Utility

Utility define as the satisfaction that a person receives from his or her economic activities.

المنفعة : عبارة عن الإشباع الذي يحصل عليه الفرد عند استهلاكه سلعة أو خدمة معينة

Utility from Consuming Two Goods

Assume that the person receives utility from consuming two goods X and Y. The utility function is given by:

$$\text{Utility} = \text{U}(\text{X}, \text{Y}\,; \text{other things})$$

This notation indicates that the utility an individual receives from consuming X and Y over some period of time depends on the quantities of X and Y consumed and on ''other things.''

يشير هذه العلاقة الى ان الاشباع الذي يحصل عليه المستهلك عند استهلاكه سلعتين X و Y خلال فترة زمنية معينة تعتمد على كميات المستهلكة من السلعة X و Y وعلى "أشياء أخرى".

These other things might include easily quantifiable items such as the amounts of other kinds of goods consumed, the number of hours worked, or the amount of time spent sleeping. They might also include such unquantifiable items as love, security, and feelings of self-worth.

تعتمد المنفقة التي يحصل عليها المستهلك من استهلاكه سلعتين X و Y خلال فترة زمنية على الكمية التي يستهلكها من السلعةX و الكمية التي يستهلكها من السلعة Y و على أشياء أخرى. تشمل الأشياء الأخرى على عناصر قابلة للقياس بسهولة مثل كميات من أنواع أخرى من السلع التي يستهلكها الشخص، وعدد ساعات العمل، أو مقدار الوقت الذي يقضيه في النوم. وقد تشمل أيضا أشياء لا يمكن تحديدها كمياً كالحب، والأمن، ومشاعر الذاتية.

These other things appear after the semicolon because we assume that they do not change (other things equal) while we look at the individual's choice between X and Y.

تم وضع الأشياء الأخرى بعد الفاصلة المنقوطة لأننا نفترض أنها لا تتغير (الأشياء الأخرى متساوية) عندما يتم دراسة الاشباع الذي يحصل عليه المستهلك من استهلاكه السلعتين X و.Y

By assuming other things equal, the utility function form become:

$$U = U(X, Y) \quad \rightarrow \textit{ Utility function}$$

U = refer to the level of satisfaction you receive from consuming the bundle of goods consisting X and Y.

Measuring Utility قياس المنفعة

A quantification of the satisfaction of wants and needs achieved through the consumption of goods and services. In principle, utility measurement can take one of two forms: (1) cardinal, which is based on numerical values (1, 2, 3, etc.) and (2) ordinal which is based on rankings (first, second, third, etc.). While the

Hypothetical instructional analysis of utility relies on cardinal utility, ordinal utility is a more realistic way to measure satisfaction.

Ordinal versus Cardinal Utility المنفعة الترتيبية والمنفعة العددية

Cardinal utility: The belief that utility can be measured and compared on a unit by unit basis, e.g. Utility measure of 200 is twice as big as a utility measure of 100.

Ordinal utility: Where you rank a bundle of goods but you cannot say how big one bundle is compared to the other.

A utility function that describes by how much one market basket is preferred to another is called cardinal utility. For example, if the consumer can say, the total utility derived from consuming good A is equal to 20, the utility is cardinal.

تقوم المنفعة العددية على التعبير عن عدد وحدت المنفعة المشتقة من سلعة ما عند استهلاكها. فعلى سبيل المثال يمكن أن نقول بأن عدد وحدات المنفعة التي يحصل عليها ماجد عند استهلاكه سلعة معينة تساوي 20 وحدة منفعة.

A utility that generates a ranking of goods or market baskets is called ordinal utility, for example, good A is preferred to good B. or ranking a utility from the same goods by comparing the 1st unit and 2nd unit from utility.

تقوم المنفعة الترتيبية على المقارنة المشتقة من السلع أو مقارنة المنفعة المشتقة من الوحدة الأولى و الوحدة الثانية من نفس السلعة. فعلى سبيل المثال يمكن القول بأن المنفعة التي يحصل عليها ماجد من تفاحة أكبر من المنفعة التي يحصل عليها من برتقالة. وكذلك يمكن القول بأن المنفعة التي يحصل عليها ماجد من كوب الشاي الأول أكبر من المنفعة التي يحصل عليها من كوب الشاي الثاني.

Consumer Preferences:

Some Basic Assumptions about Preferences:

- **Completeness**: Preferences are assumed to be complete. In other words, consumers can compare and rank all possible baskets.

  Between two consumption bundles, A and B, we might expect a person to be able to state clearly "I prefer A to B" or" I prefer B to A" or "A and B are equally attractive to me"(will be indifferent between the two).

  Note that these preferences ignore costs. A consumer might prefer steak to hamburger but buy hamburger because it is cheaper.

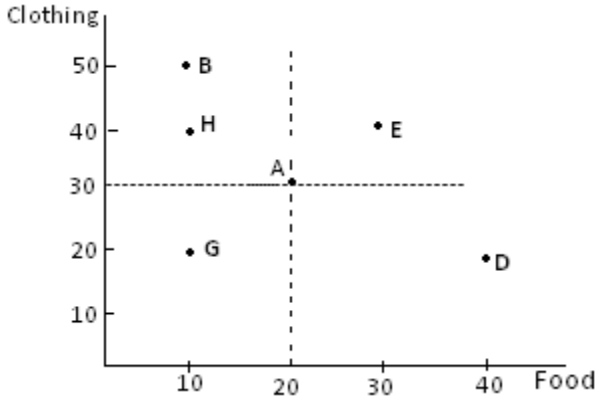
- **Transitivity**: Preferences are transitive. Transitivity means that if a consumer prefers basket A to basket B and basket B to basket C, then the consumer also prefers A to C.

  Example
  If a man prefers Tea to Pepsi and prefers Pepsi to Orange Juice, but he is indifferent between Tea and Coffee, transitivity of preferences means that he must
  a. prefer orange juice to tea
  b. prefer coffee to orange juice
  c. prefer Pepsi to coffee
  d. be indifferent between tea and orange juice
  e. be indifferent between Pepsi and coffee

- **More is better than less**: consumers always prefer more of any good to less. In addition, consumers are never satisfied or satiated; more is always better, even if just a little better. Of course, some goods, such as air pollution, may be undesirable, and consumers will always prefer less.

Market basket A, with 20 units of food and 30 units of clothing, is preferred to basket G because A contains more food and more clothing (recall our third assumption that more is better than less). Similarly, market basket E, which contains even more food and even more clothing, is preferred to A. In fact, we can easily compare all market baskets in the two shaded areas (such as F and G) to A because they contain either more or less of both food and clothing. Note, however, that B contains more clothing but less food than A. Similarly, D contains more food but less clothing than A. Therefore, comparisons of market basket A with baskets B, D, and H are not possible without more information about the consumer's ranking.

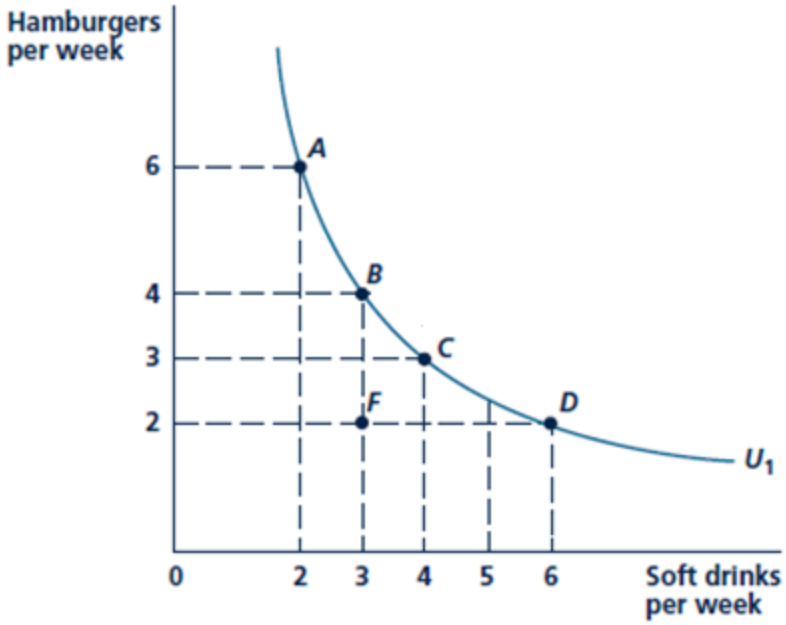
إن من إحدى البديهيات حول سلوك المستهلك أنه يفضل الأكثر على الأقل عند استهلاك. بمعنى, أنه لو حصل المستهلك على كمية أكبر من أحدى السلعتين التي يستهلكها دون أن يقلل استهلاكه من السلعة الأخرى, فأن ذلك سيزيد من منفعته. أما إذا حصل المستهلك على كمية أقل من إحدى السلعتين التي يستهلكها دون أن يزيد استهلاكه من السلعة الأخرى، فإن ذلك سوف يقلل من منفعته. فالمستهلك يفضل السلة الغذائية (point A) التي تحتوى على 20 وحدة من الطعام و 30 وحدة من الملابس على السلة الغذائية (point G) التي تحتوي على 10 وحدات طعام و20 وحدة ملابس. كذلك يفضل المستهلك النقطة E على النقطة A و ذلك لأن المستهلك يحصل على كمية اكبر من السلعتين. ولكن ليس باستطاعة المستهلك ان يقارن الخيار A مع الخيار H أو الخيار B.



Indifference Curve

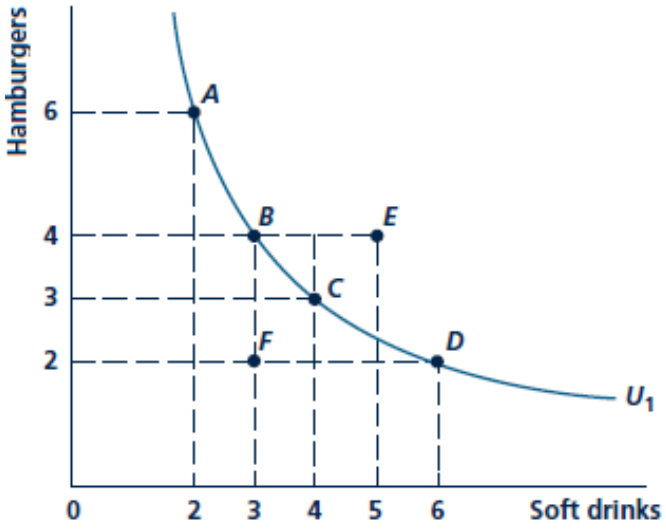
A curve that shows all the combinations of goods or services (market baskets) that provides the same level of utility.
هو ذلك المنحنى الذي يمثل المجموعات المختلفة (من سلعتين) الذي يحصل المستهلك عند استهلاكها على نفس المستوى من المنفعة.

- The curve U1 in the Figure includes all combinations of Hamburgers and Soft drink that yield the same level of utility. Point A, with 6 units of Hamburgers and 2 units of Soft drink, has the same utility as point B, 4 Hamburgers and 3 Soft drink. Since **all points on the curve (A, B,C and D) yield the same utility**, the person has no reason to prefer one point over another.



Points above and below an Indifference Curve:

In the Figure, points such as E are above of U_1. Point E has more of both goods than point C → E is preferred to A (more is better). Because of transitivity, E is preferred to A, B, and D.



Point E is preferred to A, B, C, and D

Points above an indifference curve are preferred to points on the curve.

In the Figure, points such as F are below of an indifference curve U_1. Point C is preferred to point F since it contains more of both goods. Because of transitivity, all points on U1 (A, B, C and, D) are preferred to point F.

Points A, B, C, and D are preferred to point F

Points on an indifference curve are preferred to points below it.

Example

Consider the utility function for goods X and Y as given by U(X,Y) = X Y. Which of the following bundles lie on the same indifference curve as the bundle containing 2 units of X and 4 units of Y?

a. the bundle containing 5 units of X and 1 unit of Y
b. the bundle containing 1 unit of X and 8 units of Y
c. the bundle containing 4 units of X and 2 units of Y
d. both b and c

Answer:

All the combinations of goods or services that provides the same level of utility lie on the same indifference curve

The bundle containing 2 units of X and 4 units of Y provide a U(X,Y) = 2*4 = 8
The bundle containing 5 units of X and 1 unit of Y provide a U(X,Y) = 1*5 = 5
The bundle containing 1 units of X and 8 unit of Y provide a U(X,Y) = 1*8 = 8
The bundle containing 4units of X and 2unit of Y provide a U(X,Y) = 2*4 = 8

Both the two bundle b and c provide the same level of utility as the bundle containing 2 units of X and 4 units of Y.

Example

Suppose the person has preferences for apples (A) and orange (O) given by: $Utility = \sqrt{A\ O}$

a. If A = 5 and O = 80, what will utility be?

Utility = $\sqrt{5 * 80}$ = $\sqrt{400}$ = 20

b. If A = 10, what value for O will provide that same utility as in part a?

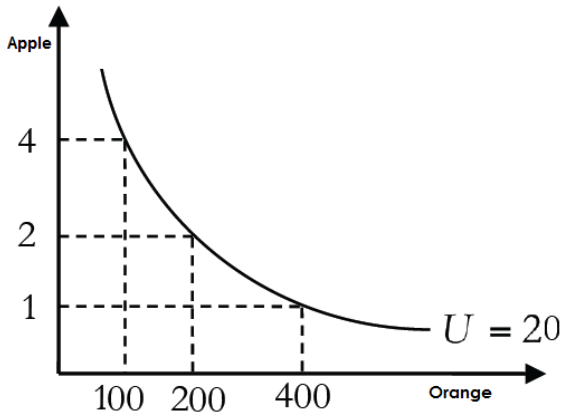
Utility = $\sqrt{A\ O}$ → 20 = $\sqrt{10\ O}$ → 10 O = 400 → O = $\frac{400}{10}$ = 40

c. Graph the indifference curve when U= 20.

When U = 20 → 20 = $\sqrt{A\ O}$

بتربيع الطرفين → 400 = A* O → O = $\frac{400}{A}$

A	O
1	400/1 = 400
2	400/2 = 200
4	400/4 = 100



Example

Suppose a consumer's preferences for two goods X and Y is given by: $U(X,Y) = \sqrt{X\ Y}$

1. Is the assumption that more is better satisfied for both goods?

   Yes, the "more is better" assumption is satisfied for both goods since as amounts of both good X and Y increases, utility increase.

   $X \uparrow \rightarrow U \uparrow \ and\ Y \uparrow \rightarrow U \uparrow$

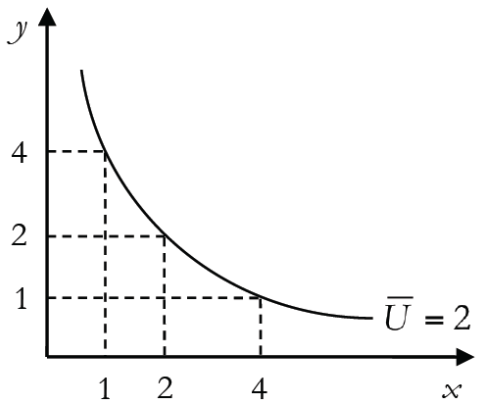
2. On a graph with x on the horizontal axis and y on the vertical axis, draw indifference curve $U = 2$

   $U(X,Y) = \sqrt{X\ Y}$

   $2 = \sqrt{X\ Y}$ بتربيع الطرفين ينتج

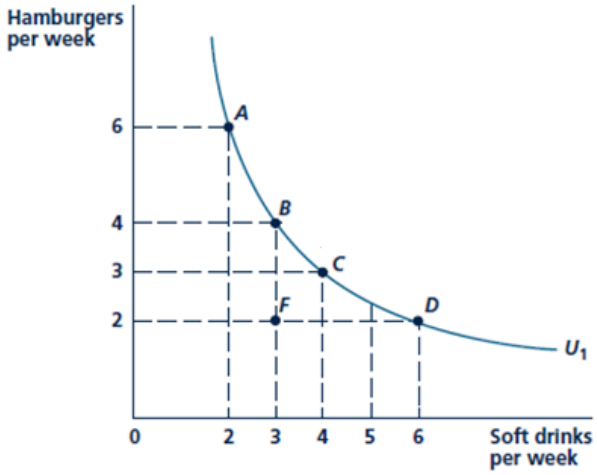
   $4 = X\ Y \rightarrow Y = \frac{4}{X}$

X	Y
1	4/1 = 4
2	4/2 = 2
4	4/4 = 1



Indifference Curve and the Marginal Rate of Substitution (MRS)

What happens when a person moves from point A (six hamburgers and two soft drinks) to point B (four hamburgers and three soft drinks)? This person remains equally well-off because the two commodity bundles lie on the same indifference curve.



This person will voluntarily give up two of the hamburgers that were being consumed at point A in exchange for one additional soft drink. The slope of the curve U1 between A and B is therefore approximately−2. That is, Y (hamburgers) declines two units in response to a one-unit increase in X (soft drinks).

Marginal rate of substitution (MRS) = The rate at which an individual is willing to reduce consumption of one good when he or she gets one more unit of another good.

هو عبارة عن عدد الوحدات التي يتخلى عنها المستهلك من سلعة في مقابل الحصول على وحدة إضافية من سلعة أخرى مع المحافظة على نفس مقدار المنفعة.

Marginal rate of substitution (MRS) = The negative of the slope of an indifference curve.

Marginal rate of substitution (MRS) = the absolute value of slope of indifference curve

The MRS (of soft drinks for hamburgers) between points A and B $= \frac{\Delta H}{\Delta S} = \left|\frac{4-6}{3-2}\right| = 2$

This person is willing to give up 2 hamburgers in order to get one more unit of soft drinks.

المستهلك مستعد التخلي عن وحدتين من هامبورجر مقابل الحصول على وحدة إضافية من المشروبات الغازية

The MRS between points B and C $= \frac{\Delta H}{\Delta S} = \left|\frac{3-4}{4-3}\right| = 1$

This person is willing to give up 1 hamburger's in order to get one more unit of soft drinks.

The MRS between points C and D $= \frac{\Delta H}{\Delta S} = \left|\frac{2-3}{6-4}\right| = ½$

This person is willing to give up ½ hamburger's in order to get one more unit of soft drinks.

Diminishing Marginal Rate of Substitution

The MRS diminishes (decreases) along an indifference curve moving from point A to point D.

The MRS between points A and B is equal 2
The MRS between points B and C is equal 1
The MRS between points C and D is equal ½

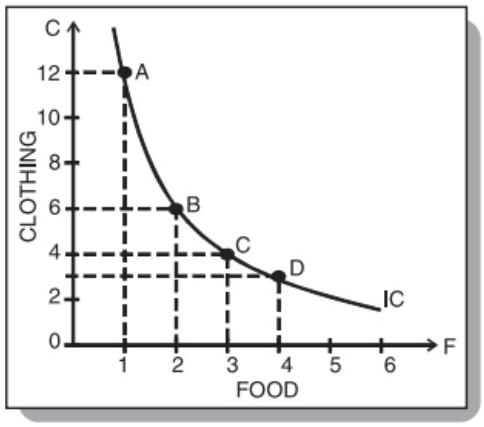
→ As we move from point A to D MRS diminishing.

Example
The price of gum is 20 cents and the price of candy is 10 cents. A student is willing to give up 2 pieces of gum (vertical axis) for 1 piece of candy (horizontal axis). Money income is $2. What is the marginal rate of substitution?

MRS: The rate at which an individual is willing to reduce consumption of one good (vertical axis) when he or she gets one more unit of another good (horizontal axis).

→ MRS = $\frac{2}{1}$ = 2

Example
The diagram shows an Indifference curve for two goods clothing's and food.



1. Calculate the MRS when the consumer increase amount of food from 1 to 2 units and interpret it in words.

   The MRS = $\frac{\Delta \text{clothing}}{\Delta \text{food}}$ = $\left|\frac{6-12}{2-1}\right|$ = 6

   The consumer is willing to give up 6 clothing's in order to get one more unit of food.
   المستهلك مستعد ان يتخلى عن 6 وحدات من الملابس مقابل الحصول على وحدة اضافية من الطعام

2. Is MRS diminishing, constant, or increasing as the consumer substitute's food for clothing along an indifference curve?

   The MRS between points A and B = $\frac{\Delta \text{clothing}}{\Delta \text{food}}$ = $\left|\frac{6-12}{2-1}\right|$ = 6

   The MRS between points B and C = $\frac{\Delta \text{clothing}}{\Delta \text{food}}$ = $\left|\frac{4-6}{3-2}\right|$ = 2

   The MRS between points C and D = $\frac{\Delta \text{clothing}}{\Delta \text{food}}$ = $\left|\frac{3-4}{4-3}\right|$ = 1

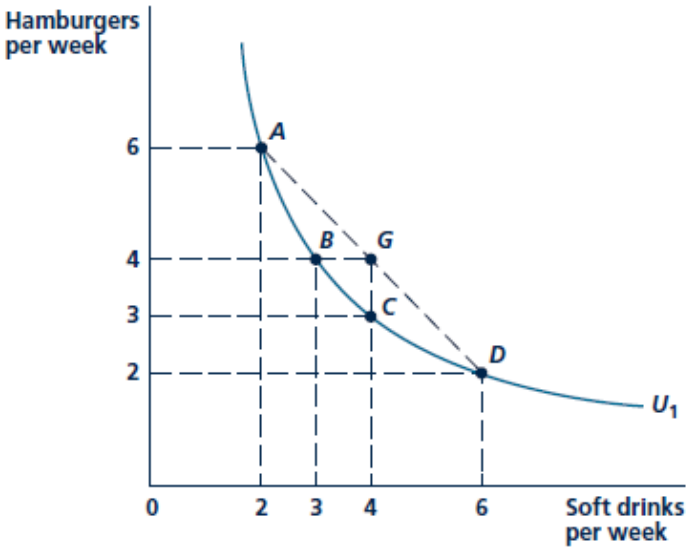
   → As we move from point A to D MRS diminishing (decreasing).

The Shape of Indifference Curves

Indifference curves are downward sloping. In our example of hamburgers and soft drinks, when the amount of soft drinks increases along an indifference curve, the amount of hamburger decreases. The fact that *indifference curves slope downward follows directly from our assumption that more of a good is better to less.* If an indifference curve sloped upward, a consumer would be indifferent between two market baskets even though one of them had more of both hamburger and soft drink.

Balance in Consumption

The conclusion of a diminishing MRS is based on the idea that people prefer balanced consumption bundles to unbalanced ones. This assumption is illustrated precisely in Figure below, where the indifference curve U1 is redrawn. Our discussion here concerns the two extreme consumption options A and D. In consuming A, this person gets six hamburgers and two soft drinks; the same satisfaction could be received by consuming D (two hamburgers and six soft drinks).



Now consider a bundle of commodities (say, G) ''between'' these extremes. With G (four hamburgers and four soft drinks), this person obtains a higher level of satisfaction (point G is northeast of the indifference curve U1) than with either of the extreme bundles A or D. The reason for this increased satisfaction should be geometrically obvious. All of the points on the straight line joining A and D lie above U1. Point G is one of these points (as the figure shows, there are many others).

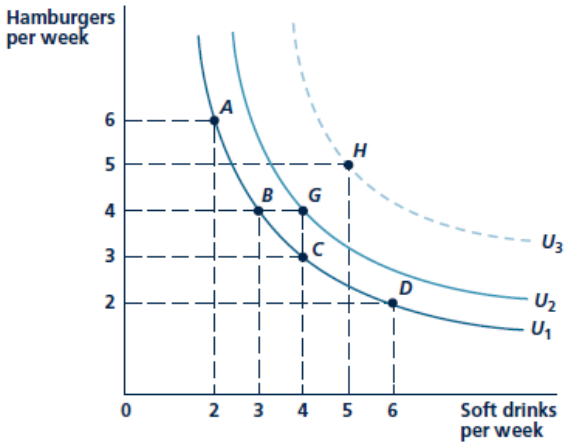
As long as the indifference curve obeys the assumption of a diminishing MRS, it will have the type of convex shape shown in Figure below. Any consumption bundle that represents an ''average'' between two equally attractive extremes will be preferred to those extremes. The assumption of a diminishing MRS (or convex indifference curves) reflects the notion that people prefer variety in their consumption choices.

Indifference Curve Maps

To describe a person's preferences for all combinations of hamburger and soft drink, we can graph a set of indifference curves called an indifference map.
Indifference map: Graph containing a set of indifference curves showing the market basket among which a consumer is indifferent.

خريطة منحنيات السواء هي مجموعة منحنيات السواء للمستهلك والتي تظهر تفضيله أو ذوقه تجاه السلع التي يستهلكها.

Any market basket on indifference curve U_3, such as basket H, is preferred to any basket on curve U_2 (e.g., basket G), which in turn is preferred to any basket on U_1, such as C.

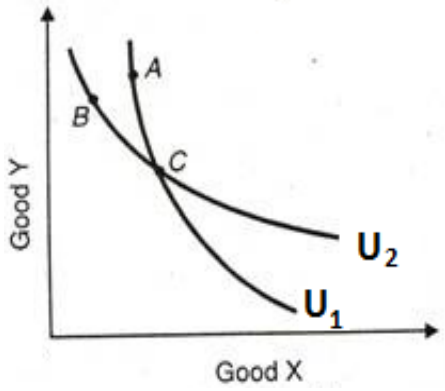


$U3 > U2 > U1$

- *Point H is preferred to point G, and point G is preferred to point C*
- *All points A, B, C, and D give the consumer the same level of satisfaction.*

Indifference Curves Cannot Intersect

If indifference curves U_1 and U_2 intersect, one of the assumptions of consumer theory is violated. According to this diagram, the consumer should be indifferent among market baskets A, B, and C. Yet A should be preferred to B because A has more of both goods. Thus, *intersecting indifference curves contradict our assumption that more is preferred to less*.



Marginal Rate of Substitution (MRS) and Marginal Utility:

Marginal utility (MU): The additional satisfaction obtained from consuming one additional unit of a good.
هو عبارة عن الاشباع الاضافي الذي يحصل عليه المستهلك من استهلاكة وحدة اضافية من سلعة

The marginal utility of good X (MUX): The extra utility obtained by consuming one more unit of good X.
هو عبارة عن الاشباع الاضافي الذي يحصل عليه المستهلك من استهلاكة وحدة اضافية من سلعة X

$$MUX = \frac{dU(X,Y)}{dx}$$

The marginal utility of good Y (MUY): The extra utility obtained by consuming one more unit of good Y.
هو عبارة عن الاشباع الاضافي الذي يحصل عليه المستهلك من استهلاكة وحدة اضافية من سلعة Y

$$MUY = \frac{dU(X,Y)}{dY}$$

$$Utility = U(X,Y)$$

$$Change\ in\ utility = MUX * \Delta X + MUY * \Delta Y$$

Since utility does not change along an indifference curve, we can derive:

$$0 = MUX * \Delta X + MUY * \Delta Y \rightarrow -\frac{\Delta Y}{\Delta X} = \frac{MUX}{MUY}$$

But $-\frac{\Delta Y}{\Delta X}$ = negative slope of the indifference curve = MRS

$$\rightarrow \quad MRS = \frac{MUX}{MUY}$$

Example

Consider the utility function $U(X,Y) = X^2 Y$

1. Calculate the marginal utility of X. Does the marginal utility of x diminish, remain constant, or increase as the consumer buys more X? Explain.

   MUX = $\frac{dU(X,Y)}{dX} = 2YX$

   As the consumer buys more X, the amount of good Y decrease (income constant)

   So as you consume more X (and less Y), the MUX diminishes

2. Is the assumption that more is better satisfied for both goods?

   Yes, the "more is better" assumption is satisfied for both goods since both marginal utilities are always positive.

3. Calculate the $MRS_{x,y}$ at the bundle (X = 2, Y= 3) and interpret it in words

   MRS = $\frac{\mathrm{MUX}}{\mathrm{MUY}} = \frac{2\mathrm{XY}}{\mathrm{X}^2} = \frac{2\mathrm{Y}}{\mathrm{X}} = \frac{2(3)}{2} = 3$

   The consumer is willing to give up 6 clothing's in order to get one more unit of food.

   المستهلك مستعد ان يتخلى عن 3 وحدات من السلعة Y مقابل الحصول على وحدة اضافية من السلعة X

4. Is $MRS_{x,y}$ diminishing, constant, or increasing as the consumer substitutes x for y along an indifference curve?

   MRS = $\frac{2\mathrm{Y}}{\mathrm{X}}$

   As X increases (and Y decreases) (البسط يقل والمقام يزيد) MRS is diminishing.

Example

Maher has $80 to spend on eggs (E) and meat (M). Eggs are $4 each and a strip of meat is $2. His utility function is given by the following: U (E, M) = E*M. What is the Maher's MRS between meat and eggs at the bundle (15 eggs and 3 meats)?

MRS between meat and eggs = $\frac{\mathrm{MUE}}{\mathrm{MUM}}$

MUE = $\frac{dU(E,M)}{dE}$ = M

MUM = $\frac{dU(E,M)}{dM}$ = E

MRS = $\frac{\mathrm{M}}{\mathrm{E}} = \frac{3}{15} = \frac{1}{5}$

This person is willing to give up 1/5 meat in order to get one more eggs).Or this person is willing to give up 1 meat in order to get 5 more eggs).

Example

A consumer spends his income to purchase two goods, X and Y. His preferences are represented by the following utility function U(X,Y) = ½X − Y^2. What is the MRS for consumer?

MRS between meat and eggs = $\frac{\text{MUX}}{\text{MUY}}$

MUX = $\frac{dU(X,Y)}{dX}$ = ½

MUY = $\frac{dU(X,Y)}{dY}$ = −2Y

MRS = $\frac{MUX}{MUY} = \frac{½}{-2\text{Y}} = \frac{1}{-4\text{Y}}$

Example

Consider the utility function $U(X,Y) = 3X + Y$

1. Calculate the marginal utility of X. Does the marginal utility of x diminish, remain constant, or increase as the consumer buys more X? Explain.

   MUX = $\frac{dU(X,Y)}{dX}$ = 3

   So as you consume more X, the MUX constant

5. Calculate the $MRS_{x,y}$. Is $MRS_{x,y}$ diminishing, constant, or increasing as the consumer substitutes x for y?

   MUX = $\frac{dU(X,Y)}{dX}$ = 3

   MUY = $\frac{dU(X,Y)}{dY}$ = 1

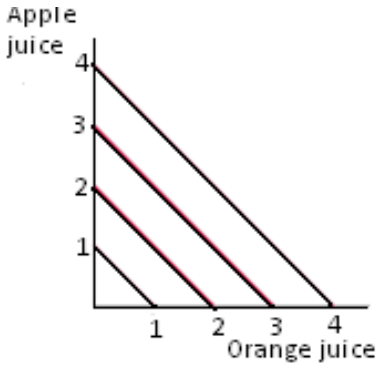
   MRS = $\frac{\text{MUX}}{\text{MUY}} = \frac{3}{1} = 3$

   As X increases (and Y decreases) MRS is constant and equal 3.

Illustrating Particular Preferences

Perfect Substitutes

The consumer views orange juice and apple juice as perfect substitutes: He is always indifferent between a glass of one and a glass of the other.



Perfect Substitutes

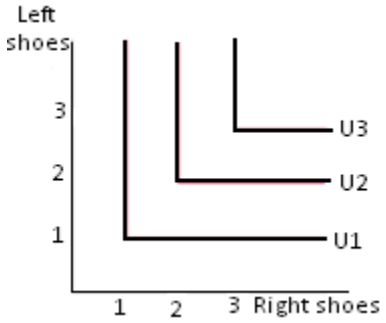
The utility function for these goods: $U\ (X,Y)\ =\ a\,X\ +\ b\,Y$

For perfect substitutes marginal rate of substitution of one for the other is a constant.

Perfect Complement

Two goods for which the MRS is infinite; the indifference curves are shaped as right angles.

Jane views left shoes and right shoes as perfect complements: An additional left shoe gives her no extra satisfaction unless she also obtains the matching right shoe.



In the Figure, the two goods are perfect complements in that they must be used together (like left and right shoes) to gain utility. If for example, he currently has three pairs of those, additional right shoes provide no more utility. Similarly, additional left shoes alone provide no additional utility, since this individual likes to consume these two goods together.

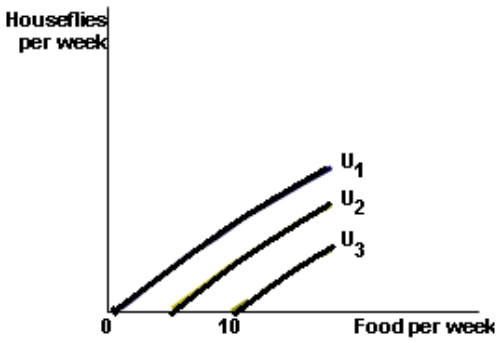
The utility function for these goods: $U\ (X,Y)\ =\ min\{\,a\,X\,,b\,Y\}$

The "min" simply means "take the minimum" of *the two values*

For example,
Suppose $U\ (X,Y)\ =\ min\{2\,X\,,Y\}$, and the consumer have 10 units of good *Y* and 3 units of good *X*. Then we will have, *U*(*X*, *Y*) = min (6, 10) = 6. The consumer derived 6 utile when he consumes 3 units of X and 10 units of Y.

Bad good:

Good for which less is preferred rather than more. Less of them are preferred to more. Air pollution is a bad; asbestos in housing insulation is another.



(b) An economic bad

Budget Constraints

Constraints that consumers face as a result of limited incomes.

The budget line: all combinations of goods for which the total amount of money spent is equal to income.
خط الميزانية هو خط مستقيم يظهر مجموعات مختلفة من سلعتين ينفق عليهما المستهلك كامل دخله.

To see how a budget constraint limits a consumer's choices, let's consider a situation in which a woman has a fixed amount of income, *I*, that can be spent on good X and good Y. Let X be the amount of good x purchases and Y be the amount of goody. We will denote the prices of the two goods P_x and P_y.

If the consumer spent all enter income on the two goods (X and Y), the combinations of good X and good Y that she can by will all lie on this line.

$$Px\,X + Py\,Y = I$$

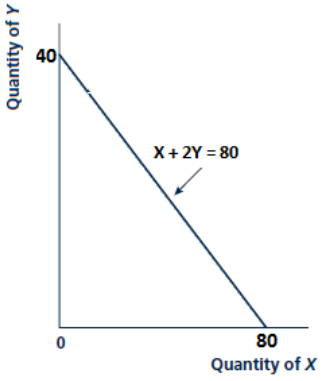
$Px\,X$: Is the amount of money spend on good X

$Py\,Y$: Is the amount of money spend on good Y

Suppose, *for example*, that our consumer has a weekly income of $80, the price of good X is $1 per unit, and the price of good Y is $2 per unit.

The table shows various combinations of good X and good Y that she can purchase each week with her $80.

Market Basket	Good X (X)	Good Y (Y)	Total Spending
A	0	40	1*0 + 2*40 = 80
B	20	30	1*20 + 2*30 = 80
D	40	20	1*40 + 2*20 = 80
E	60	10	1*60 + 2*10 = 80
G	80	0	1*80 + 2*0 = 80



- All points on the budget line are affordable, and the consumer spent all available income.
جميع النقاط التي تقع على خط الميزانية يقدر المستهلك على شرائها، وعند شرائهل يتم انفاق كامل الدخل

- Points in the area below the budget line are affordable, but these leave some portion of income unspent.
النقاط التي تقع تحت خط الميزانية يستطيع المستهلك شرائها، وعند شرائها لا يتم انفاق كامل الدخل

- Points in the area above the budget line are not affordable.
النقاط التي تقع خارج خط الميزانية لا يستطيع المستهلك شراؤها (الدخل غير كافي).

- The downward slope of the budget line shows that the individual can afford more good X only if good Y purchases are cut back.
خط الميزانية منحدر للاسفل بسبب أن المستهلك اذا زاد استهلاكة من السلعة X فإن الكمية المستهلكة من السلعة Y تقل .

The slope of the budget line:

Using the budget line equation: $P_X X + P_Y Y = I$, we can see how much of Y must be give up to consume more of X.

$Y = \frac{I}{P_Y} - \frac{P_X}{P_Y} X$: This is the equation of the budget line; its slope of ($-\frac{P_X}{P_Y}$), and intercept of $\frac{I}{P_Y}$

The slope of the budget line is the negative of the ratio of the prices of the two goods.

Example:
Suppose a person has $80 to spend only on Apples (X) and Oranges (Y). Apples cost $4 each and oranges cost $1 each.

a. Write the budget line equation.

   Budget line: $P_X X + P_Y Y = I$

   $4X + Y = 80 \quad \rightarrow$ *Budget line equation*

b. If this person buys only apples, how many can be bought?

   If this person buys only apples → the amount of oranges purchases equal zero

   $4X = 80 \rightarrow X = 80/4 = 20$ units.

c. If this person buys only oranges, how many can be bought?

   $0 + Y = 80 \rightarrow Y = 80$ units.

d. If the person were to buy 10 apples, how many oranges could be bought with the funds left over?

   $4X + Y = 80 \rightarrow 4 * 10 + Y = 80 \rightarrow 40 + Y = 80 \rightarrow Y = 80 - 40 = 40$ units.

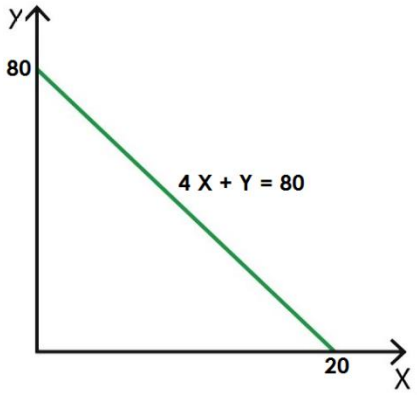
e. Find the slope of the budget line, and graph it.

   Slope = $-\frac{P_X}{P_Y} = -\frac{-4}{1} = -4$

   Vertical intercept = $\frac{I}{P_Y} = \frac{80}{1} = 80$

Horizontal intercept = $\frac{I}{P_X} = \frac{80}{4} = 20$

Graph the budget line: $4X + Y = 80$

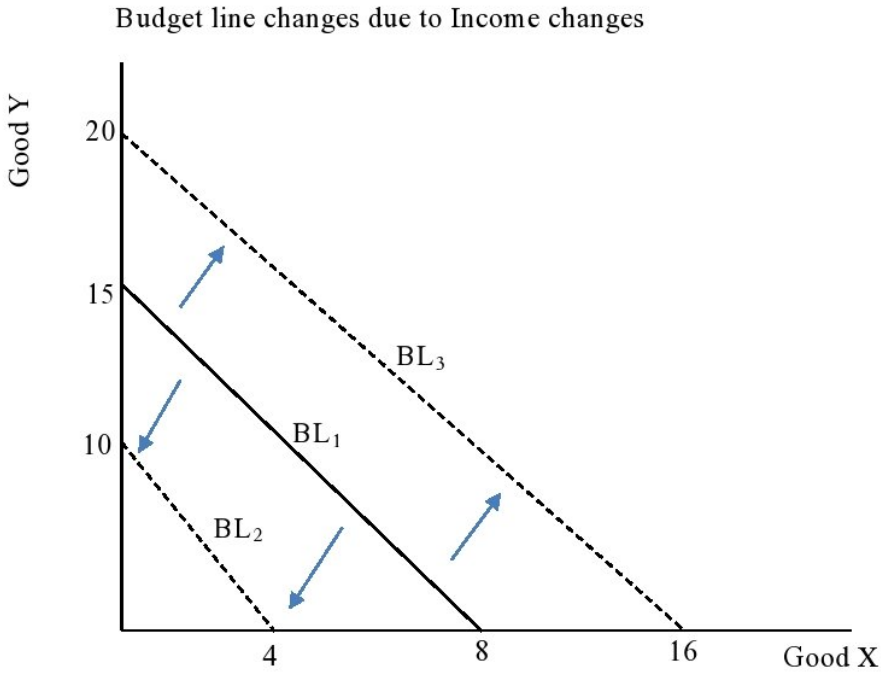


The Effects of Changes in Income and Prices

Income changes:

From the budget line equations we can see that a change in income alters the vertical intercept of the budget line but does not change the slope.

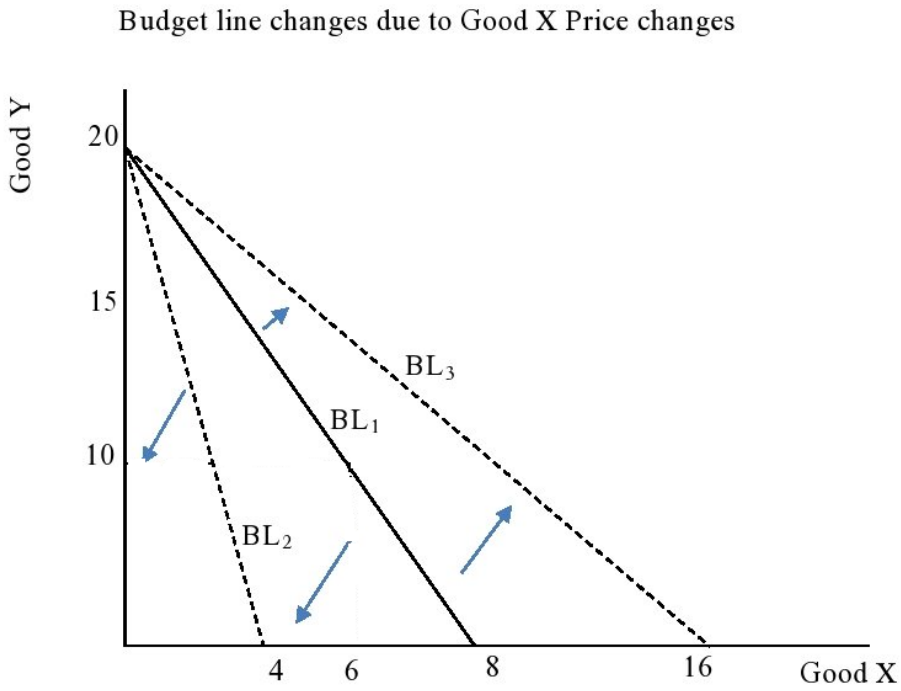
- If income increase, the budget line shifts outward (to the right), from budget line BL_1 to budget line BL_3.
- If income decreases, the budget line shifts inward (to the left), from budget line BL_1 to budget line BL_2.



Price Change:

A change in the price of one good (with income and price of another good unchanged) causes the budget line to rotate about the intercept.

- When the price of food falls, the budget line rotates outward to line BL_3.
- When the price of food increases, the budget line rotates inward to line BL_2.



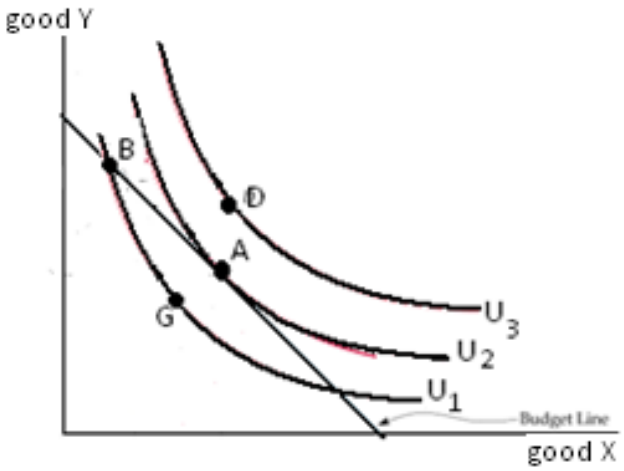
Utility Maximization (Consumer Choice)

Given preferences and budget constraint, we can determine how much consumers buy in order to maximum utility.

The maximizing market basket must satisfy two conditions:

1. It must be located on the budget line.
2. It must give the consumer the most preferred combination of goods and services.

- Point G is affordable but not all of the consumer's income would be spent.
- Point B is affordable but is not on the highest indifference curve that can be reached by the consumer.
- Point D is on a higher indifference curve than A, but is not affordable given the budget constraint.
- Point A is the point that is affordable that lies on the highest indifference curve, so it represents utility maximization.



The basket which maximizes satisfaction must lie on the highest indifference curve that touches the budget line. Point A is the point of tangency between indifference curve and the budget line.

At point A, the slope of the budget line is exactly equal to the slope of the indifference curve.

The satisfaction (utility) is maximized at the point where: MRS = $\frac{P_X}{P_Y}$

Satisfaction is maximized when the marginal rate of substitution (of good X for good Y) is equal to the ratio of the price (of X for Y).

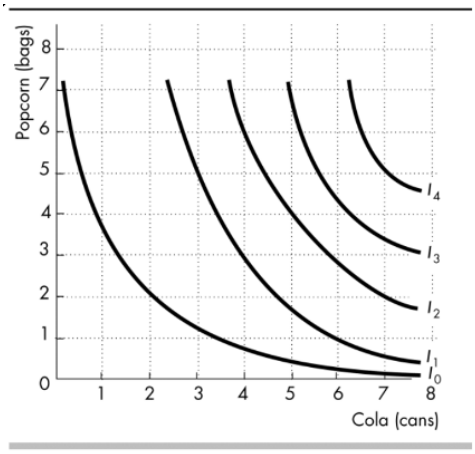
Because the MRS is also equal to the ratio of the marginal utilities of consuming good X and good Y, it follows that:

$$\frac{MUX}{MUY} = \frac{P_X}{P_Y} \quad \text{or} \quad \frac{MUX}{P_X} = \frac{MUY}{P_Y}$$

This equation tells us that utility maximization is achieved when the budget is allocated so that the *marginal utility per dollar* of expenditure is the same for each good.

Example:

Sara's income is \$12 a week. The price of popcorn is \$3 a bag, and the price of cola is \$1.5 a can. Figure below shows Sara's preference map for popcorn and cola.

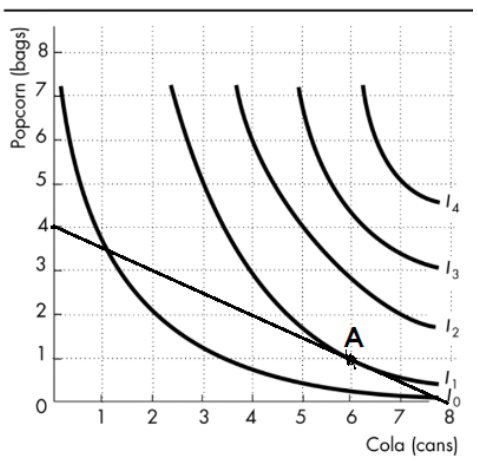


What quantities of popcorn and cola does Sara buy in order to maximize utility? What is Sara's marginal rate of substitution at the point which she consumes?

Budget line equation: $P_X X + P_Y Y = I \rightarrow 1.5X + 3Y = 12$ (X: cola ; Y: popcorn)

Graph the budget line on the same graph: Vertical intercept = $\frac{I}{P_Y} = \frac{12}{3} = 4$,horizontal intercept = $\frac{I}{P_X} = \frac{12}{1.5} = 8$

Utility maximization point is the point of tangency between indifference curve and the budget line, at point A



Popcorn = 1 bag

Cola = 6 cans

MRS = $\frac{Price\ (Cola)}{Price\ (Popcorn)} = \frac{1.5}{3} = 0.5$

Example

Assume the individual can choose between hamburgers (Y) and soft drinks (X) whose prices are PY = $1and PX=$0.5. The individual has $10to spend.

The individual gets measurable utility from X and Y as follows: $U(X,Y) = \sqrt{X\,Y}$

How much hamburgers (Y) and soft drinks (X) should he buy to maximize her utility?

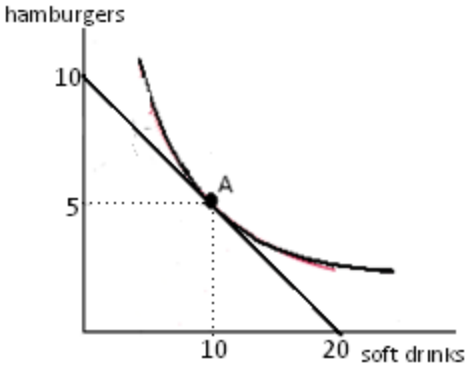
Budget line: $Px\ X + Py\ Y = I \quad \rightarrow \quad 0.5\,X + Y = 10 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$

To maximize utility: $\frac{MUX}{MUY} = \frac{P_X}{P_Y}$

$MUX = \frac{dU(X,Y)}{dx} = \frac{Y}{2\sqrt{X\,Y}}$

$MUY = \frac{dU(X,Y)}{dY} = \frac{X}{2\sqrt{X\,Y}}$

$\frac{\frac{Y}{2\sqrt{X\,Y}}}{\frac{X}{2\sqrt{X\,Y}}} = \frac{0.5}{1} \quad \rightarrow \quad \frac{Y}{X} = \frac{1}{2} \quad \rightarrow \quad X = 2Y$ -------- (2)



Solve equations (1) and (2):

$0.5(2Y) + Y = 10 \quad \rightarrow \quad Y + Y = 10 \quad \rightarrow \quad 2Y = 10 \quad \rightarrow Y = 5\ units$

From equation (2): $X = 2Y \quad \rightarrow \quad X = 2 * 5 = 10\ units$

Another solution:

Budget line equation: $0.5\,X + Y = 10$

Y	X	U(X,Y) = $\sqrt{XY}$
0	20	$\sqrt{0 * 20} = 0$
1	18	$\sqrt{1 * 18} = \sqrt{18}$
2	16	$\sqrt{2 * 16} = \sqrt{32}$
3	14	$\sqrt{3 * 14} = \sqrt{42}$
4	12	$\sqrt{4 * 12} = \sqrt{48}$
5	***10***	$\sqrt{\mathbf{5 * 10}} = \sqrt{\mathbf{50}}$ ***→ max***
6	8	$\sqrt{6 * 8} = \sqrt{48}$
7	6	$\sqrt{7 * 6} = \sqrt{42}$
8	4	$\sqrt{8 * 4} = \sqrt{32}$
9	2	$\sqrt{9 * 2} = \sqrt{18}$
10	0	$\sqrt{10 * 0} = 0$

To max utility the consumer should consume 5 units of Y and 10 units of X.

Example

Jammel receives utility from days spent traveling on vacation domestically (D), and days spend travelling on vacation in a foreign country (F), as given by the utility function $U(D,F) = 10DF$. In addition, the price of a day spend traveling on vacation domestically is \$100, the price of a day spent travelling on vacation in a foreign country is \$400, and Jammel's annual travel budget is \$4000. Find Jammel's utility maximizing choice of days spent traveling domestically and days spent in a foreign country.

Budget line: $P_D\ D + P_F\ F = I \quad \rightarrow \quad 100\ D + 400\ F = 4000 \quad \ldots\ldots\ldots\ldots\ldots\ (1)$

To maximize utility: $\frac{MUD}{MUF} = \frac{P_D}{P_F}$

$MUD = \frac{dU(D,F)}{dD} = 10F$

$MUF = \frac{dU(D,F)}{dF} = 10D$

$\frac{10F}{10D} = \frac{100}{400} \quad \rightarrow \quad \frac{F}{D} = \frac{1}{4} \quad \rightarrow$ D = 4F -------- (2)

Solve equations (1) and (2):

$100(4F) + 400F = 4000 \quad \rightarrow \quad 400F + 400F = 4000 \quad \rightarrow 800F = 4000 \quad \rightarrow F = 5\ days$

From equation (2): $D = 4F \quad \rightarrow \quad D = 4 * 5 = 20\ days$

Example (3):

Taleen consumes only grapefruits and grapes. Her utility function is $U(X,Y) = XY - 2Y$, where X is the number of grapefruits consumed and Y is the number of grapes consumed. Fanny's income is 48, and the prices of grapefruits and grapes are 1 and 3, respectively. How many grapefruits will she consume?

Budget line: $P_X * X + P_Y * Y = I \quad \rightarrow \quad X + 3Y = 48$ ---------- (1)

To maximize utility: $\frac{MUX}{MUY} = \frac{P_X}{P_Y}$

$MUX = \frac{dU(X,Y)}{dX} = Y$

$MUY = \frac{dU(D,F)}{dF} = X - 2$

$\frac{Y}{X-2} = \frac{1}{3}$ (ضرب تبادلي ينتج) $\quad 3Y = X - 2$ -------- (2)

Solve equations (1) and (2): بتعويض المعادلة 2 في المعادلة 1 ينتج

يتم تعويض محل 3Y في المعادلة رقم 1 : X - 2

$X + (X - 2) = 48 \rightarrow \quad X + X - 2 = 48 \quad \rightarrow \quad 2X = 50 \quad \rightarrow \underline{X = 25}$

بتعويض قيمة X في المعادلة رقم 2 ينتج:

From equation (2): $3Y = X - 2 \quad \rightarrow \quad 3Y = 25 - 2 = 23 \rightarrow Y = 23/2 = 11.5$

Example (4):

Cindy consumes good X and good Y. Candy has utility function U(X, Y) = 2XY. His income is \$12, the price of good X is \$2 and the price of good Y is \$1.

How much good X and good Y should he buy to maximize utility?

Budget line: $P_X * X + P_Y * Y = I \rightarrow 2X + Y = 12$ ------- (1)

To maximize utility: $\frac{MUX}{MUY} = \frac{P_X}{P_Y}$

$MUX = \frac{dU(X,Y)}{dX} = 2Y$

$MUY = \frac{dU(D,F)}{dF} = 2X$

$\frac{2Y}{2X} = \frac{2}{1}$ (ضرب تبادلي ينتج) $2Y = 4X$ -------- (2)

Solve equations (1) and (2): بتعويض المعادلة 2 في المعادلة 1 ينتج

يتم تعويض محل 2Y في المعادلة رقم 1 : 4X

$2X + 4X = 12 \rightarrow 6X = 12 \rightarrow X = 2$

بتعويض قيمة X في المعادلة رقم 2 ينتج:

From equation (2): $2Y = 4X \rightarrow 2Y = 4(2) \rightarrow 2Y = 8 \rightarrow Y = 4$

When utility maximization conditions not satisfy

- If the equation (MU per dollar of good X = MU per dollar of good Y) is not fulfilled ((غير متحقق, then some reallocation of the consumer's expenditures between good X and good Y will increase the consumer's total utility.

  If $\frac{MUX}{MUY} > \frac{P_X}{P_Y}$ *or* $\frac{MUX}{P_X} > \frac{MUY}{P_Y}$

  The last dollar spent on good X provide more utility than the last dollar spent on good Y. so *the consumer can increase total utility by purchasing (consuming) more of good X and less of good Y.*

  If $\frac{MUX}{MUY} < \frac{P_X}{P_Y}$ *or* $\frac{MUX}{P_X} < \frac{MUY}{P_Y}$

- The last dollar spent on good Y provide more utility than the last dollar spent on good X. so *the consumer can increase total utility by purchasing (consuming) more of good Y and less of good X.*

Example

Muhanad divides her consumption between orange juice (O) and bagels (B). Orange juice costs $1 per glass and bagels cost $3each. Muhanad consumes positive amounts of both goods, and has chosen consumption quantities where his marginal utility of orange juice is 10 utils per glass, his marginal utility of bagels is 25 utils per bagel, and he spends all her income. Is Muhanad maximizing utility? If no, what he do to increase his utility?

To max utility: $\frac{MUO}{MUB} = \frac{P_O}{P_B}$ ⇨ $\frac{10}{25} \neq \frac{1}{3}$ ⇨ Muhanad does not maximizing utility

But $\frac{MUO}{MUB} > \frac{P_O}{P_B}$ Since, $\frac{10}{25} > \frac{1}{3}$

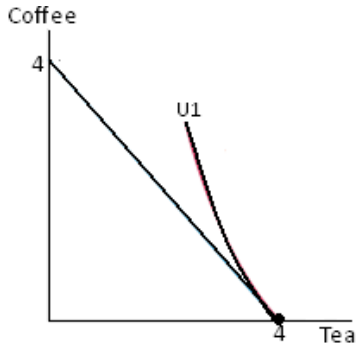
⇨ The last dollar spent on orange juice provides more utility than the last dollar spent on bagels. So the consumer can increase total utility by purchasing (consuming) *more of orange juice and less of bagels*.

Utility maximization for perfect substitute's goods case:

لمعرفة اذا كانت الحالة تمثل حالة السلع البديلة من خلال معادلة منحنى المنفعة (U(X,Y) = aX + bY)

Example

Ms. Caffeine enjoy coffee (C) and tea (T) according to the function $U(C,T) = 3C + 4T$. If coffee and tea cost $3 each and Ms. Caffeine has $12 to spend on these products, how much coffee (C) and tea (T) should she buy to maximize her utility?



Budget line: $Pc\ C + P_T\ T = I$ ⇨ $3C + 3T = 12$ ……(1)

To maximum utility: $\frac{MUC}{MUT} = \frac{P_C}{P_T}$

MUC = $\frac{dU(C,T)}{dC}$ = 3

$MUT = \frac{dU(C,T)}{dT}$ = 4

$\frac{MUC}{MUT} = \frac{P_C}{P_T}$ → $\frac{3}{4} \neq \frac{3}{3}$ ⇨ Condition fail

By using another solution:

C	T	U(C,T) = 3C + 4T
0	**4**	**3*0 + 4 * 4 = 16 ⇨ max**
4	0	3*4 + 4*0 = 12

Ms. Caffeine should consume 4 units of tea and gets 16 utile

Utility maximization for perfect complement goods case:

لمعرفة اذا كانت الحالة تمثل حالة السلع المكملة من خلال معادلة منحنى المنفعة $(U(X,Y) = \min\{X,Y\})$

Example

A consumer consume two goods movies (M) and popcorn (C) according to the utility function : $U(M,C) = min\,(2M,C)$, the price of movies ticket is \$10 and the price of popcorn is \$2.5 and consumer budget is \$30. how much movies and popcorn should she buy to maximize her utility?

Where *''Min''* means that utility is given by the smaller of the two terms in parentheses. If, for example, this person attends a movie but buys no popcorn, utility is zero. If he or she attends a movie and buys three bags of popcorn, utility is 2–the extra bag of popcorn does not raise utility.

Budget line: $PM * M + PC * C = I \quad \Rightarrow \quad 10\,M + 2.5C = 30 \quad ------(1)$

$U(M,C) = min\,(2M,C)$: this person should only consume bundles for which $C = 2M$ (that is, two bags of popcorn for each movie).

$C = 2M \quad ----------(2)$

Solve the 2 equations:

$10\,M + 2.5C = 30$

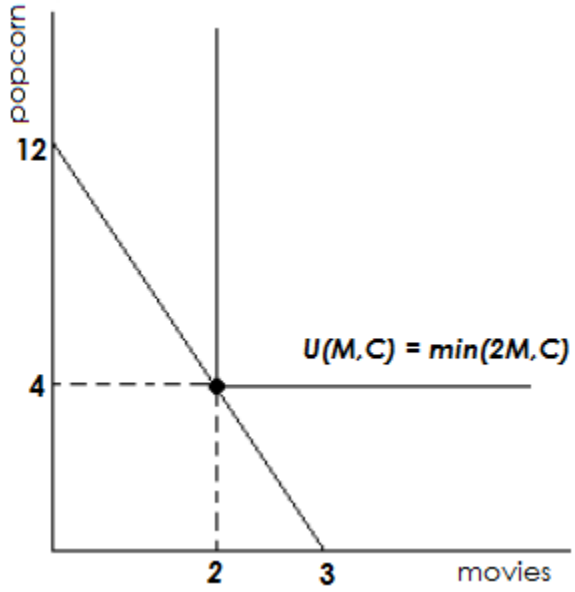
$C = 2M$

بتعويض المعادلة 2 في المعادلة 1 ينتج:

$$10\mathrm{M} + 2.5(2\mathrm{M}) = 30 \;\rightarrow\; 10\mathrm{M} + 5\mathrm{M} = 30 \;\rightarrow\; 15\mathrm{M} = 30 \;\rightarrow\; \mathrm{M} = \frac{30}{15} = 2$$

$$\mathrm{C} = 2\mathrm{M} = 2 * 2 = 4$$

$U(M,C) = min\,(2M,C) = min\,(2*2,4) = = min\,(4,4) = 4$



Example:

Shireen has a monthly income of $20 that she allocates among two goods: Meat (M) and Chickens (C). Suppose also that her utility function is given by the equation U (M,C) = 2M + C

a. Suppose meat cost $2 per pound and chickens cost $4 per pound. Derive her budget line equation and draw her budget constraint putting (M) on the vertical axes (y-axes).
b. Find her marginal rate of substitution.
c. what combination of meat and chickens should she buy to maximize her utility?
d. What is maximum utility will Shireen realized?

Solution:

a. Budget line: $P_M * M + P_C * C = I \Rightarrow 2M + 4C = 20$

b. $MRS = \frac{MUM}{MUC}$

U (M,C) = 2M + C

$MUM = \frac{dU(M.C)}{dM} = 2$

$MUC = \frac{dU(M.C)}{dC} = 1$

$MRS = \frac{MUM}{MUC} = \frac{2}{1} = 2$ (المستهلك مستعد ان يضحي (يقلل الاستهلاك) وحدتين من السلعة سي مقابل حصولة على سلعة اضافية من السلعة أم)

c. U (M,C) = 2M + C

هذا شكل منحنى المنفعة ل السلع البديلة وبالتالي طريقة الحل هي:

Budget line: $PM * M + P_C * C = I \Rightarrow 2M + 4C = 20$

نستخدم فقط قيمتين : من معادلة خط الميزانية يتم تعويض في المره الاولى قيمة Mصفر وبالتالي تكون قيمة C = 4.

$$2(0) + 4C = 20 \rightarrow C = 20/4 = 5$$

من معادلة خط الميزانية يتم تعويض في المره الثانية قيمة Cصفر وبالتالي تكون قيمة M= 10.

وبعدها بتم تعويض قيمة Mو قيمة C في منحنى المنفقة U(T,C) = 2M + C

كما في الجدول في الاسفل واعلى قيمة ل U بين القيمتين هي نقطة تعظيم المنفعة.

M	C	U(C,T) = 2M+C
0	5	3(0)+5 = 5
10	0	2(10)+0 = 20 ⇨ max

Shireen should consume 10 units of meat and 0 unit of Chicken to get maximize utility.