Chapter One Appendix

Graphs and Their Meaning

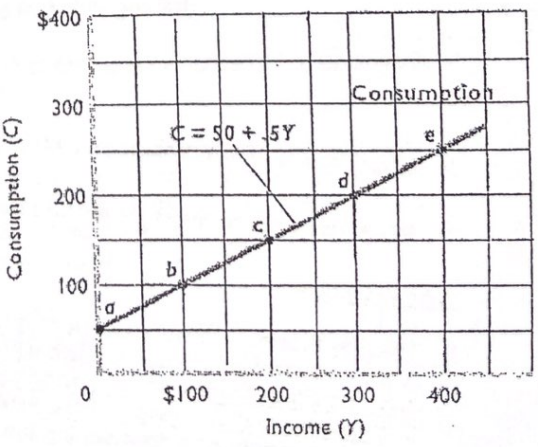
Construction of a Graph

A graph is a visual representation of the relationship between two variables.

إن الرسوم البيانية تهدف إلى تمثيل وتوضيح العلاقات بين المتغيرات الاقتصادية وخاصة عندما نريد أن نربط علاقة متغيرين مع بعضهما البعض. والخطوة الأولى في عملية التمثيل البياني هي أن نرسم محورين متعامدين يطلق على المحور الأفقي " المحور السيني" (Horizontal Axis) وعلى المحور العمودي " المحور الصادي (Vertical Axis). كما أن عملية الرسم البياني تتناسب جداً مع دراسة متغيرين, أحدهما تابع (dependent) والآخر مستقل(independent), حيث يخصص المحور الصادي (Vertical Axis) للمتغير التابع (dependent) والمحور السيني (Horizontal Axis) للمتغير المستقل(Independent).

The table in Figure 1 is a hypothetical illustration showing the relationship between income and consumption for the economy as a whole. Without even studying economics, we would logically expect that people would buy more goods and services when their incomes go up. Thus, it is not surprising to find in the table that total consumption in the economy increases as total income increases. The information in the table is expressed graphically in Figure 1. Here is how it is done: We want to show visually how consumption changes as income changes. We therefore represent income on the *horizontal axis* of the graph and consumption on the *vertical axis*.

Income per Week	Consumption per Week	Point
$ 0	$ 50	a
100	100	b
200	150	c
300	200	d
400	250	e



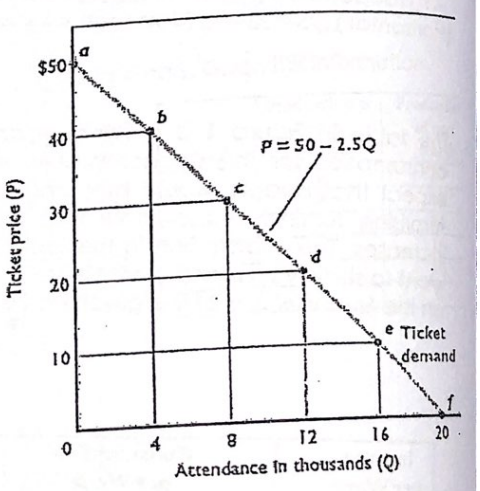
❖ If the curve is a straight line, as in Figure, we say the relationship is linear.

Direct and Inverse Relationships

The line in Figure above slopes upward to the right, so it depicts a direct relationship between income and consumption. *By a direct relationship (or positive relationship) we mean that two variables change in the same direction.* An increase in consumption is associated with an increase in income; a decrease in consumption accompanies a decrease in income. *When two sets of data are positively or directly related, they always graph as an upsloping line,* as in Figure above.

In contrast, two sets of data may be inversely related. Consider the table in Figure below, which shows the relationship between the price of basketball tickets and game attendance. Here we have an *inverse relationship (or negative relationship) because the two variables change in opposite directions.* When ticket prices decrease, attendance increases. When ticket prices increase, attendance decreases. The six data points in the table in Figure 2 are plotted in the graph. *Observe that an inverse relationship always graphs as a downsloping line.*

Ticket Price	Attendance, Thousands	Point
$50	0	a
40	4	b
30	8	c
20	12	d
10	16	e
0	20	f



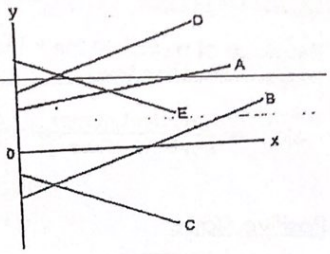
- *When two sets of variables are positively or directly related, they always graph as an upward sloping line.*
- *When two sets of variables are negatively or inversely related, they always graph as a down sloping line.*

Multiple Choices:

1. If we say that two variables are directly related, this means that:
   A. the relationship between the two is purely random.
   B. an increase in one variable is associated with a decrease in the other variable.
   C. an increase in one variable is associated with an increase in the other variable.
   D. the two graphs as a downsloping line.

2. If we say that two variables are inversely related, this means that:
   A. the two graph as an upsloping line.
   B. an increase in one variable is associated with a decrease in the other.
   C. an increase in one variable is associated with an increase in the other.
   D. the resulting relationship can be portrayed by a straight line parallel to the horizontal axis.

3. Refer to the diagram. Which line(s) show(s) a positive relationship between x and y?
   A. A only.
   B. A and D only.
   C. C. A, B, and D.
   D. both C and E.



4. Refer to the diagram. Which line(s) show(s) a negative relationship between x and y?
   A. A only.
   B. both A and D.
   C. A, B, and D.
   D. both C and E.

5. If a positive relationship exists between x and y:
   A. an increase in x will cause y to decrease.
   B. a decrease in x will cause y to increase.
   C. the relationship will graph as an upsloping line.
   D. the vertical intercept must be positive.

6. If price (*P*) and quantity (Q) are directly related, this means that:
   A. a change in Q will alter *P*, but a change in *P* will not alter Q.
   B. if *P* increases, Q will decrease.
   C. if *P* increases, Q will also increase.
   D. an increase in *P* will cause Q to change, but the direction in which Q changes cannot be predicted.

Dependent and Independent Variables

The independent variable is the cause or source; it is the variable that changes first.

Dependent variable is the effect or outcome; it is the variable that changes because of the change in the independent variable.

As in our income-consumption example, income generally is the independent variable and consumption the dependent variable. Income causes consumption to be what it is rather than the other way around. Similarly, ticket prices determine attendance basketball games; attendance at games does not determine the printed ticket prices for those games. Ticket price is the independent variable and the quantity of tickets purchased is the dependent variable.

Mathematicians put the independent variable (cause) on the horizontal axis and the dependent variable (effect) on the vertical axis.

Multiple Choices:

1. Which of the following statements is correct?
   A. The value of the independent variable is determined by the value of the dependent variable.
   B. The value of the dependent variable is determined by the value of the independent variable.
   C. The dependent variable designates the "cause" and the independent variable the "effect."
   D. Dependent variables graph as upsloping lines; independent variables graph as downsloping lines.

Slope of a line

The slope of a straight line is the ratio of the vertical change to the horizontal change between any two points of the line.

$$\text{Slope} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{\Delta y}{\Delta x}$$

Positive Slope

Slope is positive when X and Y have a direct relationship. Because consumption and income change in the same direction; that is, consumption and income are directly or positively related.

Example:

Income per Week	Consumption per Week	Point
$ 0	$ 50	a
100	100	b
200	150	c
300	200	d
400	250	e

Calculate the slope between point b and c.

Because income is the independent variable and consumption the dependent variable, when we plotted the graph we put consumption on the vertical axis and income in the horizontal axis.

$$\text{Slope} = \frac{\Delta \text{ Consumption}}{\Delta \text{ Income}} = \frac{(150-100)}{(200-100)} = \frac{50}{100} = ½$$

The slope of 0.5 tells us there will be a $0.5 increase in consumption for every $1 increase in income. Or when income increase by $2, consumption increase by $1.

Negative Slope

Slope is negative when X and Y have an inverse relationship.

Example:

The slope between point a and b is:

$$\text{Slope} = \frac{\Delta \text{ Ticket price}}{\Delta \text{ Attendence}} = \frac{(40-50)}{(4-0)} = \frac{-10}{4} = -2.5$$

Ticket Price	Attendance, Thousands	Point
$50	0	a
40	4	b
30	8	c
20	12	d
10	16	e
0	20	f

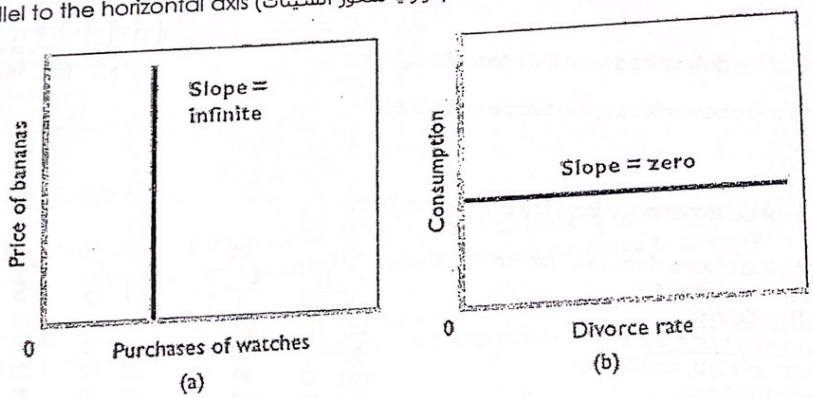
The slope of - 2.5 means that when person attendance increase by 1 person, then the ticket price reduce by $2.5

Slopes and Marginal Analysis

The concept of slope is important in economics because it reflects marginal changes-those involving 1 more (or 1 less) unit. For example, the 0.5 slope shows that $.50 of extra or marginal consumption is associated with each $1 change in income. In this example, people collectively will consume $.50 of any $1 increase in their incomes and reduce their consumption by $.50 for each $1 decline in income.

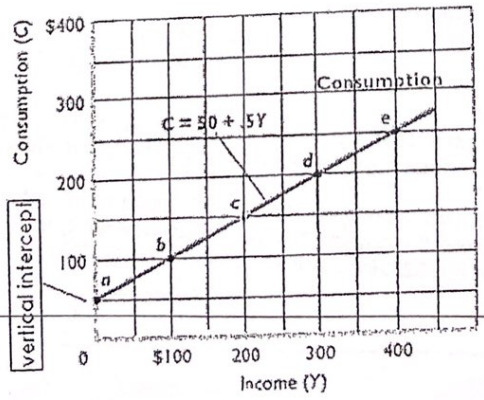
Infinite and Zero Slopes

Many variables are unrelated or independent of one another, the slope of this line is infinite or zero.
A line parallel to the vertical axis (موازي لمحور الصادات) has an infinite slope.
A line parallel to the horizontal axis (موازي لمحور السينات) has a zero slope.

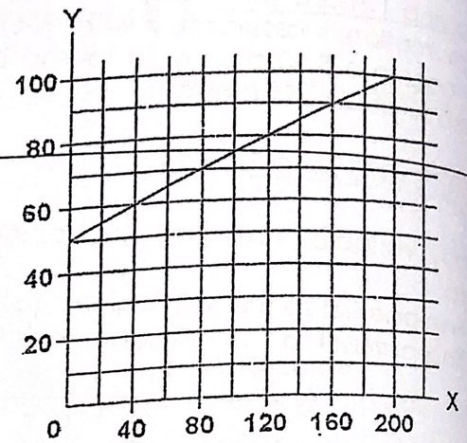


Vertical Intercept

The vertical intercept of a line is the point where the line meets the vertical axis. In Figure below the intercept is $50. *This intercept means that if current income were zero, consumers would still spend $50.* They might do this through borrowing or by selling some of their assets. Similarly, the $50 vertical intercept in Figure 2 shows that at a $50 ticket price, GSU's basketball team would be playing in an empty arena.



Multiple Choices:



1. Refer to the above diagram. The variables X and Y are:
   A. inversely related.
   B. directly related.
   C. unrelated.
   D. negatively related.

2. Refer to the above diagram. The vertical intercept:
   A. is 40.
   B. is 50.
   C. is 60.
   D. cannot be determined from the information given.

3. Refer to the above diagram. The slope of the line:
   A. is $-1/4$.
   B. is $+1/4$.
   C. is 0.40.
   D. cannot be determined from the information given.

4. Refer to the above data sets. The vertical intercept is positive for:
   A. all five data sets.
   B. data sets 1 and 3 only.
   C. data sets 1, 3, and 5 only.
   D. data set 2 only.

(1)		(2)		(3)		(4)		(5)	
J	K	L	M	N	P	R	T	U	V
0	10	0	-15	100	40	0	-15	0	0
40	20	30	-5	80	50	20	-25	5	10
80	30	60	5	60	60	40	-35	10	20
120	40	90	15	40	70	60	-45	15	30
160	50	120	25	20	80	80	-55	20	40
200	60	150	35	0	90	100	-65	25	50

Equation of a Linear Relationship

If we know the vertical intercept and slope, we can describe a line succinctly in equation form. In its general form, the equation of a straight line is $y = a + b\,x$

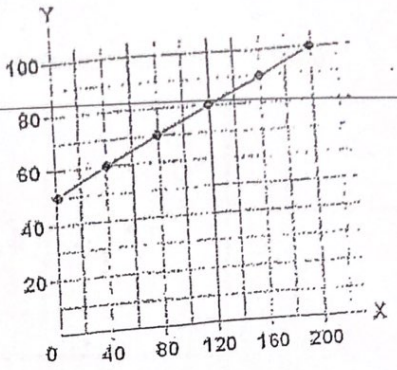
Where y = dependent variable
a =vertical intercept
b =slope of line
x =independent variable

For our income-consumption example, if C represents consumption (the dependent variable) and Y represents income (the independent variable), we can write $C = a + bY$.

By substituting the known values of the intercept and the slope, we get $C = 50 + 0.5\,Y$. This equation also allows us to determine the amount of consumption C at any specific level of income. You should use it to confirm that at the $250 income level, consumption is $175.

Example:

Refer to the diagram below write the linear equation that shows the relationship between Y and X



The linear equation form: $y = a + b\,x$

a =vertical intercept = 50

b =slope of line .

$$\text{Slope} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{\Delta y}{\Delta x} = \frac{(70-60)}{(80-40)} = \frac{10}{40} = 1/4$$

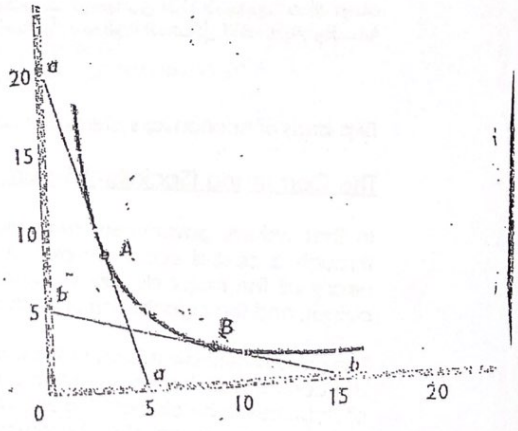
→ $\boxed{y = 50 + \frac{1}{4}x}$

Slope of a Nonlinear Curve

The slope of a straight line is the same at all its points. The slope of a line representing a nonlinear relationship changes from one point to another. Such lines are always referred to as curves.

To measure the slope at a specific point on nonlinear curve, we draw a straight line tangent (مماس) to the curve at that point. (A line is tangent at a point if it touches, but does not intersect).

To measure the slope at a specific point, we draw a straight line tangent to the curve at that point. A line is *tangent* at a point if it touches, but does not intersect; the curve at that point. Thus line aa is tangent to the curve in Figure at point A. *The slope of the curve at that point is equal to the slope of the tangent line.*



Tangent line A intercepts: (5, 0) and (0, 20)

$$\text{Slope at point A} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{(20-0)}{(0-5)} = -4$$

Line bb in Figure is tangent to the curve at point B. Following the same procedure, we find the slope at B :

Tangent line B intercepts: (15, 0) and (0, 5)

$$\text{Slope at point B} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{(5-0)}{(0-15)} = -\frac{1}{3}$$