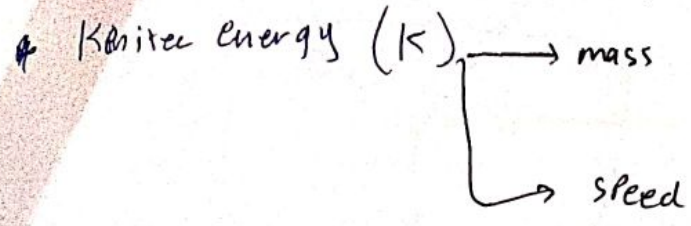


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Chapter 7

Kinetic Energy works.



$$K = \frac{1}{2} m v^2$$

$K \equiv$ scalar quantity قياسية

P5

initial

$$\left[\begin{array}{l} K_F = \frac{1}{2} K_S \\ m_S = \frac{1}{2} m_F \end{array} \right]$$

: f : father
s : son

$$(K_f)_i = \frac{1}{2} (K_f)_f$$

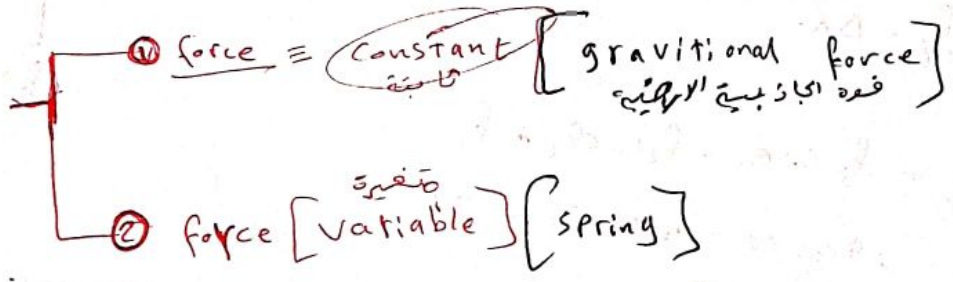
$$\frac{1}{2} m (v_f)_i^2 = \frac{1}{2} \left[\frac{1}{2} m (v_{if} + 1)^2 \right]$$

$v_{if} = ?$ \rightarrow $(v_f)_f = v_{if} + 1 \text{ m/s}$

$K_f = K_s ?$

سبب الفعل هو القوة المؤثرة

* work



$\vec{f} \equiv \text{constant} \Rightarrow W = \vec{f} \cdot \Delta \vec{x}$

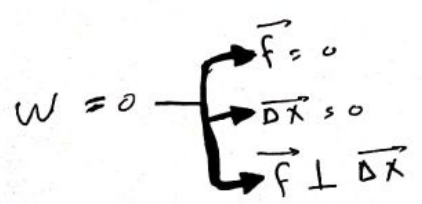
$[W] = N \cdot m = \text{Joule}$

$\vec{f} = \text{force}$
 $\Delta \vec{x} = \text{displacement}$ الازاحة
 $W = \text{work}$

work scalar quantity الكمية

$$W = |\vec{f}| |\Delta x| \cos \theta$$

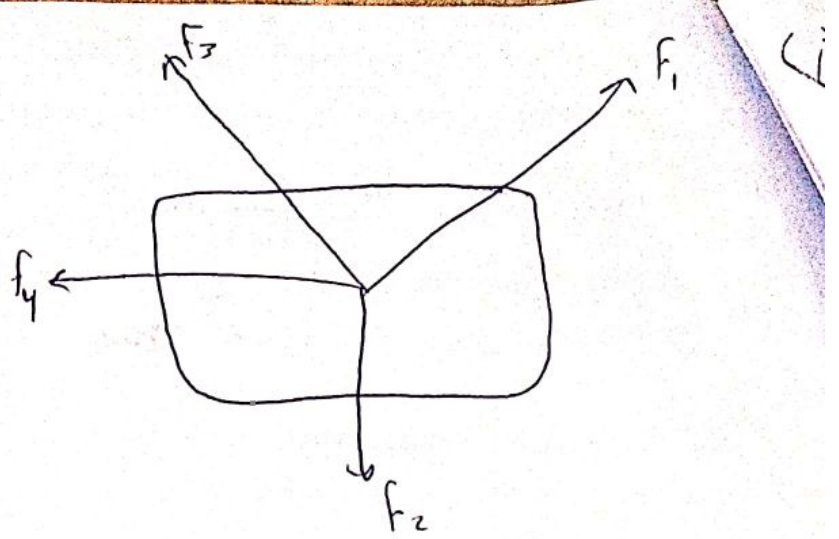
$\Delta x, f, W$



$$f = \frac{m v^2}{r}$$

$$① W_{net} = \vec{f}_{net} \cdot \Delta \vec{r}$$

$$② W_{net} = W_1 + W_2 + W_3 + \dots$$



P14

$$f_1 = 3N \quad \theta_2 = 50^\circ$$

$$f_2 = 4 \quad \theta_3 = 35^\circ$$

$$f_3 = 9N$$

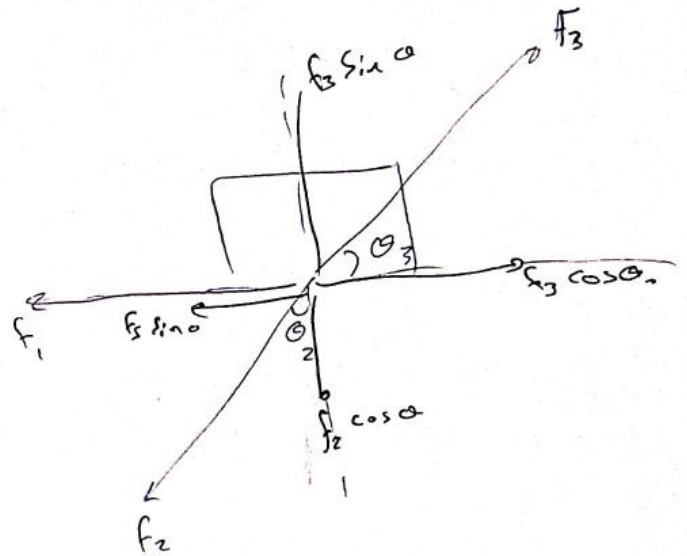
$$W = ?$$

$$W_{net} = \vec{f}_{net} \cdot \Delta \vec{r}$$

$$\begin{aligned} \sum f_x &= f_3 \cos \theta_3 - f_2 \sin \theta_2 - f_1 \\ &= 9 \cos 35 - 4 \sin 50 - 3 \\ &= \boxed{1,24 N} \end{aligned}$$

$$\begin{aligned} \sum f_y &= f_3 \sin \theta_3 - f_2 \cos \theta_2 \\ &= 9 \sin 35 - 4 \cos 50 \\ &= \boxed{2,6 N} \end{aligned}$$

من
نقطة



$$f_{net} = \sqrt{(1,24)^2 + (2,6)^2} = \boxed{2,88}$$

$$\begin{aligned} W &= f \cdot \Delta r \\ &= 2,88 \cdot 4 = \boxed{11,5 \text{ Joule}} \end{aligned}$$

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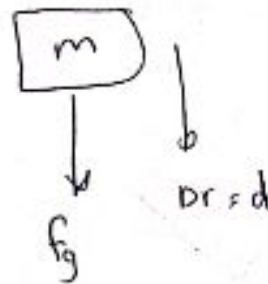
Exp: gravitational force

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① $\vec{f}_g = mg$

① $W_{fg} = \vec{f}_g \cdot \vec{\Delta r}$
 $= f_g d \cos \alpha$

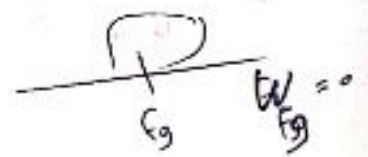
$W_{fg} = f_g d$



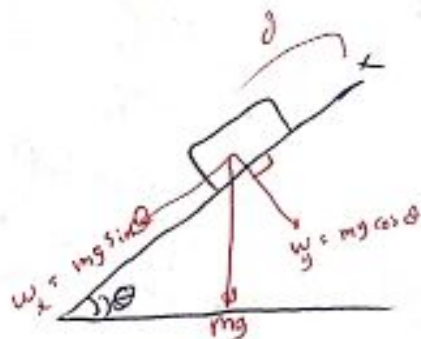
② $W_{fg} = -f_g d$



$f_g \perp \Delta r$



Case 3



① downward

$W_{Wg} = 0$

$W_{Wx} = W_x \cdot d$

② upward $W_{Wx} = -W_x \cdot d$

constant speed (up) \Rightarrow

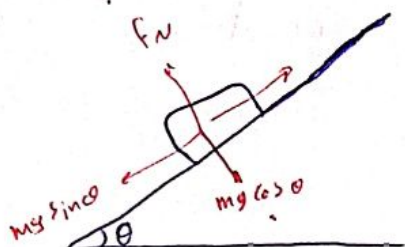
تارة فترق

⊛ Kientic energy - work theorem :-

$$W_{\text{net}} = \Delta K$$

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P 20

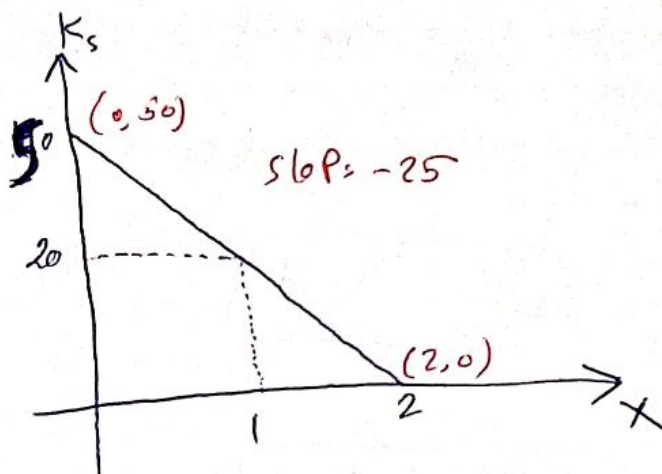


$$v_i = 5 \text{ m/s}$$

$$F_N = ??$$

$$F_N = mg \cos \theta$$

$$= 4(9.8) \cos 39.6^\circ$$



$$W_{\text{net}} = f_{\text{net}} \cdot \Delta x = \Delta K$$

$$\text{slope} = \frac{\Delta K}{\Delta x} = f_{\text{net}} \cdot x = -mg \sin \theta$$

$$25 = 4(9.8) \sin \theta$$

$$\theta = 39.6$$

$$K_i = 50 \text{ J}$$

$$50 = \frac{1}{2} m (5)^2$$

$$m = 4 \text{ kg}$$

⊛ work done by variable Part 2

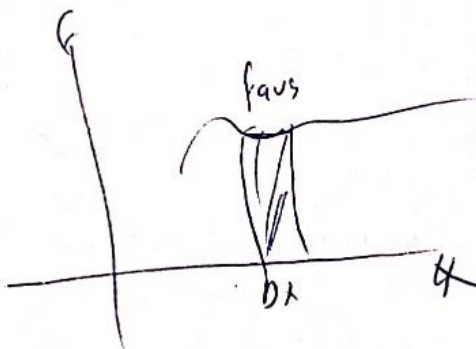
$$\Delta x \rightarrow f_{\text{avg}}$$

$$W_i = f_{\text{avg}} \cdot \Delta x$$

$$W = \sum_i f_{\text{avg}} \cdot \Delta x$$

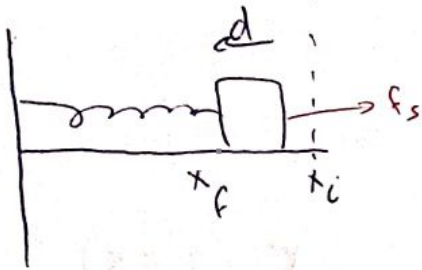
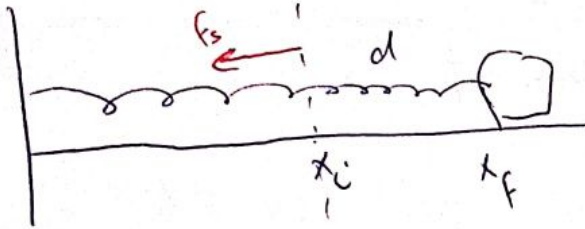
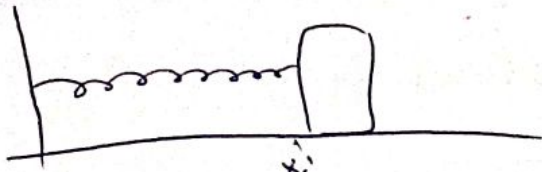
$$W = \lim_{\Delta x \rightarrow 0} \sum f_{\text{avg}} \Delta x$$

$$W = \int_{x_i}^{x_f} F dx$$



قوة اللف و Spring force

ف، و، و، و



Hook's law,

$$\vec{f} \propto \vec{d}$$

$$f_s = -Kd$$

K ≡ spring constant
ثابت اللف

$$W_{\text{spring}} = \int_{x_i}^{x_f} f_s \cdot dx$$

$$= \int_{x_i}^{x_f} -Kx \cdot dx$$

$$= \frac{1}{2} Kx \Big|_{x_f}^{x_i}$$

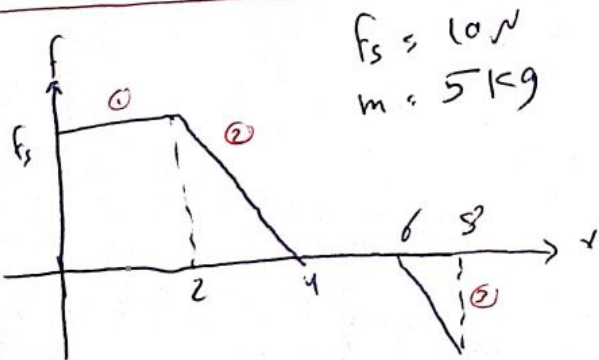
$$= \frac{1}{2} Kx_i^2 - \frac{1}{2} Kx_f^2$$

$$= \frac{1}{2} K(x_i^2 - x_f^2)$$

$$W_{\text{applied}} = -W_{\text{spring}}$$

$$= \frac{1}{2} K(x_f^2 - x_i^2)$$

P36

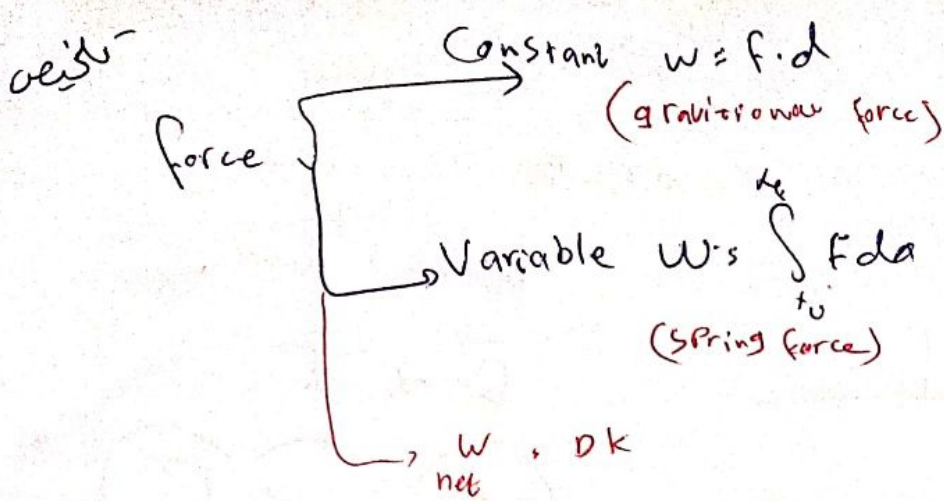


$f_s = 10 \text{ N}$
 $m = 5 \text{ kg}$

$$W = \int_{x_i}^{x_f} f \cdot dx = \text{Area}(f-x)$$

$$x_f = \sqrt{\frac{mv_i^2}{K}}$$

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* \vec{F} in 2D, 3D

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

EXPL $\vec{F} = 2x \hat{i} - 3y \hat{j} + \hat{k}$ $\vec{r}_1 = (0, 2, 4) \rightarrow \vec{r}_2 = (2, 3, 5)$

$$W = \int_0^2 2x dx - \int_2^3 3y dy + \int_4^5 dz$$

$$= x^2 \Big|_0^2 - \frac{3y^2}{2} \Big|_2^3 + z \Big|_4^5$$

* Work - Kinetic theorem

$$W_{\text{net}} = \int F_{\text{net}} dx, F_{\text{net}} = ma, a = \frac{dv}{dt}$$

$$= \int m a dx = \int m \frac{dv}{dt} dx$$

$$\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v \frac{dv}{dx}$$

$$W_{\text{net}} = m \left[\frac{v^2}{2} \Big|_{v_i}^{v_f} \right]$$

$$= \frac{1}{2} m (v_f^2 - v_i^2) = \Delta K$$

$$\int m v \frac{dv}{dx} dx$$

$$= m \int_{v_i}^{v_f} v dv$$

Power (P)

$$P_{avg} = \frac{\Delta W}{\Delta t} = \frac{J}{s}$$

$$[P] = \frac{J}{s} = \text{watt}$$

$$W = mgh$$

$$\Rightarrow P_{inst} = \frac{dW}{dt}$$

$$P_{avg} = \vec{F} \cdot \vec{v}$$

$$P = F \cdot v \text{ (if } \vec{F} \parallel \vec{v} \text{)}$$

Discussion

$$P_i: m_p = 1,67 \times 10^{-27} \text{ kg}$$

$$\alpha = 3,6 \times 10^{15} \text{ m/s}^2$$

$$v_i = 2,4 \times 10^7 \text{ m/s}$$

$$\Delta x = 3,5 \text{ cm}$$

(a) final speed (b) ΔK

$$(b) \Delta K = K_f - K_i = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$(a) v_f^2 = v_i^2 + 2a \Delta x$$

$$v_f = \sqrt{(2,4 \times 10^7)^2 + 2(3,6 \times 10^{15})(3,5 \times 10^{-2})} = 2,9 \times 10^7 \text{ m/s}$$

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$$= \frac{1}{2} [1,67 \times 10^{-27}] [(2,9 \times 10^7)^2 - (2,4 \times 10^7)^2] = 2,1 \times 10^{-18} \text{ J}$$

P15

(a) $W_{net} = ?$

$$W_{net} = \vec{F} \cdot \Delta \vec{x}$$

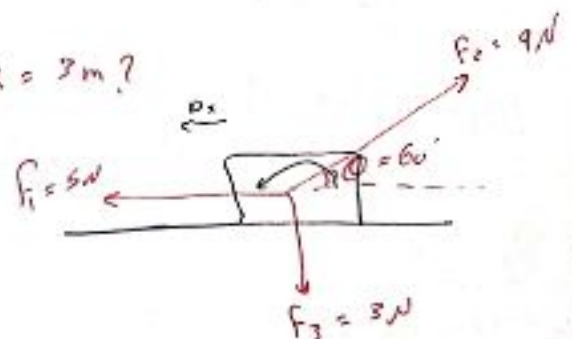
$$W_1 = \vec{F} \cdot \Delta \vec{x} = |F| |\Delta x| \cos \theta = 5(3) \cos 0 = 15 \text{ J}$$

$$W_2 = fd \cos \theta = 9(3) \cos 120 = -13,5 \text{ J}$$

$$W_3 = fd \cos \theta = 3(3) \cos 90 = 0 \text{ J}$$

$$W_{net} = 1,5 \text{ J}$$

$\Delta x = 3 \text{ m}$?



(b) $\Delta K = W_{\text{net}}$
 $= 1,5 \text{ J}$

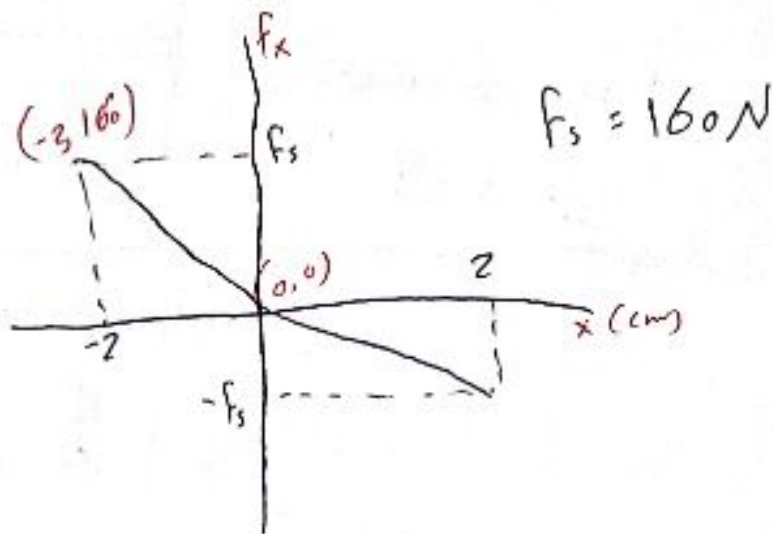
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137

P32

how much work does the spring do on the block moves from $x_i = 8 \text{ cm}$ to

- (a) $x_f = 5 \text{ cm}$
- (b) $x_f = -5 \text{ cm}$
- (c) $x_f = -8 \text{ cm}$



$W_s = \frac{1}{2} K (x_i^2 - x_f^2)$

$F_s = -kx$
 slope = $\frac{y}{x}$

slope = $-k = \frac{160 - 0}{-2 - 0} = -80$

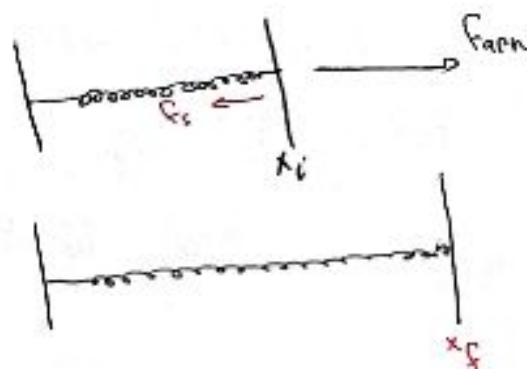
$K = 80 \text{ N/cm} \times 10^{-2}$

$K = 8000 \frac{\text{N}}{\text{m}}$

(a) $W_s = \frac{1}{2} (8000) [(8 \times 10^{-2})^2 - (5 \times 10^{-2})^2] = 15,6 \text{ J}$

(b) $W_s = 15,6$

(c) $W_s = 0$



$F_s = -kx$

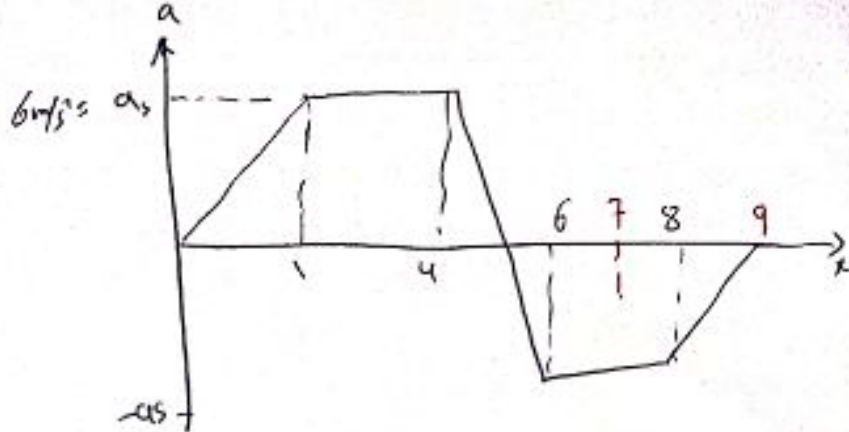
$F_a = -F_s$

$W_s = \int_{x_i}^{x_f} F dx$

$W_s = -W_a$

P 37

m = 2 kg



$$W_{net} = \int_{net} f(x) dx$$

$$= \int m a(x) dx$$

$$W_{net} = m \int a(x) dx \rightarrow \text{area}$$

a) $W_{net(4)} = 2 \left[\frac{1}{2}(1)(6) + (3)(6) \right] = 42 \text{ J}$

b) $W_{net(7)} = 30 \text{ J}$

c) $W_{net(9)} = 12 \text{ J}$

d) v at x = 4

$$W_{net} = \Delta K$$

$$= K_f - K_i$$

$$42 = \frac{1}{2} m v_f^2 \Rightarrow v_f = \boxed{6.5 \text{ m/s}}$$

v at x = 7

$$30 = K_f - K_i \Rightarrow 30 = \frac{1}{2} (2) v_f^2 \Rightarrow v_f = \boxed{5.5}$$

v at x = 9

$$12 = \frac{1}{2} (2) v_f^2 \Rightarrow v_f = \boxed{3.5 \text{ m/s}}$$

$$P_{avg} = \frac{dW}{dt}, \quad P_{inst} = \frac{dW}{dt}, \quad P = \vec{F} \cdot \vec{v}$$

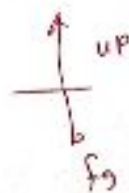
P 46 m = $3 \times 10^3 \text{ kg}$, $\Delta y = 210 \text{ m}$ up, $\Delta t = 23 \text{ sec}$

$$P = \vec{F} \cdot \vec{v} = F v \cos \theta$$

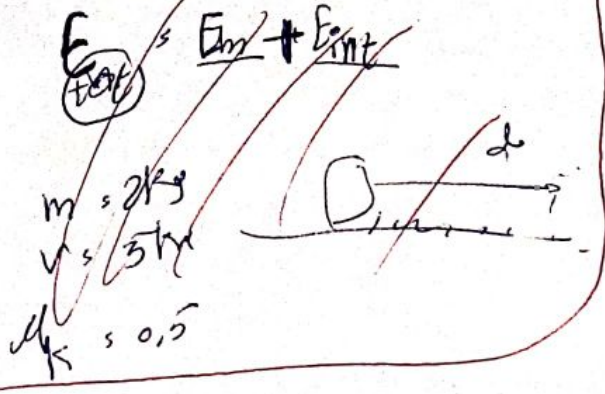
$$= mg v$$

$$|P| = mg \frac{\Delta y}{\Delta t}$$

$$= 3 \times 10^3 (9.8) \left(\frac{210}{23} \right) = 2.68 \times 10^3$$

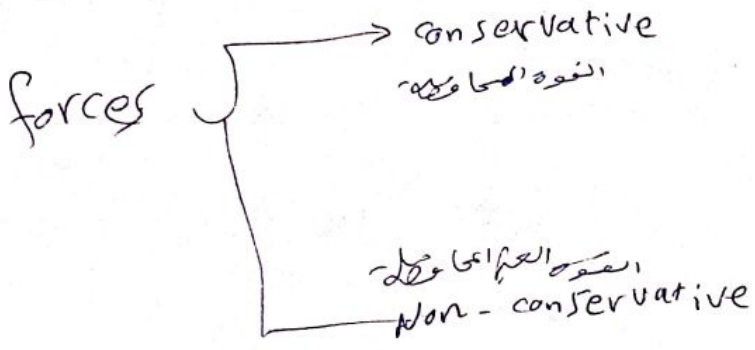


Good Luck
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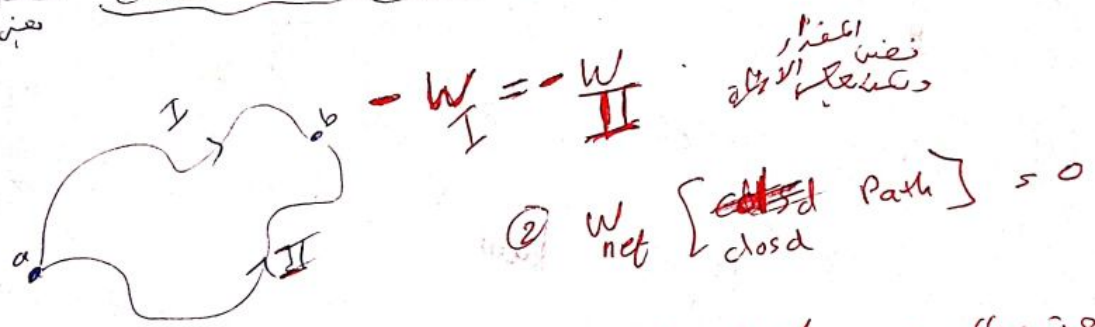


Potential energy (U)
 $\Delta U = U_2 - U_1$ ($U_1 < 0$)
 $\Delta U = -W$

- ① Isolated system
- ② Conservative force



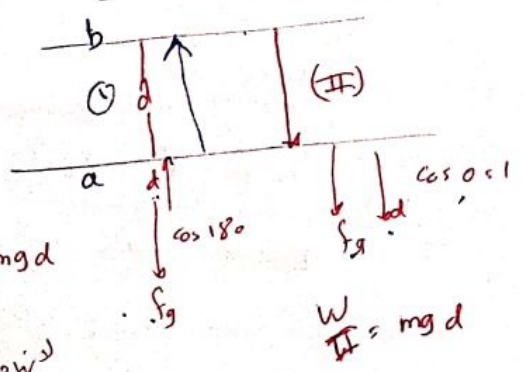
Conservative force: ① Work does not depend on the path
 (القوة الحافظة: العمل لا يعتمد على المسار)



- Non-Conservative forces: work depends on the path
 (القوة غير الحافظة: العمل يعتمد على المسار)

انواع Conservative forces

① gravitational force



$W_I = -mgd$

$W_{II} = mgd$

conservative $W_{net} = 0$

- K. friction
- Drag force

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$$W = \int_{x_i}^{x_f} f(x) dx$$

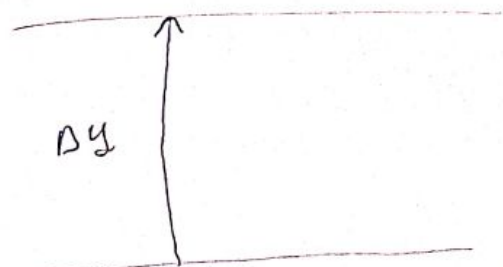
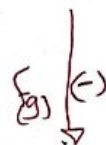
$$\Delta U = -W = - \int_{x_i}^{x_f} f(x) dx$$

$$U_2 - U_1 = - \int_{x_0}^{x_f} f(x) dx$$

Case (I) : gravitational force

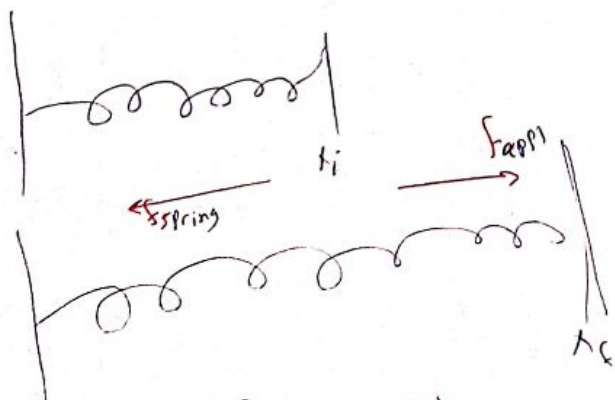
$$\Delta U = - \int_{y_i}^{y_f} f(y) dy$$

$$= \int_{y_i}^{y_f} mg dy \Rightarrow \Delta U = mg(y_f - y_i)$$



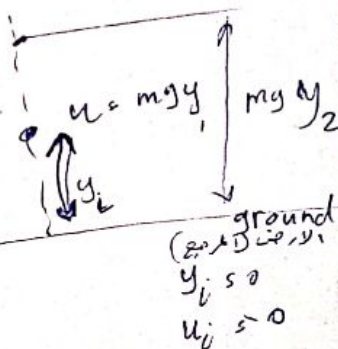
Case (II) spring force

$F_{spring} = -Kx$ Hook's law



$$W_s = \frac{1}{2}K(x_i^2 - x_f^2)$$

$$W_{app} = -W_s$$



$$\Delta U = - \int_{x_i}^{x_f} f(x) dx$$

$$= \int_{x_i}^{x_f} Kx dx$$

$$\Delta U = \frac{1}{2}K(x_f^2 - x_i^2)$$

$$\Delta U = -W_s$$

Mechanical Energy (E)

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$$E_{mec} = K + U$$

- Isolated [no external forces]

- Conservative forces

E_{mec} = Conserved *المحفوظ*

$$W = \Delta K \Rightarrow \Delta K = -\Delta U$$

$$W = -\Delta U$$

$$K_f - K_i = -(U_f - U_i)$$

$$K_i + U_i = K_f + U_f$$

$$E_i = E_f \quad \Delta E_{mec} = 0$$

$$E = \frac{1}{2} m \omega^2 a^2$$

$$\omega = \sqrt{\frac{K}{m}}$$

$$E = \frac{1}{2} K a^2$$

$$a = \sqrt{\frac{2E}{K}}$$

$$v = \omega a$$

$$h = \frac{v^2 \sin^2 \theta}{2g}$$

$$v = \sqrt{2gh} = \frac{h \omega}{2} \cdot 5 \pi$$

P(29)

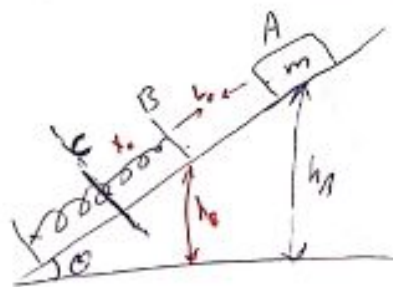
$$m = 12 \text{ kg}$$

$$v_i = 0$$

$$\theta = 30^\circ, x_0 = 5,5 \text{ m}$$

$$x = 2 \text{ cm}, F = 270 \text{ N}$$

$$l_0 + x_0 = ??$$



$$F = -Kx \Rightarrow K = \frac{F}{x}$$

$$= \frac{270}{2 \times 10^{-2}}$$

$$= 1,35 \times 10^4$$

$$\text{Since } = \frac{h_A}{l_0 + x_0}$$

$$\sin 30 = \frac{0,174}{l_0 + 5,5 \times 10^{-2}}$$

$$l_0 = 0,292 \text{ m}$$

$$E_A = E_C$$

$$K_A + U_A = K_C + U_C$$

$$0 + mgh_A = 0 + \frac{1}{2} K x^2$$

$$mgh_A = \frac{1}{2} K x^2$$

$$12(9,8)h_A = \frac{1}{2} (1,35 \times 10^4)(5,5)$$

$$h_A = 0,174 \text{ m}$$

$$E_A = E_B$$

$$\cancel{K_A} + u_A = K_B + u_B$$

$$mgh_{(A)} = \frac{1}{2} m v_B^2 + mgh_{(B)}$$

$$\Delta y = h_A - h_B$$

$$L_0 \sin \theta = \Delta y$$

$$L_0 \sin \theta = h_A - h_B$$

$$h_B =$$

ويعني موقف وشو فيه (م)

$$v_B = 1.7 \text{ m/s}$$

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Reading a potential energy curve

$\Delta u(x)$

$\Delta u(x) = -W = -f(x) \cdot \Delta x$

$f(x) = -\frac{\Delta u}{\Delta x}$

$f(x) = -\frac{du(x)}{dx}$

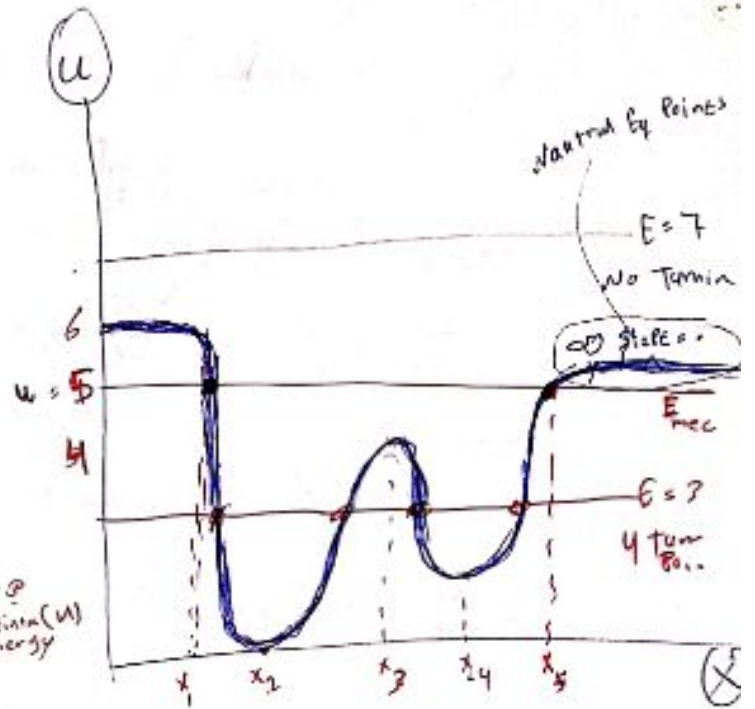
$E_{mec} = \bar{v} \bar{v}$

$E = K(x) + u(x)$

* turning point

$E = u$
 $K(x) = 0$

نقطة التوقف
عندما تكون الطاقة الحركية $K=0$



$x_1, x_5 =$ turning points
 $u = E$
 $K(x) = 0$
 $V(x) = 0$
 $V = 0$

* Equilibrium Points

نقطة التوازن
عندما تكون القوة صفر
في نقطة التوازن

$F_{net} = 0$, $f = -\frac{du}{dx}$
slope(u-x)

$-\frac{du}{dx} = 0 \Rightarrow$ [local min, local max]

if $F = 0$, slope $= 0$
 $x_1, x_3, x_4 \Rightarrow E_f$ points

Eq. Points

ثبات
نقطة التوازن

① Stable Eq Points

x_2, x_4 (stable)

② un-stable Eq Points,
 x_3 (un-stable)

Exo $F(x) = x^2 - 2x$

$\left. \begin{array}{l} \frac{dF}{dx} > 0 \text{ --- max} \\ \frac{dF}{dx} < 0 \text{ --- min} \end{array} \right\} \text{at } x_0$

نقطة التوازن
 $u(x)$

① Eq. Points ($f=0 \Rightarrow \frac{du}{dx} = 0$)

stable

unstable

② turning Points
($K=0$) ($u=0$)

$E = u(x)$; $x \in \text{turning points}$

Exp: if $u(x) = x^2 - 2x$, and $E_{mec} = 4$ Find

① Turning Points, ② Eq. Points.

① $E_{mec} = u(x)$

$4 = x^2 - 2x$

$x^2 - 2x - 4 = 0$

$x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{2 \pm \sqrt{4 - 4(1)(-4)}}{2}$

$\frac{2 \pm \sqrt{20}}{2} \Rightarrow x_1 = \frac{2 + \sqrt{20}}{2} = 3, 2m$

$x_2 = \frac{2 - \sqrt{20}}{2} = -1, 2m$

② Eq. Points

$f=0 \Rightarrow \frac{du}{dx} = 0$

$2x - 2 = 0 \Rightarrow x = 1m$



local min [Stable Points]

4
او بتجوز في
صانوج

Anan Elayan

$$f(x) = -\frac{du}{dx}$$

$$\Delta u = \int f dx = -W$$

$$\boxed{\Delta u = -W} \text{ - Isolated conservative forces.}$$

Exp $u(x,y) = x^2 y + 2x$

$$f(x) = -\frac{du}{dx} = -(2xy + 2)$$

$$\vec{F} = (-2xy + 2)\hat{i} - x^2\hat{j}$$

$$f(y) = -\frac{du}{dy} = -(x^2 + 0)$$

at $(x=1, y=2)$

$$\vec{F} = -4\hat{i} - \hat{j}$$

Anan Elayan

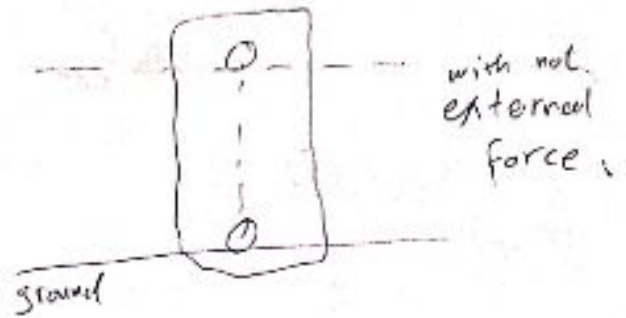
review

* Isolated system

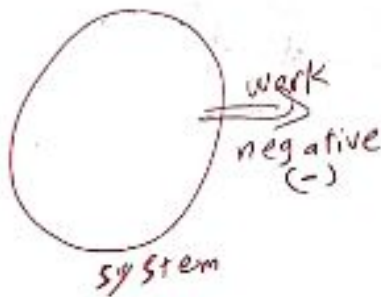
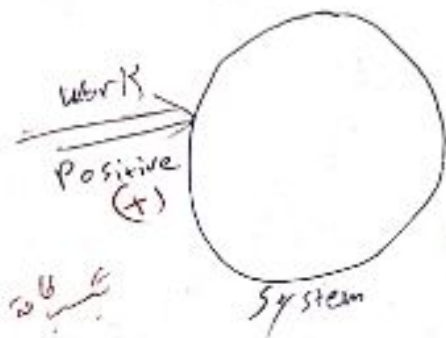
[conservative force]

$$E_{mec} = K + U \quad w.s.o$$

$$\Delta E_{mec} = \Delta K + \Delta U = 0$$

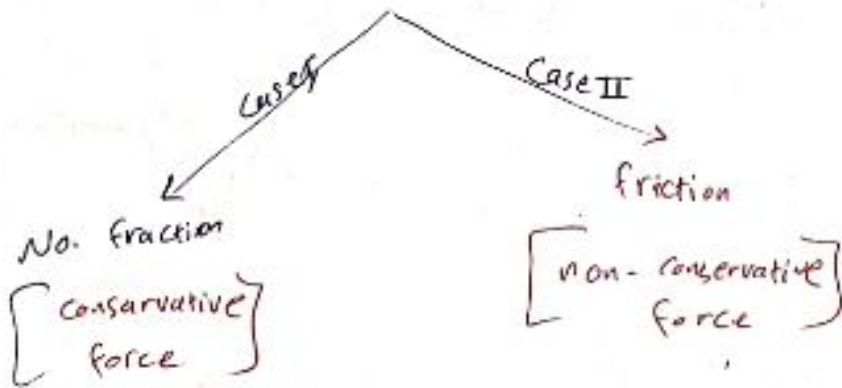


* work : is energy transferred to or from the system when external force acts on that system:

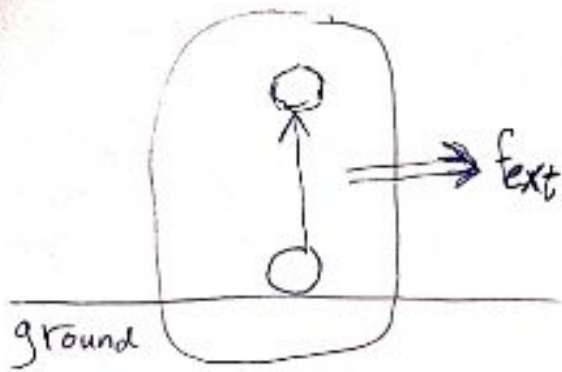


* non-isolated system

(external)



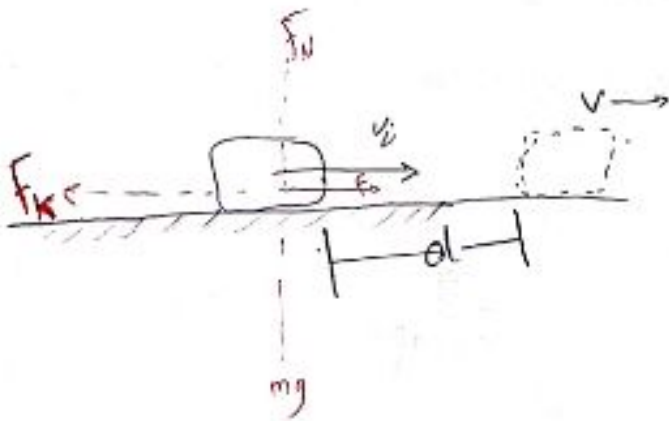
Non-Isolated system (without friction):
غير معزولة / غير معزولة



$$W = \Delta E_{mec} \neq 0$$

$$= \Delta K + \Delta U$$

Non-Isolated system with friction:



$$F_{net} = ma$$

$$F - F_k = ma$$

to find $a \rightarrow$ $\frac{v_f - v_0}{\Delta t}$

$$v^2 = v_0^2 + 2ad$$

$$2ad = v^2 - v_0^2$$

$$a = \frac{1}{2} \frac{(v_f^2 - v_0^2)}{d}$$

$$F d = \left(\frac{1}{2} m v^2 - m v_0^2 \right) + F_k d$$

$$F d = \Delta K + F_k d$$

$$\Delta E_{mec} = \Delta K$$

$$F d = \Delta E_{mec} + F_k d$$

$$W = \Delta E_{mec} + \frac{F_k d}{\text{friction}}$$

thermal energy

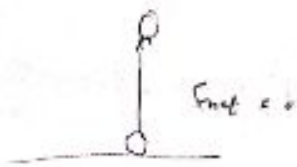
Uesib~

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Isolated system ($w = 0$)

No friction

$$\Delta E_{mec} = 0$$



friction

$$\Delta E_{mec} + \Delta E_{th} = 0$$

Non-Isolated system

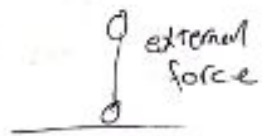
$$w \neq 0$$

$\Delta E_{th} = 0$

No friction

$$w = \Delta E_{mec}$$

$$= \Delta K + \Delta U$$



friction

$$w = \Delta E_{mec} + \Delta E_{th}$$

* Conservation of energy

$$w = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}$$

* Isolated system (friction)

$$w = 0$$

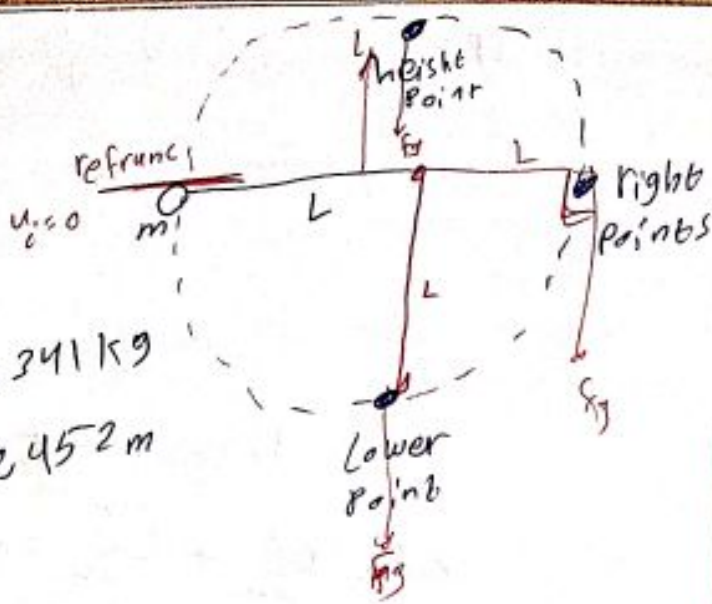
$$\Delta E_{mec} + \Delta E_{th} = 0$$

$$\Delta U + \Delta K + f_k d = 0$$

$$u_f - u_i + K_f - K_i + f_k d = 0$$

$$u_f + K_f = u_i + K_i - f_k d \rightarrow \text{friction}$$

$m = 0,341 \text{ kg}$
 $L = 0,452 \text{ m}$



a) $W_{\text{lower}} = f_g \cdot d = mgL = 1,51 \text{ J}$
 $W_H = -mgh = -1,51 \text{ J}$
 $W_A = 0$

b) $\Delta u = -W$
 $\Delta u_L = -1,51 \text{ J}$
 $\Delta u_A = 0$
 $\Delta u_H = 1,51 \text{ J}$

$K_i + u_i = K_f + u_f$
 $K_i = \frac{1}{2}(0,9)(7)^2 = 22,1 \text{ J}$
 $22,1 + 15 = \frac{1}{2}(0,9)(v_f^2) + 35$
 $v_f^2 = \frac{(37,1 - 35) \times 2}{0,9}$

$v_f = 2,1 \text{ m/s}$

b) $f_x = -\frac{\Delta u}{\Delta x} = -\text{slope}$

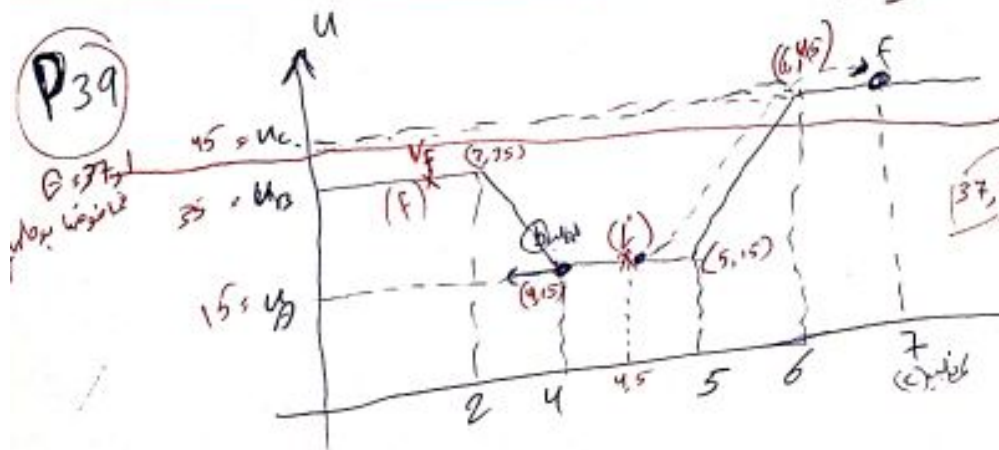
from $x=0$ to $x=2$ $F=0$

$f_x = -\left(\frac{15 - 35}{4 - 2}\right) = 10 \text{ N}$

c) $K_i + u_i = K_f + u_f$
 $37,1 = \frac{1}{2}(0,9)(v_f^2) + 45$
 $v_f^2 = \frac{(37,1 - 45) \times 2}{0,9} < 0$

~~v_f~~
 ليس له نقطة عطف

P39



turning points ($v=0, K=0$)

$E = K + u$
 $E = u(x)$
 $= 15 + 30(x-5)$
 $37,1 = 15 + 30(x-5)$

37,1

$x = 5,7 \text{ m}$

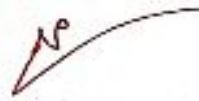
$u_A = 15 \text{ J}$
 $u_B = 35 \text{ J}$
 $u_C = 45 \text{ J}$

$m = 0,9 \text{ kg}$
 at $x = 4,5 \Rightarrow v_i = 7 \text{ m/s}$

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Isolated system ($F_{ext} = 0$)

$$W_{ext} = 0$$



No-friction

$$W = \Delta E_{mec} = 0$$

$$K_i + U_i = K_f + U_f$$

friction $f_k \cdot d$

$$W = \Delta E_{mec} + \Delta E_{th} = 0$$

الميكانيكية

$$K_i + U_i = K_f + U_f + f_k d$$

Non Isolated ($F_{ext} \neq 0$)

$$W_{ext} \neq 0$$

No friction

$$W = \Delta E_{mec}$$

$$= \Delta K + \Delta U$$

friction

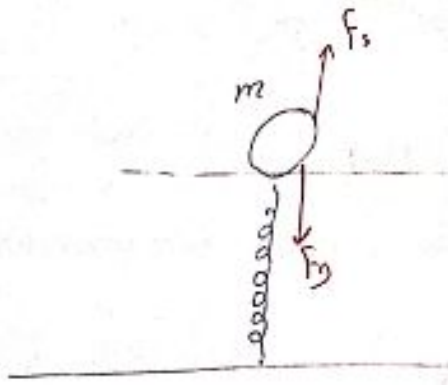
$$W = \Delta E_{mec} + \Delta E_{th}$$

$$= \Delta K + \Delta U + f_k d$$

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P19

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$m = 8 \text{ Kg}$,

$v_i = 0$ (at rest at 0)

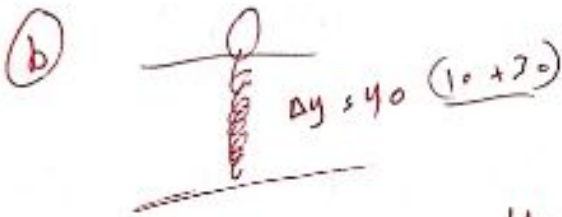
$\Delta y = 10 \text{ cm}$

a) $K = ?$

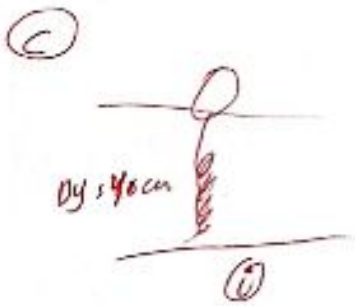
$F_s - F_g = 0$

$K \Delta y = mg$

$K = \frac{mg}{\Delta y} = 784 \text{ N/m}$



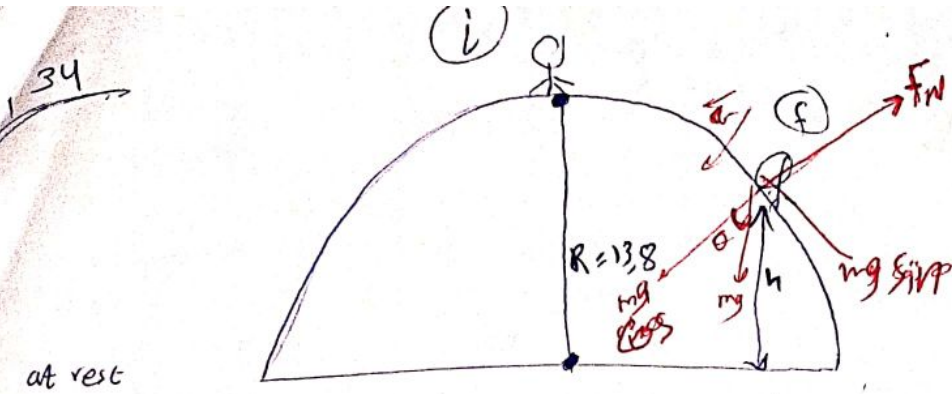
$U_s = \frac{1}{2} K \Delta y^2 = 62.7 \text{ J}$



$K \Delta y_i + u_i = K \Delta y_f + u_f$

$62.7 = mgh$

$h = \frac{62.7}{mg} = 9.8 \text{ m}$



$$K_i + U_i = K_f + U_f$$

$$0 + mgR = \frac{1}{2}mv_f^2 + mg h \quad ??$$

$$R = \frac{1}{2}v_f^2 / g + h$$

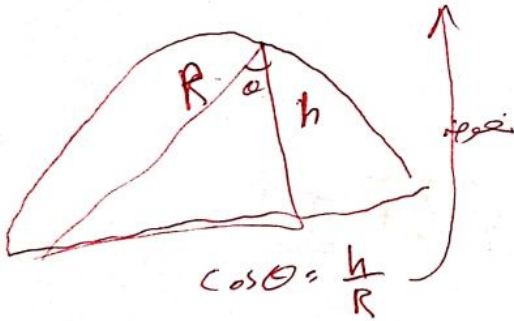
$$R = \frac{1}{2}R \cos \theta + h$$

or $\frac{v^2}{R}$ (f=0) بعد سقوط

$$mg \cos \theta - F_N = \frac{mv^2}{R}$$

$$mg \cos \theta = \frac{mv^2}{R}$$

$$v^2 = Rg \cos \theta$$

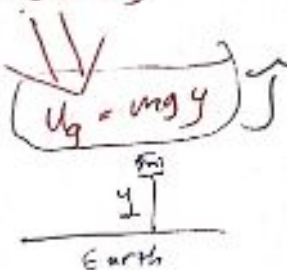


$$h = 9,2 \text{ m}$$

Anan Elayan

Potential Energy & Conservation of Energy

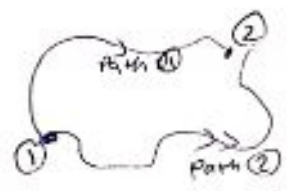
Forces in Nature :-

<p>① Gravitational force = mg</p> <p>② Spring force = $-kx$</p>	<p>قوة الجاذبية</p> <p>Conservative forces</p>	<p>U Potential Energy</p> <p>طاقة الوضع</p>
<p>③ Normal force = $F_n = N$</p> <p>④ Friction force $f_s = \mu_s N$ $f_k = \mu_k N$</p> <p>⑤ Air Drag force = $\frac{1}{2} C_D \rho A v^2$</p>	<p>Non conservative force</p>	<p>$U_s = \frac{1}{2} kx^2$ Joule</p> <p>$U_g = mgy$</p> 

Properties of conservative forces :-

① Work done by conservative is Path independent:

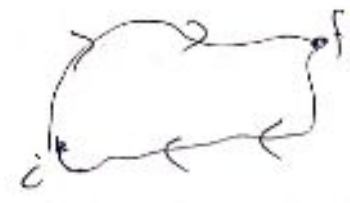
$$W_{con1} = W_{con2}$$



توجد القوة المحافضة لا تعتمد على المسار

② Work done by $\oint_{con} F = 0$

$W_{cons} = 0$ around a closed path



③ Work done against $F_{conservative}$ do not ~~lost~~ but it stored as Energy called Potential Energy.

العمل المبذور ضد القوة المحافضة لا يضيع بل يتم تخزينه في شكل طاقة محافضة

$$W_{app} = \Delta U = U_f - U_i$$

④ W done by $F_{\text{cons}} = -\Delta U$

$$W_{\text{cons}} = -\Delta U$$

$$i \rightarrow f = -[U_f - U_i]$$

Gravitational Potential Energy.

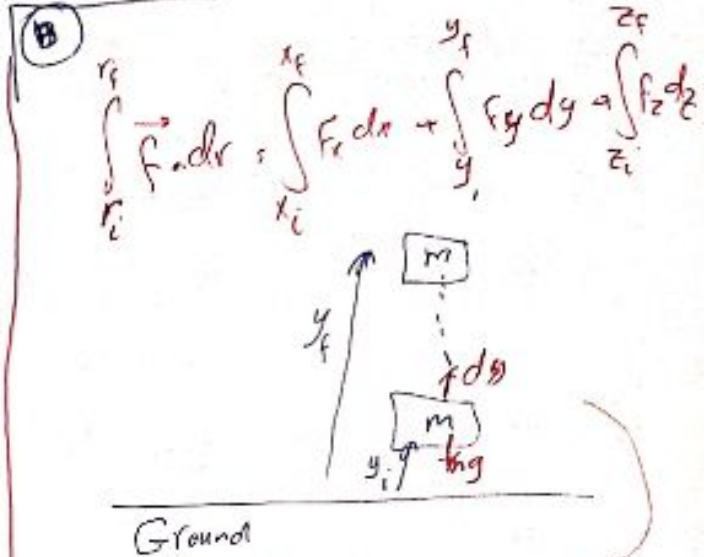
$$W_{\text{cons}} = -\Delta U = -[U_f - U_i]$$

$$\int_{\text{cons}} \vec{F} \cdot d\vec{r} = -\Delta U$$

تغير الطاقة (الكامنة)
تعمل القوة المتماثلة نقطة

$$\Delta U = - \int_{\text{cons}} \vec{F} \cdot d\vec{r}$$

$$U_f - U_i = - \int_{r_i}^{r_f} \vec{F}_{\text{cons}} \cdot d\vec{r}$$



$$U_f - U_i = - \int_{r_i}^{r_f} \vec{F}_g \cdot d\vec{r}$$

$$= - \int_{y_i}^{y_f} (-mg dy)$$

$$= mg \int_{y_i}^{y_f} dy$$

$$U_f - U_i = mgy_f - mgy_i$$

let $y_i = 0, U_i = 0$

$$U_g = mgy$$

Joules

y is above zero level

⑥ Spring Potential Energy is

$$U_f - U_i = - \int_{r_i}^{r_f} \vec{F}_s \cdot d\vec{r}$$

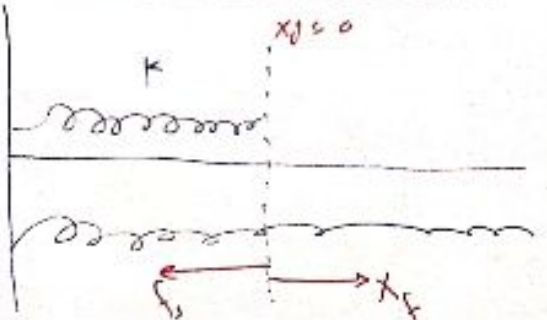
$$U_f - U_i = - \int_{x_i}^{x_f} (-Kx) dx = K \int_{x_i}^{x_f} x dx$$

$$U_f - U_i = \frac{1}{2} K x_f^2 - \frac{1}{2} K x_i^2$$

let $x_i = 0 \Rightarrow U_i = 0$

$$U_s = \frac{1}{2} K x^2$$

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$$E_{\text{mechanical}} = \text{Kinetic Energy} + \text{Potential Energy}$$

$$E_m = \frac{1}{2}mv^2 + U \quad \text{Anan Elayan}$$

★ Conservation of Mechanical Energy :-
 قانون حفظ الطاقة الميكانيكية

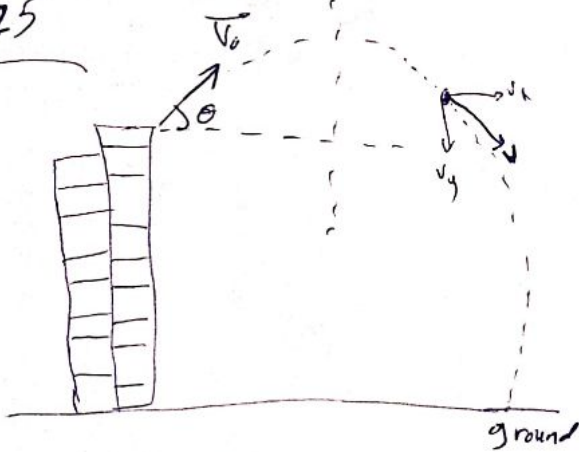
If the only force Acting on the system is conservative force.

$$\begin{aligned} W_{\text{Cons}} &= -\Delta U \\ W_{\text{net}} &= \Delta K \end{aligned}$$

$$\begin{aligned} \Delta K &= -\Delta U \\ \Delta K + \Delta U &= 0 \end{aligned}$$

$$\begin{aligned} \Delta(K+U) &= 0 \quad \text{قانون حفظ الطاقة الميكانيكية} \\ K+U &= \text{constant} \\ (K+U)_i &= (K+U)_f \end{aligned}$$

Q25



at 6s

$$\begin{aligned} v_x &= 18 \text{ m/s} \\ v_y &= v_{0y} + a_y t \\ &= 24 + (-10)(6) \\ &= 24 - 60 \\ &= -36 \text{ m/s} \end{aligned}$$

$$v_6 = 18\hat{i} - 36\hat{j} \text{ m/s}$$

$m = 1 \text{ kg}$ At $t=0$, $\vec{v}_0 = 18\hat{i} + 24\hat{j} \text{ m/s}$
 After 6s find ΔU ?

$$\begin{aligned} (K+U)_i &= (K+U)_f = K_0 + U_0 = K_6 + U_6 \\ &= K_0 - K_6 = U_6 - U_0 \implies U_6 - U_0 = 450 - 768 = -318 \text{ J} \end{aligned}$$

$$\begin{aligned} K_0 &= \frac{1}{2}mv_0^2 \\ &= \frac{1}{2}(1)(18^2 + 24^2) = 450 \text{ J} \\ K_6 &= \frac{1}{2}mv_6^2 \\ &= \frac{1}{2}(1)(18^2 + (-36)^2) = 768 \text{ J} \end{aligned}$$

العمل المبذوب $W_{\text{nc}} = -\Delta U$
 $0 \rightarrow 6 = 318 \text{ J}$

Chapter (8) Lecture (2)

F Conservation

$$\rightarrow mg = U_g = mgy \quad (y)$$

$$-kx \Rightarrow U_s = \frac{1}{2} kx^2 \quad (y)$$

$$E_{\text{initial}} = E_{\text{Final}}$$

$$\frac{1}{2} m v_i^2 + U_i = \frac{1}{2} m v_f^2 + U_f$$

F is conservative

$$W_{\text{conservative}} = -\Delta U$$

* Finding F_{cons} From U :-

$$W_{\text{con}} = -\Delta U$$

$$dU = -F_x dx$$

$$F_x = -\frac{dU}{dx}$$

a) $W_g (P \rightarrow Q)$

$$W_g (P \rightarrow Q) = -\Delta U$$

$$= -[U_Q - U_P]$$

$$= -[mgR - mg(5R)]$$

$$= +4mgR = \boxed{0.150 \text{ J}}$$

Problem 8, 6

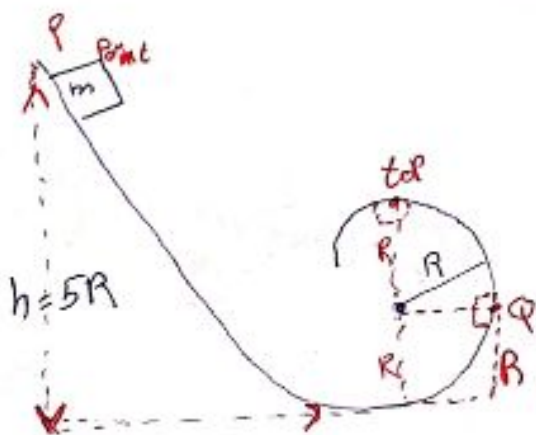
b) $W_g (P \rightarrow Q) = -\Delta U$

$$= -[U_{top} - U_P]$$

$$= -[mg(2R) - mg(5R)]$$

$$= +3mgR = \boxed{0.113 \text{ J}}$$

$m = 0.32 \text{ kg}$
 $R = 12 \text{ cm}$



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Problem 8-17

مسألة 8-17

(a) At Q find \vec{F}_N ? \vec{F}_V ?

$$F_y = -mg$$

$$K = \frac{1}{2} m v_p^2 + U_p = \frac{1}{2} m v_q^2 + U_q$$

$$0 + 5mgR = \frac{1}{2} m v_q^2 + mgR$$

$$\frac{1}{2} v_q^2 = 4Rg \Rightarrow v_q = \sqrt{8Rg}$$

$$F_N = \frac{m v_q^2}{R} = \frac{m(8Rg)}{R} = 8mg \text{ () (-i)}$$

$$\vec{F}_Q = -8mg \hat{i} - mg \hat{j}$$

find the normal force acting on (m) at the top point at the top

$$F_N + mg = \frac{m v_t^2}{R}$$

$$(K + U)_p = (K + U)_t$$

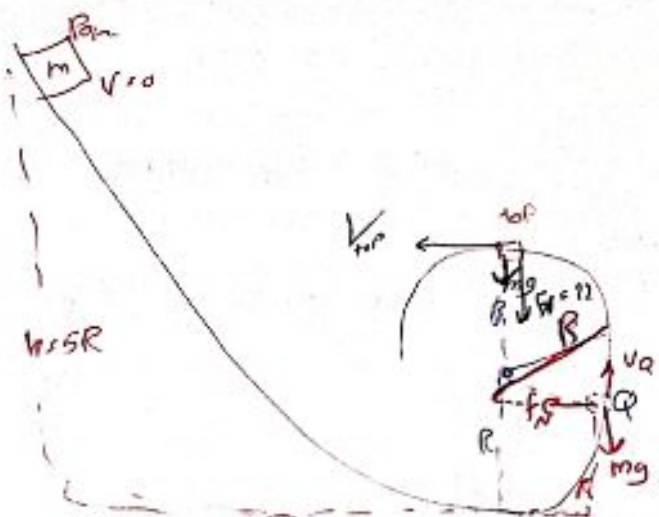
$$(0 + 5mgR) = \frac{1}{2} m v_t^2 + mg(2R) \Rightarrow v_t = \sqrt{6Rg}$$

at the top

$$F_N = \frac{m v_t^2}{R} - mg$$

$$F_N = \frac{m}{R} 6Rg - mg \Rightarrow F_N = 5mg \text{ at the top}$$

$$F_N = -5mg \hat{j}$$

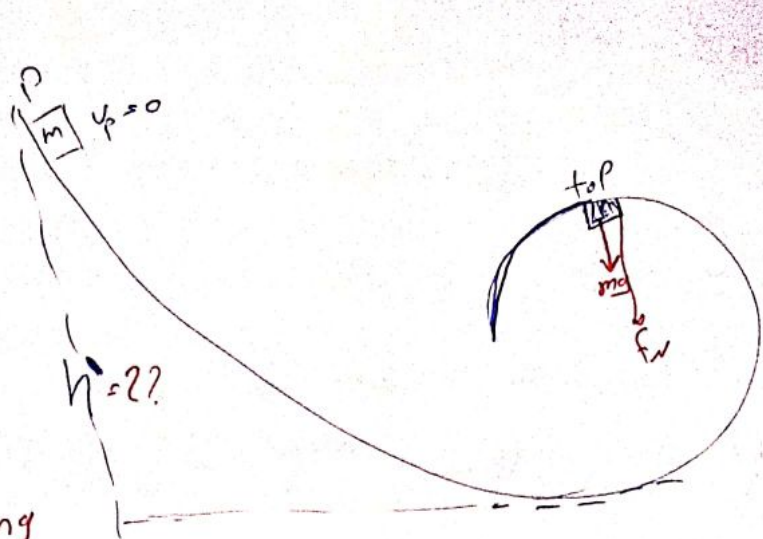


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© find h ??

At the top

$$\frac{mv^2}{R} = mg + F_N$$



on the verge of losing

contact at the top

means $F_N \rightarrow 0$

$$\frac{mv_t^2}{R} = mg \Rightarrow v_t = \sqrt{Rg}$$

$$K_p + U_p = K_t + U_t$$

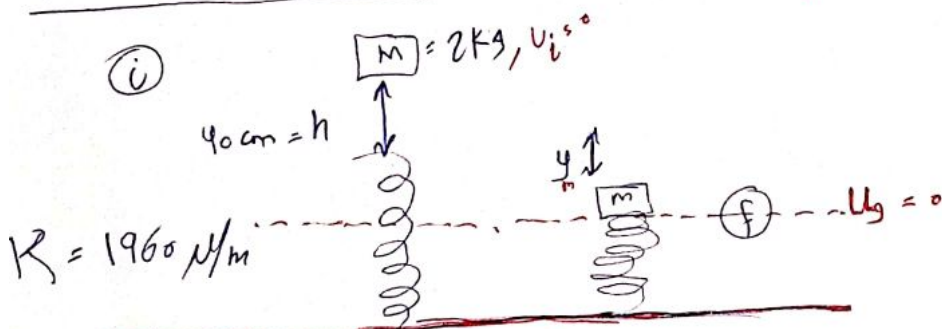
$$0 + mgh' = \frac{1}{2}mv_t^2 + 2mgR$$

$$mgh' = \frac{1}{2}m(Rg) + 2mgR$$

$$mgh' = 2.5mgR \Rightarrow h' = 2.5R$$

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Problem 8-24



$$(K + U)_i = (K + U)_f$$

$$(0 + mg(h + y_m)) = (0 + \frac{1}{2}ky_m^2)$$

$$2(9.8)[0.4 + y_m] = \frac{1}{2}(1960)y_m^2 = \square$$

Problem 8 - 104

$$m = 20 \text{ kg}, \quad F_{\text{con}} = -3x - 5x^2$$

$$\text{at } x=0, \quad U_0 = 0$$

a) Find U at $x = 2 \text{ m}$?

$$W_{\text{con}} = -\Delta U \implies \Delta U = - \int_i^f F_{\text{con}} dx$$

$$U_f - U_i = - \int (-3x - 5x^2) dx$$

$$U = \frac{3}{2}x^2 + \frac{5}{3}x^3 + C$$

$$0 = 0 + 0 + C$$

$$\boxed{C = 0}$$

$$U = \frac{3}{2}x^2 + \frac{5}{3}x^3$$

$$u(2) = \frac{3}{2}(2)^2 + \frac{5}{3}(2)^3 = 19,6 \text{ J}$$

b) At $x = 5 \text{ m}$, $U_x = -4 \text{ m/s}$

find V_x at $x = 0$??

$$(K+U)_{x=0} = (K+U)_{x=5}$$

$$\frac{1}{2}mV_0^2 + 0 = \frac{1}{2}m(-4)^2 + \left[\frac{3}{2}(5)^2 + \frac{5}{3}(5)^3 \right]$$

$$V_0 = \sqrt{\quad}$$
$$V_0 = -6,37 \text{ m/s}$$

c) Repeat a & b for $U = -8x^2$ at $x = 0$

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Problem $m = 0,2 \text{ kg}$, $U(x) = 8x^2 + 2x^4$ Joule
at $x = 1 \text{ m}$, $v = 5 \text{ m/s}$

Find v at the origin?

$$(K + U)_1 = (K + U)_0$$

$$\frac{1}{2}(0,2)(5)^2 + (8(1) + 2(1)) = \left(\frac{1}{2}(0,2)v_0^2 + 0\right)$$

b) Find F_{cons} ?

$$F_{\text{cons}} = -\frac{dU}{dx} = -[6x + 8x^3]$$

$$F_{\text{cons}} = -16x - 8x^3$$

End ch8

Good Luck

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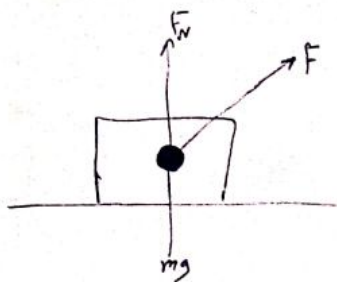
Chapter 9

Center of mass, linear momentum:
 مركز الكتلة كمية التمرار الخطية (الزخم)

* center of mass: is the point that moves as though, all of the system's mass concentrated there, and

all external force applied there.

نقطة مركز الكتلة
 - جميع القوة
 - على النقطة
 - كأنها متركزة في تلك النقطة



Two cases → ^{مجموعة من الاجسام} system of Particles
 → Rigid (solid) bodies.

Case I \circ system of Particles

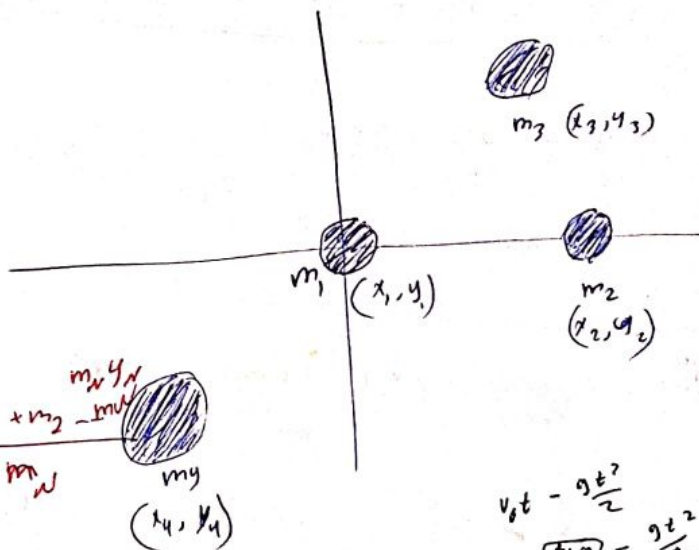
$$X_{Com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4}{m_1 + m_2 + m_3 + m_4}$$

$$\Rightarrow X_{Com} = \frac{1}{M} \sum_{i=1}^N m_i x_i$$

total mass

$$\Rightarrow Y_{Com} = \frac{1}{M} \sum_{i=1}^N m_i y_i = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4}{m_1 + m_2 + m_3 + m_4}$$

$$\Rightarrow Z_{Com} = \frac{1}{M} \sum_{i=1}^N m_i z_i$$



$$v_{gt} - \frac{gt^2}{2}$$

$$25 + \boxed{+0} - \frac{9t^2}{2}$$

(13) $v = gt$

$$\vec{r}_{Com} = x_{Com} \hat{i} + y_{Com} \hat{j} + z_{Com} \hat{k}$$

$$\vec{r}_{Com} = \frac{1}{M} \sum_{i=1}^N m_i \vec{r}_i$$

$P_1 \quad m_1 = 2 \text{ kg} \rightarrow (-1, 5) \rightarrow r_1 = -\hat{i} + 5\hat{j}$

$m_2 = 4 \text{ kg} \rightarrow (6, -7, 5)$

$m_3 = 3 \text{ kg} \rightarrow ?? (x_3, y_3)$

$r_{\text{com}} = (-0,5, -0,7)$
 $\downarrow \quad \downarrow$
 $x_{\text{com}} \quad y_{\text{com}}$

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$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$

$\frac{-0,5}{1} = \frac{-2 + 24 + 3x_3}{2 + 4 + 3} \Rightarrow x_3 = -1,5 \text{ m}$
 $\frac{-4,5}{-2,2} = \frac{22 + 3x_3}{-2,2}$

$y_{\text{com}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{9}$

$-0,7 = \frac{10 + 30 + 3y_3}{9} \Rightarrow y_3 = -1,43 \text{ m}$

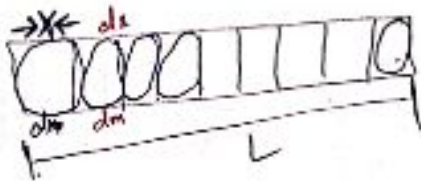
$\frac{-6,3}{-40} = \frac{40 + 3y_3}{-40}$
 $\frac{-46,3}{3} = \frac{3y_3}{3}$
 $y_3 = -15,43$

OB
 $r_{\text{com}} = \frac{m_1 r_1 + m_2 r_2 + m_3 r_3}{M}$

$-0,5\hat{i} + 0,7\hat{j} = \frac{2(-\hat{i} + 5\hat{j}) + 4(6\hat{i} - 7,5\hat{j}) + 3(x_3\hat{i} + y_3\hat{j})}{9}$

Case II: Solid bodies (Rigid)

$$X_{com} = \frac{1}{M} \int x \, dm, \quad Y_{com} = \frac{1}{M} \int y \, dm, \quad Z_{com} = \frac{1}{M} \int z \, dm$$



Uniform λ $\Rightarrow \lambda = \text{constant}$
 إذا كان الكثافة λ متساوية في كل مكان، فإن $\frac{dm}{dx} = \lambda$

$$\frac{dm}{dx} = \frac{M}{L} = \lambda$$

Uniform linear density

تساوي الكثافة في كل مكان

$$\frac{dm}{dx} = \frac{M}{L} \Rightarrow dm = \frac{m}{L} dx$$

$$dm = \lambda dx$$

1-D
تساوي الكثافة في كل مكان

$$\left. \begin{aligned} X_{com} &= \frac{1}{M} \int x \, dm \\ Y_{com} &= \frac{1}{M} \int y \, dm \\ Z_{com} &= \frac{1}{M} \int z \, dm \end{aligned} \right\} \Rightarrow r_{com} = \frac{1}{M} \int \vec{r} \, dm$$

$$dm = \lambda dx, \quad \lambda = \text{uniform}, \quad \lambda = \frac{M}{L}, \quad \lambda = \text{linear density}$$

2-D
الكثافة

$$dm = \sigma \, dA, \quad \sigma = \text{surface density}$$

$$\sigma = \frac{M}{A}$$

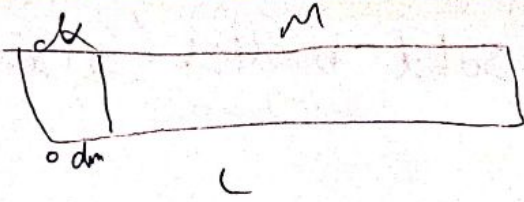
الكثافة السطحية

3-D

$$dm = \rho \, dV, \quad \rho = \text{Volume density} = \frac{M}{V_{\text{Volume}}}$$

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$$x_{com} = \frac{1}{M} \int x dm$$



$$dm = \lambda dx = \frac{M}{L} dx$$

$$\Rightarrow x_{com} = \frac{1}{M} \int_0^L x \left(\frac{M}{L}\right) dx = \frac{1}{L} \int_0^L x dx$$

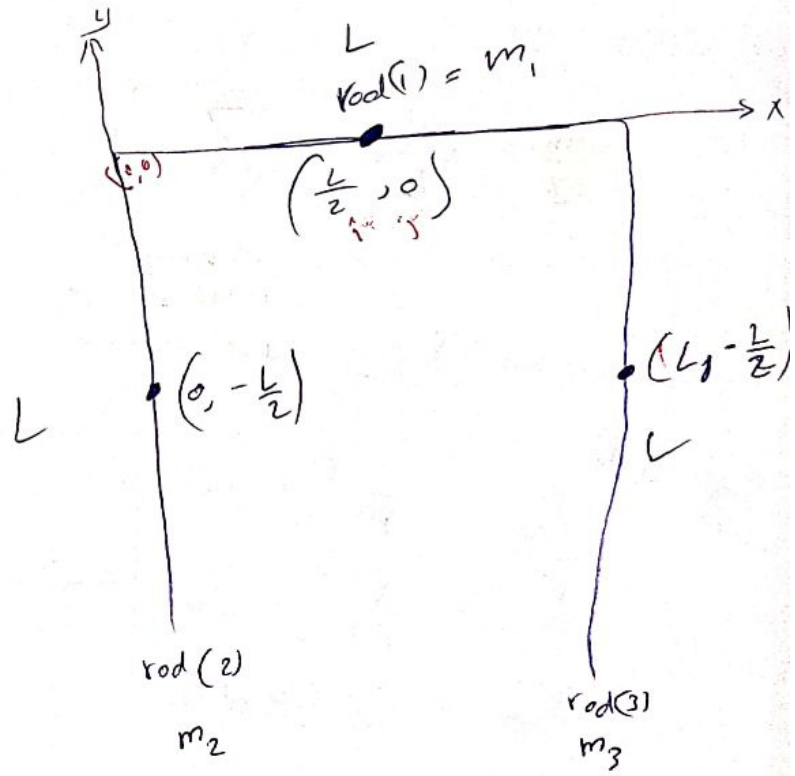
rod is
(uniform rod)

$$= \frac{1}{L} \left[\frac{x^2}{2} \right]_0^L = \frac{1}{L} \cdot \frac{L^2}{2} = \frac{L}{2}$$

L = locus

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$y_{com} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$



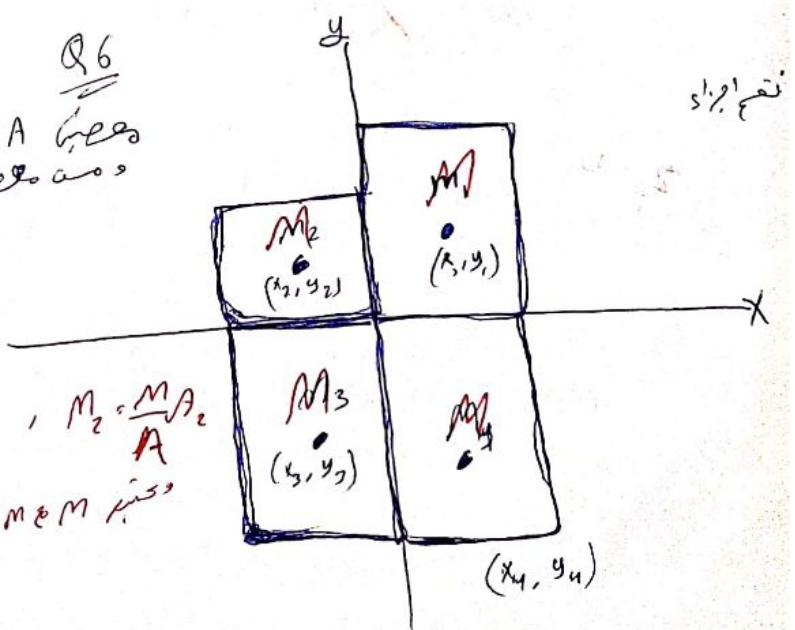
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Q6
A Area
mass

$$\frac{M_1}{A_1} = \frac{M}{A}$$

$$M_1 = \frac{M}{A} A_1, \quad M_2 = \frac{M}{A} A_2$$

M & M mass



تقسيم المساحة

Newton's 2nd Law for system of particles.

$$\vec{F}_{net} = M \vec{a}_{com}$$

$$\vec{r}_{com} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_n \vec{r}_n}{M}$$

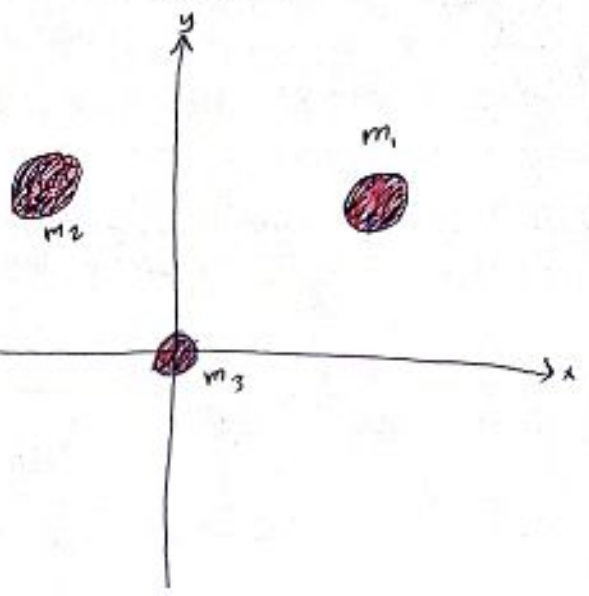
$$M \vec{r}_{com} = m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_n \vec{r}_n$$

$$M \vec{v}_{com} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_n \vec{v}_n$$

$$M \vec{a}_{com} = m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots + m_n \vec{a}_n$$

$$M \vec{a}_{com} = \vec{F}_{net}$$

system



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Chapter 9, 2

Center of mass

System of Particles

$$x_{com} = \frac{1}{M} \sum_{i=1}^n m_i x_i$$

$$y_{com} = \frac{1}{M} \sum_{i=1}^n m_i y_i$$

$$z_{com} = \frac{1}{M} \sum_{i=1}^n m_i z_i$$

$$\vec{r}_{com} = \frac{1}{M} \sum_{i=1}^n m_i \vec{r}_i$$

Rigid bodies

$$x_{com} = \frac{1}{M} \int x \, dm$$

$$dm = \lambda \, dx$$

$$\lambda = \frac{M}{L}$$

$$\rho = \frac{M}{V}$$

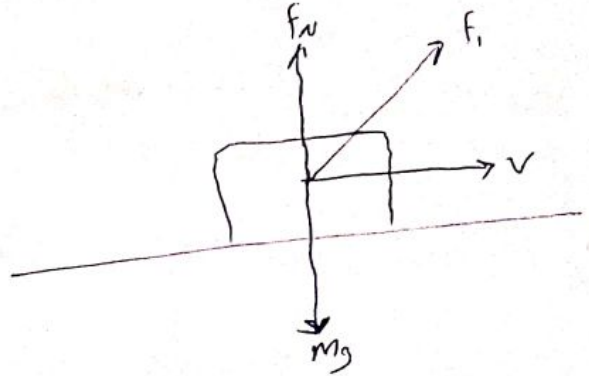
* linear momentum كمية التمرک (\vec{P})

* $\vec{P} = m \vec{v}$, $m = \text{mass}$
 $v = \text{velocity}$

* P in the same direction of \vec{v} .
 بنفس اتجاه (v) يكون P .

$$\frac{dP}{dt} = F_{\text{net}}$$

قوة القوة



* Newton's 2nd law

$$F_{\text{net}} = \frac{dP}{dt} \quad , \quad \vec{P} = m\vec{v}$$

$$F_{\text{net}} = m\vec{a}$$

$m = \text{constant}$

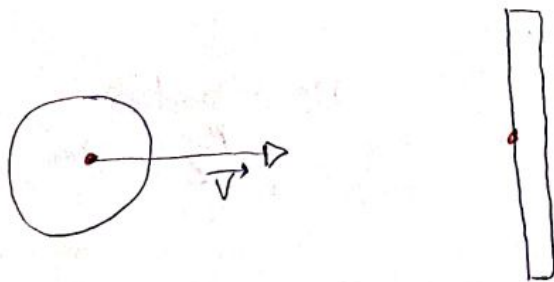
$$= \frac{d}{dt} [mv] = m \frac{dv}{dt} + v \frac{dm}{dt}$$

if $m = \text{constant}$ $\Rightarrow \frac{dm}{dt} = 0$

$$F_{\text{net}} = m \frac{dv}{dt} = \boxed{m\vec{a}}$$

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* Collision and Impulse ^{الاصطدام}



* Impulse (\vec{J})

$$\vec{J} = \vec{F}_{\text{Avg}} \cdot \Delta t$$

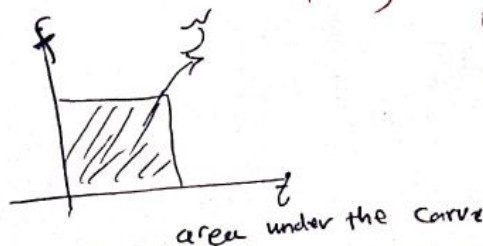
, \vec{F} constant \rightarrow

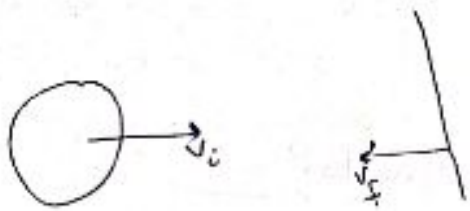
المنطقة

$$\vec{J} = \int_{t_1}^{t_2} f(t) dt$$

(f variable) متغيرة

depend on time يعتمد على الزمن





$$J = \Delta P$$

$$J = m \Delta V$$

$$= m (\vec{v}_f - \vec{v}_c)$$

$$F_{\text{max}} = \frac{dP}{dt}$$

$$F_{\text{avg}} = \frac{\Delta P}{\Delta t}$$

$$\vec{J} = F_{\text{avg}} \cdot \Delta t = \Delta P = m(v_f - v_c), \quad F \text{ constant}$$

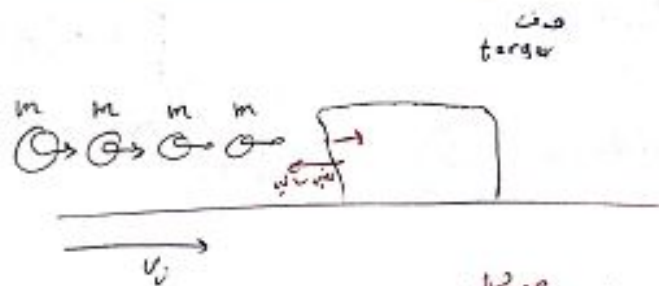
$$J = \int F dt = \Delta P, \quad \text{not constant.}$$

Special case

$$\vec{J} = -n \Delta \vec{P}$$

$n = \# \text{ of particles}$

$\Delta P = \text{change in linear momentum of 1. Particle.}$



تاراج
السرعة
والزخم

$$* J = -n \Delta P = n m \Delta V$$

$$F_{\text{avg}} \Delta t = -n m \Delta V$$

$$F_{\text{avg}} = \frac{-n m \Delta V}{\Delta t}$$

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حفظ كمية الزخم
 Conservation of linear momentum

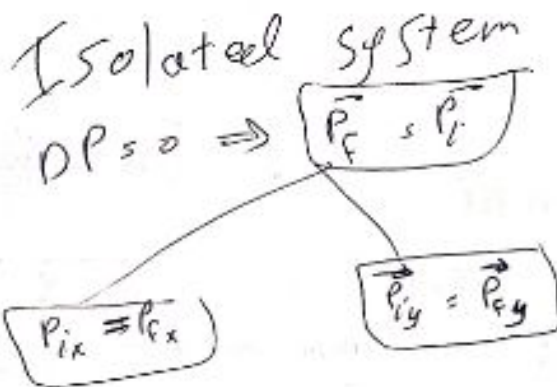
From Newton's 2nd law

$F_{net} = \frac{dP}{dt}$ \therefore P is conserved if $\Delta P = 0$

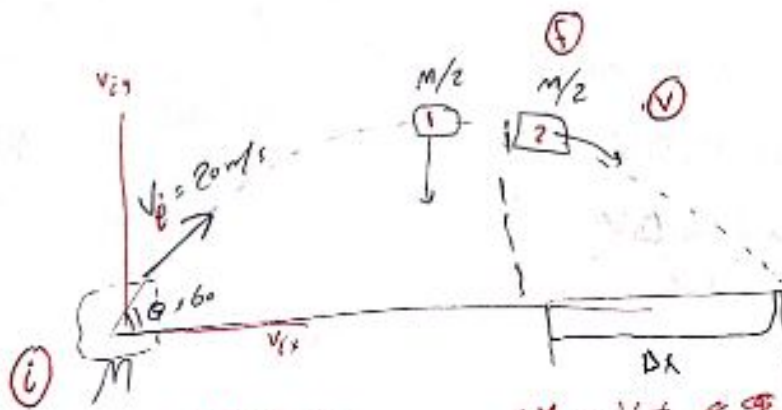
\vec{P} is conserved ($\Delta \vec{P} = 0$), if the system is isolated and closed

$\Delta P = 0$
 $\vec{P}_f - \vec{P}_i = 0$
 $\vec{P}_f = \vec{P}_i$

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P13



$P_{ix} = P_{f,x(c)} + P_{f,x(p)}$
 $M v_x = M v + \frac{M}{2} v$

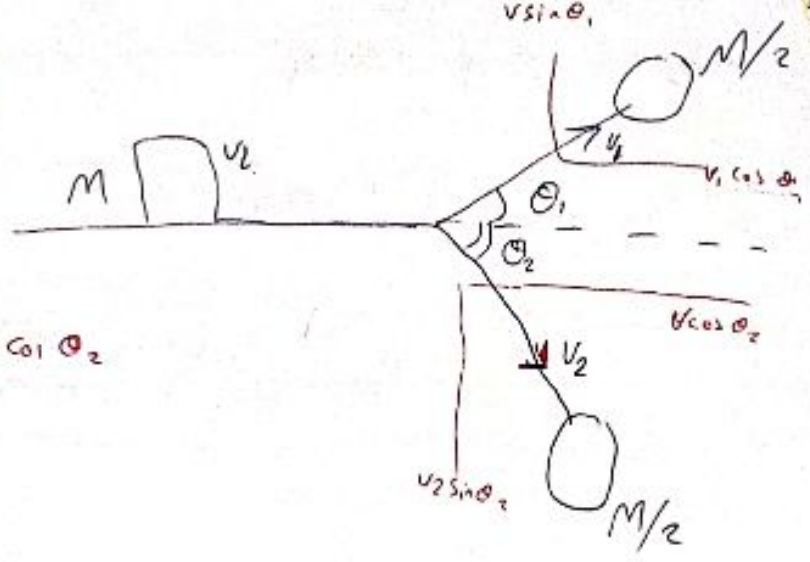
$v = 2 v_i \cos \theta$
 $= 2(2) \cos 60 = 20 \text{ m/s}$

$\Delta x = v_i t$

$v_f^2 = v_{iy}^2 + 2a \Delta y$
 $0 = v_0^2 \sin^2 \theta + 2g \Delta y$
 $\Delta y = \frac{-v_0^2 \sin^2 \theta}{2g} = 15.3$

$\Delta y = v_{iy} t + \frac{1}{2} g t^2$
 $15.3 = \frac{1}{2} g t^2 \Rightarrow t = 1.75 \text{ s}$

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$$\vec{P}_{ix}, \vec{P}_{if}$$

$$M v_i = \frac{M}{2} v_1 \cos \theta_1 + \frac{M}{2} v_2 \cos \theta_2$$

$$\vec{P}_{iy} = \vec{P}_{if}$$

$$0 = \frac{M}{2} v_1 \sin \theta_1 - \frac{M}{2} v_2 \sin \theta_2$$

chapter 9, 3

momentum and kinetic energy in collision

النوع التصادمات
* type of collisions.

① elastic collision • $\vec{P}_i = \vec{P}_f$ ($\Delta P = 0$) ($\Delta K = 0$)

② inelastic collision ($\Delta K \neq 0$)

③ completely inelastic collision. ($\Delta K \neq 0$)

1 Elastic collision

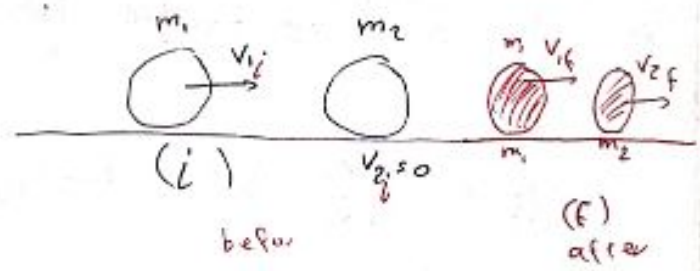
$$\vec{P}_i = \vec{P}_f$$

$$K_i = K_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \rightarrow \text{①}$$

$$\frac{1}{2} m_1 v_{1i}^2 + 0 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \rightarrow \text{②}$$



before

after

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{--- (1)}$$

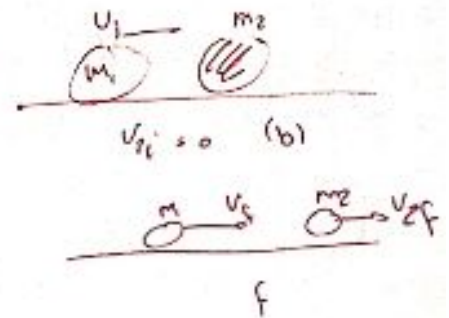
since v_{2f} from eqn (1)
substitute in eqn (2)

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad \text{--- (2)}$$

at $t = 0$, $v_{2i} = 0$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$



$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

از جهت m_2 تکرار

thrust (F) = mass rate of fuel ejection velocity
 $F = \dot{m} v_e$

if $m_1 = m_2 \Rightarrow v_{1f} = 0$

$\Rightarrow v_{2f} = \frac{2m}{m_1 + m_2} v_{1i}$

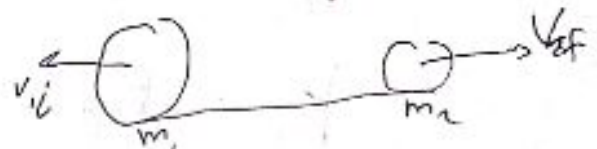
$$v_{2f} = v_{1i}$$

در این حالت که $m_1 = m_2$ پس از برخورد هر دو جرم با سرعت v_{1i} حرکت می کنند

(2) if massive target ($m_2 \gg m_1$) neglect $\frac{m_1}{m_2}$

$$v_{1f} = \frac{-m_2}{m_2} v_{1i} = -v_{1i}$$

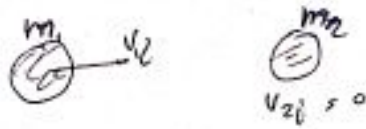
$$v_{2f} = \frac{2m_1}{m_2} v_{1i}$$



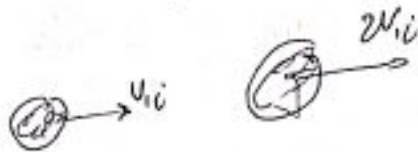
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5) massive projectile ($m_1 \gg m_2$)

$$v_{1f} = v_{1i}$$



$$v_{2f} = \frac{2m_1 v_{1i}}{m_1} = 2v_{1i}$$



حالت -

* target at rest ($v_{2i} = 0$)

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

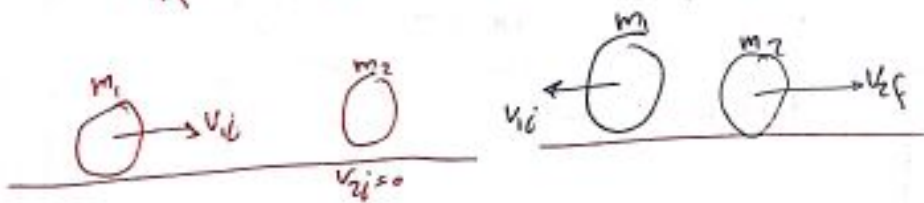
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Case 1) $m_1 = m_2$ ($v_{1f} = 0$, $v_{2f} = v_{1i}$)



Case 2) massive target ($m_2 \gg m_1$)

$$v_{1f} = -v_{1i}, \quad v_{2f} \approx \frac{2m_1}{m_2} v_{1i}$$



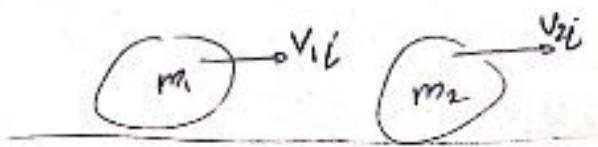
Case 3) massive projectile ($m_1 \gg m_2$)

$$v_{1f} \approx v_{1i}, \quad v_{2f} \approx 2v_{1i}$$

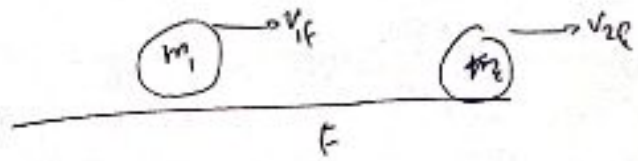


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المصادم المتحرك
 * moving target



(i)



$$* \vec{P}_i = \vec{P}_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$* K_i = K_f$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

moving target

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

* Elastic collision

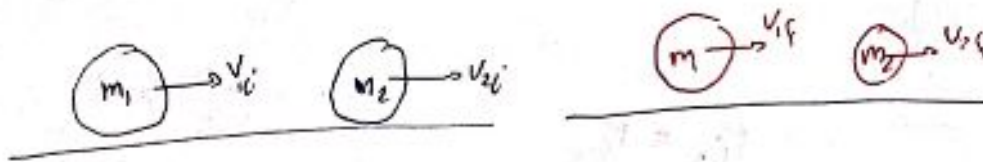
- └─ target at rest
- └─ moving target

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2) Inelastic collision

$$\vec{P}_i = \vec{P}_f, \quad K_i \neq K_f \Rightarrow K_f \neq K_i + E_{\text{dissip}}$$

$$\Delta K = K_i - K_f$$

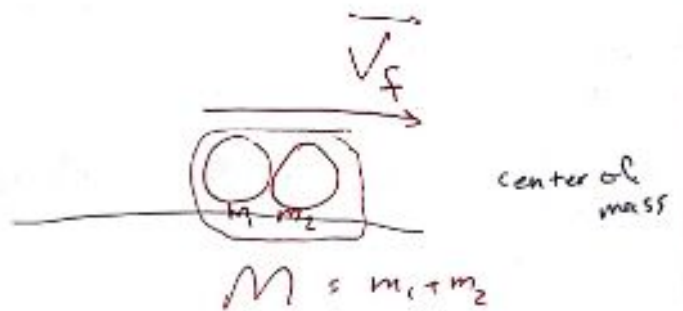
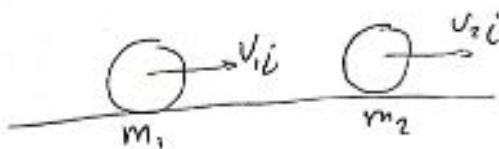


$$P_i = P_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\Delta E = \left(\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 \right) - \left(\frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \right)$$

3) Completely inelastic collision



$$\vec{P}_i = \vec{P}_f \Rightarrow m_1 v_{1i} + m_2 v_{2i} = M v_f$$

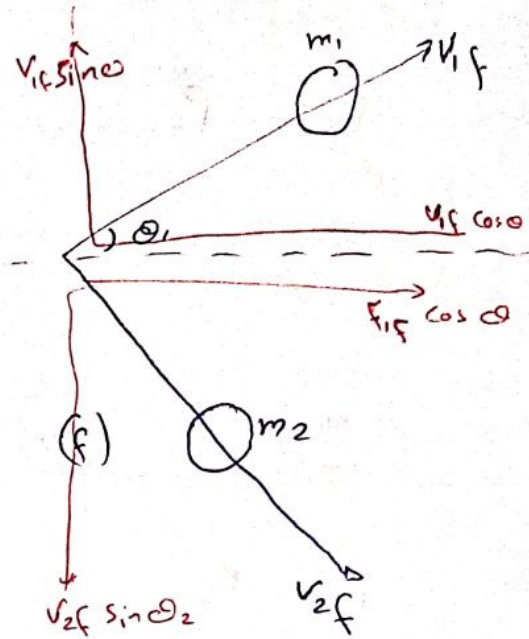
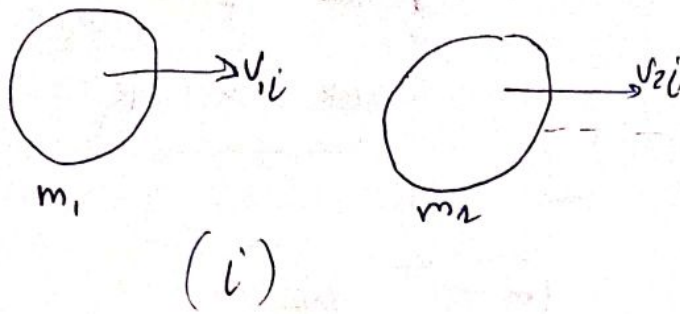
$$\Delta E = K_i - K_f \Rightarrow \Delta E = \left(\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 \right) - \frac{1}{2} M v_f^2$$

$$P = M \vec{v}_{\text{com}}$$

$$v_{\text{com}} = \frac{P}{M} = \frac{m_1 v_{1i} + m_2 v_{2i}}{M}$$

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* Collision in 2D :



$$\vec{P}_i = \vec{P}_f$$

$$P_{ix} = P_{fx}$$

$$P_{iy} = P_{fy}$$

لا ينفصل

$$P_{ix} = P_{fx}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} \cos \theta_1 + m_2 v_{2f} \cos \theta_2 \quad \text{--- (1)}$$

$$P_{iy} = P_{fy}$$

$$0 = m_1 v_{1f} \sin \theta_1 - m_2 v_{2f} \sin \theta_2 \quad \text{--- (2)}$$

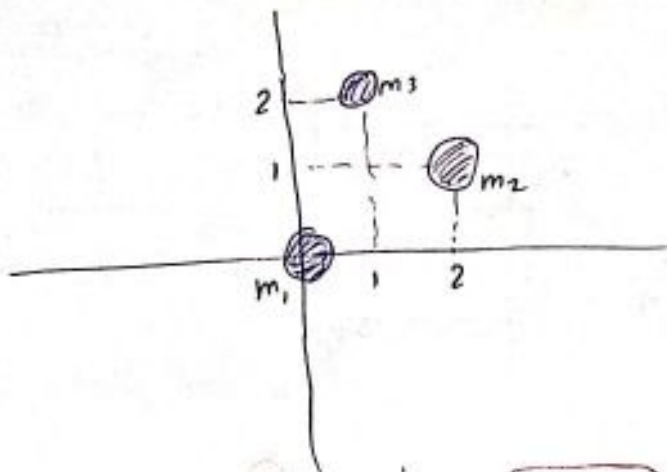
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P ②

$$m_1 = 3 \text{ kg}$$

$$m_2 = 4 \text{ kg}$$

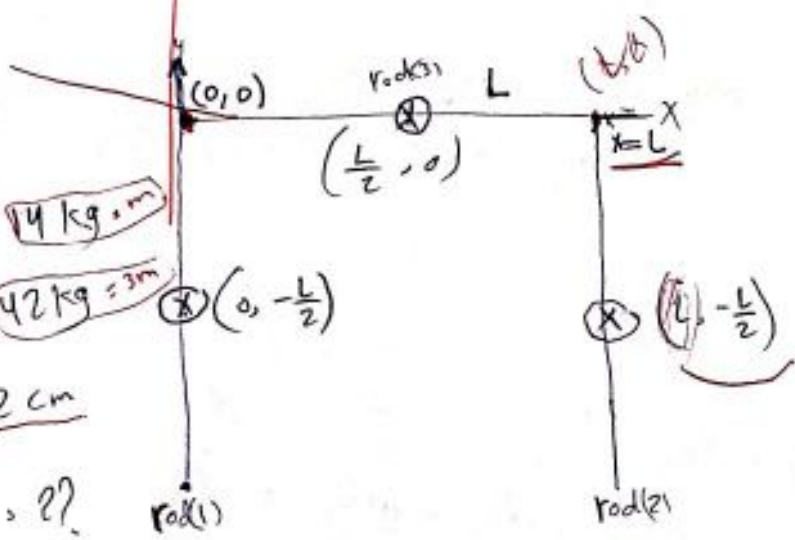
$$m_3 = 8 \text{ kg}$$



$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = 1,1 \text{ m}$$

$$y_{com} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = 1,3 \text{ m}$$

P ④



$$m_1 = m_2 = 4 \text{ kg} \cdot \text{m}$$

$$m_3 = 42 \text{ kg} = 3 \text{ m}$$

$$L = 22 \text{ cm}$$

$$y_{com}, x_{com} = ??$$

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{M} = \frac{4(0) + 4(2) + 3(2)}{5} = \frac{14}{5} = 2,8$$

$$= \frac{L}{2} = \frac{22}{2} = 11$$

$$y_{com} = \frac{m_1(-\frac{L}{2}) + m_2(-\frac{L}{2}) + m_3(0)}{5} = \frac{-4(11) - 4(11)}{5} = \frac{-88}{5} = -17,6 \text{ cm}$$

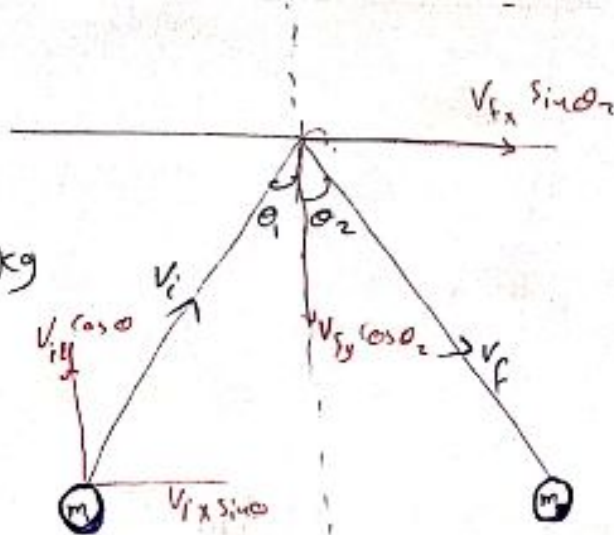
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P22

$\theta_1 = 30^\circ$
 $m = 0.165 \text{ kg}$
 $v_i = 2 \text{ m/s}$

$V_{xi} = V_{xf}$

y = variable



@ angle $\theta_2 = ?$

(b) $\Delta \vec{P} = ??$

(a) $\vec{P}_x = m\vec{v}_x$
 $\vec{P}_{x(i)} = \vec{P}_{x(f)}$

$m v_i \sin \theta_1 = m v_f \sin \theta_2$

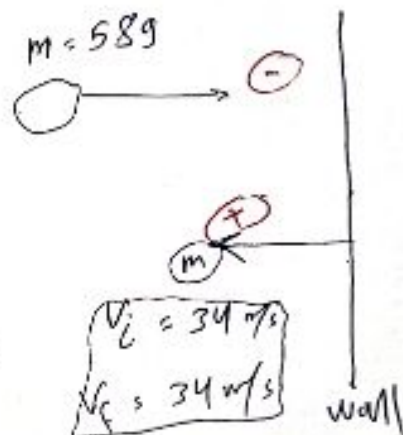
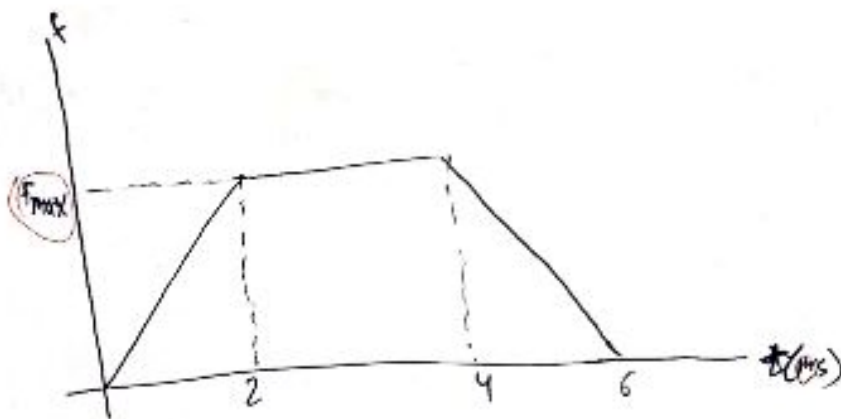
$\sin \theta_1 = \sin \theta_2 \Rightarrow \theta_2 = \theta_1 = 30^\circ$

(b) ΔP
 $\Delta P_x = 0$
 $\Delta P_y = ??$

$\Delta P_y = P_{fy} - P_{iy}$
 $= -m v_f \cos \theta_2 - m v_i \cos \theta_1$

$= -2m v_i \cos \theta_1$
 $\Delta P = -0.572 \text{ kg} \cdot \text{m/s}$

P35



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$$\vec{J} = \Delta P = \int \vec{F} dt$$

المتوسط الزمني
المتوسط الزمني
 $t(n)$

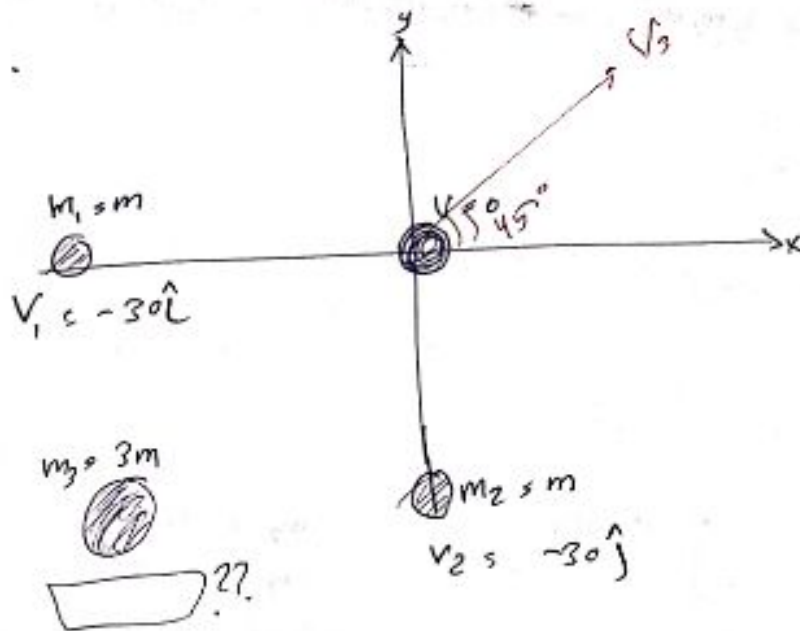
$$\begin{aligned} \Delta P &= \vec{P}_f - \vec{P}_i \\ &= m\vec{V}_f - m\vec{V}_i \\ \Delta P &= 2mV_i \end{aligned}$$

$$\begin{aligned} \int F dt &= \frac{1}{2}(0,002)F_{max} + 0,002 F_{max} \\ &+ \frac{1}{2}(0,002) F_{max} \\ &= 0,004 F_{max} \end{aligned}$$

$$2mV_i = 0,004 F_{max}$$

$$F_{max} = 9,9 \times 10^2 \text{ N}$$

P(47)



$$\vec{P}_i = \vec{P}_f$$

$$0 = m_1 \vec{V}_1 + m_2 \vec{V}_2 + m_3 \vec{V}_3$$

$$\vec{V}_3 = \frac{-m_1 \vec{V}_1 - m_2 \vec{V}_2}{m_3} = \frac{30 m \hat{i} + 30 m \hat{j}}{3m} = 10 \hat{i} + 10 \hat{j} = \vec{V}_3$$

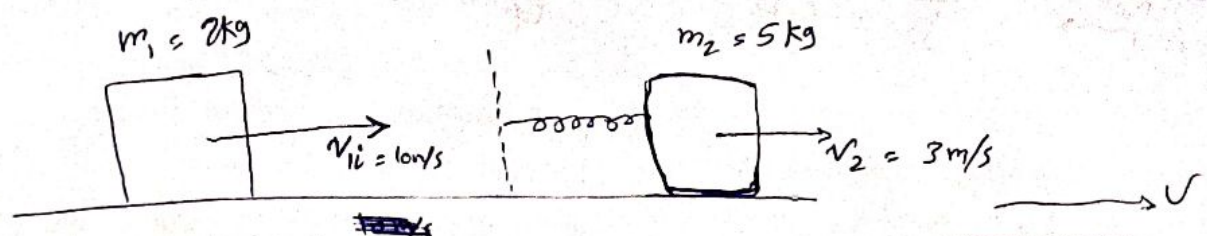
$$\tan \alpha = \frac{10}{10} = 1 = 45^\circ$$

$\alpha = 45^\circ$

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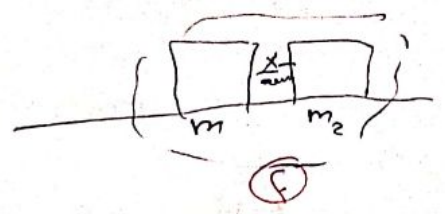
P 59

$k = 1120 \text{ N/m}$



$x_s \text{ max} = ??$

"completely inelastic collision"
تصادم غير مرنة



$K_i + U_{s0} = K_f + U_{s \text{ spring}} \quad U_s = \frac{1}{2} k x^2$

$K_i - K_f = U_s$

$(\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2) - \frac{1}{2} (m_1 + m_2) v_f^2 = \frac{1}{2} k x^2 \rightarrow x = 0.25 \text{ m}$

المعادن

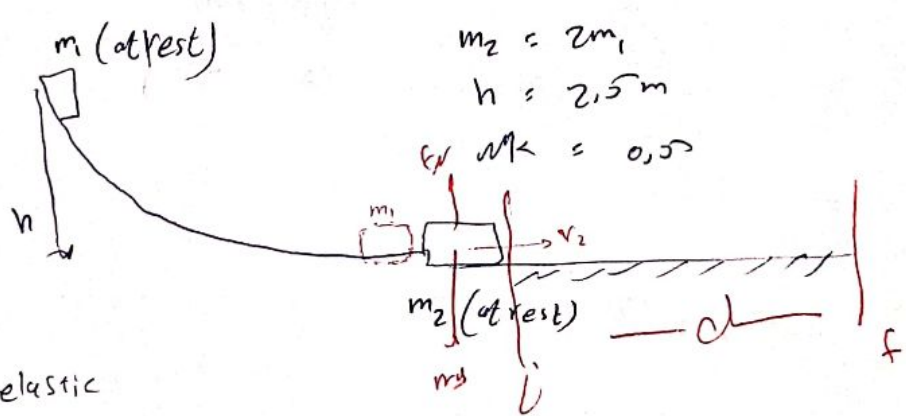
$P_i = P_f$

$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$

$v_f = \boxed{}$

Anan Elayan

P 68



$m_2 = 2m_1$
 $h = 2.5 \text{ m}$
 $\mu_k = 0.5$

a) elastic

b) Completely inelastic

$v_2 = \frac{2m_1}{m_1 + m_2} v_{1i}$

$v_{1i} = \sqrt{2gh} = \sqrt{2(9.8)(2.5)} = 7 \text{ m/s}$

$\frac{2m_1}{m_1 + 2m_2} (7) = \frac{2}{3} (7) = \boxed{4.67 \text{ m/s}}$

$$K_i + U_i = K_f + U_f + \Delta E_{th} \rightarrow W_{friction}$$

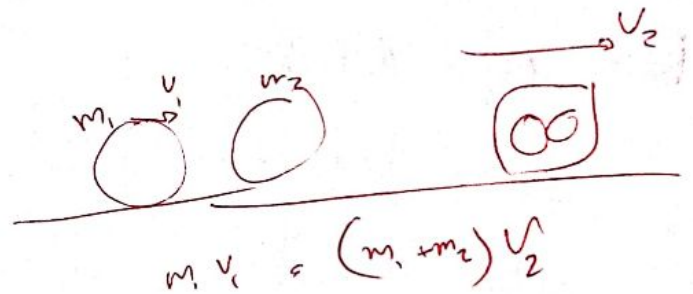
$$\frac{1}{2} m v_2^2 = \Delta E_{th}$$

$$\frac{1}{2} m v_2^2 = f_k \cdot d$$

$$\frac{1}{2} m v_2^2 = \mu_k (mg) d$$

$$d = 3,22 \text{ m}$$

$$(2) \quad v_2 = \frac{m_1}{m_1 + m_2} v_{1i} \Rightarrow d = 0,556 \text{ m}$$



$$F_{net} = \frac{dP}{dt} \Rightarrow F_{net} = m \vec{a}$$

$$P = mv$$

$$F_{net} = \frac{d}{dt} [mv] = m \frac{dv}{dt} + v \frac{dm}{dt}$$

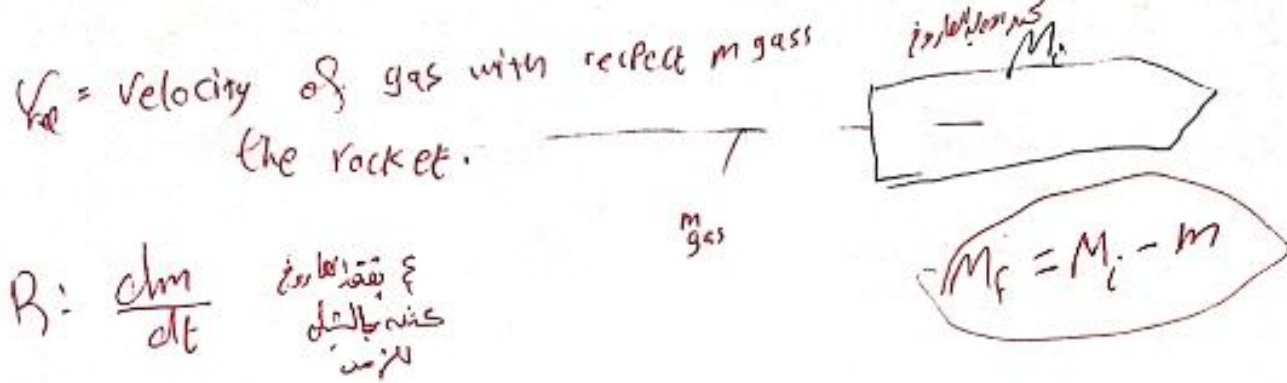
$$= ma + v \frac{dm}{dt} \rightarrow \text{if } m \text{ constant}$$

$$F_{net} = ma$$

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ϕ $M a = R V_{rel}$ 'is equation of rocket'

* $V_f - V_i = V_{rel} \ln \frac{M_i}{M_f}$ '2nd rocket equation'
ibidax/ jax/ax/abx/abx



R: $\frac{dm}{dt}$ *تغير الكتلة في الزمن*

- P76
- $M_i = 6090 \text{ kg}$
 - $V_i = 105 \text{ m/s}$
 - $m_{\text{gas}} = 80 \text{ kg}$
 - $V_{\text{rel}} = 253$
 - $V_f = ??$

??

$$V_f - V_i = V_{rel} \ln \frac{M_i}{M_f}$$

$$V_f - 105 = 253 \ln \frac{6090}{6090 - 80}$$

$V_f = \square$

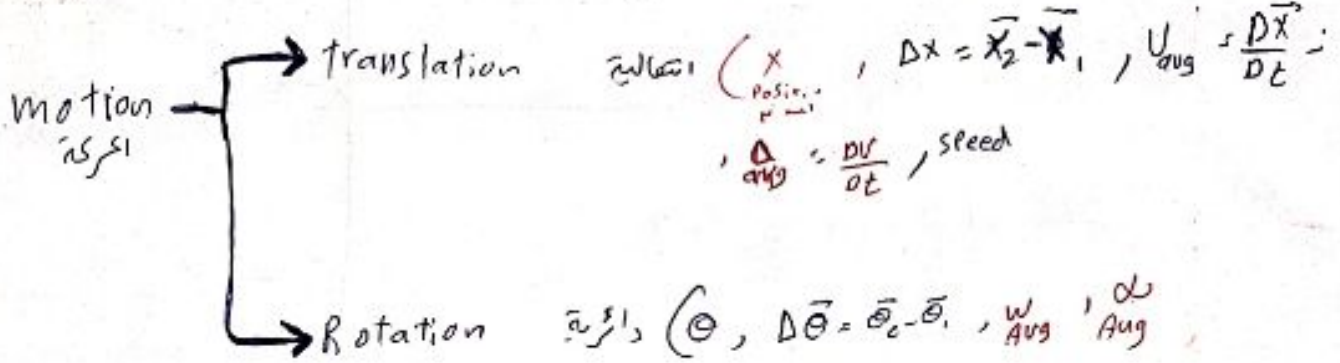
$6090 - 80 = \square$

End ch9
 Good Luck
 Anan Elayan

Anan Elayan

Chapter 10

Rotation الحركة الدورانية



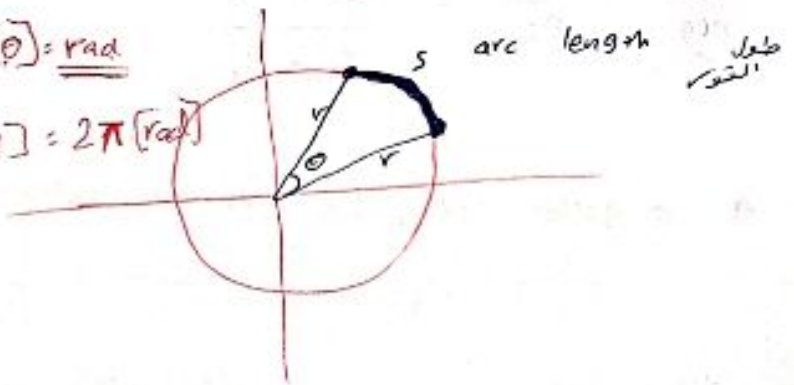
* Rotational Variables : متغيرات الحركة الدورانية

① angular position : الموقع الزاوي / الزاوي (θ)

$$\theta = \frac{s}{r}$$

$$[\theta] = \text{rad}$$

$$1 [r.v] = 2\pi [\text{rad}]$$

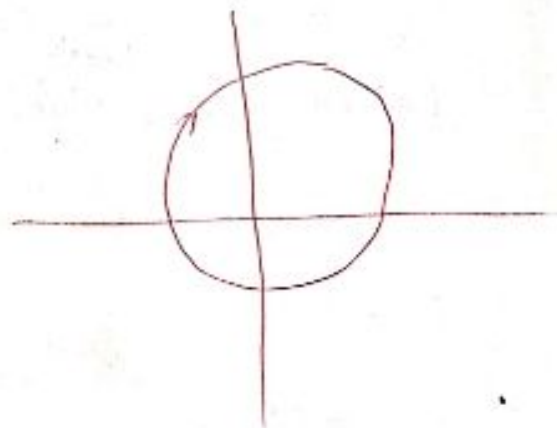


② angular displacement (الزاوية الزاوية) $(\Delta \theta)$

$$\Delta \theta = \theta_2 - \theta_1$$

تغير في الزاوية الزاوية
وإنها زاوية

$$[\Delta \theta] = \text{rad}$$



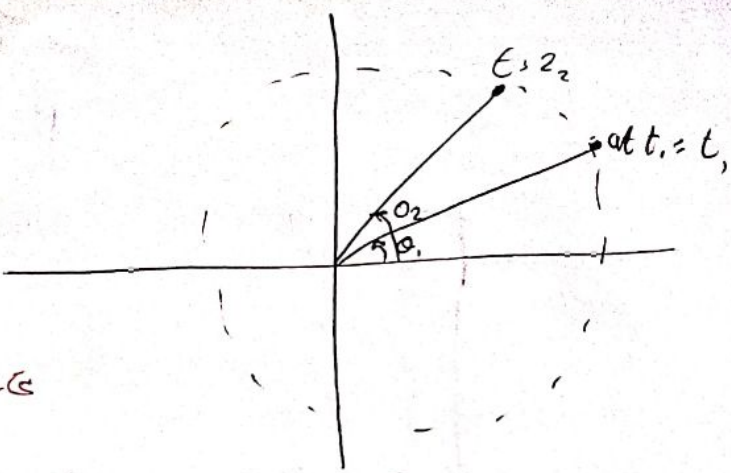
* Clock wise $\Rightarrow \Delta \theta$ is negative (-)

* Counter clock wise $\Rightarrow \Delta \theta$ is positive (+)

تحدد الإشارة
بصفاً للزاوية

3 angular Velocity (ω_{avg}) السرعة الزاوية

$$\omega_{Avg} = \frac{\Delta \theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$



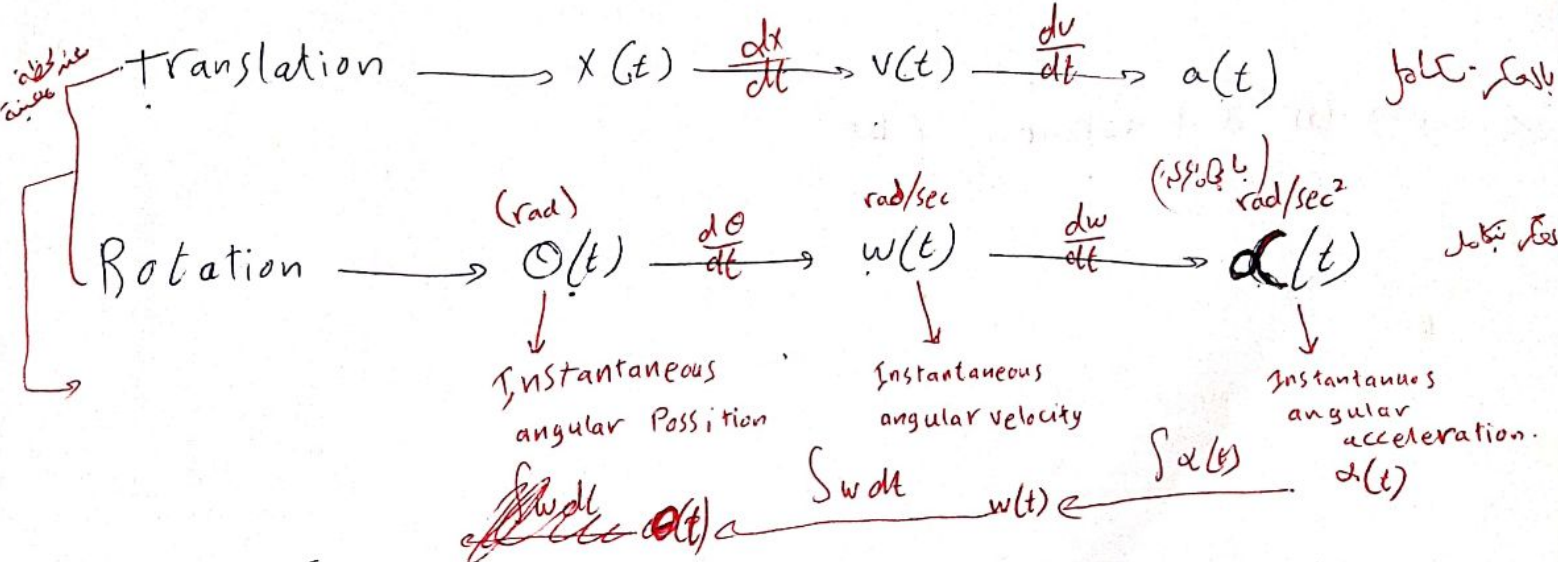
$\omega_{avg} \rightarrow \oplus$ counter clockwise عكس عقارب الساعة

$\omega_{avg} \rightarrow \ominus$ clockwise عقارب الساعة

4 angular acceleration (α_{avg}) التسارع الزاوي

$$\alpha = \frac{\Delta \omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

angular speed = $\left| \overset{(\omega)}{\text{angular velocity}} \right|$



ω direction (Right hand rule).

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$P(u)_2 \quad \theta(t) = 2 + 4t + 2t^3$

at $t=0 \Rightarrow$ (a) angular position

(b) angular velocity

(c) ω at $t = 4$ sec

(d) α at $t = 2$ sec?

$\theta|_{t=0} = 2 \text{ rad}$

(b) $\omega(t) = \frac{d\theta}{dt} = 4 + 6t^2 = \omega(0) = 4 \text{ rad/sec}$

(c) $\omega|_{t=4} = 4 + 6(4)^2 = 100 \text{ rad/s}$

(d) $\alpha = \frac{d\omega}{dt} = 12t = \alpha|_{t=2} = 24 \text{ rad/sec}^2$

w_{avg} $t=1$
 $t=4$

$w_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$

Rotation motion at constant α :- (w) *proof*

translation ($a \equiv \text{cons}$)

$V = v_0 + at$

$v^2 = v_0^2 + 2a \Delta x$

$\Delta x = v_0 t + \frac{1}{2} a t^2$

$\Delta x, v, a$

Rotation ($\alpha \equiv \text{cons}$)

$\omega = \omega_0 + \alpha t$

$\omega^2 = \omega_0^2 + 2\alpha \Delta \theta$

$\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$

$\Delta \theta, \omega, \alpha$

$v = \frac{2\pi R}{T}$
 $\omega = \frac{v}{r}$
 $\alpha = \frac{dv}{dt}$

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P(9)

$$\omega_0 = 12,6 \text{ rad/sec}, \quad \alpha = -4,2 \text{ rad/sec}^2$$

(a) t until t stop ?

$$\omega = \omega_0 + \alpha t$$

$$t = \frac{-\omega_0}{\alpha} = \frac{-12,6}{-4,2} = \boxed{3 \text{ sec}}$$

(b) $\omega^2 = \omega_0^2 + 2\alpha \Delta\theta$

$$\Delta\theta = \frac{-\omega_0^2}{2\alpha} = \frac{-(12,6)^2}{-2(4,2)} = \boxed{18,9 \text{ rad}}$$

$1/r \rightarrow 2\pi \text{ rad}$
 $\times \rightarrow 18,9 \text{ rad}$

translation

Rotation

$x, v, a \leftarrow r \rightarrow \ominus, \omega, \alpha$

$s = r\theta$ → angular position.

linear position $\frac{ds}{dt} = r \frac{d\theta}{dt} \Rightarrow \boxed{v = r\omega}$

$$\frac{ds^2}{dt^2} = r \frac{d\omega^2}{dt^2}$$

$$\boxed{a = r\alpha}$$

$$\boxed{\frac{a}{v} = \frac{\alpha}{\omega}}$$

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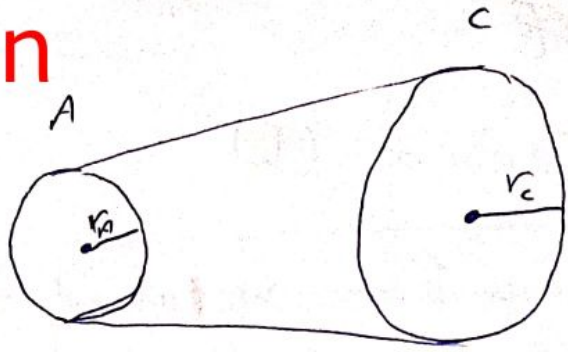
P 28

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$$r_A = 10 \text{ cm}, \quad r_C = 25 \text{ cm}$$

$$\omega_{A(i)} = 0, \quad \alpha_A = 1,6 \text{ rad/sec}^2$$

$$v_a = v_c$$



(a) $t \rightarrow \omega_C = 100 \text{ rev/min} \quad ?? \quad \rightarrow \quad 100 \times \frac{2\pi}{60} = 10,67 \text{ rad/s}$

$v_A = v_C$

$$\omega_A r_A = \omega_C r_C$$

$$\omega_A = \frac{\omega_C r_C}{r_A} = (10,67) \left(\frac{25}{10} \right) = \boxed{26,16 \text{ rad/sec}}$$

$$\omega_A = \omega_{0A} + \alpha t$$

$$26,16 = 0 + 1,6 (t) \Rightarrow \boxed{t = 16 \text{ sec}}$$

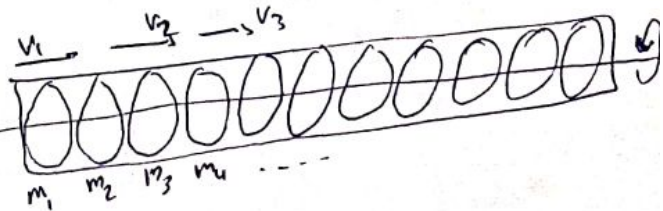
* Kinetic energy (K.E) in Rotation :-

$$K.E = \sum_{i=1}^N \frac{1}{2} m v_i^2$$

$$v_i = r_i \omega_i \Rightarrow K.E = \sum_{i=1}^N \frac{1}{2} m_i (r_i \omega_i)^2$$

$$= \frac{1}{2} \left(\sum_{i=1}^N m_i r_i^2 \right) \omega_i^2 = \boxed{\frac{1}{2} I \omega_i^2} \quad (m \propto I)$$

↓
Inertia (I)



~~KE~~

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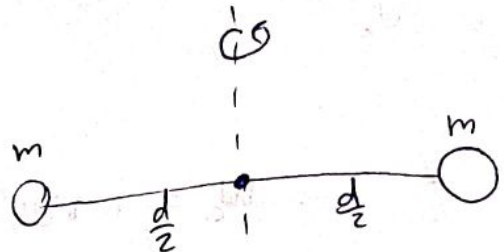
* Inertia (I)

Case I → ^{محور الدوران يمر من مركز الكتلة} Rotational axis through the center of mass

$$I_{com} = \sum_i m_i r_i^2$$

$$= m \left(\frac{d}{2}\right)^2 + m \left(\frac{d}{2}\right)^2$$

$$= \frac{m d^2}{2}$$



$6ma^2$

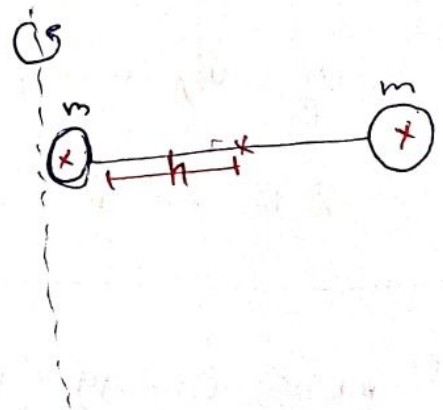
→ Rotational axis through a part from the com

$$I = I_{com} + Mh^2$$

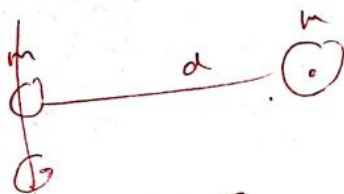
total mass
 distance of center of mass
 Rotational axis.
^{مسافة الكتلة من محور الدوران}

$$= \frac{m d^2}{2} + 2m \left(\frac{d}{2}\right)^2$$

$$= \boxed{m d^2} \text{ for } \left(\frac{d}{2}\right)^2$$



OR



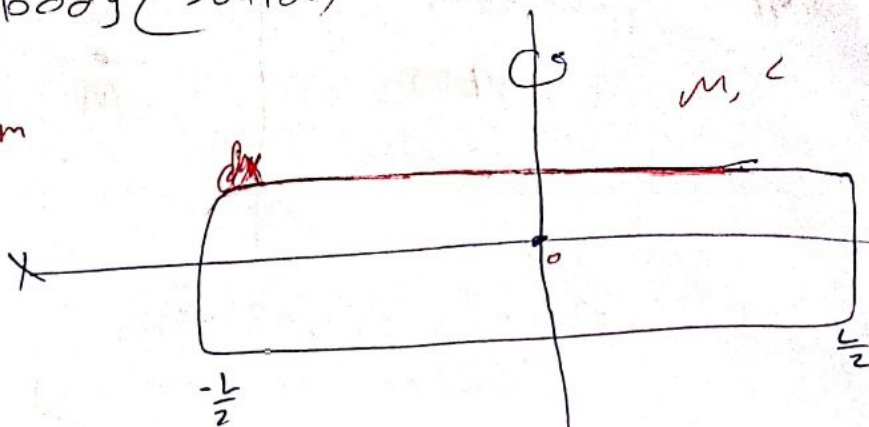
$$I = I_1 + I_2$$

$$= 0 + m d^2 = \boxed{m d^2}$$

Case II \Rightarrow Rigid body (solid)

$$I = \int r^2 dm = \int x^2 dm$$

$$\lambda = \frac{dm}{dx} = \frac{M}{L}$$



$$I = \int_{-L/2}^{L/2} x^2 \frac{M}{L} dx = \frac{M}{L} \int_{-L/2}^{L/2} x^2 dx = \boxed{\frac{1}{12} M L^2}$$

$$I = \sum m_i r_i^2$$

$$\boxed{I_{rod} = \frac{1}{12} m L^2}$$

بسط المساحة
بمربع مركز
المساحة

OR

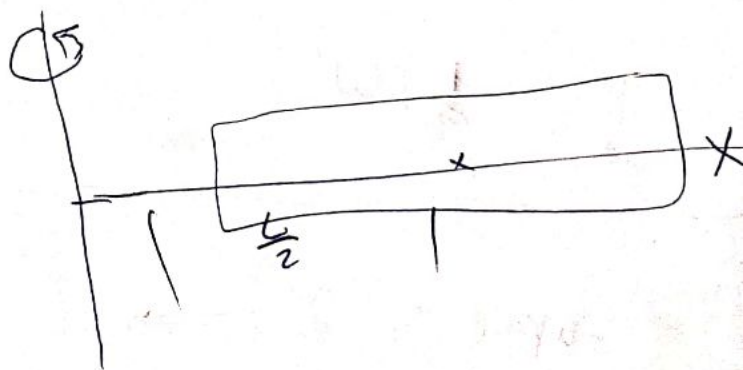
Parallel-axis theorem

$$I = I_{com} + Mh^2$$

$$= \frac{1}{12} mL^2 + M\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{12} mL^2 + \frac{ML^2}{4} = \frac{4ML^2 + 12ML^2}{12 \times 4} = \frac{16ML^2}{48}$$

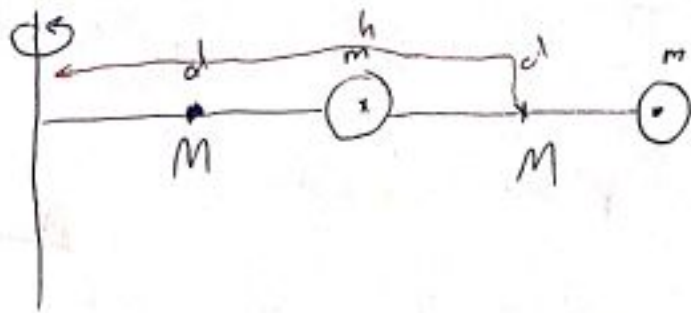
$$= \boxed{\frac{ML^2}{3}}$$



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P41

$m = 0,85 \text{ kg}$
 $M = 1,2 \text{ kg}$
 $\omega = 0,3 \text{ rad/sec}$



$I = I_1 + I_2 + I_3 + I_4 = 0,023 \text{ kg/m}^2$

$I_1 = I_{\text{cm}} + Mh^2 = \frac{1}{12} mL^2 + M\left(\frac{d}{2}\right)^2 =$

$I_2 = \frac{1}{12} M L^2 + M\left(\frac{3d}{2}\right)^2 =$

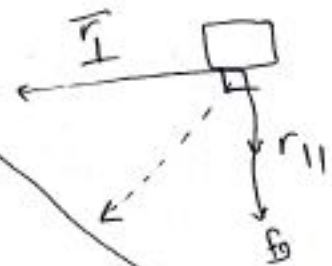
$I_3 = m d^2 =$

$I_4 = m(2d)^2 =$

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$K_{\text{rot}} = \frac{1}{2} I \omega^2$

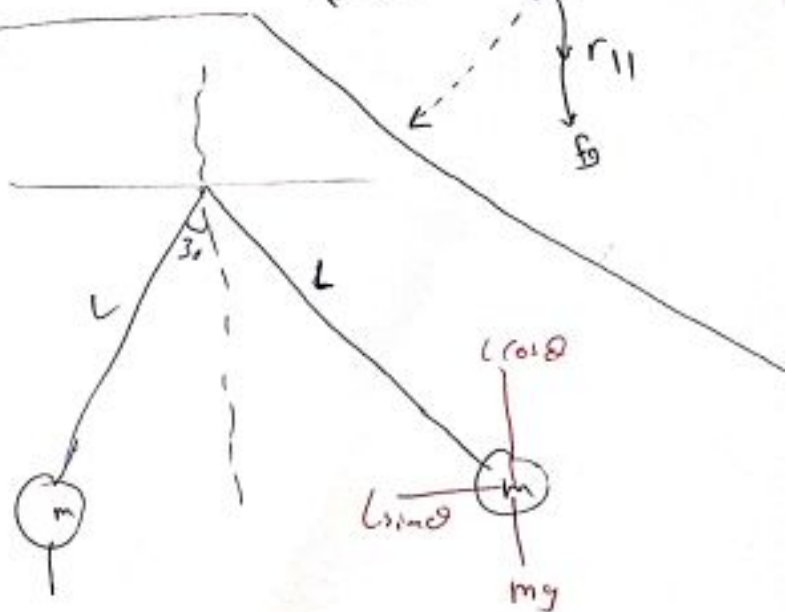
$\vec{\tau}_{\text{ouge}} : (\vec{r}) \Rightarrow \vec{\tau} = \vec{r} \times \vec{F} = |\vec{r}| |\vec{F}| \sin \theta$
 $[E] = \text{N} \cdot \text{m}$



P47 $m = 0,75 \text{ kg}$
 $L = 1,25 \text{ m}$

$\tau \text{ of } F_g = ?$

$\tau = mg d \sin \theta = 4,6 \text{ N} \cdot \text{m}$

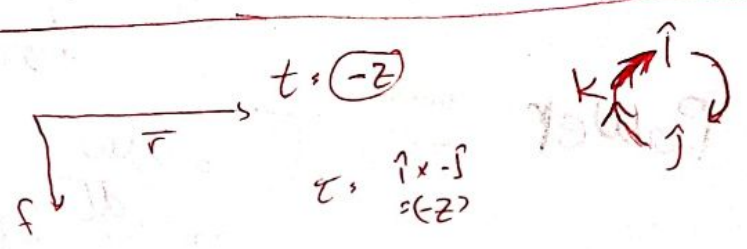
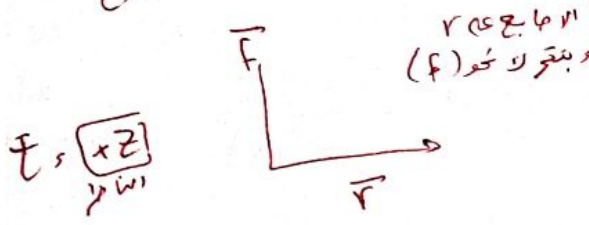


Newton's second law: $f_{net} = m \vec{a}$

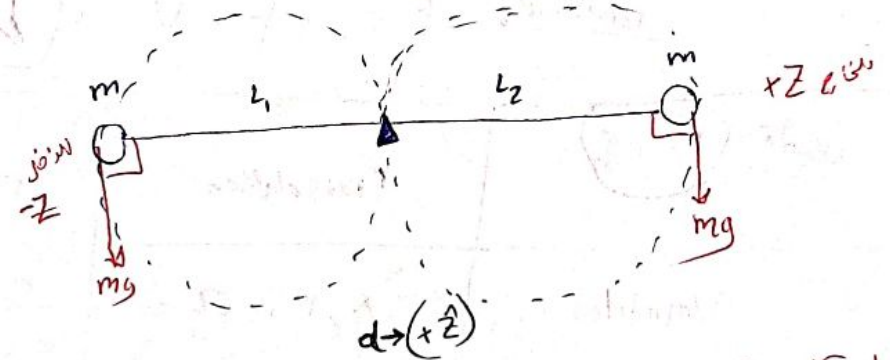
$\tau_{net} = I \alpha$, $\vec{\tau}_{net} \equiv$ net torque

$\vec{\tau} = \vec{r} \times \vec{F}$

$I \equiv$ Inertia
 $\alpha \equiv$ angular acceleration.



P56) $l_1 = 20 \text{ cm}$, $l_2 = 80 \text{ cm}$
 a_1, a_2 ??



$\tau_{net} = I \alpha$
 $\tau_{net} = \tau_1 + \tau_2 \Rightarrow \tau_1 = mg l_1$
 $\tau_2 = mg l_2 \Rightarrow \tau_{net} = mg (l_2 - l_1)$

$a = r \alpha$
 $a_1 = l_1 \alpha$
 $a_2 = l_2 \alpha$

$I = m l_1^2 + m l_2^2$
 $mg (l_2 - l_1) = (m l_1^2 + m l_2^2) \alpha \Rightarrow \alpha = 8.65 \text{ rad/sec}^2$

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$$W_{net} = \Delta K \implies K = \frac{1}{2} I \omega^2$$

⊕ Translation motion : $W = \int_{x_0}^{x_f} F dx$

Rotational $\implies W = \int_{\theta_i}^{\theta_f} \tau d\theta$
 $\tau(\theta) = \tau$

$$W = \tau_{avg} \cdot \Delta\theta$$

⊕ Power $\xrightarrow{translating} \vec{P} = \frac{dW}{dt}$

Rotational $\vec{P} = \vec{F} \cdot \vec{v} = \tau \cdot \omega$

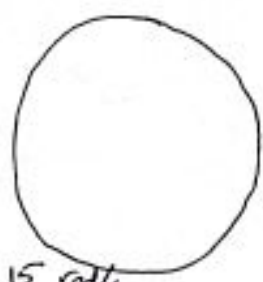
ceiri (Part I)	Translation	Rotation
Variables	x, v, a	r, ω, α
Constant acceleration	$v = v_0 + at$ $v^2 = v_0^2 + 2ax$ $\Delta x = v_0 t + \frac{1}{2} at^2$	$\omega = \omega_0 + \alpha t$ $\omega^2 = \omega_0^2 + 2\alpha \Delta\theta$ $\Delta\theta = \omega_0 t + \frac{1}{2} \alpha t^2$
(Part 2) K	$K = \frac{1}{2} mv^2$	$K = \frac{1}{2} I \omega^2$ (Part 3)
(Part 4)	$W = \int F dx$ $W_{net} = \Delta K$ $P = \vec{F} \cdot \vec{v}$ $F_{net} = ma$	$W = \int \tau d\theta$ $W_{net} = \Delta K$ $P = \tau \cdot \omega$ $F_{net} = I \alpha$ $\omega_f^2 - \omega_i^2 = 2\alpha \theta$

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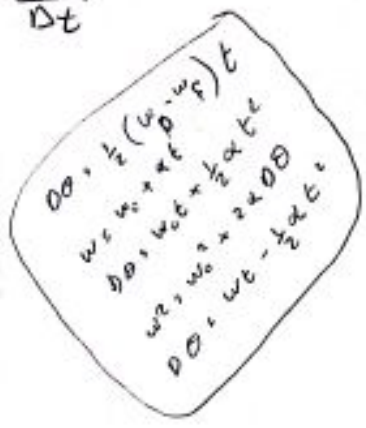
23
34

P2

ω - angular velocity



$$\omega = \frac{d\theta}{dt}$$



$\omega_{\text{second hand}} = \frac{2\pi}{60} = 0,15 \text{ rad/s}$

$\omega_{\text{min hand}} = \frac{2\pi}{60 \times 60} = 1,75 \times 10^{-3} \text{ rad/s}$

$\omega_{\text{hour hand}} = \frac{2\pi}{60 \times 60 \times 60} = 1,45 \times 10^{-4} \text{ rad/sec}$

P13: $\theta_0 = 0$, $\omega_0 = 1,5 \text{ rad/sec}$, $\theta = 40 \text{ rev}$, $\omega_f = 0$

1 rev $\rightarrow 2\pi \text{ rad}$
40 rev $\rightarrow \times$

a) t b) α

a) $\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} \Rightarrow \Delta t = \frac{\Delta\theta}{\omega_{\text{avg}}} = \frac{2\Delta\theta}{\omega_0} = 3,4 \times 10^2 \text{ sec}$

$\omega_{\text{avg}} = \frac{1}{2}(\omega_0 + \omega_f) = \frac{\omega_0}{2}$

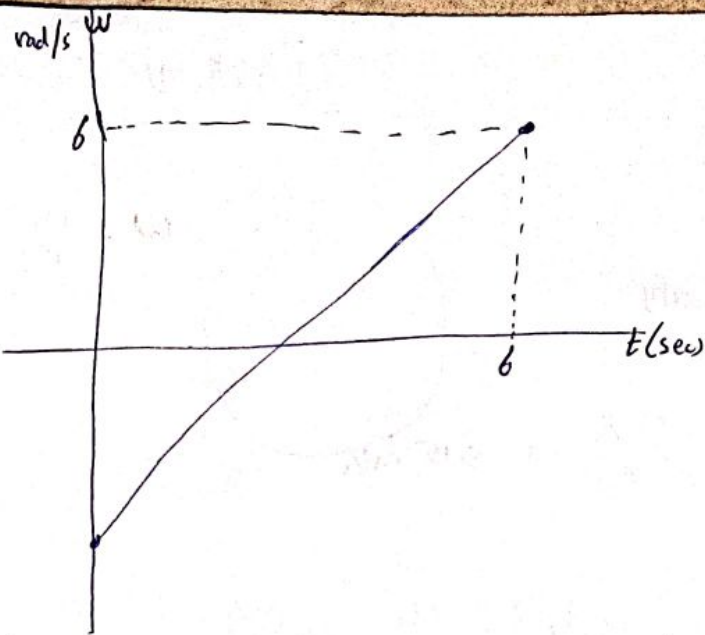
b) $\omega = \omega_0 + \alpha t$
 $\alpha = \frac{-\omega_0}{t} = -7,12 \times 10^{-4} \text{ rad/s}^2$

1 rev $\rightarrow 2\pi \text{ rad}$
20 rev $\rightarrow \times$

c) t for first 20 rev
 $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
 $t = 98 \text{ sec} \checkmark$
 $t = 572 \text{ sec} \times$

Anan Elayan

P34



a) α rod

$$\alpha_{rod} = \frac{d\omega}{dt} = \text{slope} = \frac{4-1}{4-2} = 1.5 \text{ rad/s}^2$$

$\odot \rightarrow \omega \xrightarrow{\frac{d\omega}{dt}} \alpha$

b) $t = 4 \text{ sec} \rightarrow K_{(4)} = 1.6 \text{ J}$ ω^{2-2}

$K_0 \rightarrow (t=0) = ??$

$$K = \frac{1}{2} I \omega^2$$

$$\frac{K_0}{K_4} = \frac{\frac{1}{2} I \omega_0^2}{\frac{1}{2} I \omega_{(4)}^2} =$$

$$K_0 = \frac{\omega_0^2}{\omega_4^2} K_{(4)} = \boxed{0.4 \text{ J}}$$

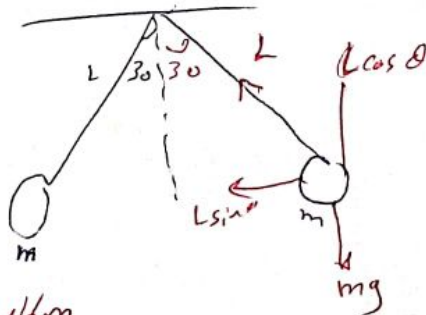
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P47

$m = 0.75 \text{ kg}$

$L = 0.25 \text{ m}$

τ (Tareq gravitintonal force)



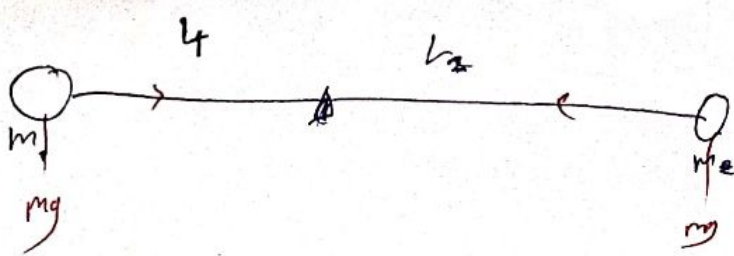
$$\tau = mgL \sin \theta = 4.6 \text{ Nm}$$

العزمي

$$\tau = r \times F = r f \sin \theta$$

wha $\theta = 90$
 $180 \rightarrow \tau = 0$

P56



$L_1 = 20 \text{ cm}$
 $L_2 = 80 \text{ cm}$

$\alpha_1, \alpha_2 = ??$

$a = \alpha r$
 $v = r\omega$
 $s = r\theta$

$$\alpha_1 = \alpha L_1$$

$$\alpha_2 = \alpha L_2$$

$$\tau_{\text{net}} = I \alpha = (m_1 L_1^2 + m_2 L_2^2) \alpha$$

$$m_1 g L_1 - m_2 g L_2 = (m_1 L_1^2 + m_2 L_2^2) \alpha$$

$\alpha = \square$

P23

$D = 1.2$ $r = 0.6$ $\omega_0 = 200 \text{ rev/min}$

A) $\omega_0 = \frac{200 \text{ rev} \times 2\pi}{60} = 20.9 \text{ rad/sec}$

B) $v = \omega_0 r = 20.9(0.6) = 12.5 \text{ m/s}$

C) $\omega = 1000 \text{ rev/min}$, $t = 1 \text{ min}$

12 rev/sec

$\omega = \omega_0 + \alpha t$

$1000 = 200 + \alpha(1) \rightarrow \alpha = 800 \text{ rev/min}^2$

$\theta - \theta_0 = 15 (\omega_0 + \omega) t$

$\Delta\theta = 15 (200 + 1000)(1) \rightarrow \Delta\theta = 600 \text{ rev}$

End ch10
 Good Luck
 Anan Elayan