Center of Mass

- The center of mass of a system of particles is the point that moves as though :-

1- all of the system's mass were concentrated there
2- all external forces were applied there

- for a discrete system

1 Dimen → the location of com :- $X_{com} = \dfrac{m_1 X_1 + m_2 X_2}{m_1 + m_2}$

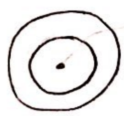
3 Dime → the location of com :- $\vec{r}_{com} = X_{com}\,\hat{i} + y_{com}\,\hat{j} + Z_{com}\,\hat{k}$

- for a continuous system (solid body) uniform objects

3 Dimen → $\vec{r}_{com} = \dfrac{\int \vec{r}\,dm}{m_{tot}}$

- A uniform object has a uniform density or mass per unit volume which is:- $\rho = \dfrac{M}{V}$ (M → mass, V → volume)

PS :- the center of Mass does not necessarely lies within the object : exp: a doughnut



← com

- If an external force acts on a Com

$$\boxed{\vec{F}_{net} = M\,\vec{a}_{com}}$$

M → the total Mass of the system (constant)

$\vec{a}_{com}$ → acceleration of Com (not the particles)

$\vec{F}_{net}$ → net force of all external forces that acts on system
→ internal forces are not included

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linear Momentum

↳ of a particle

→ • $\vec{P} = m\vec{V}$ of a particle

Mass ↵ ↳ velocity

• $F_{net} = \frac{d\vec{P}}{dt}$ Newton's 2nd law in terms of momentum

external

P changes when there is a net external force only

= The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force *

↳ of a system of particles

• $\vec{P} = \vec{P_1} + \vec{P_2}$ (→ of particle 1, → of particle 2)

• $\vec{P} = M\vec{V}_{cm}$

• $\vec{F}_{net} = \frac{d\vec{P}}{dt}$

Collision

brief collision :- in a small duration

P is conserved and impulse

$\vec{P}_i = \vec{P}_f$ $J = \Delta P$

• in a single collision :- $\Delta P = J$

• in series of collisions :- n is the number of collisions

$J = -n\Delta P$ Ps: the minus sign indicates that J and ΔP has opposite Directions

$F_{avg} = \frac{n}{\Delta t} m\Delta V$

$= \frac{-\Delta m}{\Delta t} \Delta V$ $(\Delta m = nm)$

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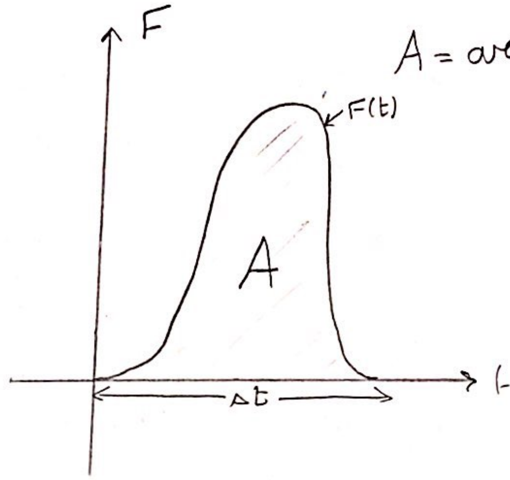
impulse

$$J = \int_{t_i}^{t_f} \vec{F}(t)\,dt$$

$$\Delta P = J$$

$$J = F_{avg}\,\Delta t$$



$A = \text{area} = J$

Kinetic Energy in collisions

elastic collision

- K.E conserved

$K.E_i = K.E_f$

↳ we use this equation

$(v_1 i - v_2 i) = -(\vec{v}_1 f - \vec{v}_2 f)$

Completely inelastic collision

- greatest loss of K.E
- the bodies stick together

$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$

inelastic collision

- K.E is not conserved

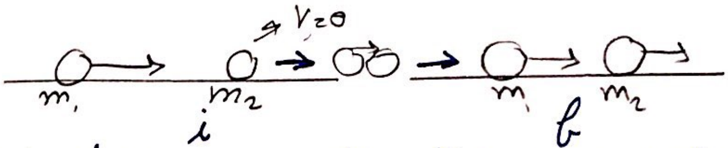
$K.E_f \neq K.E_i$

P.S :- $\vec{V}_{com}$ is constant before and after a collision because $\Sigma F_{external} = 0$

Alaa Etaimy

problems with a Projectile and an object :-

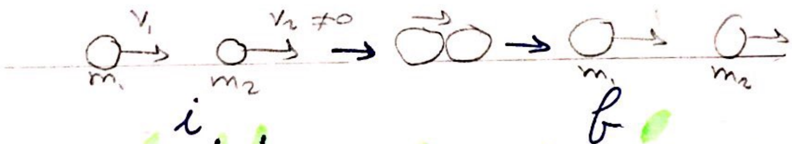
- The object (Target) is stationary :- $V_{2i} = 0$
  - If $m_1 > m_2 \Rightarrow m_1$ moves forward
  - if $m_1 < m_2 \Rightarrow m_1$ bounces (يرتد للخلف)
  - if $m_1 = m_2 \Rightarrow$ body 2 stops after collision and body 1 moves with the same velocity as body 1



- The object (Target) is moving :- $V_{2i} \neq 0$

$$(m_1 v_1 + m_2 v_2)_i = (m_1 v_1 + m_2 v_2)_f$$

$$\left(\tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2\right)_f = \left(\tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2\right)_f$$



Systems with Varying Mass : A Rocket

to find a:- $R\, v_{rel} = M a_{Total}$ $\Rightarrow$ $T = M a_{Total}$

↳ Thrust

R ← positive mass rate $= \frac{dM}{dt}$

to find v:- $V_f - V_i = V_{rel} \ln \frac{M_i}{M_f}$

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How to solve Problems

Center of Mass

Those are The main ideas

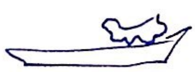
1- If the problem is about a system of particles we use

$X_{com} = \frac{x_1 m_2 + x_2 m_2}{(m_1 + m_2)}$ to find the position of com

2- If the system is a uniform body then it's probably in the center

3- in some question X_{com} doesn't change cause there is no horizontal or vertical Force so we put $X_{com} = 0$

Example : P17 Page 231



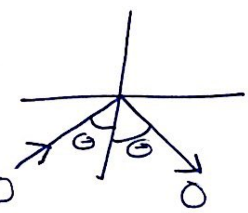
→ Dog moves But $X_{com} = 0$

4- If 2 bodies are projected the com will be moving in a projectile motion So we use equation of constant acceleration to find V_f for each body and then we can find V_{com} and a_{com}

5 ## linear Momentum

1- If the problem is about finding ΔP Then find V_f and V_i

2- in these problems you should pay attention to θ's and Directions, The main trick is in it



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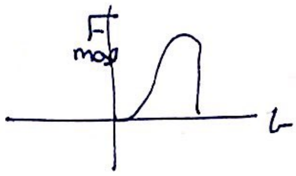
3- in problems with such plots remember that



Area under (F, t) curve $= J = \Delta P$

4- in explosions :-

$$\Delta P = 0$$
$$P_i = P_f$$

$$\Sigma m (v) = m_1 v_1 + m_2 v_2 \ldots$$

5- sometimes you need to use $(K+U)_f = (K+U)_i$ specially if the question is talking about h or when the question is about releasing a ball

Or if the question is talking about a spring

Then $K = U$

$$\frac{1}{2} m v^2 = \frac{k x^2}{2}$$

6- in Rockets problems :

you use : $v_f - v_i = v_{rel} \ln \frac{M_i}{M_f}$

or $R \, v_{rel} = M_{Tot}$

$\frac{dM}{dt}$

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