ, $r = R_E\cos\theta$, so $v = \omega_E r = (463\text{ m/s})\cos\theta$.

ASSESS The angle $\theta = 0$ corresponds to the equator, so the result found in (b) agrees with (a). In addition, if we take $\theta = 90°$, then we are at the poles, and the linear speed is zero.

15. **INTERPRET** This problem involves converting angular speed from various units to radians/s (which is the same as s^{-1}, or frequency).

DEVELOP Use the appropriate conversion factors to convert each angular speed to units of rad/s.

EVALUATE (**a**) $(720\text{ rev/min})(2\pi\text{ rad/rev})(\text{min}/60\text{ s}) = 24\pi\text{ rad/s} = 75\text{ rad/s}$, to two significant figures.

(**b**) $(50°/h)(\pi\text{ rad}/180°)(\text{h}/3600\text{ s}) = 2.4\times10^{-4}\text{ rad/s}$, to two significant figures.

(**c**) $(1000\text{ rev/s})(2\pi\text{ rad/rev}) = 2000\pi\text{ s}^{-1} = 6\times10^{3}\text{ rad/s}$ to a single significant figure.

(**d**) $(1\text{ rev/y}) = 2\pi\text{ rad}/(\pi\times10^{7}\text{ s}) = 2\times10^{-7}\text{ rad/s}$, to a single significant figure.

ASSESS Note that radians are a dimensionless angular measure, i.e., pure numbers; therefore angular speed can be expressed in units of inverse seconds. The approximate value for 1 y used in part (d) is often handy for estimates, and is fairly accurate; see Chapter 1, Problem 20.)

16. **INTERPRET** The problem asks you to find the linear speed at the outer edge of a 25-cm diameter circular saw.

DEVELOP We first convert the angular speed to rad/s:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{3500\text{ rev}}{1\text{ min}} = \frac{2\pi(3500)\text{ rad}}{60\text{ s}} = 367\text{ rad/s}$$

The linear speed can then be computed using Equation 10.3, $v = \omega r$. $v = \omega r$.

EVALUATE The radius of the circular saw is $r = d/2 = 12.5\text{ cm} = 0.125\text{ m}$. From Equation 10.3 the linear speed of the outer edge of the saw is

$$v = \omega r = (367\text{ rad/s})(0.125\text{ m}) = 45.8\text{ m/s}$$

ASSESS The linear speed of the saw is over 100 mi/h!

17. **INTERPRET** For this problem, we are asked to find the average angular acceleration given the initial and final acceleration and the time interval.

DEVELOP Apply Equation 10.4 (before the limit is taken), $\bar{\alpha} = \Delta\omega/\Delta t$. The change in the angular velocity is $\Delta\omega = \omega_f - \omega_i$ = 500 rpm – 200 rpm = 300 rpm and the time interval is Δt = (74 min)(60 s/min) = 4400 s.

EVALUATE Inserting the given quantities gives an average angular acceleration of

$$\bar{\alpha} = \frac{300\text{ rpm}}{4400\text{ s}}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60\text{ s}}\right) = 7\times10^{-2}\text{ s}^{-1}$$

ASSESS Note that the units cancel out to leave units of frequency, as expected.

18. **INTERPRET** In this problem we are given the angular acceleration of a turbine and asked how long it takes to reach its operating speed and the number of revolutions that occur during this start-up period. The key to this type of rotational problem is to understand the analogous situation for linear motion, and apply the appropriate equation The analogies are summarized in Table 10.1.

DEVELOP Given a constant angular acceleration α, the angular velocity and angular position at a later time t can be found using Equations 10.7 and 10.8, respectively:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

The initial and final angular velocities are $\omega_0 = 0$ and

$$\omega = 3600\text{ rpm} = \frac{3600\text{ rev}}{1\text{ min}} = \frac{2\pi(3600)\text{ rad}}{60\text{ s}} = 377\text{ rad/s}$$

EVALUATE (**a**) The amount of time it takes to reach its operating speed is

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{377\text{ rad/s} - 0}{0.52\text{ rad/s}^2} = 725\text{ s} = 12\text{ min}$$

(**b**) Using Equation 10.8, we find the number of turns made during this time interval to be

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha t^2 = \frac{1}{2}(0.52\text{ rad/s}^2)(725\text{ s})^2 = 1.37\times10^5\text{ rad} = 2.2\times10^4\text{ rev}$$

ASSESS The responses are given to two significant figures to reflect the precision of the data. The turbine turns very fast. After 12.1 min, it has reached an angular speed of 377 rad/s, or 60 rev/s!

19. **INTERPRET** This problem is an exercise in angular kinematics. We are given an angular acceleration and the acceleration period, and are asked to find the revolutions made in this time and the average angular speed.

DEVELOP Apply the formulas in Table 10.1. To find the number of revolutions, we find the total angular displacement θ from Equation 10.8, then divide this by 2π (= 1 revolution) to find the number of revolutions. The linear analog to this can be thought of as finding a distance, then dividing it by a given distance (say, 10-km segments) to find the number of 10-km segments traveled. In both cases, we end up with a dimensionless number. To find the average angular speed, use Equation 10.1.

EVALUATE (**a**) Inserting $\alpha = 0.010\text{ rad/s}^2$ and t = 14 s into Equation 10.8 gives a final rotational distance θ of

$$\Delta\theta = \theta - \theta_0 = \overbrace{\omega_0}^{=0} t + \frac{1}{2}\alpha t^2$$

$$\Delta\theta = \frac{1}{2}(0.010\text{ rad/s}^2)(14\text{ s})^2 = 0.98\text{ rad}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = 0.16\text{ rev}$$

(**b**) From Equation 10.7, with $\theta_0 = 0$, the final angular speed is

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{\theta - \theta_0}{\Delta t} = \frac{0.98\text{ rad}}{14\text{ s}} = 0.07\text{ rad/s}$$

ASSESS The final angular speed of the merry-go-round is, from Equation 10.7,

$$\omega = \overbrace{\omega_0}^{=0} + \alpha t = \left(0.010\text{ rad/s}^2\right)\left(14\text{ s}\right) = 0.14\text{ rad/s}$$

which is twice the average speed. This is expected because we start from zero speed and accelerate at a constant rate, so the average speed is attained at half the acceleration period, at which point the object in question is rotating at half the angular speed.

Section 10.2 Torque

20. **INTERPRET** In this problem we are asked to find the torque produced by the frictional force about a wheel's axis. We will apply the concept of torque, which is the force applied perpendicular to the radial direction from an axis of rotation.

DEVELOP The torque produced by a force is given by Equation 10.10:

$$\tau = rF\sin\theta$$

where $r_\perp = r\sin\theta$ is the perpendicular distance between the rotation axis and the line of action of the force F. The frictional force acts tangent to the circumference of the wheel and thus is perpendicular to the radius at the point of contact. Thus $\sin\theta = \sin(90°) = 1$.

EVALUATE Using $r = d/2 = (1.0\text{ m})/1 = 0.50\text{ m}$, Equation 10.10 gives the torque as

$$\tau = rf = \left(0.50\text{ m}\right)\left(320\text{ N}\right) = 160\text{ N}\cdot\text{m}$$

opposite to the direction of rotation.

ASSESS Since frictional force opposes motion, the torque it produces tends to slow down the rotation.

21. **INTERPRET** This problem involves the concept of torque. We are given the force applied (by the child) and are asked to find the lever arm needed to produce a given torque.

DEVELOP Solve Equation 10.10 for the lever arm r, assuming the child pushes perpendicular to the door (so $\theta =$ 90°).

EVALUATE Solving Equation 10.10 for r gives

$$r = \frac{\tau}{F\sin\theta} = \frac{110\text{ N}\cdot\text{m}}{\left(90\text{ N}\right)\sin\left(90^\circ\right)} = 1.2\text{ m}$$

so the child must push 1.2 m from the center of the door.

ASSESS This seems like a reasonable distance, given that the typical width of revolving doors is greater than 1.2 m.

22. **INTERPRET** This problem involves a force that is applied at different angles to the lever arm. For each angle, we are asked to find the resulting torque.

DEVELOP The torque produced by a force is given by Equation 10.10:

$$\tau = rF\sin\theta = r_\perp F = rF_\perp$$

where $r_\perp = r\sin\theta$ is the perpendicular distance (lever arm) between the rotation axis and the line of action of the force F. Alternatively, one can think of $F_\perp = F\sin\theta$ as the effective force.

EVALUATE Using Equation 10.10, the magnitude of the applied force may be obtained. The forces are **(a)**

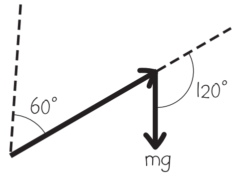
$$F = \frac{\tau}{r\sin\theta} = \frac{35.0\text{ N}\cdot\text{m}}{\left(0.240\text{ m}\right)\sin\left(90°\right)} = 146\text{ N}$$

and **(b)**

$$F = \frac{\tau}{r\sin\theta} = \frac{35.0\text{ N}\cdot\text{m}}{\left(0.240\text{ m}\right)\sin\left(110°\right)} = 155\text{ N}$$

ASSESS As expected, we have to push harder if we do not push perpendicular to the lever arm. To produce a specific torque most effectively, the applied force should be at the right angle to $\vec{r}$, the position vector from the axis of rotation to the point where the force is applied. This would yield the maximum effective force $F_{\perp} = F\sin(90^{\circ}) = F$.

23. **INTERPRET** This problem involves torque, which we are asked to calculate given a force (gravity on the mouse), the radial distance at which it is applied, and the angle at which it is applied (from the geometry of a clock).
DEVELOP Draw a diagram of the situation (see figure below). From the geometry of a clock, we know that the angle the minute hand makes with the vertical is $\phi = 180°/3 = 60°$. The angle between the force and the radial position vector from the axis of rotation to the point where the force is applied is therefore $\theta = 180° - 60° = 120°$. The force applied by the mouse is simply its weight, so $F = mg$, and the lever arm is $r = 17$ cm.

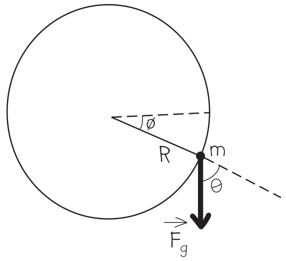


EVALUATE Inserting the given quantities into Equation 10.10 gives

$$\tau = rF\sin\theta = (0.17\ \text{m})(0.055\ \text{kg})(9.8\ \text{m/s}^2)\sin(120^{\circ}) = 7.9\times10^{-2}\ \text{N}\cdot\text{m}$$

ASSESS In 5 minutes, the torque applied by the mouse (assuming it doesn't move) will be $\tau = (0.17\ \text{m})(0.055\ \text{kg})(9.8\ \text{m/s}^2) = 9.2\times10^{-2}$ N·m, or 16% more torque.

24. **INTERPRET** This problem involves torque, which we are asked to calculate given a force (gravity on the valve stem), the radial distance at which it is applied, and the angle at which it is applied (from the given geometry).
DEVELOP Apply Equation 10.10, $\tau = rF\sin\theta$. Make a sketch of the system, as shown in the figure below. From this, we see that the angle between the force and the position vector $\vec{r}$ is $\theta = 90° - 24° = 66°$. The valve stem is located a distance $r = 0.32$ m from the center, and has mass $m = 0.025$ kg. The force applied is thus $F = mg$.



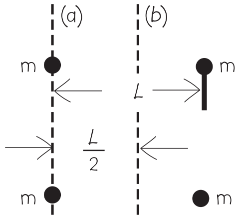
EVALUATE Inserting the given quantities into Equation 10.10 gives the torque as

$$\tau = rF\sin\theta = rmg\sin\theta = (0.32\ \text{m})(0.025\ \text{kg})(9.8\ \text{m/s}^2)\sin(66^{\circ}) = 0.072\ \text{N}\cdot\text{m}$$

ASSESS We see that the torque would be zero if the angle ϕ were 90° because sin(90° − 90°) = 0. This makes sense because the valve stem would be directly below the axis of rotation, so its weight would apply no torque.

Section 10.3 Rotational Inertia and the Analog of Newton's Law

25. **INTERPRET** This problem involves rotational inertia, which include both mass and the spatial distribution of that mass. Rotation inertia is the rotational analog of mass in linear motion. Because the spatial mass distribution enters into rotational inertia, the position of the axis of rotation is important. We are asked to find the rotational inertia of an arrangement of 4 masses about two different axes of rotation.
DEVELOP Draw a diagram of the situation (see figure below), and apply Equation 10.12.



EVALUATE (a) For the axis labeled (a), two masses have $r = 0$, and the other two masses have $r = L$. Inserting these quantities into Equation 10.12 gives

$$I = \sum_i m_i r_i^2 = m(0)^2 + m(0)^2 + mL^2 + mL^2 = 2mL^2$$

(b) For the axis labeled (b), each mass has $r = L/2$, so Equation 10.12 gives

$$I = \sum_i m_i r_i^2 = m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = mL^2$$

ASSESS Thus, there is more rotational inertia when the axis of rotation is at the edge of the object than when it is at the center of the object, as expected.

26. **INTERPRET** We want to find the moment of inertia of a shaft that has a shape of a solid cylinder.
DEVELOP The rotational inertia of a solid cylinder or disk about its axis is $I = \frac{1}{2}MR^2$ (see Table 10.2). The radius is half the diameter and the mass, in more familiar units, is $6.8\times10^3\,\text{kg}$.

EVALUATE The rotational inertia of the shaft is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}\left(6.8\times10^3\,\text{kg}\right)\left(\tfrac{1}{2}0.85\text{ m}\right)^2 = 610\text{ kg}\cdot\text{m}^2$$

ASSESS The numerical value is reasonable, given its mass and radius, and the units $\left(\text{kg}\cdot\text{m}^2\right)$ are correct.

27. **INTERPRET** This problem involves combining the rotational inertia of several objects to find the overall rotation inertia of the combined object. In addition, we are asked to find the torque needed to give the object the given angular acceleration.
DEVELOP Because both the cylinder and the end caps rotate about the same axis, we can sum the rotational inertia of each object to find the total rotational inertia: $I_{\text{tot}} = I_{\text{cyl}} + 2I_{\text{cap}}$. The rotational inertia of the individual components are given in Table 10.2, and are $I_{\text{cyl}} = M_{\text{cyl}}R^2$ and $I_{\text{cap}} = M_{\text{cap}}R^2/2$. To find the torque, apply the rotational analog of Newton's second law (for constant mass), Equation 10.11: $\tau = I\alpha$.
EVALUATE (a) The total rotational inertia of the capped cylinder is

$$I_{\text{tot}} = I_{\text{cyl}} + 2I_{\text{cap}} = M_{\text{cyl}}R^2 + 2\left(\frac{1}{2}M_{\text{cap}}R^2\right) = (0.071\text{ m})^2\left(0.065\text{ kg} + 0.022\text{ kg}\right) = 4.4\times10^{-4}\text{ kg}\cdot\text{m}^2$$

(b) The torque needed to accelerate the capped cylinder is

$$\tau = I_{\text{tot}}\alpha = \left(4.39\times10^{-4}\text{ kg}\cdot\text{m}^2\right)\left(3.4\text{ rad/s}^2\right) = 1.5\times10^{-3}\text{ N}\cdot\text{m}$$

ASSESS Notice that we used more significant figures for the total rotational inertia in part (b) because it was an intermediate result in this case.

28. **INTERPRET** In this problem we are asked to find the minimum total mass of a wheel, given its diameter and rotational inertia.
DEVELOP Every part of the wheel has a distance from the center less than or equal to the maximum radius. Therefore, using Equation 10.12, we obtain the following inequality:

$$I = \sum m_i r_i^2 \le \left(\sum m_i\right) r_{\text{max}}^2$$

EVALUATE **(a)** The above equation implies that

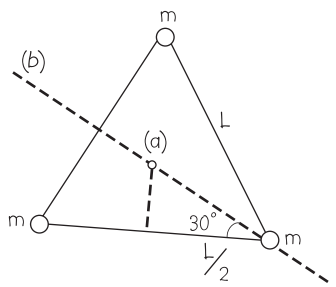
$$M = \sum m_i \ge \frac{I}{r_{\text{max}}^2}$$

Therefore, the minimum total mass is

$$M_{\text{min}} = \frac{I}{r_{\text{max}}^2} = \frac{7.8\text{ kg}\cdot\text{m}^2}{(0.92\text{ m/2})^2} = 37\text{ kg}$$

(b) If not all the mass of the wheel is concentrated at the rim, the total mass is greater than this minimum.
ASSESS To have the same rotational inertia I, we can have some of the mass of wheel concentrated near the axis of rotation. Its contribution to I would be small because $I = \sum m_i r_i^2$.

29. **INTERPRET** We are asked to find the rotational inertia I of three point masses located at the corners of an equilateral triangle. We find I for the axis through the center perpendicular to the plane, and for the axis along the line through one corner and the midpoint of the opposite side, as shown in the figure below.



DEVELOP We use the equation for rotational inertia of a collection of point masses: $I = \sum m_i r_i^2$. Therefore, we will need the distances of the three particles from the appropriate axis. In part (a), the distance of each particle from the center of the triangle is $r_a = (L/2)/\cos 30°$. In part (b), one mass is on the axis where the distance is zero. The other two masses are at a distance of $r_b = L/2$.

EVALUATE (a) The rotational inertia around the center of the triangle is

$$I = \sum m_i r_i^2 = 3\left[mr_a^2\right] = \frac{3mL^2}{4\cos^2 30°} = mL^2$$

(b) The rotational inertia around a mid-line of the triangle is

$$I = \sum m_i r_i^2 = 2\left[mr_b^2\right] = \frac{2mL^2}{4} = \tfrac{1}{2} mL^2$$

ASSESS Both of these axes pass through the center of mass of the triangle. But the axis in part (a) maximizes the distance to the three masses, so it has a larger moment of inertia than the axis in part (b).

30. **INTERPRET** By assuming Earth to be a solid sphere with uniform mass distribution, we want to estimate its rotational inertia, and the torque needed to change the length of the day by one second every century.

DEVELOP From Table 10.2, the rotational inertia of a solid sphere of radius R and mass M is

$$I = \frac{2}{5}MR^2$$

Once I is known, the torque needed to slow down the rotation can be found by using Equation 10.11: $\tau = I\alpha$.

EVALUATE **(a)** For a uniform solid sphere with an axis through the center,

$$I_E = \frac{2}{5}M_E R_E^2 = \frac{2}{5}\left(5.97\times 10^{24}\ \text{kg}\right)\left(6.37\times 10^6\ \text{m}\right)^2 = 9.69\times 10^{37}\ \text{kg}\cdot\text{m}^2$$

(b) The angular speed of rotation of Earth is $\omega = 2\pi/T$, where the period is T = 1 d = 86,400 s. If the period were to change by 1s per century,

$$\frac{dT}{dt} = \frac{1\ \text{s}}{(100\ \text{y})\left(3.16\times 10^7\ \text{s/y}\right)} = 3.16\times 10^{-10}$$

This would correspond to an angular acceleration of

$$\alpha = \frac{d\omega(T)}{dt} = \frac{d}{dt}\left(\frac{2\pi}{T}\right) = -\frac{2\pi}{T^2}\frac{dT}{dt}$$

Therefore, to change the length of a day by ±1 s would require a torque of magnitude

$$\tau = I\left|\alpha\right| = \frac{2\pi I}{T^2}\frac{dT}{dt} = \frac{2\pi\left(9.69\times 10^{37}\ \text{kg}\cdot\text{m}^2\right)}{(86{,}400\ \text{s})^2}\left(3.16\times 10^{-10}\right) = 2.58\times 10^{19}\ \text{N}\cdot\text{m}$$

would be required.

ASSESS The torque in (b) is actually generated by tidal friction between the Moon and the Earth. Note that the Earth has a core of denser material, so its actual rotational inertia is less than that obtained in (a).

31. **INTERPRET** This problem involves finding the rotational inertia of a sphere of constant density that rotates about an axis through its geometric center. In addition, we are to find the torque needed to impart the given acceleration to this sphere.

DEVELOP Use the formula $I = 2MR^2/5$ from Table 10.2 to calculate the rotational inertia of the neutron star. The radius R_{NS} of the neutron star is

$$M_{NS} = \rho V = \rho\left(\frac{4}{3}\pi R_{NS}^3\right) = 1.8M_S$$

$$R_{NS} = \sqrt[3]{\frac{3(1.8M_S)}{4\pi\rho}}$$

where (from Appendix E) $M_S = 1.99 \times 10^{30}$ kg. The torque may be found by inserting the resulting inertia and the given angular acceleration into Equation 10.11.

EVALUATE (**a**) The rotational inertia of the neutron star is

$$I_{NS} = \frac{2}{5}M_{NS}R_{NS}^2 = \frac{2}{5}(1.8M_S)\left[\frac{3(1.8M_S)}{4\pi\rho}\right]^{2/3}$$

$$= \frac{2}{5}\left(1.8\times 1.99\times 10^{30}\ \text{kg}\right)^{5/3}\left(\frac{3}{4\pi\left(1\times 10^{18}\ \text{kg}\cdot\text{m}^{-3}\right)}\right)^{2/3} = 1\times 10^{38}\ \text{kg}\cdot\text{m}^2$$

(**b**) To achieve a spin-down rate of −5 × 10−5 s−2, the torque needed is

$$\tau = I_{NS}\alpha = \left(1.29\times 10^{38}\ \text{kg}\cdot\text{m}^2\right)\left(-5\times 10^{-5}\ \text{rad/s}^2\right) = -6\times 10^{33}\ \text{N}\cdot\text{m}$$

ASSESS The results are reported to a single significant figure to reflect the precision of the data. Checking the units for part (a), we find $(\text{kg})^{5/3}(\text{m}^3)^{2/3}(\text{kg})^{-2/3} = (\text{kg})^{5/3-2/3}(\text{m})^{3\times 2/3} = \text{kg m}^2$, as expected.

32. **INTERPRET** We are asked about the rotational inertia of a Frisbee, given its mass distribution, and the torque required to generate the rotation.

DEVELOP The Frisbee rotates around an axis through its center and perpendicular to its flat surface. Its rotational inertia is the sum from a disk $\left(I_d = \frac{1}{2}M_dR^2\right)$ and a ring $\left(I_r = M_rR^2\right)$, each accounting for half the mass of the Frisbee $\left(M_d = M_r = \frac{1}{2}M_f\right)$. In part (b), the torque exerted by the student can be found from the rotational analog of Newton's second law: $\tau = I_f\alpha$ (Equation 10.11). We don't have the angular acceleration, but it can be determined from Equation 10.9: $\omega^2 = \omega_0^2 + 2\alpha(\theta-\theta_0)$, given that the Frisbee goes from rest $(\omega_0 = 0)$ to 550 rpm after a quarter-turn given by the student $\left(\theta-\theta_0 = \frac{1}{4}\text{rev}\right)$.

EVALUATE (a) The Frisbee's rotational inertia is the sum of inertia from the disk and the ring:

$$I_f = I_d + I_r = \tfrac{3}{4}M_fR^2 = \tfrac{3}{4}(0.108\ \text{kg})\left(\tfrac{1}{2}0.24\ \text{m}\right)^2 = 1.17\times 10^{-3}\,\text{kg}\cdot\text{m}^2$$

(b) To find the torque, we first calculate the angular acceleration:

$$\alpha = \frac{\omega^2-\omega_0^2}{2(\theta-\theta_0)} = \frac{(550\ \text{rpm})^2}{2\left(\frac{1}{4}\text{rev}\right)}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)^2 = 1056\ \text{rad/s}^2$$

The torque given by the student is then

$$\tau = I_f\alpha = \left(1.17\times 10^{-3}\,\text{kg}\cdot\text{m}^2\right)\left(1056\ \text{rad/s}^2\right) = 1.24\ \text{N}\cdot\text{m}$$

ASSESS The units are all correct and the numerical values seem reasonable. Using Equation 10.10, we can estimate that the force exerted by the student when flicking the Frisbee is roughly: $F \sim \tau/r = 10$ N, which is well within the strength limits of a human wrist.

33. **INTERPRET** This problem involves calculating the torque that results from a frictional force applied about a 41-cm shaft, and the angular acceleration this engenders. We are then asked to find the time it takes the shaft (and the accompanying flywheel) to stop, given their initial rotational speed.

DEVELOP From Equation 10.10, the torque applied to the flywheel is

$$\tau = rF\sin\theta = R_{\text{shaft}} f_{\text{k}}$$

where $\theta = 90°$, $f_{\text{k}} = 34$ kN, and $R_{\text{shaft}} = (41\text{ cm})/2 = 0.205$ m. Inserting this torque into the rotational analog of Newton's second law (for constant mass), we can find the angular acceleration. We find $\alpha = -\tau/I_{\text{fw}}$, where the negative sign indicates that the acceleration is directed opposite to the motion. Use Table 10.2 to find the formulas for the rotational inertia of the flywheel (which we take to be a solid disk). This is

$$I_{\text{fw}} = \frac{1}{2}M_{\text{fw}}R_{\text{fw}}^2$$

where $M_{\text{fw}} = 7.7\times10^4$ kg and $R_{\text{fw}} = 2.4$ m. The time it will take the flywheel to stop is, from Equation 10.7 with $\omega = 0$,

$$0 = \omega_0 + \alpha t$$

$$t = -\frac{\omega_0}{\alpha} = \frac{\omega_0 I_{\text{fw}}}{\tau} = \frac{\omega_0 M_{\text{fw}}R_{\text{fw}}^2}{2R_{\text{shaft}} f_{\text{k}}}$$

EVALUATE Inserting the given quantities into the expression for the time gives

$$t = \frac{\omega_0 M_{\text{fw}}R_{\text{fw}}^2}{2 f_{\text{k}} R_{\text{shaft}}} = \frac{(360\text{ rpm})(7.7\times10^4\text{ kg})(2.4\text{ m})^2}{2(34\times10^3\text{ N})(0.205\text{ m})}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60\text{ s}}\right) = 1200\text{ s} = 20\text{ min}$$

ASSESS The exact rotational inertia for the flywheel is $M\left(R_{\text{shaft}}^2 + R_{\text{fw}}^2\right)/2$, which is just 0.7% different from $MR_{\text{fw}}^2/2$ for the given radii.

Section 10.4 Rotational Energy

34. **INTERPRET** The problem asks about the rotational kinetic energy of the blade of a circular saw. In addition, we also want to find the power required for the saw to start from rest and reach a given angular speed, which involves the work-energy theorem (see Equation 6.14).

DEVELOP The rotational kinetic energy of the saw can be found by using Equation 10.18:

$$K = \frac{1}{2}I\omega^2$$

where $I = MR^2/2$ for a disk (see Table 10.2). The average power required is the work done on the saw divided by the time over which the work is done ($\bar{P} = W/\Delta t$). The work done on the saw may be found by using the work-energy theorem (Equation 6.14): $W_{\text{net}} = \Delta K$.

EVALUATE **(a)** With $\omega = 3500\text{ rpm} = 2\pi(3500\text{ rpm})/(60\text{ s}) = 367\text{ rad/s}$, the final rotational kinetic energy is

$$K_{\text{f}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{4}(0.85\text{ kg})(0.125\text{ m})^2(367\text{ rad/s})^2 = 450\text{ J}$$

(b) The average power required is

$$\bar{P} = \frac{W_{\text{net}}}{\Delta t} = \frac{\Delta K}{\Delta t} = \frac{K_{\text{f}} - \overbrace{K_{\text{i}}}^{=0}}{\Delta t} = \frac{446\text{ J}}{3.2\text{ s}} = 140\text{ W}$$

where we have used the result from part (a) to three significant figures because it is an intermediate result.

ASSESS To check the answer, we present an alternative approach to computing K. In this problem, the angular acceleration is

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{367\text{ rad/s}}{3.2} = 115\text{ rad/s}^2$$

and the angular displacement is (using Equation 10.9)

$$\theta = \theta_0 + \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(367\text{ rad/s})^2}{2(115\text{ rad/s}^2)} = 586\text{ rad}$$

With these quantities, the rotational energy may be calculated as

$$K = W = \tau\theta = I\alpha\theta = \frac{1}{2}MR^2\alpha\theta = \frac{1}{2}(0.85\ \text{kg})(0.125\ \text{m})^2(115\ \text{rad/s}^2)(586\ \text{rad}) = 450\ \text{J}$$

which is same as before.

35. **Interpret** We're asked to imagine extracting energy from the Earth's rotational kinetic energy. We want to know how long it would take to slow the rotation rate enough for the day to increase by 1 minute.

Develop We imagine that rotational kinetic energy is extracted from the Earth at a rate of $P = 15\times10^{12}$ W. The rotational kinetic energy will correspondingly decrease, manifesting itself as a slowdown in the rotational speed. Over sufficient time, *t*, the rotational speed will decrease from its current value of $\omega_0 = 2\pi/1\,\text{d}$ to a value for which the day is one minute longer: $\omega_f = 2\pi/(1\,\text{d} + 1\ \text{min})$. Equating the change in rotational kinetic energy to the energy extracted gives:

$$\tfrac{1}{2}I_E\left(\omega_0^2 - \omega_f^2\right) = Pt$$

Evaluate From Problem 30, the rotational inertia of the Earth can be estimated as $I_E = 9.69\times10^{37}\ \text{kg}\cdot\text{m}^2$. Because $1\,\text{min} \ll 1\,\text{d}$, we can approximate the change in the rotational velocity squared as:

$$\left(\omega_0^2 - \omega_f^2\right) = \left(\frac{2\pi}{1\text{d}}\right)^2\left[1 - \left(1 + \frac{1\,\text{min}}{1\text{d}}\right)^{-2}\right] \simeq \left(\frac{2\pi}{1\text{d}}\right)^2\left[2\left(\frac{1\,\text{min}}{1\text{d}}\right)\right]$$

Plugging this into the above energy equation and solving for the time gives:

$$t = \frac{\frac{1}{2}I_E\left(\omega_0^2 - \omega_f^2\right)}{P} = \frac{\left(9.69\times10^{37}\ \text{kg}\cdot\text{m}^2\right)(2\pi)^2\left(\frac{1}{24\cdot60}\right)}{\left(1.5\times10^{13}\ \text{W}\right)(24\cdot60\cdot60\ \text{s})^2} = 2.4\times10^{13}\,\text{s} = 750{,}000\ \text{y}$$

Assess This is nearly one million years, which simply shows how much kinetic energy there is in the Earth's rotation. Of course, the computed time may be an underestimate, since humankind is continuously increasing its power consumption.

36. **Interpret** The kinetic energy of the baseball consists of two parts: the kinetic energy of the center of mass, K_{cm}, and the rotational kinetic energy, K_{rot}. We want to find the fraction of total kinetic energy that is due to K_{rot}.

Develop The total kinetic energy has center-of-mass energy and internal rotational energy associated with spin about the center of mass (see Equation 10.20):

$$K_{tot} = K_{cm} + K_{rot} = \frac{1}{2}Mv^2 + \frac{1}{2}I_{cm}\omega^2$$

Evaluate For a solid sphere, $I_{cm} = 2MR^2/5$ (see Table 10.2). Therefore, the rotational fraction of the total kinetic energy is

$$\frac{K_{rot}}{K_{tot}} = \frac{I_{cm}\omega^2}{Mv^2 + I_{cm}\omega^2} = \frac{\left(2MR^2/5\right)\omega^2}{Mv^2 + \left(2MR^2/5\right)\omega^2} = \frac{2R^2\omega^2}{5v^2 + 2R^2\omega^2}$$

$$= \frac{2(0.037\ \text{m/s})^2(42\ \text{rad/s})^2}{5(33\ \text{m/s})^2 + 2(0.037\ \text{m/s})^2(42\ \text{rad/s})^2}$$

$$= 8.86\times10^{-4} = 0.089\%$$

to two significant figures.

Assess Rotational kinetic energy constitutes a very small fraction of the total kinetic energy, which is reasonable because the linear speed at a point on the surface of the baseball due to rotation is only $\omega R = (42\ \text{rad/s})(0.037\ \text{m}) = 1.55\ \text{m/s}$, which is much less than the linear velocity of 33 m/s.

37. **Interpret** We are asked to find the energy stored in a the flywheel of Problem 10.33, so we will use the concepts of rotational inertia and kinetic energy of rotation. We also need to find the power output of a generator if the speed of the flywheel changes a given amount in a given time.

DEVELOP Apply Equation 10.18, $K = I\omega^2/2$, to calculate the kinetic energy stored in the flywheel. We will need to convert the angular speed in rpm to rad/s, and calculate the rotational inertia of the flywheel disk using $I = mR^2/2$ (from Table 10.2). From the work-energy theorem (see Equation 10.19) and using $\bar{P} = W/\Delta t$, we have

$$\bar{P} = \frac{W}{\Delta t} = \frac{\Delta K}{\Delta t}$$

where $\omega_i = 360$ rpm, $\omega_f = 300$ rpm, and $\Delta t = 3$s.

The mass of the flywheel is m = 7.7×10^4 kg, the radius is $R = 2.4$ m, and the initial rotation rate is 360 rpm.

EVALUATE

(a) The energy stored in the flywheel is

$$K = \frac{1}{2}I\omega^2 = \frac{1}{4}mR^2\omega^2$$
$$= \frac{1}{4}\left(7.7\times10^4\ \text{kg}\right)\left(2.4\ \text{m}\right)^2\left(360\ \frac{\text{rev}}{\text{min}}\right)^2\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)^2\left(\frac{1\ \text{min}}{60\ \text{s}}\right)^2 = 1.6\times10^8\ \text{J}$$

(b) The average power output during the deceleration of the flywheel is

$$\bar{P} = \frac{\Delta K}{\Delta t} = \frac{mR^2}{4\Delta t}\left(\omega_f^2 - \omega_i^2\right)$$
$$= \frac{\left(7.7\times10^4\ \text{kg}\right)\left(2.4\ \text{m}\right)^2}{4\left(3\ \text{s}\right)}\left[\left(300\ \frac{\text{rev}}{\text{min}}\right)^2 - \left(360\ \frac{\text{rev}}{\text{min}}\right)^2\right]\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)^2\left(\frac{1\ \text{min}}{60\ \text{s}}\right)^2 = 16\ \text{MW}$$

ASSESS This is a good way of generating enormous power pulses.

Section 10.5 Rolling Motion

38. **INTERPRET** We are asked to find the translational and rotational kinetic energy of a rolling sphere, given its translational speed and the mass of the sphere.

DEVELOP Translational kinetic energy is $K_t = mv^2/2$, and rotational kinetic energy is $K_r = I\omega^2/2$. From Table 10.2, the rotational inertia of a sphere is $I = 2mR^2/5$. The velocity is $v = r\omega = 5.0$ m/s. The mass of the sphere is $m = 2.4$ kg.

EVALUATE

(a) $K_t = \frac{1}{2}mv^2 = \frac{1}{2}(2.4\ \text{kg})(5.0\ \text{m/s})^2 = 30\ \text{J}$

(b) $K_r = \frac{1}{2}I\omega^2 = \frac{1}{\not{2}}\left(\frac{\not{2}}{5}mR^{\not{2}}\right)\left(\frac{v}{\not{R}}\right)^2 = \frac{mv^2}{5} = \frac{2}{5}K_t = \frac{2}{5}(30\ \text{J}) = 12\ \text{J}.$

ASSESS Note that the rotational kinetic energy is 2/5 the translational kinetic energy, in this case. Does the 2/5 look familiar? How would the two answers be related if instead of a solid sphere it was a hollow sphere?

39. **INTERPRET** This problem involves comparing the rotational and translational kinetic energy, so we will use the relationship between ω and v ($v = r\omega$).

DEVELOP The total kinetic energy is

$$K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

where the first term is the translational kinetic energy and the second term is the rotational kinetic energy. From Table 10.2, we find that the rotational inertia of a solid disk is $I = mr^2/2$. Recalling that $v = r\omega$, we can calculate the ratio $f = K_{rot}/K$.

EVALUATE The ratio of rotational kinetic energy to total kinetic energy is

$$f = \frac{\not{\tfrac{1}{2}}I\omega^2}{\not{\tfrac{1}{2}}Mv^2 + \not{\tfrac{1}{2}}I\omega^2} = \frac{\left(\frac{1}{2}MR^2\right)\omega^2}{M(\omega R)^2 + \left(\frac{1}{2}MR^2\right)\omega^2}$$
$$= \frac{\frac{1}{2}}{1+\frac{1}{2}} = \frac{1}{3}$$

ASSESS This is consistent with what we noted in the previous problem: the rotational inertia is $mR^2/2$ so the rotational kinetic energy is ½ the translational kinetic energy when it rolls without slipping.

40. **INTERPRET** This problem involves rotational kinetic energy and rotational inertia. Knowing the fraction of kinetic energy due to rotation, we are to determine whether the ball is solid or hollow.

DEVELOP The given fraction of kinetic energy due to rotation is

$$f = \frac{K_{\text{rot}}}{K_{\text{total}}} = \frac{40}{100} = \frac{2}{5}$$

We know that $K_{\text{tot}} = mv^2/2 + K_{\text{rot}}$, and $K_{\text{rot}} = I\omega^2/2$. From Table 10.2, we find that the rotational inertia for a hollow sphere is $I = 2MR^2/3$, whereas for a solid sphere it is $I = 2MR^2/5$. Use these formulas to calculate the ratio of rotational to total kinetic energy to see which one corresponds to the ratio give ($f = 2/5$).

EVALUATE For the solid sphere,

$$f = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2} = \frac{\left(\frac{2}{5}MR^2\right)\omega^2}{M(\omega R)^2 + \left(\frac{2}{5}MR^2\right)\omega^2} = \frac{\frac{2}{5}}{1+\frac{2}{5}} = \frac{2}{7}.$$

For the hollow sphere,

$$f = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2} = \frac{\left(\frac{2}{3}MR^2\right)\omega^2}{M(\omega R)^2 + \left(\frac{2}{3}MR^2\right)\omega^2} = \frac{\frac{2}{3}}{1+\frac{2}{3}} = \frac{2}{5}$$

Therefore, the sphere must be hollow.

ASSESS Notice that the rotation kinetic energy comprises a larger fraction of the total kinetic energy for a hollow sphere because more of its mass is concentrated away from the axis of rotation, so the rotational inertia is greater.

PROBLEMS

41. **INTERPRET** The problem is about the rotational motion of the wheel. By identifying the analogous situation for linear motion (see Table 10.1), we can apply the correct formula.

DEVELOP We are given the angular displacement and the angular acceleration, and the initial angular speed (= 0). To find the final angular speed, we can apply Equation 10.9, which relates all these quantities:

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

To find the time it takes for the wheel to make 2 turns, apply Equation 10.8:

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}at^2$$

For the calculation, we will convert the angular acceleration to s^{-2}

$$\alpha = 18\left(\frac{\text{rev}}{\text{min}\cdot\text{s}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \frac{6\pi}{10}\text{ rad}\cdot\text{s}^{-2}$$

EVALUATE **(a)** Inserting the angular acceleration and the angular displacement $\theta - \theta_0 = 4\pi$ into Equation 10.9, we find the final angular velocity is

$$\omega = \pm\sqrt{\omega_0^2 + 2\alpha(\theta - \theta_0)}$$
$$= \sqrt{0 + 2\left(\frac{6\pi}{10}\right)(4\pi\text{ rad})} = \pi\sqrt{4.8} = 6.9\text{ rad/s}$$

where the two signs indicate that the wheel could turn either clockwise or counter clockwise (we arbitrarily chose the positive sign).

(b) Inserting the acceleration and the angular displacement $\theta - \theta_0 = 4\pi$ rad into Equation 10.8 gives

$$\theta - \theta_0 - \overbrace{\omega_0}^{=0} t = \frac{1}{2}\alpha t^2$$

$$t = \pm\sqrt{\frac{2(\theta - \theta_0)}{\alpha}} = \sqrt{\frac{2(4\pi \text{ rad})}{6\pi/10 \text{ rad/s}^2}} = \sqrt{\frac{40}{3}} \text{ s} = 3.7 \text{ s}$$

Assess Another way to answer **(b)** is to use Equation 10.7:

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad t = \frac{\omega - \overbrace{\omega_0}^{=0}}{\alpha} = \frac{\pi\sqrt{4.8} \text{ rad/s}}{6\pi/10 \text{ rad/s}^2} = 3.7 \text{ s}$$

where we have used $\omega = \pi\sqrt{4.8}$ rad/s because it is more precise than 6.9 rad/s, which is only precise to two significant figures.

42. **Interpret** You need to determine how many revolutions (i.e., the angular displacement) the blender makes while accelerating between the two speeds. You can assume that the angular acceleration is constant.

Develop You're not given the angular acceleration, but this is not a problem since you can combine Equations 10.1 and 10.6 to find the angular displacement: $\Delta\theta = \frac{1}{2}(\omega_0 + \omega)\Delta t$.

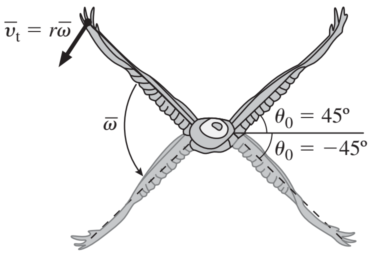
Evaluate Plugging in the known values gives:

$$\Delta\theta = \tfrac{1}{2}(3600 \text{ rpm} + 1800 \text{ rpm})\left(\frac{1\text{min}}{60\text{s}}\right)(1.4 \text{ s}) = 63 \text{ rev}$$

The blender does not meet the specs, since it makes 3 revolutions too many.

Assess A blender that does meet the specs would need an angular acceleration of at least: $\alpha = (\omega^2 - \omega_0^2)/2\Delta\theta = 23 \text{ rev/s}^2$. Therefore, the maximum time to switch between speeds would be: $t = (\omega - \omega_0)/\alpha = 1.3$ s, which explains why the blender above is unable to meet the specs.

43. **Interpret** We're asked to characterize one of the eagle's downstrokes, in which case the top of the stroke is θ_0 and the bottom of the stroke is θ, as shown in the figure below.



Develop If the eagle flaps 20 times per minute, then it makes a full flap every 3 seconds. A full flap consists of an upstroke and a downstroke, so a single downstroke takes $\Delta t = 1.5$ s . We can plug this time into Equation 10.1 to determine the average angular velocity $(\bar{\omega} = \Delta\theta/\Delta t)$. The tangential velocity at the tip can be found using Equation 10.3, $\bar{v} = \bar{\omega} r$. For the radius, *r*, we assume it's roughly half the wingspan, which is by definition the distance between the two wing tips.

Evaluate (a) Let's first convert the angles from degrees to radians:

$$\theta_0 = 45^\circ\left(\tfrac{2\pi \text{ rad}}{360^\circ}\right) = 0.785 \text{ rad} \;; \quad \theta = -45^\circ\left(\tfrac{2\pi \text{ rad}}{360^\circ}\right) = -0.785 \text{ rad}$$

So the average angular velocity is:

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{(0.785 \text{ rad}) - (-0.785 \text{ rad})}{1.5 \text{ s}} = 1.05 \text{ rad/s} \simeq 1.1 \text{ rad/s}$$

(b) As for the tangential velocity at the tip of the wing:

$$\bar{v} = \bar{\omega} r = (1.05 \text{ rad/s})\left(\tfrac{1}{2}\cdot 2.1 \text{ m}\right) = 1.1 \text{ m/s}$$

Assess The eagle makes a single downstroke in 1.5s, which seems reasonable. And since its wings are about a meter long each, it makes sense that the tangential velocity is approximately one meter per second.

44. **INTERPRET** The problem is about the angular velocity (rotation rate) of the CD, which we are asked to calculate given the requisite linear speed and the distance to the center of the CD (the radial distance).

DEVELOP Equation 10.3 gives the relation between linear speed and angular speed,

$$\omega = \frac{v}{r}$$

where r is the distance from the center of rotation. With v = 130 cm/s being the requisite constant linear speed, the angular speed can be found once r is specified.

EVALUATE **(a)** With $r = d/2 = 12.0/2.00 = 6.00$ cm, we have

$$\omega = \frac{v}{r} = \frac{130 \text{ cm/s}}{6.00 \text{ cm}} = 21.7 \text{ rad/s}$$

$$\left(21.7 \ \frac{\text{rad}}{\text{s}}\right)\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 207 \text{ rpm}$$

for a point on the CD's outer edge.

(b) With r = 3.75 cm, we have

$$\omega = \frac{v}{r} = \frac{130 \text{ cm/s}}{3.75 \text{ cm}} = 34.7 \text{ rad/s}$$

$$\left(34.7 \ \frac{\text{rad}}{\text{s}}\right)\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 331 \text{ rpm}$$

ASSESS Note that radians are a dimensionless angular measure, i.e., pure numbers; therefore angular speed can be expressed in units of inverse seconds (s^{-1}). Also, notice that there is roughly a factor of 10 difference between rad/s and rpm.

45. **INTERPRET** This problem involves angular acceleration, which we shall assume is constant. We are provided the initial and final angular speed of the motor and the time interval over which the motor accelerates and are asked to find several characteristics of the rotational kinematics of the engine.

DEVELOP Because we have no information about the variation in time of the acceleration, we can only calculate the average acceleration over the given time interval. This is given by Equation 10.4 in the form

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

To find the tangential (i.e., linear) acceleration, differentiate Equation 10.3 with respect to time to find

$$a = \frac{dv}{dt} = \frac{d}{dt}(\omega r) = \frac{d\omega}{dt} r = \alpha r$$

(note that this result holds only for constant radius). Finally, knowing the angular acceleration and the initial and final angular velocities, we can apply Equation 10.9 to find the number of revolutions made during the given time interval.

EVALUATE **(a)** The average angular acceleration is

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{(5500 - 1200) \text{ rpm}}{(2.7 \text{ s})}\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 170 \text{ s}^{-2}$$

to two significant figures.

(b) With d = 3.75 cm, we find an average linear acceleration of

$$\bar{a} = \bar{\alpha} r = (167 \text{ s}^{-2})\left(\frac{3.5 \text{ cm}}{2}\right) = 2.9 \text{ m/s}^2$$

(c) The engine makes

$$\Delta\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(5500 \text{ rpm})^2 - (1200 \text{ rpm})^2}{2(167 \text{ s}^{-2})(60 \text{ s/min})^2} = 150 \text{ revolutions}$$

during this 2.7-s time interval (to two significant figures).

ASSESS Note that dimensional analysis would lead us to the proper formula for part (b), where we needed to multiply the angular acceleration by a length to recover linear acceleration.

46. INTERPRET You have to determine if a saw stops within the required time limit. You assume the saw has constant angular acceleration, so you can use the equations in Table 10.1.

DEVELOP You know the initial angular velocity, ω_0, and the number of revolutions, $\Delta\theta$, before the saw stops. What you're looking for is the how long this takes. Combining Equations 10.1 and 10.6, you have an expression for the stopping time: $\Delta t = \Delta\theta / \frac{1}{2}(\omega_0 + \omega)$.

EVALUATE The time to stop the saw $(\omega = 0)$ is

$$\Delta t = \frac{2\Delta\theta}{\omega_0} = \frac{2(75 \text{ rev})}{(5500 \text{ rpm})} = 1.6 \text{ s}$$

The saw therefore meets its specs with 0.4 s to spare.

ASSESS The expression that we derived for the time can be arrived at in other ways. For example, Equation 10.7 says the angular acceleration during stopping is $\alpha = -\omega_0 / \Delta t$. Plugging this into Equation 10.8 gives $\Delta\theta = \frac{1}{2}\omega_0 \Delta t$, which works out to the same expression as above.

47. INTERPRET The problem concerns the *E. coli* bacteria, whose linear motion is related directly to the rotational motion of its flagellum.

DEVELOP The time that it takes for the bacteria to cross the microscope's field of view is simply the linear distance divided by the linear velocity: $t = \Delta x / v$. Over the same time, the flagellum completes a number of revolutions given by: $\Delta\theta = \omega t$.

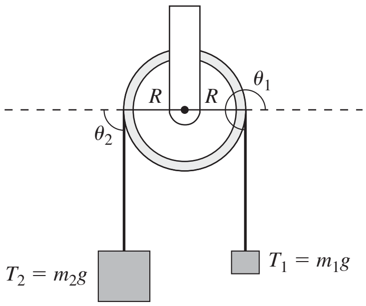
EVALUATE Combining the two equations from above gives

$$\Delta\theta = \frac{\omega\Delta x}{v} = \frac{(600 \text{ rad/s})(150\ \mu\text{m})}{(25\ \mu\text{m/s})}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 570 \text{ rev}$$

ASSESS The units all work out, and the answer seems reasonable. Note that the v we use here is not necessarily the same as the v in Equation 10.3 $(v = \omega r)$, which is the tangential speed of the rotating object.

48. INTERPRET This problem involves the angular counterpart to Newton's second law (compare $\tau = I\alpha$ to $F = ma$), which we can apply to find the torque required to prevent the pulley from turning.

DEVELOP Draw a picture of the situation (see figure below).



If the pulley and string are not moving (no rotation and no slipping), the net torque on the pulley is zero, and the tensions in the string on either side are equal to the weights tied on either end, as shown in the figure. Each tension force is applied perpendicular to the radius of the pulley. Notice that the angles are both measured in the same direction (we chose arbitrarily to measure the angles in the counter-clockwise direction). Therefore, from Equation 10.10, the torques due to the tensions have magnitudes

$$\tau_1 = RT_1 m_1 \sin\theta_1 = gR\sin(270^\circ) = -m_1 gR$$
$$\tau_2 = Rm_2 g \sin\theta_2 = m_2 gR\sin(90^\circ) = m_2 gR$$

and are applied in opposite directions, as expected. Thus, applying the rotational analog of Newton's second law (Equation 10.11) with zero angular acceleration gives

$$\tau_{\rm net} = \tau_{\rm app} - m_1 gR + m_2 gR = I \overbrace{\alpha}^{=0} = 0$$
$$\tau_{\rm app} = -m_1 gR + m_2 gR$$

where $\tau_{\rm app}$ is the torque we apply to prevent the pulley from moving.

EVALUATE Inserting the values given in the problem statement (note that $R = d/2 = (12\ \text{cm})/2 = 6.0\ \text{cm} = 0.060\ \text{m}$) into the expression for applied torque, we find

$$\tau_{\rm app} = (-m_1 + m_2)\, gR = (-0.22\ \text{kg} + 0.47\ \text{kg})(9.8\ \text{m/s}^2)(0.060\ \text{m}) = 0.15\ \text{N}\cdot\text{m}$$

ASSESS The torque $\tau_{\rm app}$ equals the torque which would be produced by balancing the pulley by adding 250 g to the side with lesser mass, as we can see by calculating the torque that such a mass would apply:

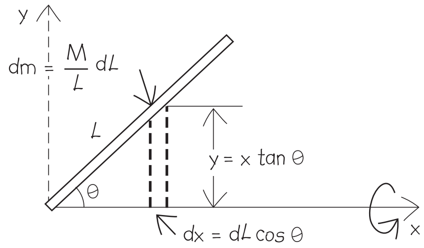
$$\tau_{\rm bal} = mgR = (0.25\ \text{kg})(9.8\ \text{m/s}^2)(0.06\ \text{m}) = 0.15\ \text{N}\cdot\text{m}$$

49. **INTERPRET** This problem involves finding the rotational inertia of an object about several different axes. For some axes, the object can be decomposed into objects for which the rotational inertia is given in Table 10.2, whereas for the other axes we must apply Equation 10.13 to find the rotational inertia for the various axes.

DEVELOP For part (a), the square frame can be decomposed into two rods parallel and two rods perpendicular to the axis. For the parallel rods, we can treat them as if all the mass were concentrated at the center of mass, so $I_{\rm par} = Mr^2 = M(L/2)^2 = ML^2/4$. The rotational inertia of the perpendicular rods can be found from Table 10.2, and is $I_{\rm per} = ML^2/12$. For part (b), we apply Equation 10.13 first to a single rod. Using the coordinate system drawn in the figure below, the integral of Equation 10.13 becomes

$$I = \int_0^{L\cos\theta} y^2 dm = \frac{M}{L}\frac{\tan^2\theta}{\cos\theta}\int_0^{L\cos\theta} x^2 dx = \frac{M}{L}\frac{\tan^2\theta}{\cos\theta}\left(\frac{x^3}{3}\right)_0^{L\cos\theta} = \frac{ML^2}{3}\sin^2\theta$$

Because all four rods are symmetric, the total rotational inertial will be four times this result.



For part (c), apply the parallel axis theorem. From Table 10.2 we find the rotational inertia of a rod rotating about an axis through its center of mass is $I_{\rm cm} = ML^2/12$. The parallel-axis theorem tells us the rotational inertia about a parallel axis a distance L/2 from the center-of-mass axis is

$$I = I_{\rm cm} + M\left(\frac{L}{2}\right)^2 = ML^2\left(\frac{1}{12} + \frac{1}{4}\right) = \frac{1}{3}ML^2$$

EVALUATE (a) Because we have two rods parallel to the axis and two rods perpendicular to the axis, the total rotational inertia is

$$I_{\rm a} = 2I_{\rm par} + 2I_{\rm per} = 2\frac{ML^2}{4} + 2\frac{ML^2}{12} = \frac{2}{3}ML^2$$

(b) Given that we have four rods, each with the rotational inertia given by the expression above, we can sum them to find the total rotational inertia. The result is

$$I_{\rm b} = 4I_{\rm rod} = \frac{4}{3}ML^2\sin^2\left(\frac{\pi}{4}\right) = \frac{4}{3}ML^2\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{2}{3}ML^2$$

(c) Again, we have four rods, each with the rotational inertial derived above. Therefore, the total rotational inertia is

$$I_c = \frac{4}{3}ML^2$$

ASSESS Notice how we used symmetry to simplify the calculations.

50. **INTERPRET** We are asked to find the rotational inertia of a thick ring with inner and outer radii R_1 and R_2. The mass distribution is continuous, so we need to do an integral.

DEVELOP For a thick ring, the ring-shaped mass elements used in Example 10.7 have mass

$$dm = \sigma dA = \frac{M}{\pi\left(R_2^2 - R_1^2\right)} 2\pi r dr$$

where $\sigma = M/A$ is the mass density (units: kg/m^2). Note that the ring only extends in radius from R_1 to R_2. The rotational inertia can then be obtained by integrating over

$$I = \int_{R_1}^{R_2} r^2 dm$$

EVALUATE Upon carrying out the integration, the rotational inertia about an axis perpendicular to the ring and through its center is

$$I = \int_{R_1}^{R_2} r^2 dm = M\int_{R_1}^{R_2} \frac{2\pi r^3 dr}{\pi\left(R_2^2 - R_1^2\right)} = \frac{M\left(R_2^4 - R_1^4\right)}{2\left(R_2^2 - R_1^2\right)} = \frac{M}{2}\left(R_1^2 + R_2^2\right)$$

ASSESS To see that the result makes sense, let's consider the following limits: **(i)** $R_1 \to 0$: In this case, we have a disk with radius R_2 and $I = MR_2^2/2$ **(ii)** $R_1 \to R_2$: In this limit, we have a thin ring with $I = MR_2^2$.

51. **INTERPRET** This problem involves applying the parallel axis theorem to find the rotational inertia of an object. We can use Table 10.2 to find the expression for the rotational inertia for an axis through the center of mass of the object.

DEVELOP The object is a flat plate that is rotating about one of its long edges (of length b). Therefore, if we displace the axis of rotation to go through the center of the plate, we have the situation depicted in the last entry of Table 10.2, so $I_{cm} = Ma^2/12$. The displacement of the axis of rotation is $d = a/2$.

EVALUATE Applying the parallel-axis theorem (Equation 10.17), gives

$$I = I_{cm} + Md^2 = Ma^2\left(\frac{1}{12} + \frac{1}{4}\right) = \frac{1}{3}Ma^2$$

ASSESS Notice the length b of the long side does not enter into the result. This makes sense because a longer plate will simply have more mass than a shorter one, but the distribution of the mass will not have changed.

52. **INTERPRET** We are asked about the rotational inertia of a propeller, treating it as a uniform thin rod. In addition, we want to find the time it takes to change its angular speed, given the torque.

DEVELOP The rotational inertia of a thin rod of length L is (from Table 10.2)

$$I = \frac{1}{3}ML^2$$

where the axis of the rotation passes through one of the endpoints of the rod. For part **(b)**, we note that the average torque is related to the average rate of change of angular speed (angular acceleration) as

$$\bar{\tau} = I\bar{\alpha} = I\frac{\Delta\omega}{\Delta t}$$

EVALUATE **(a)** The rotational inertia of one blade is $\frac{1}{3}ML^2$ (see Table 10.2). The propeller has three such blades, so

$$I = 3\left(\frac{1}{3}ML^2\right) = ML^2 = (10\text{ kg})(1.25\text{ m})^2 = 15.6\text{ kg}\cdot\text{m}^2$$

(b) The engine torque is the only one considered. Therefore, with $\omega_0 = 1400\text{ rpm} = 147\text{ rad/s}$ and $\omega = 1900\text{ rpm} = 199\text{ rad/s}$, we have

$$\Delta t = \frac{I\Delta\omega}{\bar{\tau}} = \frac{(15.6\text{ kg}\cdot\text{m}^2)(199\text{ rad/s} - 147\text{ rad/s})}{2700\text{ N}\cdot\text{m}} = 0.303\text{ s}$$

ASSESS Our result shows that Δt is inversely proportional to the applied torque. Increasing the torque reduces the time required to speed up the propeller.

53. **INTERPRET** The problem concerns the cellular motor that drives the flagellum of the *E. coli* bacteria. We are asked to find the force exerted by this motor, given the torque and the radius at which the force is applied.

DEVELOP We're told that the force is applied tangentially, so $\theta = 90^\circ$, and Equation 10.10 reduces to: $\tau = rF$.

EVALUATE Solving for the motor's applied force:

$$F = \frac{\tau}{r} = \frac{400 \text{ pN}\cdot\text{nm}}{12 \text{ nm}} = 33 \text{ pN}$$

ASSESS This is a very small force, but it's rather impressive that an *E. coli*, with a typical mass of about 10^{-15} kg, can exert a force that is over 1000 times its own weight.

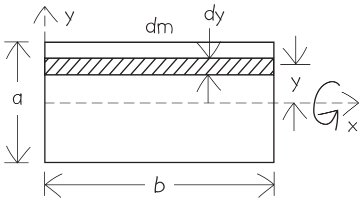
54. **INTERPRET** This problem is an exercise in finding the rotational inertia of an object. The object in question is a flat plane that rotates about a central axis (i.e., the last entry in Table 10.2).

DEVELOP Following the hint, we divide the plane up into strips parallel to the central axis (see figure below). For a uniform plate

$$dm/M = b\left[dy/(ab)\right]$$

$$dm = \frac{M}{a}dy$$

Insert this result into Equation 10.13 to and integrate from $y = -a/2$ to $y = a/2$ to find the rotational inertial.



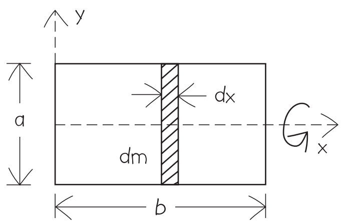
EVALUATE Evaluating the integral gives the rotational inertia as

$$I = \int y^2 dm = \frac{M}{a}\int_{-a/2}^{a/2} y^2 dy = \frac{M}{a}\left|\frac{y^3}{3}\right|_{-a/2}^{a/2} = \frac{Ma^2}{12}$$

which agrees with Table 10.2.

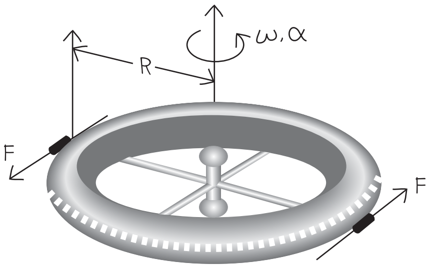
ASSESS To double check this result, we can divide the plate into thin strips perpendicular to the axis of rotation (see figure below). The rotational inertia of each strip is $dI = \left(a^2/12\right)dm$, where $dm/M = \left[a/(ab)\right]dx = dx/b$. Therefore,

$$I = \int dI = \int \frac{a^2}{12}\,dm = \frac{Ma^2}{12}\int_0^b \frac{dx}{b} = \frac{Ma^2}{12}$$



55. **INTERPRET** You are asked to find the time it takes for the space station to start from rest and reach a certain angular speed, with a given thrust.

DEVELOP The space station is essentially a ring with radius $R = 11\,\text{m}$ and rotational inertia $I = MR^2$ (from Table 10.1). The two rockets provide a net torque of $\tau = 2FR$, as can be seen from the figure below.



This torque causes an angular acceleration, $\alpha = \tau / I = 2F / MR,$ that spins up the station from rest to an angular velocity, ω. This final rotation speed is chosen such that the centripetal acceleration at the rim is equal to the gravitational acceleration on the surface of the Earth:

$$a_c = \frac{v^2}{R} = \frac{(\omega R)^2}{R} = \omega^2 R = g \quad \rightarrow \quad \omega = \sqrt{\frac{g}{R}}$$

Your job is to determine how long the rockets must fire to reach this angular velocity and how many rotations does the station make during this time period.

EVALUATE (a) The time can be found with Equation 10.7:

$$t = \frac{\omega}{\alpha} = \frac{\sqrt{g/R}}{2F/MR} = \frac{M\sqrt{gR}}{2F} = \frac{(5.0\times10^5\,\text{kg})\sqrt{(9.8\text{ m/s}^2)(11\text{ m})}}{2(100\text{ N})} = 2.60\times10^4\,\text{s} = 7.2\text{ h}$$

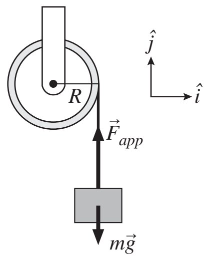
(b) We could use Equation 10.8 to find the number of revolutions completed in this time, but Equation 10.9 provides a simple formula with the weight of the space station:

$$\Delta\theta = \frac{\omega^2}{2\alpha} = \frac{Mg}{4F} = \frac{(5.0\times10^5\,\text{kg})(9.8\text{ m/s}^2)}{4(100\text{ N})} = \frac{12,250\text{ rad}}{2\pi\text{ rad/rev}} = 1900\text{ rev}$$

ASSESS These are relatively small rockets, so it takes a fair amount of time to reach the desired rotational velocity. Since $t \sim 1/F$, a larger thrust will shorten this spin up time.

56. **INTERPRET** This problem involves the concepts of rotational inertia, torque, and the rotational analog of Newton's second law, $F = ma$. We can apply this law to find the force needed to raise the mass with the given acceleration, then use the definition of torque to find the torque necessary to deliver this force.

DEVELOP Draw a diagram of the situation (see figure below).



Applying Newton's second law (for constant mass, Equation 4.3) to the hanging mass, we find

$$\vec{F}_{\text{net}} = m\vec{a}$$
$$F_{\text{app}} - mg = ma$$
$$F_{\text{app}} = m(a+g)$$

Applying the rotational analog of Newton's second law to the drum gives

$$\tau_{\text{net}} = I\alpha$$
$$\tau_{\text{app}} + \tau_{\text{mass}} = \frac{1}{2}MR^2\alpha$$

where τ_{mass} is the torque on the drum due to the hanging mass, $\tau_{\text{mass}} = -RF_{\text{app}}$ (see Equation 10).The negative sign comes from Newton's third law (see Chapter 4), which says that if F_{app} is pulling the mass upward, than a force of equal magnitude but opposite in direction must act on the drum.

Evaluate Evaluating the expression for the applied torque, and using the definition of angular acceleration (Equation 10.4), gives

$$\tau_{\text{app}} + \tau_{\text{mass}} = \frac{1}{2}MR^2\alpha$$

$$\tau_{\text{app}} - RF_{app} = \frac{1}{2}MR^2 \overbrace{\alpha}^{=a/R}$$

$$\tau_{\text{app}} = Rm(a+g) + \frac{1}{2}MR^2\left(\frac{a}{R}\right)$$

$$= R\left[a\left(m+\frac{M}{2}\right)+mg\right]$$

$$= (0.60\text{ m})\left[(1.1\text{ m/s}^2)\left(38\text{ kg}+\frac{51\text{ kg}}{2}\right)+(38\text{ kg})(9.8\text{ m/s}^2)\right] = 270\text{ N}\cdot\text{m}$$

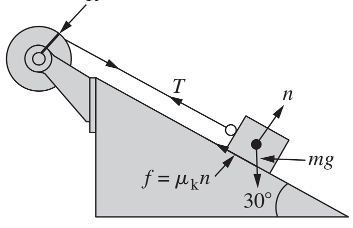
Assess If the drum radius goes to zero, we find that the applied torque goes to zero. Conversely, if the radius goes to infinity, the applied torque goes to infinity.

57. **Interpret** This problem involves Newton's second law in both linear and rotational form, which we can apply to find the coefficient of friction between block and slope, given the acceleration of the block. We will also need to consider the rotational inertia of the wheel in this problem.

Develop Draw a diagram of the situation (see figure below). Applying Newton's second law (Equation 4.3) to the mass gives

$$\left.\begin{array}{r} mg\sin\theta - f_{\text{k}} - T = ma \\ n - mg\cos\theta = 0 \end{array}\right\} mg\sin\theta - \mu_{\text{k}} mg\cos\theta - T = ma$$

where we have used the Equation 5.3 to express the force due to kinetic friction, $f_{\text{k}} = \mu_{\text{k}} n$.



Likewise, applying the rotational analog of Newton's second law (Equation 10.11) to the wheel gives

$$\tau_{\text{net}} = I\alpha$$
$$TR = I\alpha$$

where $t_{\text{net}} = TR$ because the tension is the only torsional force acting on the wheel, $I = MR^2/2$ (from Table 10.2) and $a = \alpha R$ (Equation 10.4). These equations allow us to determine μ_{k}.

Evaluate Solving first for the tension from the rotational application of Newton's second law gives

$$T = \frac{I\alpha}{R} = \frac{(MR^2/2)(a/R)}{R} = \frac{1}{2}Ma$$

Insert this into the equation derived from Newton's second law applied to the block, and solve for μ_{k}:

$$\mu_{\text{k}} = \frac{mg\sin\theta - ma - Ma/2}{mg\cos\theta}$$

$$= \frac{(2.4\text{ kg})(9.8\text{ m/s}^2)\sin(30°) - (2.4\text{ kg} + 0.425\text{ kg})(1.6\text{ m/s}^2)}{(2.4\text{ kg})(9.8\text{ m/s}^2)\cos(30°)} = 0.36$$

Assess To see that our expression for μ_{k} makes sense, let's check some limits: **(i)** If $a = 0$, then $\mu_{\text{k}} = \tan\theta$. This is precisely the equation we obtained in Chapter 5 (see Example 5.10). **(ii)** M = 0 and $\mu_{\text{k}} = 0$. The situation corresponds to a block of mass m sliding down a frictionless slope with acceleration $a = g\sin\theta$.

58. **INTERPRET** This problem combines Newton's second law for rotational motion and the concept of torque. Combining these with the rotational kinematic equations (Equations 10.6–10.9), we can find the final angular speed of the wheel.

DEVELOP Assuming the wheel spins about an essentially frictionless axis, the only torsional force acting on the wheel is due to the wrench, so Newton's second law (Equation 10.11) gives

$$\tau_{\text{net}} = \tau_{\text{wrench}} = I\alpha$$

From Example 10.6, the rotational inertia of the bicycle wheel is I = MR2, and from Equation 10.10, the torque applied by the wrench is $\tau_{\text{wrench}} = -f_k R = -\mu_k F_{\text{app}} R$. Note that $\theta = 90°$ in this case for Equation 10.10 because the frictional force is applied tangentially to the wheel, and we have used Equation 5.3 to express the frictional force. This gives us the angular acceleration, which we can use in Equation 10.7 to find the final angular speed ω.

EVALUATE Inserting the given quantities into the expression derived above using Newton's second law gives

$$\omega = \omega_0 + \alpha t = \omega_0 + \frac{\tau_{\text{wrench}}}{I} t = \omega_0 - \frac{\mu_k F_{\text{app}} R}{MR^2} t = \omega_0 - \frac{\mu_k F_{\text{app}} t}{MR}$$

$$= \left(230\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) - \frac{0.46(2.7\ \text{N})(3.1\ \text{s})}{(1.9\ \text{kg})(0.33\ \text{m})} = 18\ \text{rad/s} = 170\ \text{rev/min}$$

ASSESS Notice that the greater the applied force, the smaller will be the final angular momentum. One might think the wheel will reverse direction if the applied force is great enough, but this will not happen because friction only acts to counter the motion, not to create motion. Once the wheel stops, the friction force will be static and will not create motion. (It could, however, prevent another force from turning the wheel.)

59. **INTERPRET** In this problem we want to find the angular speed of the potter's wheel after he exerts a tangential force to the edge of the wheel. We can address this problem in several ways, either through the work-energy theorem, or through Newton's second law (Equation 10.11). The force produces a torque that causes the wheel to rotate.

DEVELOP We will apply the work-energy theorem for constant torque (Equation 10.19). This gives

$$W = \tau\Delta\theta = \Delta K = K_f - K_0 = \frac{1}{2} I\omega^2$$

because the wheel starts from rest. The equation allows us to determine the angular velocity ω.

EVALUATE Because the force acting on the wheel is tangential to the wheel circumference, $\theta = 90°$ in Equation 10.10, so $\tau = FR$. In addition, From Table 10.2, we know that the rotational inertia of a disk is I = MR2/2. Inserting $\Delta\theta = \frac{1}{8}\ \text{rev} = \frac{\pi}{4}\ \text{rad}$, we have

$$\omega^2 = \frac{2\tau\Delta\theta}{I} = \frac{2FR\Delta\theta}{MR^2/2} = \frac{4F\Delta\theta}{MR}$$

or

$$\omega = \pm\sqrt{\frac{4F\Delta\theta}{MR}} = \pm\sqrt{\frac{4(75\ \text{N})(\pi\ \text{rad}/4)}{(120\ \text{kg})(0.45\ \text{m})}} = \pm 2.1\ \text{rad/s}$$

ASSESS The two signs indicate that the potter may spin the wheel either clockwise or counter clockwise. The greater the force exerted on the wheel, the larger the angular speed. On the other hand, larger *M* and *R* result in a larger rotational inertia, and smaller angular speed (if the same force is applied). If we apply Newton's second law to this problem, we find

$$\tau_{\text{net}} = FR = Ia = MR^2\alpha/2$$

$$\alpha = \frac{2F}{MR}$$

Inserting this result into Equation 10.9 and solving for the final angular velocity gives

$$\omega^2 = \overbrace{\omega_0^2}^{=0} + 2\alpha\overbrace{(\theta - \theta_0)}^{\Delta\theta}$$

$$\omega = \pm\sqrt{\frac{4F\Delta\theta}{MR}}$$

which is the same expression found using the work-energy theorem.

60. **INTERPRET** This problem involves conservation of total mechanical energy, which we can use to find the angular speed of the hollow drum when the anchor hits the water.

DEVELOP By conservation of total mechanical energy, we can equate the initial and final mechanical energy. The initial energy is just the gravitational potential energy of the anchor, $E_0 = U = mgh$. The final energy is the sum of the kinetic energies of the rotating hollow drum and the dropping anchor, or

$$E_f = K_{\rm drum} + K_{\rm anchor} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

From Table 10.2, the rotational inertia of the hollow drum is $I = MR^2$.

EVALUATE Equating the initial and final total mechanical energies gives

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}m(\omega R)^2$$

$$\omega = \pm\sqrt{\frac{2mgh}{R^2(M+m)}} = \pm\sqrt{\frac{2(5000\text{ N})(16\text{ m})}{(1.1\text{ m/s})^2\left[380\text{ kg} + (5000\text{ N})/(9.8\text{ m/s}^2)\right]}} = \pm 12\text{ rad/s}$$

ASSESS The result has two signs because we cannot tell if the hollow drum rotates clockwise or counter clockwise.

61. **INTERPRET** This problem involves conservation of energy: gravitational potential energy is converted to center-of-mass kinetic energy and rotational kinetic energy.

DEVELOP By conservation of energy, the sum of the gravitational potential energy and the total kinetic energy (Equation 10.20) is a constant. If we assume the gravitational potential is zero where the ball is at rest, then this constant is zero, or in other words:

$$K_{\rm cm} + K_{\rm rot} = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2 = -U$$

As it rolls down the incline, the potential decreases: $U = -Mgh$, where the height is related to the distance rolled down the incline by: $h = d\sin\theta$. The ball is hollow, so its rotational inertia is $I = \tfrac{2}{3}MR^2$, and we assume that it rolls without slipping, so $v = \omega R$ (Equation 10.21).

EVALUATE Plugging in the various expressions into the energy conservation equation gives:

$$\tfrac{1}{2}Mv^2 + \tfrac{1}{3}Mv^2 = Mgd\sin\theta$$

Solving for the speed,

$$v = \sqrt{\tfrac{6}{5}gd\sin\theta}$$

ASSESS If the ball were sliding down the incline without friction, the speed would have been $v = \sqrt{2gd\sin\theta}$. The fact that the ball is rolling means it will go slower down the incline.

62. **INTERPRET** This problem involves conservation of total mechanical energy, which we can apply to find the height to which the ball rolls up the incline.

DEVELOP If the ball rolls without slipping and we define potential energy to be zero at the incline's base, then the initial energy is only kinetic energy, and is given by

$$E_{\rm i} = K_{\rm cm} + K_{\rm rot} = \frac{1}{2}Mv_{\rm cm}^2 + \frac{1}{2}I_{\rm cm}\omega^2$$

The final energy is only potential energy, and is $E_{\rm f} = Mgh$. Conservation of total mechanical energy allows us to equate the initial and final mechanical energy, which we can then solve for the height h.

EVALUATE Setting the initial and final energies equal gives

$$Mgh = \tfrac{1}{2} I_{\rm cm}\omega^2 + \tfrac{1}{2} M v_{\rm cm}^2.$$

With $I_{\rm cm} = 2MR^2/3$ (from Table 10.2) and $\omega = v_{\rm cm}/R$ this becomes

$$Mgh = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\left(v_{\rm cm}/R\right)^2 + \frac{1}{2}Mv_{\rm cm}^2 = \frac{5}{6}Mv_{\rm cm}^2$$

$$h = \frac{5v_{\rm cm}^2}{g} = \frac{5(3.7\ \text{m/s})^2}{(9.8\ \text{m/s}^2)} = 1.2\ \text{m}$$

ASSESS The height attained is proportional to the linear speed of the ball squared.

63. **INTERPRET** The kinetic energy of the wheel consists of two parts: the kinetic energy of the center of mass, $K_{\rm cm}$, and the rotational kinetic energy, $K_{\rm rot}$. We want to find how changing the moment of inertia and mass of the wheel affects the total kinetic energy.

DEVELOP The total kinetic energy of the wheel consists of center-of-mass energy and internal rotational energy associated with the spin about the center of mass (see Equation 10.20):

$$K_{\rm tot} = K_{\rm cm} + K_{\rm rot} = \frac{1}{2}Mv_{\rm cm}^2 + \frac{1}{2}I_{\rm cm}\omega^2$$

With the condition for rolling without slipping, $v = \omega R$, the total kinetic energy can be rewritten as

$$K_{\rm tot} = K_{\rm cm} + K_{\rm rot} = \frac{1}{2}Mv_{\rm cm}^2 + \frac{1}{2}I_{\rm cm}\left(\frac{v_{\rm cm}}{R}\right)^2 = \frac{1}{2}Mv_{\rm cm}^2\left(1 + \frac{I_{\rm cm}}{MR^2}\right)$$

The initial condition is $I_{\rm cm}/\left(MR^2\right) = 0.40 = 40\%$. After the redesign,

$$\frac{I'_{\rm cm}}{M'R^2} = \frac{0.9I_{\rm cm}}{(0.8M)R^2} = 1.125\frac{I_{\rm cm}}{MR^2} = (1.125)(0.40) = 0.45$$

EVALUATE The fractional decrease in kinetic energy is

$$\frac{K - K'}{K} = 1 - \frac{K'}{K} = 1 - \frac{M'\left[1 + I'_{\rm cm}/\left(M'R^2\right)\right]}{M\left[1 + I_{\rm cm}/\left(MR^2\right)\right]} = 1 - \frac{0.8M}{M}\frac{(1+0.45)}{(1+0.40)} = 0.171 = 17\%$$

to two significant figures.

ASSESS Initially, $K_{\rm cm}$ accounts for 1/14 = 71% of the total kinetic energy, while $K_{\rm rot}$ accounts for the remaining 0.4/14 = 29%. After the redesign $M \to M' = 0.8M$, so the translational kinetic energy decreases by 20%, while the rotational kinetic energy goes down by 10% $(I_{\rm cm} \to I'_{\rm cm} = 0.9I_{\rm cm})$. Therefore, the total kinetic energy is now

$$(0.8)\frac{1}{1.4} + (0.9)\frac{0.4}{1.4} = 0.829 = 83\%$$

of the original. This is a 17% decrease.

64. **INTERPRET** This problem involves conservation of total mechanical energy, which is composed in this case of rotational kinetic, translational kinetic, and gravitational potential energies. We shall take the bottom of the trajectory to be the zero of gravitational potential energy. Using these concepts, we can find the height to which the ball rises on the frictionless trajectory. Note that Newton's second law also applies, because when the ball enters the frictionless surface, no tangential force will act on it, so its rotational speed will remain constant.

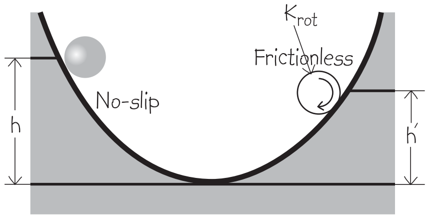
DEVELOP To apply conservation of total mechanical energy, we must express the initial and final total mechanical energies. The initial mechanical energy is $E_{\rm i} = Mgh$. At the bottom of the trajectory, the total mechanical energy is purely kinetic and consists of rotational and translational kinetic energy:

$$E_{\rm b} = K_{\rm rot} + K_{\rm cm} = \frac{1}{2}I_{\rm cm}\omega_{\rm b}^2 + \frac{1}{2}Mv_{\rm cm}^2$$

At the right top of the trajectory, the final total mechanical energy is

$$E_{\rm f} = U + K_{\rm rot} = Mgh' + \frac{1}{2}I\omega_{\rm b}^2$$

because the ball continues to spin at the constant angular speed ω_b once it enters the frictionless surface (see figure below—by Newton's second law, because no tangential forces act on the ball, its angular acceleration is zero, so its angular speed is constant). By conservation of total mechanical energies, all three of these expressions for the total mechanical energy must give the same result, which allows us to solve for h'.



EVALUATE Equating E_0 and E_b, we find

$$Mgh = \frac{1}{2}I_{cm}\omega_b^2 + \frac{1}{2}Mv_{cm}^2$$

Using $\omega_b R = v_{cm}$, and $I_{cm} = 2MR^2/5$ (from Table 10.2), and solving for ω_b gives

$$Mgh = \frac{1}{2}\left(\frac{2MR^2}{5}\right)\omega_b^2 + \frac{1}{2}M\left(\omega_b R\right)^2$$

$$\omega_b^2 = \frac{10gh}{7R^2}$$

Equating now the initial and final energies, and using this result for ω_b, we find

$$Mgh = Mgh' + \frac{1}{2}I_{cm}\omega_b^2 = Mgh' + \frac{1}{2}\left(\frac{2MR^2}{5}\right)\left(\frac{10gh}{7R^2}\right)$$

$$h' = h - \frac{2h}{7} = \frac{5}{7}h$$

ASSESS Notice that neither the size of the ball does nor its mass enters into the final result.

65. **INTERPRET** This problem involves finding the rotational inertia of a circular disk after an off-center hole has been drilled through it. The parallel-axis theorem is likely to be useful here.

DEVELOP Equation 10.12 shows that the rotational inertia of an object is the sum of the rotational inertias of its pieces, so

$$I_{disk} = I_{hole} + I_{remainder}$$

The hint expresses this fact as $I_{remainder} = I_{disk} - I_{hole}$. Here $I_{disk} = MR^2/2$ is the rotational inertia of the whole disk about an axis perpendicular to the disk and through the disk center (see Example 10.7). Use the parallel-axis theorem to find the rotational inertia of the hole, I_{hole}. This gives

$$I_{hole} = M_{hole}\left(\frac{R}{4}\right)^2 + I_{cm} = M_{hole}\frac{R^2}{16} + M_{hole}\frac{R^2}{32} = \frac{3}{32}M_{hole}R^2$$

where $R/4$ is the distance of the hole's center of mass from the axis of the disk, and we have $I_{cm} = M_{hole}\left(R/4\right)^2/2$ as the rotational inertia of the hole material about a parallel axis through its center of mass (see Example 10.7). With these equations, we can determine $I_{remainder}$.

EVALUATE Because the planar mass density of the disk (assumed to be uniform) is $\sigma = M/\pi R^2$, the mass of the hole material is

$$M_{hole} = \sigma A_{hole} = \frac{M}{\pi R^2}\pi\left(\frac{R}{4}\right)^2 = \frac{M}{16}$$

Therefore, the rotational inertia of the hole is

$$I_{hole} = \left(\frac{3}{32}\right)\left(\frac{M}{16}\right)R^2 = \frac{3}{512}MR^2$$

and

$$I_{\text{remainder}} = I_{\text{disk}} - I_{\text{hole}} = \frac{1}{2}MR^2 - \frac{3}{512}MR^2 = \frac{253}{512}MR^2 = 0.494MR^2$$

ASSESS If the hole drilled were concentric with the disk, we would have

$$I'_{\text{hole}} = I_{\text{cm}} = \frac{1}{2}\frac{M}{16}\left(\frac{R}{4}\right)^2 = \frac{1}{512}MR^2$$

and

$$I'_{\text{remainder}} = I_{\text{disk}} - I'_{\text{hole}} = \frac{1}{2}MR^2 - \frac{1}{512}MR^2 = \frac{255}{512}MR^2 = 0.498MR^2$$

The same result is obtained if we use the formula $M'\left(R_1^2 + R_2^2\right)/2$ derived in Problem 51, with $M' = \pi R^2 - \pi(R/4)^2 = (15/16)\pi R^2 = (15/16)M$, $R_1 = R$ and $R_2 = R/4$.

66. **INTERPRET** This problem involves Newton's second law and the rotational inertia of a solid cylinder. We are given a mass that hangs from a cylindrical drum by a massless rope that is wrapped around the drum. The mass is allowed to fall, but is restrained by the rope that makes the drum spin (causing an angular acceleration of the drum). We are to find the tension in the rope as the mass falls and the drum's mass.

DEVELOP The situation is similar to Example 10.9. Applying Newton's second law to the falling mass gives $mg - T = ma$ (where we have taken the downward direction to be positive), which we can solve for the tension T. For part (b), apply the rotational analog of Newton's second law, Equation 10.11 $\tau_{\text{net}} = I\alpha$, which gives $\tau_{\text{net}} = RT = I\alpha$, where the rotational inertia is $I = MR^2/2$ (from Table 10.2) and $\alpha R = a$.

EVALUATE (**a**) Solving for the tension gives $T = mg - ma = (50\text{ kg})(9.8\text{ m/s}^2 - 3.7\text{ m/s}^2) = 310\text{ N}$ (to two significant figures).

(**b**) Inserting the known quantities into the expression for the net torque gives

$$RT = \frac{MR^2}{2}\frac{a}{R}$$
$$M = \frac{2T}{a} = \frac{2(305\text{ N})}{3.7\text{ m/s}^2} = 170\text{ kg}$$

to two significant figures.

ASSESS Notice that we retained three significant figures for the tension in part (b) because the tension is an intermediate result in this case.

67. **INTERPRET** This problem involves conservation of total mechanical energy, which we can use to find how high up the hill the motorcyclist can go.

DEVELOP If all possible losses are neglected, the total mechanical energy of the motorcycle and rider is conserved as it coasts uphill, so the total kinetic energy at the bottom equals the total potential energy at the highest point,

$$K_{\text{trans}} + K_{\text{rot}} = M_{\text{tot}}gh$$

The translation kinetic energy of the cycle and rider (including the wheels) and the rotational kinetic energy of the wheels (about their center of mass) are, assuming rolling without slipping,

$$K_{\text{trans}} = \frac{1}{2}M_{\text{tot}}v^2, \qquad K_{\text{rot}} = 2\left(\frac{1}{2}I\omega^2\right) = I\left(\frac{v}{R}\right)^2$$

These expressions can be combined to solve for h.

EVALUATE Substituting the second equation into the first, and using $v = 85\text{ km/h} = 23.6\text{ m/s}$, we find the maximum vertical height reached is

$$h = \frac{v^2}{2g}\left(1 + \frac{2I}{M_{\text{tot}}R^2}\right) = \frac{(23.6\text{ m/s})^2}{2(9.8\text{ m/s}^2)}\left(1 + \frac{2(2.1\text{ kg}\cdot\text{m}^2)}{(395\text{ kg})(0.26\text{ m})^2}\right) = 33\text{ m}$$

ASSESS If the rolling motion is ignored, the result would be $h = v^2/2g$, which is what we expect from considering only the linear motion.

68. INTERPRET This problem involves conservation of total mechanical energy and Newton's second law. The former allows us to find the speed at the top of the loop, and the latter allows us to find the minimum speed necessary to stay on the track.

DEVELOP The center of mass of the marble travels in a circle of radius $R - r$ inside the loop, so at the top, $mg + N = mv^2/(R - r)$. To remain in contact with the track, Newton's second law tells us that $n \geq 0$, so that the track causes the ball to accelerate (otherwise it would be in free-fall). Thus, we have

$$F_{\text{net}} = -mg - n = ma = -m\frac{v^2}{R-r}$$

$$v^2 = \frac{mg+n}{m}(R-r) \geq g(R-r)$$

where we have taken the downward direction to be negative. By conservation of total mechanical energy, we can equate the total mechanical energies at point A and B. For point A, the energy is just the gravitational potential energy, which is

$$E_A = mg(h+r)$$

For point B, the energy is the sum of the gravitational potential energy and the kinetic energies do to rotation and translation, or

$$E_B = mg(2R-r) + \frac{1}{2}I_{\text{cm}}\omega^2 + \frac{1}{2}mv^2$$

For a sphere, $I_{\text{cm}} = 2MR^2/5$, and $\omega R = v$, so

$$E_B = mg(2R-r) + \frac{1}{2}\left(\frac{2mR^2}{5}\right)\left(\frac{v}{R}\right)^2 + \frac{1}{2}mv^2 = mg(2R-r) + \frac{7}{10}mv^2$$

Equating these two energies (by conservation of energy), we can find the minimum height h.

EVALUATE Equating the E_A and E_B gives

$$mg(h+r) = mg(2R-r) + \frac{7}{10}mv^2$$

Inserting the minimum value for v^2 from above gives the minimum height h:

$$mg(h+r) \geq mg(2R-r) + \frac{7}{10}mg(R-r)$$

$$h \geq 2.7(R-r)$$

ASSESS If we let r → 0, then we would have $mgh' = mg(2R) + mv^2/2$ from conservation of total mechanical energy and $v^2 \geq gR$ from Newton's second law. Combing these gives $h' \geq 2.5R$, so we see that $h' < h$, even if we insert $r = 0$ in the result for h. This is because the rotational inertia of a finite-sized ball consumes some mechanical energy, so any finite-sized ball will have to start higher than a ball with zero size (a point particle).

69. INTERPRET In this problem we are given a disk with non-uniform mass density, and asked to find its total mass and rotational inertia. We will therefore need to use the integral expression to calculate the rotational inertia.

DEVELOP As mass elements, choose thin rings of width dr and radius r (as in Example 10.7) so that

$$dm = \rho(r)\,dV = \left(\frac{\rho_0 r}{R}\right)2\pi rw\,dr = \frac{2\pi\rho_0 w}{R}r^2 dr$$

The total mass is $M = \int_0^R dm$ and the rotational inertia about the disk axis is $I = \int_0^R r^2 dm$ (see Equation 10.13).

EVALUATE **(a)** The disk's total mass is

$$M = \int_0^R dm = \frac{2\pi\rho_0 w}{R}\int_0^R r^2 dr = \frac{2\pi\rho_0 wR^2}{3}$$

(b) The disk's rotational inertia about a perpendicular axis through its center is

$$I = \int_0^R r^2 dm = \frac{2\pi\rho_0 w}{R}\int_0^R r^4 dr = \frac{2\pi\rho_0 wR^4}{5} = \frac{3}{5}\left(\frac{2\pi\rho_0 wR^2}{3}\right)R^2 = \frac{3}{5}MR^2$$

ASSESS Our result for I is intermediary between a disk of uniform density and a ring; $\frac{1}{2}MR^2 < I < MR^2,$ if expressed in terms of the total mass M, but is less than a disk of uniform density ρ_0; $I < \frac{1}{2}\rho_0 \pi R^4 w,$ because ρ_0 is the maximum density.

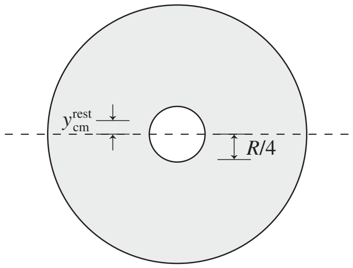
70. **INTERPRET** As the hint implies, this problem involves conservation of total mechanical energy. We can use this principle to relate the angular speed of the unbalanced disk when the void is at the bottom to that when the void is at the top.

DEVELOP Find the center of mass of the disk and compare its total mechanical energy when the void is at the top and when it is at the bottom. The vertical position of the center of mass of the complete disk (i.e., with no void, see figure below) is

$$0 = \frac{m_{\text{void}} y_{\text{cm}}^{\text{void}} + m_{\text{rem}} y_{\text{cm}}^{\text{rem}}}{M}$$

so the center of mass of the solid piece (the "remainder") is

$$y_{\text{cm}}^{\text{rem}} = -\frac{m_{\text{void}} y_{\text{cm}}^{\text{void}}}{m_{\text{rem}}}$$



From Problem 10.65, we know that $m_{\text{void}}/m_{\text{rem}} = 1/15$, and $y_{\text{cm}}^{\text{void}} = R/4$, so $y_{\text{cm}}^{\text{rem}} = -R/60$. Equating the total mechanical energy when the void is at the top and when its at the bottom gives

$$K_{\text{bot}} + U_{\text{bot}} = K_{\text{top}} + U_{\text{top}}$$
$$\tfrac{1}{2}I\omega_{\text{min}}^2 = \tfrac{1}{2}I\omega_{\text{max}}^2 - (U_{\text{bot}} - U_{\text{top}})$$

where the minimum angular speed occurs when the void is at the bottom and the maximum angular speed occurs when the void is at the top. From Problem 10.65, we know that $I = 253MR^2/512$, where M is the mass of the complete disk ($M = m_{\text{void}} + m_{\text{rem}}$). The difference in height of the center of mass between positions with the hole at the bottom and at the top is $\Delta y_{\text{cm}} = R/30$, so the change in potential energy is

$$U_{\text{bot}} - U_{\text{top}} = m_{\text{rem}} g\ \Delta y_{\text{cm}} = (15/16)Mg(R/30) = MgR/32$$

With these results we can solve for the minimum angular speed in terms of the maximum angular speed.

EVALUATE Solving the system of equations derived above for ω_{min} gives

$$\tfrac{1}{2}I\omega_{\text{min}}^2 = \tfrac{1}{2}I\omega_{\text{max}}^2 - (U_{\text{bot}} - U_{\text{top}})$$
$$\omega_{\text{min}}^2 = \omega_{\text{max}}^2 - \frac{2MgR/32}{I} = \omega_{\text{max}}^2 - \frac{MgR}{16}\left(\frac{512}{253MR^2}\right)$$
$$\omega_{\text{min}} = \pm\sqrt{\omega_{\text{max}}^2 - \frac{32g}{253R}}$$

ASSESS The positive and negative signs in the result reflect the fact that the expression cannot differentiate between clockwise and counter-clockwise rotation—both directions are equally valid. Notice that if $R \to \infty$, then $\omega_{\text{min}} = \omega_{\text{max}}$ because the void becomes negligible.

71. **INTERPRET** We are asked to show that the rotational inertia of a planar object around an axis perpendicular to the plane of the object is equal to the sum of the rotational inertias around two perpendicular axes within the plane of the object. We will use the integral form of rotational inertia, since the mass is distributed continuously.

DEVELOP The rotational inertia is $I = \int r^2 dm.$ We will set up our coordinate system such that the two rotational axes within the plane of the object are the *x* and *y* coordinate axes.

EVALUATE The rotational inertia around the *x* axis is $I_x = \int r^2 dm = \int y^2 dm.$ The rotational inertia around the *y* axis is $I_y = \int r^2 dm = \int x^2 dm.$ The sum of the two is $I_x + I_y = \int x^2 dm + \int y^2 dm = \int \left(x^2 + y^2\right) dm,$ but $\left(x^2 + y^2\right)$ is just the distance *r* from the perpendicular *z* axis, so $I_x + I_y = \int r^2 dm = I_z.$

ASSESS We have proven what was requested.

72. **INTERPRET** This problem involves the perpendicular-axis theorem (see Problem 10.71), which we are to use to check the rotational inertia of a flat rectangular plate given in Table 10.2 around an axis perpendicular to the plane of the plate. We are also asked to apply it to find the rotational inertia of a thin disk around an axis along the disk's diameter.

DEVELOP The rotational inertia of a flat plate around a central axis in the plane of the plate is given in Table 10.2 as $I = Ma^2/12$. The rotational inertia of a flat plate around a central axis perpendicular to the plate is given in the same table as $I = M\left(a^2 + b^2\right)/12$. We will see that the first, taken along two perpendicular axes, adds to give the second. In the second part, we will work backward from $I = MR^2/2$ with the perpendicular-axis theorem to find the rotational inertia of a thin disk around an axis in the plane of the disk.

EVALUATE **(a)** With respect to the axis shown for the first case, $I_1 = Ma^2/12$. With respect to a second, perpendicular axis in the plane of the plate, $I_2 = Mb^2/12$. The sum of these two is $I = M\left(a^2 + b^2\right)/12$, which is the equation given for the plate around an axis perpendicular to the plate.

(b) By the perpendicular-axis theorem, the rotational inertia of a disk about an axis through the center is $I = MR^2/2$ is equal to the sum of the rotational inertias around two perpendicular diameters. Because the rotational inertia around any diameter has the same value,

$$I_{\text{center}} = 2I_{\text{diameter}}$$

$$I_{\text{diameter}} = \tfrac{1}{2} I_{\text{center}} = \tfrac{1}{2}\left(\tfrac{1}{2} MR^2\right) = \tfrac{1}{4} MR^2$$

ASSESS We have shown that Table 10.2 is consistent, and we have used this theorem to find a new rotational inertia value not shown in the table.

73. **INTERPRET** This problem is an exercise in calculating the rotational inertia of an object (in this case, a right-circular cone). Because the mass is distributed continuously throughout the cone, we will apply the integral formula Equation 10.13 to find the rotational inertia.

DEVELOP Divide the cone into circular slices parallel to the base of the cone, and integrate over all these slices to find the total rotational inertia of the cone. The height of the cone is *h* and the base radius is *R*, so the radius of each slice is $r = Rx/h$, where *x* is the distance from the apex. The volume of the cone is $V = Ah/3$, where *A* is the area of the base, $A = \pi R^2$, so $V = \pi R^3 h/3$. The volume of each disk-shaped slice is $dV = \pi r^2 dx$. The cone has uniform mass density M/V, so each disk has mass $dm = MdV/V = 3M(\pi r^2 dx)/(\pi r^2 h)$. From Table 10.2, the rotational inertia of each disk is $dI = r^2 dm/2$.

EVALUATE Evaluating the integral gives

$$I = \int_0^h dI = \frac{1}{2}\int_0^h r^2 dm = \frac{1}{2}\int_0^h \left(\frac{\cancel{R}x}{h}\right)^2 \frac{3M}{\cancel{\pi}\cancel{R^2}h} \cancel{\pi}R^2 d = \frac{3M}{2h^3}\int_0^h x^2 \left(\frac{Rx}{h}\right)^2 dx$$

$$I = \frac{3MR^2}{2h^5}\int_0^h x^4 dx = \frac{3MR^2}{2\cancel{h^5}}\left[\frac{1}{5}\cancel{h^5}\right] = \frac{3}{10}MR^2$$

ASSESS The units are correct. The value of *I* is less than that of a cylinder, since a greater proportion of the mass is concentrated along the axis of the cone.

74. **INTERPRET** We are to show that the rotational inertia of a uniform solid spheroid, about its axis of revolution, is $2MR^2/5$ regardless whether the spheroid is prolate, oblate, or spherical. R in this case is the semi-axis perpendicular to the axis of revolution.

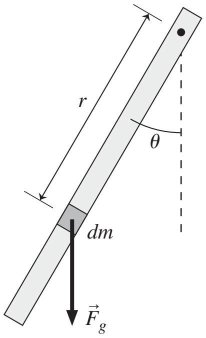
DEVELOP We imagine slicing a sphere into a large number of thin slices, cut perpendicularly to the rotation axis. If we stretch the sphere into a prolate spheroid, it's the same as increasing the thickness of each slice, without changing the mass or mass distribution of that slice. We have not moved any mass toward or away from the axis of rotation, though, so the rotational inertia of each disk stays the same. Since the rotational inertia of the spheroid is just the sum of the rotational inertias of the disks, which don't change, the rotational inertia of the spheroid is the same as that of the sphere.

EVALUATE We're actually done already. For the oblate spheroid, we do the same thing but compress each disk. Again, the rotational inertia of each disk does not change, so the rotational inertia of the spheroid does not change.

ASSESS Following this same reasoning, the rotational inertia of a rectangular plate, through an axis centered in the plane of the plate, should be the same as for a thin rod through the center. We can check this with Table 10.2, and we find that it is the case.

75. **INTERPRET** We are asked to find the torque due to gravity on a rod hanging from one end.

DEVELOP We can break the rod into infinitesimal mass elements, $dm = \mu dr$, where the mass per unit length is $\mu = M/L$. The gravitational force on each element is $F_g = g\,dm$, which means the torque (Equation 10.10) on each mass element is $d\tau = r(gdm)\sin\theta$, see the figure below.



EVALUATE The torque on the full rod is just the integral of the infinitesimal torques over the length of the rod:

$$\tau = \int d\tau = \int_0^L r\mu g\sin\theta dr = \mu g\sin\theta\left[\tfrac{1}{2}r^2\right]_0^L = \tfrac{1}{2}MGL\sin\theta$$

ASSESS We could have gotten the same answer by considering just the torque on the center of mass. The rod's center of mass is located at a distance of $r = L/2$ from the pivot point. The gravitational force, Mg, applied at the center of mass, creates a torque of $\tau = (L/2)(MG)\sin\theta$, which is what we obtained from integrating over the individual mass elements.

76. **INTERPRET** This problem involves force and torque, which are related by Equation 10.10. We can use this to find the force given the torque, the distance from the axis at which the torque is applied, and the angle between the force and the vector from the axis of rotation to the point at which the force is applied.. Use the definition of torque, and since no angular information is given, assume that the force is applied at 90°.

DEVELOP The force is applied at 90° to the vector from the axis of rotation to the point at which the force is applied, so Equation 10.10 reduces to $\tau = RF$, which we can solve for F.

EVALUATE Inserting the given quantities into the expression for torque and solving for the force F gives

$$F = \frac{\tau}{R} = \frac{10.1\ \text{kN}\cdot\text{m}}{0.95\ \text{m}} = 10.6\ \text{kN} = 11\ \text{kN}$$

to two significant figures.

ASSESS Since the distance is approximately 1 meter, the torque and the force have nearly the same numeric value. Useless. The specs are wildly incorrect!

77. **INTERPRET** You want to know if a rotating flywheel has as much energy as its manufacturer claims.

DEVELOP The flywheel can be modeled as a ring with rotational inertia $I = MR^2$. Its rotational kinetic energy is $\frac{1}{2}I\omega^2$, from Equation 10.18.

EVALUATE We have to divide the given diameter by 2 to get the radius, and we have to convert the non-SI unit of rpm to rad/s. Following that, the flywheel's kinetic energy is:

$$K_{\text{rot}} = \tfrac{1}{2}MR^2\omega^2 = \tfrac{1}{2}(48\text{ kg})\left(\tfrac{1}{2}0.39\text{ cm}\right)^2(30{,}000\text{ rpm})^2\left(\frac{\frac{2\pi}{60}\text{ rad/s}}{1\text{ rpm}}\right)^2 = 9.0\text{ MJ}$$

The specs are incorrect. The flywheel's storage capacity is 3 MJ below what the manufacturer claims.

ASSESS A flywheel is like a battery that stores energy as kinetic rotational energy. It has a high rotational inertia and presumably very little friction, so it will spin freely for a long time without slowing down appreciably. When the need arises, the flywheel can be connected to an electric generator, where its rotational energy is converted to electricity.

78. **INTERPRET** We're asked to derive the parallel axis theorem for an object of arbitrary shape.

DEVELOP The law of cosines from Appendix A says that the three sides (A, B, C) of a triangle obey:

$$C^2 = A^2 + B^2 - 2AB\cos\gamma$$

where γ is the angle between line segments A and B.

EVALUATE (a) If we choose θ to be the angle between the vectors $\vec{r}_{\text{cm}}$ and $\vec{h}$, then the law of cosines stipulates

$$r^2 = r_{\text{cm}}^2 + h^2 - 2r_{\text{cm}}h\cos\theta$$

But recall that the scalar product between $\vec{h}$ and $\vec{r}_{\text{cm}}$ is equal to $\vec{h}\cdot\vec{r}_{\text{cm}} = r_{\text{cm}}h\cos\theta.$ Therefore, as expected:

$$r^2 = r_{\text{cm}}^2 + h^2 - 2\vec{h}\cdot\vec{r}_{\text{cm}}$$

(b) Plugging the above relation into Equation 10.13 $\left(I = \int r^2 dm\right)$ gives us three separate integrals. The first is the integral of $r_{\text{cm}}^2 dm,$ which is the rotational inertia of the mass element dm around the center of mass:

$$\int r_{\text{cm}}^2 dm = I_{\text{cm}}$$

The second term involves h^2, which is constant since it is the square of the distance between the fixed points CM and A. So the integral reduces to an integral over the mass elements, which is just the total mass:

$$\int h^2 dm = h^2\int dm = Mh^2$$

For the third term, we can again use the fact that $\vec{h}$ is constant to rewrite the integral as

$$\int 2\vec{h}\cdot\vec{r}_{\text{cm}}\,dm = 2\vec{h}\cdot\int\vec{r}_{\text{cm}}\,dm$$

The integral $\int\vec{r}_{\text{cm}}\,dm$ is like the integral in Equation 9.4 for the center of mass. In fact, $\frac{1}{M}\int\vec{r}_{\text{cm}}\,dm$ gives the location of the center of mass in a coordinate system where the origin is already at the center of mass. Since the distance *to* the center of mass *from* the center of mass is zero, the integral $\int\vec{r}_{\text{cm}}\,dm$ must be zero.

ASSESS In summary, for an arbitrary object: $\int r^2 dm = I_{\text{cm}} + Mh^2$, which is the parallel axis theorem from Equation 10.17.

79. **INTERPRET** We must compare two centrifuges with slightly different designs.

DEVELOP We're told that the two centrifuges have the same mass and radius. But design A looks like a thin ring, while design B looks like a flat disk.

EVALUATE Design A should have approximately a rotational inertia of $I_{\text{A}} \approx MR^2$, compared to the design B with $I_{\text{B}} \approx \frac{1}{2}MR^2$.

The answer is (a).

ASSESS The rotational kinetic energy is proportional to rotational inertia $\left(K_{\text{rot}} = \frac{1}{2}I\omega^2\right)$. Therefore, it will take twice the work $(W = \Delta K)$ to spin up centrifuge A to the same rotational speed as centrifuge B.

80. **Interpret** We must compare two centrifuges with slightly different designs.

Develop If design A were made thicker, it would start to resemble a hollow cylinder more than a thin ring. If design B were made thicker, it would start to resemble a solid cylinder more than a flat disk.

Evaluate The rotational inertia is the same for both rings and hollow cylinders, as well as for solid cylinders and disks.

The answer is (a).

Assess Solid and hollow cylinders are symmetric about their central axes, so their rotational inertias do not depend on whether they're flat or thick, as long as the mass stays the same.

81. **Interpret** We must compare two centrifuges with slightly different designs.

Develop The sample tubes do not rest vertically, but instead tilt outwards. The bottom of the tubes are therefore at a radius greater than the radius of the centrifuges themselves.

Evaluate If the tubes are made longer, the bottom of the tubes will extend to a greater radius, so the rotational inertia will increase.

The answer is (b).

Assess Some centrifuges have a fixed angle (e.g. 45°), at which the tubes are placed. Others have a hinge that lets the tubes swing out when the device starts to turn.

82. **Interpret** We must compare two centrifuges with slightly different designs.

Develop When the centrifuges are spinning, the samples in the tubes are in uniform circular motion, so there must be a centripetal force acting on them.

Evaluate The centripetal force points inward.

The answer is (b).

Assess The walls of the sample tube provide a normal force that acts as the centripetal force on the sample. Since the sample material is "falling" towards the bottom of the tubes, the net effect is like gravity. As such, denser material will settle farther down in the tubes than less dense material. This separation is exactly what a centrifuge is designed for.

83. **Interpret** We must compare two centrifuges with slightly different designs.

Develop For both designs, the rotational inertia is proportional to the mass times the radius squared: $I \propto MR^2$.

Evaluate Doubling both the mass and the radius will change the rotational inertia by

$$\frac{I'}{I} = \frac{(2M)(2R)^2}{MR^2} = 8$$

The answer is (c).

Assess For a given rotational speed, the centripetal force is proportional to the radius: $F = m\omega^2 r$. So making the centrifuge bigger will presumably improve its ability to separate materials by their density.