

UTILITY

Utility define as the satisfaction that a person receives from his or her economic activities.

المنفعة : عبارة عن الإشباع الذي يحصل عليه الفرد عند استهلاكه سلعة أو خدمة معينة

Utility from Consuming Two Goods

Assume that the person receives utility from consuming two goods X and Y. The utility function is given by:

$$Utility = U(X, Y ; other things)$$

This notation indicates that the utility an individual receives from consuming X and Y over some period of time depends on the quantities of X and Y consumed and on "other things."

These other things might include easily quantifiable items such as the amounts of other kinds of goods consumed, the number of hours worked, or the amount of time spent sleeping. They might also include such unquantifiable items as love, security, and feelings of self-worth.

تعتمد المنفعة التي يحصل عليها المستهلك من استهلاكه سلعتين X و Y خلال فترة زمنية على الكمية التي يستهلكها من السلعة X و الكمية التي يستهلكها من السلعة Y و على أشياء أخرى. تشمل الأشياء الأخرى على عناصر قابلة للقياس بسهولة مثل كميات من أنواع أخرى من السلع التي يستهلكها الشخص، وعدد ساعات العمل، أو مقدار الوقت الذي يقضيه في النوم. وقد تشمل أيضا أشياء لا يمكن تحديدها كمياً كالحب، والأمن، ومشاعر الذاتية.

These other things appear after the semicolon (بعد الفاصلة المنقوطة) because we assume that they do not change (*other things equal*) while we look at the individual's choice between X and Y.

By assuming other things equal, the utility function form become:

$$Utility = U(X, Y) \quad \text{-----} \rightarrow \text{Utility function}$$

Example:

Suppose that Ahmad utility function for consuming to goods; food (F) and clothing (C) is given by the equation: $U(F, C) = F + 2C$. In that case, if Ahmad consume 8 units of food and 3 units of clothing, he gets a utility of $U(F, C) = 8 + 2(3) = 14$.

Measuring Utility قياس المنفعة

A quantification of the satisfaction of wants and needs achieved through the consumption of goods and services. In principle, utility measurement can take one of two forms: (1) cardinal, which is based on numerical values (1, 2, 3, etc.) and (2) ordinal which is based on rankings (first, second, third, etc.). While the

hypothetical instructional analysis of utility relies on cardinal utility; ordinal utility is a more realistic way to measure satisfaction.

Ordinal versus Cardinal Utility المنفعة الترتيبية والمنفعة العددية

A utility that generates a ranking of goods or market baskets is called ordinal utility, for example, good A is preferred to good B. or ranking a utility from the same goods by comparing the 1st unit and 2nd unit from utility.

تقوم المنفعة الترتيبية على المقارنة المشتقة من السلع أو مقارنة المنفعة المشتقة من الوحدة الأولى و الوحدة الثانية من نفس السلعة. فعلى سبيل المثال يمكن القول بأن المنفعة التي يحصل عليها ماجد من تفاحة أكبر من المنفعة التي يحصل عليها من برتقالة. وكذلك يمكن القول بأن المنفعة التي يحصل عليها ماجد من كوب الشاي الأول أكبر من المنفعة التي يحصل عليها من كوب الشاي الثاني.

A utility function that describes by how much one market basket is preferred to another is called cardinal utility. For example, if the consumer can say, the total utility derived from consuming good A is equal to 20, the utility is cardinal.

تقوم المنفعة العددية على التعبير عن عدد وحدات المنفعة المشتقة من سلعة ما عند استهلاكها. فعلى سبيل المثال يمكن أن نقول بأن عدد وحدات المنفعة التي يحصل عليها ماجد عند استهلاكه سلعة معينة تساوي 20 وحدة منفعة.

Consumer Preferences تفضيلات المستهلك

Some Basic Assumptions about Preferences:

- *Completeness*: Preferences are assumed to be *complete*. In other words, consumers can compare and rank all possible baskets.

Between two consumption bundles, A and B, we might expect a person to be able to state clearly "I prefer A to B" or "I prefer B to A" or "A and B are equally attractive to me"(will be indifferent between the two).

Note that these preferences ignore costs. A consumer might prefer steak to hamburger but buy hamburger because it is cheaper.

- *Transitivity*: Preferences are *transitive*. Transitivity means that if a consumer prefers basket A to basket B and basket B to basket C, then the consumer also prefers A to C.

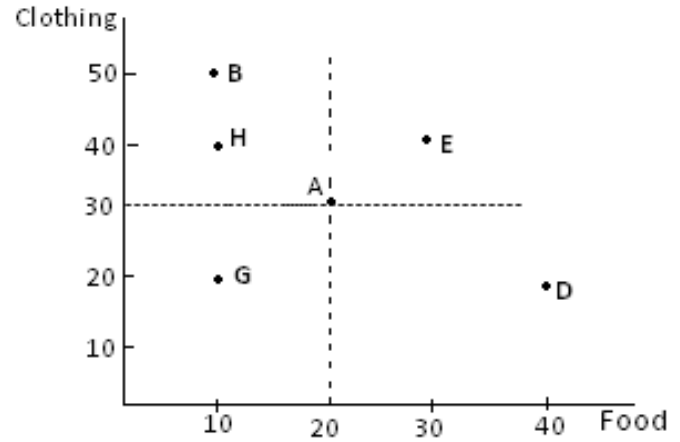
Example

If a man prefers Tea to Pepsi and prefers Pepsi to Orange Juice, but he is indifferent between Tea and Coffee, transitivity of preferences means that he must

- prefer orange juice to tea
- prefer coffee to orange juice**
- prefer Pepsi to coffee
- be indifferent between tea and orange juice
- be indifferent between Pepsi and coffee

- *More is better than less*: consumers always prefer more of any good to less. In addition, consumers are never satisfied or satiated; more is always better, even if just a little better. Of course, some goods, such as air pollution, may be undesirable, and consumers will always prefer less.

Market basket A, with 20 units of food and 30 units of clothing, is preferred to basket G because A contains more food and more clothing (recall our third assumption that more is better than less). Similarly, market basket E, which contains even more food and even more clothing, is preferred to A. In fact, we can easily compare all market baskets in the two shaded areas (such as F and G) to A because they contain either more or less of both food and clothing. Note, however, that B contains more clothing but less food than A. Similarly, D contains more food but less clothing than A. Therefore, comparisons of market basket A with baskets B, D, and H are not possible without more information about the consumer's ranking.



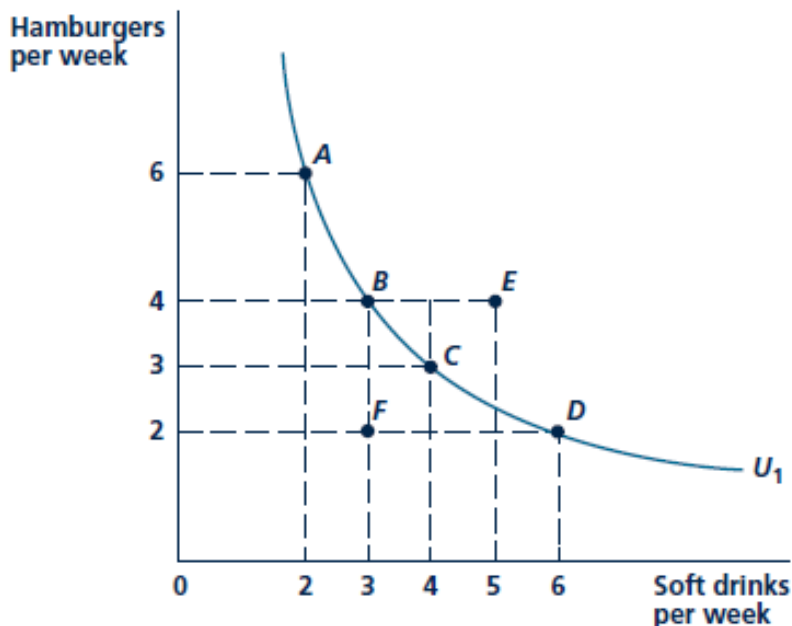
إن من إحدى البديهيات حول سلوك المستهلك أنه يفضل الأكثر على الأقل عند استهلاكه. بمعنى، أنه لو حصل المستهلك على كمية أكبر من إحدى السلعتين التي يستهلكها دون أن يقلل استهلاكه من السلعة الأخرى، فإن ذلك سيزيد من منفعته. أما إذا حصل المستهلك على كمية أقل من إحدى السلعتين التي يستهلكها دون أن يزيد استهلاكه من السلعة الأخرى، فإن ذلك سوف يقلل من منفعته. فالمستهلك يفضل السلة الغذائية (point A) التي تحتوي على 20 وحدة من الطعام و 30 وحدة من الملابس على السلة الغذائية (point G) التي تحتوي على 10 وحدات طعام و 20 وحدة ملابس. كذلك يفضل المستهلك النقطة E على النقطة A وذلك لأن المستهلك يحصل على كمية أكبر من السلعتين. ولكن ليس باستطاعة المستهلك أن يقارن الخيار A مع الخيار H أو الخيار B.

Indifference Curve

A curve that shows all the combinations of goods or services (market baskets) that provides the same level of utility.

هو ذلك المنحنى الذي يمثل المجموعات المختلفة (من سلعتين) الذي يحصل المستهلك عند استهلاكها على نفس المستوى من المنفعة.

- The curve U_1 in the Figure includes all combinations of Hamburgers and Soft drink that yield the same level of utility.
- Point A, with 6 units of Hamburgers and 2 units of Soft drink, has the same utility as point B, 4 Hamburgers and 3 Soft drink. Since all points on the curve (A, B, C and D) yield the same utility, the person has no reason to prefer one point over another.

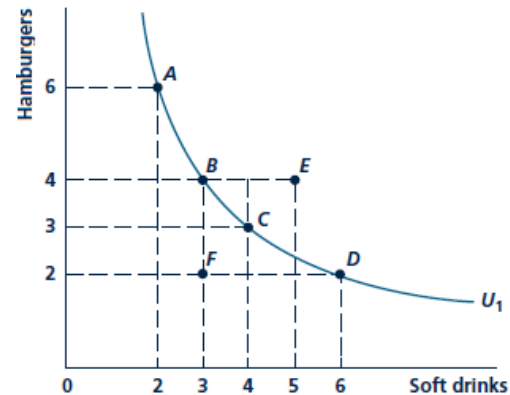


Points above and below an Indifference Curve:

In the Figure, points such as E are above of U_1 . Point E has more of both goods than point C \rightarrow E is preferred to A (more is better). Because of transitivity, E is preferred to A, B, and D.

Points above an indifference curve are preferred to points on the curve.

In the Figure, points such as F are below of an indifference curve U_1 . Point C is preferred to point F since it contains more of both goods. Because of transitivity, all points on U_1 (A, B, C and, D) are preferred to point F.



Points on an indifference curve are preferred to points below it.

Example

Consider the utility function for goods X and Y as given by $U(X, Y) = XY$. Which of the following bundles lie on the same indifference curve as the bundle containing 2 units of X and 4 units of Y?

- the bundle containing 5 units of X and 1 unit of Y
- the bundle containing 1 unit of X and 8 units of Y
- the bundle containing 4 units of X and 2 units of Y
- both b and c

Answer:

All the combinations of goods or services that provides the same level of utility lie on the same indifference curve

The bundle containing 2 units of X and 4 units of Y provide a $U(X, Y) = 2 * 4 = 8$

The bundle containing 5 units of X and 1 unit of Y provide a $U(X, Y) = 1 * 5 = 5$

The bundle containing 1 units of X and 8 unit of Y provide a $U(X, Y) = 1 * 8 = 8$

The bundle containing 4units of X and 2unit of Y provide a $U(X, Y) = 2 * 4 = 8$

Both the two bundle b and c provide the same level of utility as the bundle containing 2 units of X and 4 units of Y.

Example

Suppose the person has preferences for apples (A) and orange (O) given by: $Utility = \sqrt{A O}$

- If A = 5 and O = 80, what will utility be?

$$Utility = \sqrt{5 * 80} = \sqrt{400} = 20$$

- If A = 10, what value for O will provide that same utility as in part a?

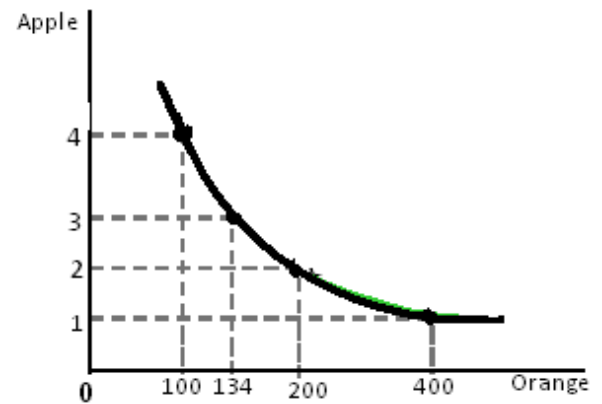
$$Utility = \sqrt{A O} \rightarrow 20 = \sqrt{10 O} \rightarrow 10 O = 400 \rightarrow O = \frac{400}{10} = 40$$

c. Graph the indifference curve when $U = 20$.

$$\text{When } U = 20 \rightarrow 20 = \sqrt{A \cdot O}$$

$$\text{بتربيع الطرفين} \rightarrow 400 = A \cdot O \rightarrow O = \frac{400}{A}$$

A	O
1	$400/1 = 400$
2	$400/2 = 200$
3	$400/3 = 133.4$
4	$400/4 = 100$

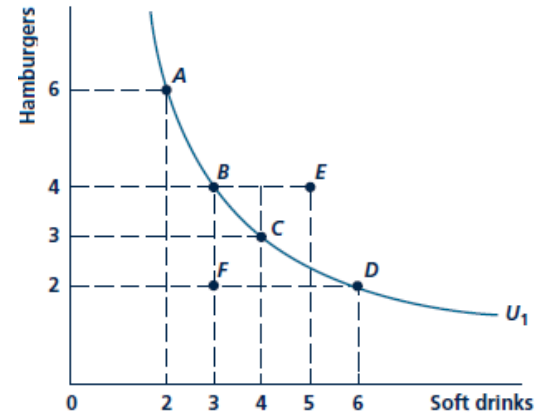


Indifference Curve and the Marginal Rate of Substitution (MRS)

What happens when a person moves from point A (six hamburgers and two soft drinks) to point B (four hamburgers and three soft drinks)? This person remains equally well-off because the two commodity bundles lie on the same indifference curve.

This person will voluntarily give up two of the hamburgers that were being consumed at point A in exchange for one additional soft drink. The slope of the curve U_1 between A and B is therefore approximately -2 . That is, Y (hamburgers) declines two units in response to a one-unit increase in X (soft drinks).

The absolute value of indifference curve slope = marginal rate of substitution (MRS)



Marginal rate of substitution (MRS) = The rate at which an individual is willing to reduce consumption of one good when he or she gets one more unit of another good. The negative of the slope of an indifference curve.

هو عبارة عن عدد الوحدات التي يتخلى عنها المستهلك من سلعة في مقابل الحصول على وحدة إضافية من سلعة أخرى مع المحافظة على نفس مقدار المنفعة.

Marginal rate of substitution (MRS) = the absolute value of slope of indifference curve

$$\text{The MRS (of soft drinks for hamburgers) between points A and B} = \frac{\Delta H}{\Delta S} = \left| \frac{4-6}{3-2} \right| = 2$$

This person is willing to give up 2 hamburgers' in order to get one more unit of soft drinks.

المستهلك مستعد التخلي عن وحدتين من هامبورجر مقابل الحصول على وحدة إضافية من المشروبات الغازية

$$\text{The MRS between points B and C} = \frac{\Delta H}{\Delta S} = \left| \frac{3-4}{4-3} \right| = 1$$

This person is willing to give up 1 hamburger's in order to get one more unit of soft drinks.

$$\text{The MRS between points C and D} = \frac{\Delta H}{\Delta S} = \left| \frac{2-3}{6-4} \right| = \frac{1}{2}$$

This person is willing to give up ½ hamburger's in order to get one more unit of soft drinks.

Diminishing Marginal Rate of Substitution

The MRS diminishes (decreases) along an indifference curve moving from point A to point D.

The MRS between points A and B is equal 2

The MRS between points B and C is equal 1

The MRS between points C and D is equal ½

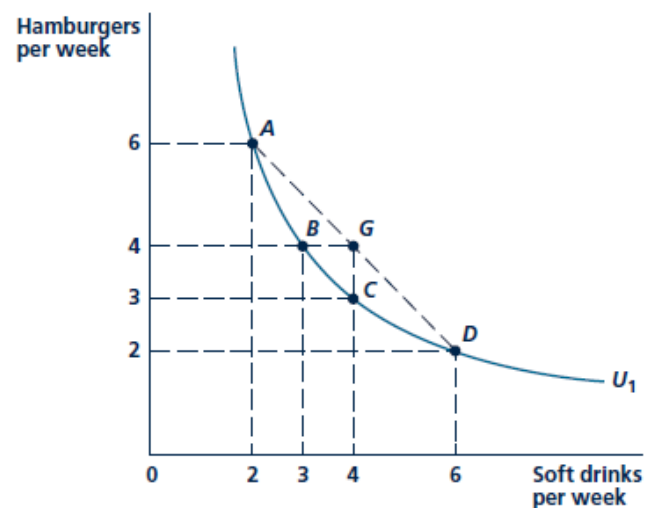
→ As we move from point A to D MRS diminishing.

The Shape of Indifference Curves

Indifference curves are downward sloping. In our example of hamburgers and soft drinks, when the amount of soft drinks increases along an indifference curve, the amount of hamburger decreases. The fact that indifference curves slope downward follows directly from our assumption that more of a good is better to less. If an indifference curve sloped upward, a consumer would be indifferent between two market baskets even though one of them had more of both hamburger and soft drink.

Balance in Consumption

The conclusion of a diminishing MRS is based on the idea that people prefer balanced consumption bundles to unbalanced ones. This assumption is illustrated precisely in Figure below, where the indifference curve U₁ is redrawn. Our discussion here concerns the two extreme consumption options A and D. In consuming A, this person gets six hamburgers and two soft drinks; the same satisfaction could be received by consuming D (two hamburgers and six soft drinks). Now consider a bundle of commodities (say, G) “between” these extremes. With G (four hamburgers and four soft drinks), this person obtains a higher level of satisfaction (point G is northeast of the indifference curve U₁) than with either of the extreme bundles A or D. The reason for this increased satisfaction should be geometrically obvious.



All of the points on the straight line joining A and D lie above U₁. Point G is one of these points (as the figure shows, there are many others). As long as the indifference curve obeys the assumption of a diminishing MRS, it will have the type of convex shape shown in Figure below. Any consumption bundle that represents an “average” between two equally attractive extremes will be preferred to those extremes. The assumption of a diminishing MRS (or convex indifference curves) reflects the notion that people prefer variety in their consumption choices.

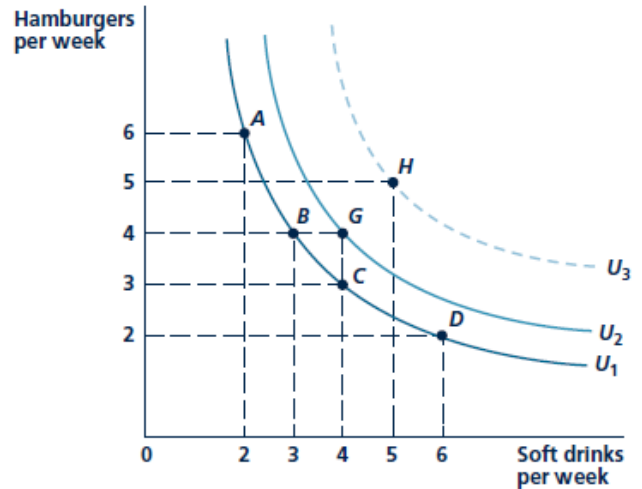
Indifference Curve Maps

To describe a person's preferences for all combinations of hamburger and soft drink, we can graph a set of indifference curves called an indifference map.

Indifference map: Graph containing a set of indifference curves showing the market basket among which a consumer is indifferent.

خريطة منحنيات السواء هي مجموعة منحنيات السواء للمستهلك والتي تظهر تفضيله أو ذوقه تجاه السلع التي يستهلكها.

Any market basket on indifference curve U_3 , such as basket H, is preferred to any basket on curve U_2 (e.g., basket G), which in turn is preferred to any basket on U_1 , such as C.

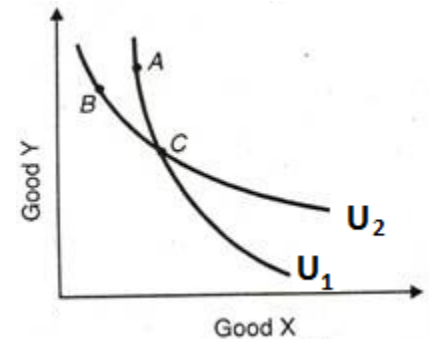


$$U_3 > U_2 > U_1$$

➤ Point H is preferred to point G, and point G is preferred to point C

Indifference Curves Cannot Intersect

If indifference curves U_1 and U_2 intersect, one of the assumptions of consumer theory is violated. According to this diagram, the consumer should be indifferent among market baskets A, B, and C. Yet A should be preferred to B because A has more of both goods. Thus, intersecting indifference curves contradict our assumption that more is preferred to less.



Marginal Rate of Substitution (MRS) and Marginal Utility:

Marginal utility (MU): The additional satisfaction obtained from consuming one additional unit of a good.

The marginal utility of good X (MUX): The extra utility obtained by consuming one more unit of good X.

$$MUX = \frac{dU(X,Y)}{dx}$$

The marginal utility of good Y (MUY): The extra utility obtained by consuming one more unit of good Y.

$$MUY = \frac{dU(X,Y)}{dY}$$

$$MRS = \frac{MUX}{MUY}$$

Example

Maher has \$80 to spend on eggs (E) and meat (M). Eggs are \$4 each and a strip of meat is \$2. His utility function is given by the following: $U(E, M) = E * M$. What is the Maher's MRS between meat and eggs at the bundle (15 eggs and 3 meats)?

$$\text{MRS between meat and eggs} = \frac{MUE}{MUM}$$

$$MUE = \frac{dU(E, M)}{dE} = M$$

$$MUM = \frac{dU(E, M)}{dM} = E$$

$$\text{MRS} = \frac{M}{E} = \frac{3}{15} = \frac{1}{5}$$

This person is willing to give up 1/5 meat in order to get one more eggs). Or this person is willing to give up 1 meat in order to get 5 more eggs).

المستهلك مستعد للتخلي عن 1/5 كيلو من اللحم مقابل الحصول على وحدة إضافية من البيض (او المستهلك مستعد للتخلي عن وحدة واحدة من اللحم مقابل الحصول على 5 وحدات إضافية من البيض)

Example

The price of gum is 20 cents and the price of candy is 10 cents. A student is willing to give up 2 pieces of gum (vertical axis) for 1 piece of candy (horizontal axis). Money income is \$2. What is the marginal rate of substitution?

MRS: The rate at which an individual is willing to reduce consumption of one good (vertical axis) when he or she gets one more unit of another good (horizontal axis).

$$\rightarrow \text{MRS} = 2/1 = 2$$

Example

A consumer spends his income to purchase two goods, X and Y. His preferences are represented by the following utility function $U(X, Y) = \frac{1}{2}X - Y^2$. What is the MRS when the consumer consume 10 units of X and 2 units of Y?

$$\text{MRS} = \frac{MUX}{MUY}$$

$$MUX = \frac{dU(X, Y)}{dX} = \frac{1}{2}$$

$$MUY = \frac{dU(X, Y)}{dY} = -2Y$$

$$\text{MRS} = \frac{MUX}{MUY} = \frac{\frac{1}{2}}{-2Y} = \frac{1}{-4Y}$$

$$\text{MRS} = \frac{1}{-4Y} = \frac{1}{-4(2)} = \frac{-1}{8}$$

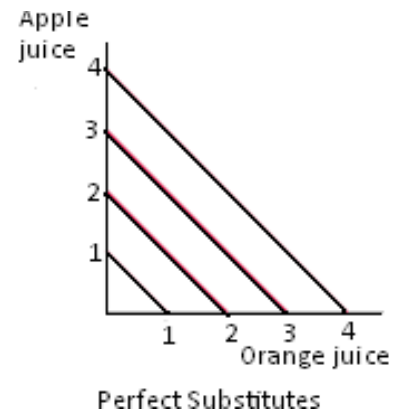
Illustrating Particular Preferences

Perfect Substitutions and Perfect Complement

The consumer views orange juice and apple juice as perfect substitutes: He is always indifferent between a glass of one and a glass of the other.

The utility function for these goods: $U(X, Y) = aX + bY$

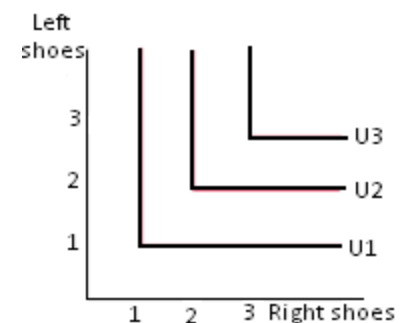
Perfect substitutes: Two goods for which the marginal rate of substitution of one for the other is a constant.



Perfect complements: Two goods for which the MRS is infinite; the indifference curves are shaped as right angles.

Jane views left shoes and right shoes as perfect complements: An additional left shoe gives her no extra satisfaction unless she also obtains the matching right shoe.

In the Figure, the two goods are perfect complements in that they must be used together (like left and right shoes) to gain utility. If for example, he currently has three pairs of those, additional right shoes provide no more utility. Similarly, additional left shoes alone provide no additional utility, since this individual likes to consume these two goods together.

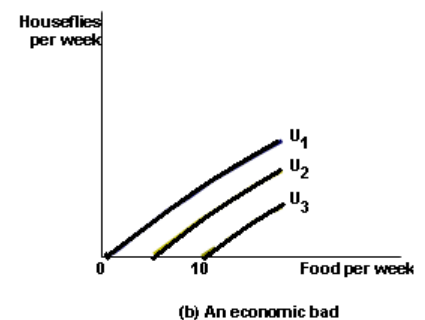


The utility function for these goods: $U(X, Y) = \min \{ aX, bY \}$

The “min” simply means “take the minimum” of *the two values*

For example, suppose $U(X, Y) = \min \{ 2X, Y \}$, and the consumer have 10 units of good Y and 3 units of good X . Then we will have, $U(X, Y) = \min(6, 10) = 6$. The consumer derived 6 utility when he consumes 3 units of X and 10 units of Y .

Bad good: Good for which less is preferred rather than more. Less of them are preferred to more. Air pollution is a bad; asbestos in housing insulation is another.



Budget Constraints

Constraints that consumers face as a result of limited incomes.

The budget line: all combinations of goods for which the total amount of money spent is equal to income.
خط الميزانية هو خط مستقيم يظهر مجموعات مختلفة من سلعتين ينفق عليهما المستهلك كامل دخله.

To see how a budget constraint limits a consumer's choices, let's consider a situation in which a woman has a fixed amount of income, I , that can be spent on good X and good Y. Let X be the amount of good X purchases and Y be the amount of good Y. We will denote the prices of the two goods P_x and P_y .

If the consumer spent all her income on the two goods (X and Y), the combinations of good X and good Y that she can buy will all lie on this line. $P_x X + P_y Y = I$

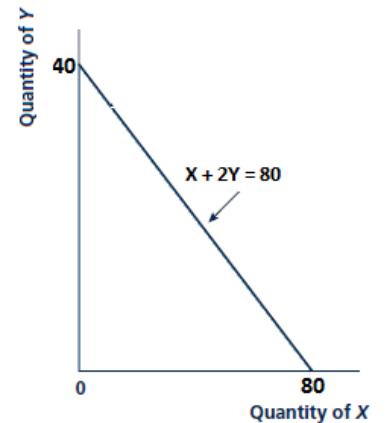
$P_x X$ = is the amount of money spent on good X

$P_y Y$ = is the amount of money spent on good Y

Suppose, *for example*, that our consumer has a weekly income of \$80, the price of good X is \$1 per unit, and the price of good Y is \$2 per unit.

The table shows various combinations of good X and good Y that she can purchase each week with her \$80.

Market Basket	good X (X)	good Y (Y)	Total Spending
A	0	40	$1 \cdot 0 + 2 \cdot 40 = 80$
B	20	30	$1 \cdot 20 + 2 \cdot 30 = 80$
D	40	20	$1 \cdot 40 + 2 \cdot 20 = 80$
E	60	10	$1 \cdot 60 + 2 \cdot 10 = 80$
G	80	0	$1 \cdot 80 + 2 \cdot 0 = 80$



- All points on the budget line are affordable, and the consumer spent all available income.
كل النقاط التي تقع على خط الميزانية يقدر المستهلك على شرائها، كذلك ينفق المستهلك على شرائها كامل دخله.
- Points in the area below the budget line are affordable, but these leave some portion of income unspent.
النقاط التي تقع تحت خط الميزانية (داخل المثلث) يستطيع المستهلك شرائها، ولكن لا ينفق عليها كامل دخله.
- Points in the area above the budget line are not affordable.
النقاط التي تقع فوق خط الميزانية لا يستطيع المستهلك شرائها (دخلة غير كافية).
- The downward slope of the budget line shows that the individual can afford more good X only if good Y purchases are cut back.

يظهر خط الميزانية بشكل منحدر للأسفل وذلك لأن المستهلك يستطيع زيادة استهلاكه من السلعة X فقط إذا قل استهلاكه من السلعة Y

The slope of the budget line:

Using the budget line equation: $P_X X + P_Y Y = I$, we can see how much of Y must be give up to consume more of X.

$Y = \frac{I}{P_Y} - \frac{P_X}{P_Y} X$: This is the equation of the budget line; its slope of $(-\frac{P_X}{P_Y})$, and intercept of $\frac{I}{P_Y}$

→ *The slope of the budget line is the negative of the ratio of the prices of the two goods.*

Example:

Suppose a person has \$80 to spend only on apples and oranges. Apples cost \$4 each and oranges cost \$1 each.

a. Write the budget line equation.

$$\text{Budget line: } P_A * A + P_O * O = I$$

$$4A + O = 80 \quad \text{----- budget line equation}$$

b. If this person buys only apples, how many can be bought?

If this person buys only apples → the amount of oranges purchases equal zero

$$\rightarrow 4A = 80 \quad \rightarrow A = 80/4 = 20 \text{ units.}$$

c. If this person buys only oranges, how many can be bought?

$$0 + O = 80 \quad \rightarrow \quad O = 80 \text{ units.}$$

d. If the person were to buy 10 apples, how many oranges could be bought with the funds left over?

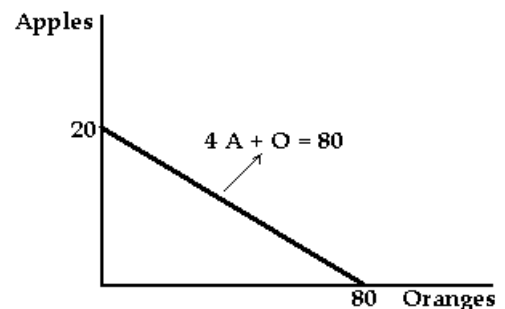
$$4A + O = 80 \quad \rightarrow 4 * 10 + O = 80 \quad \rightarrow 40 + O = 80 \quad \rightarrow O = 80 - 40 = 40 \text{ units.}$$

e. Find the slope of the budget line, and graph it (put apples on the vertical axis and orange in the horizontal axis).

$$\text{Slope} = -P_A / P_O = -4/1 = -4$$

$$\text{Vertical intercept} = I / P_A = 80/4 = 20$$

$$\text{Horizontal intercept} = I / P_O = 80/1 = 80$$

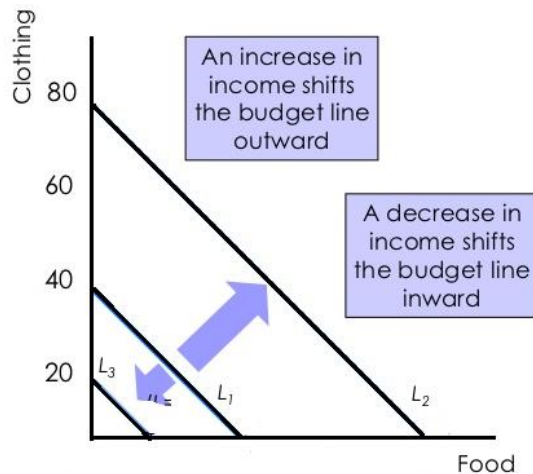


The Effects of Changes in Income and Prices

Income changes:

From the budget line equations, we can see that a change in income alters the vertical intercept of the budget line but does not change the slope.

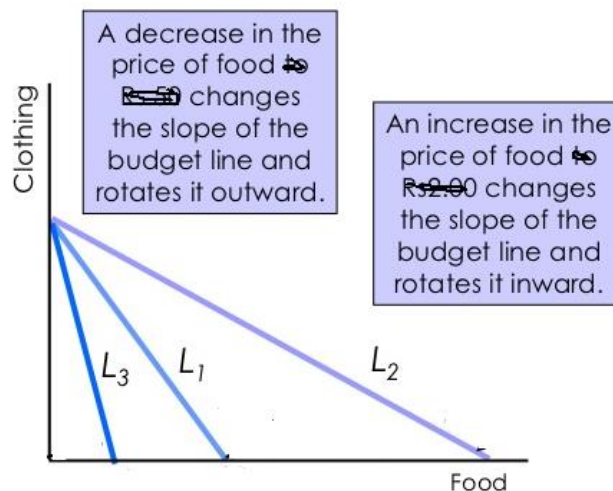
- *If income increase, the budget line shifts outward (to the right), from budget line L_1 to budget line L_2 .*
- *If income decreases, the budget line shifts inward (to the left), from budget line L_1 to budget line L_3 .*



Price Change:

A change in the price of one good (with income and price of another good unchanged) causes the budget line to rotate about the intercept.

- *When the price of food falls, the budget line rotates outward to line L_2 .*
- *When the price of food increases, the budget line rotates inward to line L_3 .*



Marginal Rate of Substitution (MRS) and Marginal Utility:

Marginal utility (MU): The additional satisfaction obtained from consuming one additional unit of a good.

The marginal utility of good X (MUX): The extra utility obtained by consuming one more unit of good X.

$$MUX = \frac{dU(X,Y)}{dx}$$

The marginal utility of good Y (MUY): The extra utility obtained by consuming one more unit of good Y.

$$MUY = \frac{dU(X,Y)}{dY}$$

$$MRS = \frac{MUX}{MUY}$$

Example

Maher has \$80 to spend on eggs (E) and meat (M). Eggs are \$4 each and a strip of meat is \$2. His utility function is given by the following: $U(E, M) = E * M$. What is the Maher's MRS between meat and eggs at the bundle (15 eggs and 3 meats)?

$$\text{MRS between meat and eggs} = \frac{MUE}{MUM}$$

$$MUE = \frac{dU(E,M)}{dE} = M$$

$$MUM = \frac{dU(E,M)}{dM} = E$$

$$MRS = \frac{M}{E} = \frac{3}{15} = \frac{1}{5}$$

This person is willing to give up 1/5 meat in order to get one more eggs). Or this person is willing to give up 1 meat in order to get 5 more eggs).

المستهلك مستعد للتخلي عن 1/5 كيلو من اللحم مقابل الحصول على وحدة إضافية من البيض (او المستهلك مستعد للتخلي عن وحدة واحدة من اللحم مقابل الحصول على 5 وحدات إضافية من البيض)

Example

The price of gum is 20 cents and the price of candy is 10 cents. A student is willing to give up 2 pieces of gum (vertical axis) for 1 piece of candy (horizontal axis). Money income is \$2. What is the marginal rate of substitution?

MRS: The rate at which an individual is willing to reduce consumption of one good (vertical axis) when he or she gets one more unit of another good (horizontal axis).

$$\rightarrow MRS = 2/1 = 2$$

Example

A consumer spends his income to purchase two goods, X and Y. His preferences are represented by the following utility function $U(X, Y) = \frac{1}{2}X - Y^2$. What is the MRS when the consumer consume 10 units of X and 2 units of Y?

$$MRS = \frac{MUX}{MUY}$$

$$MUX = \frac{dU(X,Y)}{dX} = \frac{1}{2}$$

$$MUY = \frac{dU(X,Y)}{dY} = -2Y$$

$$MRS = \frac{MUX}{MUY} = \frac{\frac{1}{2}}{-2Y} = \frac{1}{-4Y}$$

$$MRS = \frac{1}{-4Y} = \frac{1}{-4(2)} = \frac{-1}{8}$$

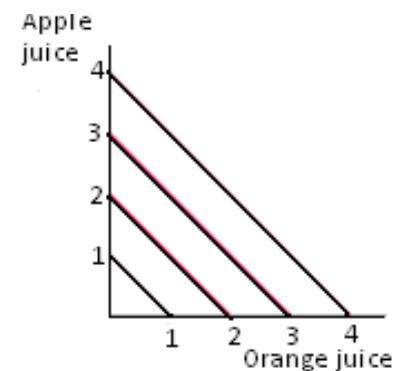
Illustrating Particular Preferences

Perfect Substitutions and Perfect Complement

The consumer views orange juice and apple juice as perfect substitutes: He is always indifferent between a glass of one and a glass of the other.

The utility function for these goods: $U(X, Y) = aX + bY$

Perfect substitutes: Two goods for which the marginal rate of substitution of one for the other is a constant.



Perfect Substitutes

Perfect complements: Two goods for which the MRS is infinite; the indifference curves are shaped as right angles.

Jane views left shoes and right shoes as perfect complements: An additional left shoe gives her no extra satisfaction unless she also obtains the matching right shoe.

In the Figure, the two goods are perfect complements in that they must be used together (like left and right shoes) to gain utility. If for example, he currently has three pairs of those, additional right shoes provide no more utility. Similarly, additional left shoes alone provide no additional utility, since this individual likes to consume these two goods together.

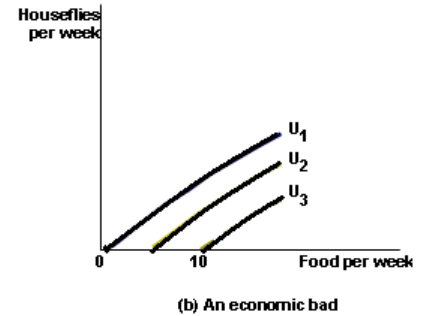


The utility function for these goods: $U(X, Y) = \min \{ aX, bY \}$

The “min” simply means “take the minimum” of *the two values*

For example, suppose $U(X, Y) = \min\{2X, Y\}$, and the consumer have 10 units of good Y and 3 units of good X . Then we will have, $U(X, Y) = \min(6, 10) = 6$. The consumer derived 6 utility when he consumes 3 units of X and 10 units of Y .

Bad good: Good for which less is preferred rather than more. Less of them are preferred to more. Air pollution is a bad; asbestos in housing insulation is another.



Budget Constraints

Constraints that consumers face as a result of limited incomes.

The budget line: all combinations of goods for which the total amount of money spent is equal to income.
خط الميزانية هو خط مستقيم يظهر مجموعات مختلفة من سلعتين ينفق عليهما المستهلك كامل دخله.

To see how a budget constraint limits a consumer's choices, let's consider a situation in which a woman has a fixed amount of income, I , that can be spent on good X and good Y . Let X be the amount of good X purchases and Y be the amount of good Y . We will denote the prices of the two goods P_x and P_y .

If the consumer spent all her income on the two goods (X and Y), the combinations of good X and good Y that she can buy will all lie on this line. $P_x X + P_y Y = I$

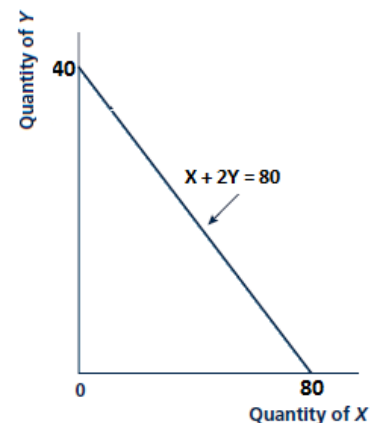
$P_x X =$ is the amount of money spent on good X

$P_y Y =$ is the amount of money spent on good Y

Suppose, *for example*, that our consumer has a weekly income of \$80, the price of good X is \$1 per unit, and the price of good Y is \$2 per unit.

The table shows various combinations of good X and good Y that she can purchase each week with her \$80.

Market Basket	good X (X)	good Y (Y)	Total Spending
A	0	40	$1 \cdot 0 + 2 \cdot 40 = 80$
B	20	30	$1 \cdot 20 + 2 \cdot 30 = 80$
D	40	20	$1 \cdot 40 + 2 \cdot 20 = 80$
E	60	10	$1 \cdot 60 + 2 \cdot 10 = 80$
G	80	0	$1 \cdot 80 + 2 \cdot 0 = 80$



- All points on the budget line are affordable, and the consumer spent all available income.
كل النقاط التي تقع على خط الميزانية يقدر المستهلك على شرائها، كذلك ينفق المستهلك على شرائها كامل دخله.
- Points in the area below the budget line are affordable, but these leave some portion of income unspent.

النقاط التي تقع تحت خط الميزانية (داخل المثلث) يستطيع المستهلك شرائها، ولكن لا ينفق عليها كامل دخله.

- Points in the area above the budget line are not affordable.
النقاط التي تقع فوق خط الميزانية لا يستطيع المستهلك شرائها (دخله غير كافي).
- The downward slope of the budget line shows that the individual can afford more good X only if good Y purchases are cut back.

يظهر خط الميزانية بشكل منحدر للأسفل وذلك لأن المستهلك يستطيع زيادة استهلاكه من السلعة X فقط اذا قلل استهلاكه من السلعة Y

The slope of the budget line:

Using the budget line equation: $P_X X + P_Y Y = I$, we can see how much of Y must be give up to consume more of X.

$Y = \frac{I}{P_Y} - \frac{P_X}{P_Y} X$: This is the equation of the budget line; its slope of $(-\frac{P_X}{P_Y})$, and intercept of $\frac{I}{P_Y}$

→ *The slope of the budget line is the negative of the ratio of the prices of the two goods.*

Example:

Suppose a person has \$80 to spend only on apples and oranges. Apples cost \$4 each and oranges cost \$1 each.

- f. Write the budget line equation.

$$\text{Budget line: } P_A * A + P_O * O = I$$

$$4 A + O = 80 \text{ ----- budget line equation}$$

- g. If this person buys only apples, how many can be bought?

If this person buys only apples → the amount of oranges purchases equal zero

$$\rightarrow 4 A = 80 \rightarrow A = 80/4 = 20 \text{ units.}$$

- h. If this person buys only oranges, how many can be bought?

$$0 + O = 80 \rightarrow O = 80 \text{ units.}$$

- i. If the person were to buy 10 apples, how many oranges could be bought with the funds left over?

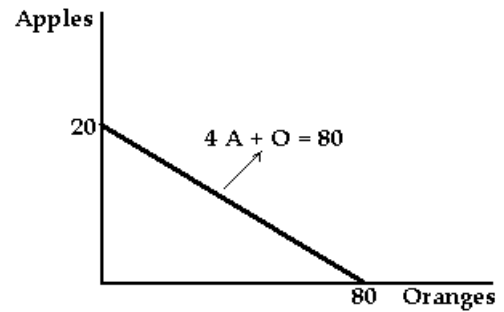
$$4 A + O = 80 \rightarrow 4 * 10 + O = 80 \rightarrow 40 + O = 80 \rightarrow O = 80 - 40 = 40 \text{ units.}$$

- j. Find the slope of the budget line, and graph it (put apples on the vertical axis and orange in the horizontal axis).

$$\text{Slope} = -P_A / P_O = -4/1 = -4$$

$$\text{Vertical intercept} = I / P_A = 80/4 = 20$$

$$\text{Horizontal intercept} = I / P_O = 80/1 = 80$$

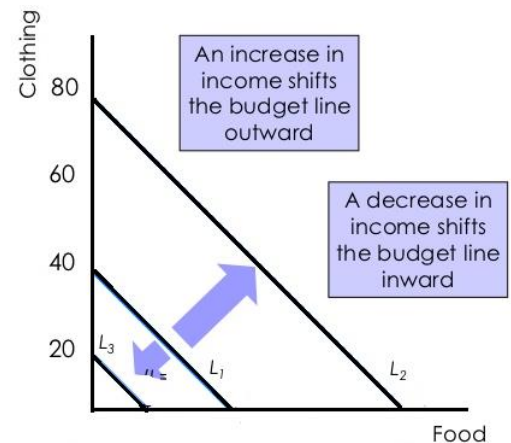


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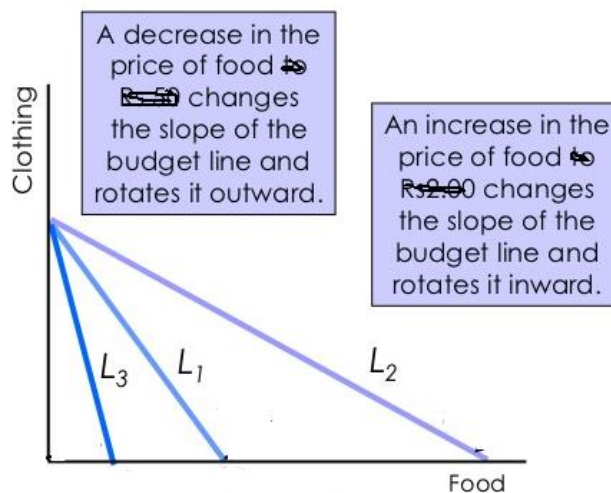
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Utility Maximization (Consumer Choice)

The maximizing market basket must satisfy two conditions:

1. It must be located on the budget line.
2. It must give the consumer the most preferred combination of goods and services.

- Point G is affordable (يقدر على شرائها) but not all of the consumer's income would be spent (لا يتم انفاق كامل الدخل).

- Point B is affordable but is not on the highest indifference curve that can be reached by the consumer.

النقطة B يستطيع المستهلك شرائها وينفق عليها كامل دخله لكن لا تعطي المستهلك اكبر منفعة

(يوجد خيارات اخرى تعطي المستهلك منفعة أكبر مثل النقطة A او النقطة D)

- Point D is on a higher indifference curve than A, but is not affordable given the budget constraint. النقطة D تقع على منحنى أعلى من النقطة A لكن لا يستطيع المستهلك شرائها (تقع خارج خط الميزانية)

- Point A is the point that is affordable that lies on the highest indifference curve, so it represents utility maximization.

النقطة A هي نقطة يستطيع المستهلك شرائها وينفق عليها كامل الدخل وتقع على منحنى أعلى لذلك تمثل هذه النقطة نقطة أعلى اشباع يحصل عليه المستهلك. (Utility maximization)

➤ The satisfaction (utility) is maximized at the point where: $MRS = \frac{P_X}{P_Y}$

➤ Satisfaction is maximized when the marginal rate of substitution (of good X for good Y) is equal to the ratio of the price (of X for Y).

➤ Because the MRS is also equal to the ratio of the marginal utilities of consuming good X and good Y, it follows that:

$$\frac{MUX}{MUY} = \frac{P_X}{P_Y} \quad \text{or} \quad \frac{MUX}{P_X} = \frac{MUY}{P_Y}$$

This equation tells us that utility maximization is achieved when the budget is allocated so that the marginal utility per dollar ($\frac{MUX}{P_X}$) of expenditure is the same for each good.

Example (1):

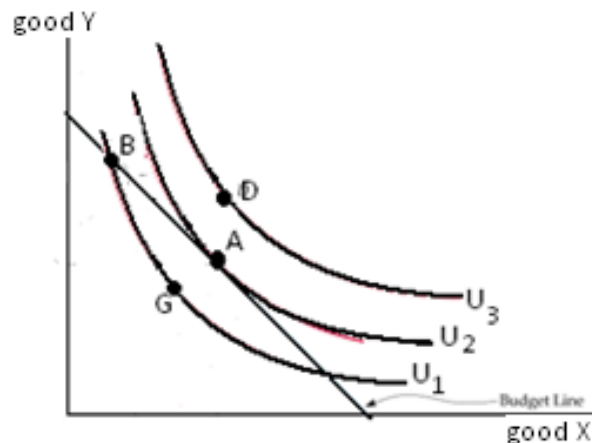
Assume the individual can choose between hamburgers (Y) and soft drinks (X) whose prices are $P_Y = \$1$ and $P_X = \$0.5$. The individual has \$10 to spend.

The individual gets measurable utility from X and Y as follows: $U(X, Y) = \sqrt{X Y}$

How much hamburgers (Y) and soft drinks (X) should he buy to maximize her utility?

$$\text{Budget line: } P_X * X + P_Y * Y = I \quad \rightarrow \quad 0.5 X + Y = 10 \quad \text{----- (1)}$$

$$\text{To maximize utility: } \frac{MUX}{MUY} = \frac{P_X}{P_Y}$$



$$MUX = \frac{dU(X,Y)}{dx} = \frac{Y}{2\sqrt{XY}} \quad (\text{مشتقة منحنى المنفعة بالنسبة لـ أكس})$$

مشتقة الجذر = مشتقة ما داخل الجذر ÷ 2 * الجذر

$$MUY = \frac{dU(X,Y)}{dY} = \frac{X}{2\sqrt{XY}} \quad (\text{مشتقة منحنى المنفعة بالنسبة لـ واي})$$

$$\frac{\frac{Y}{2\sqrt{XY}}}{\frac{X}{2\sqrt{XY}}} = \frac{0.5}{1} \rightarrow \frac{Y}{X} = \frac{1}{2} \rightarrow X = 2Y \quad \text{----- (2)}$$

Solve equations (1) and (2):

$$0.5(2Y) + Y = 10 \rightarrow Y + Y = 10 \rightarrow 2Y = 10 \rightarrow \underline{Y = 5 \text{ units}}$$

$$\text{From equation (2): } X = 2Y \rightarrow \underline{X = 2*5 = 10 \text{ units}}$$

Another solution:

$$\text{Budget line equation: } 0.5X + Y = 10$$

في معادلة خط الميزانية يتم وضع قيمة X صفر: $Y = 10 \leftarrow 0.5(0) + Y = 10$

فتكون قيم Y من صفر الى 10
بعدها يتم تعويض قيم Y في معادلة خط الميزانية لحساب قيم X

مثلاً: عندما تكون قيمة Y تساوي 3 فإن قيمة X تساوي:

$$X = 14 \leftarrow 0.5X = 7 \leftarrow 0.5X + 3 = 10$$

بعد حساب قيم X يتم تعويض قيم X و Y في $U(X,Y) = \sqrt{XY}$ وهي *Utility maximization* أعلى قيمة لـ Utility

Y	X	$U(X,Y) = \sqrt{XY}$
0	20	$\sqrt{0 * 20} = 0$
1	18	$\sqrt{1 * 18} = \sqrt{18}$
2	16	$\sqrt{2 * 16} = \sqrt{32}$
3	14	$\sqrt{3 * 14} = \sqrt{42}$
4	12	$\sqrt{4 * 12} = \sqrt{48}$
5	10	$\sqrt{5 * 10} = \sqrt{50} \rightarrow \text{max}$
6	8	$\sqrt{6 * 8} = \sqrt{48}$
7	6	$\sqrt{7 * 6} = \sqrt{42}$
8	4	$\sqrt{8 * 4} = \sqrt{32}$
9	2	$\sqrt{9 * 2} = \sqrt{18}$
10	0	$\sqrt{10 * 0} = 0$

To max utility the consumer should consume 5 units of Y and 10 units of X.

Example (2):

Jammel receives utility from days spent traveling on vacation domestically (D), and days spend travelling on vacation in a foreign country (F), as given by the utility function $U(D, F) = 10DF$. In addition, the price of a day spends traveling on vacation domestically is \$100, the price of a day spent travelling on vacation in a foreign country is \$400, and Jammel's annual travel budget is \$4000. Find Jammel's utility maximizing choice of days spent traveling domestically and days spent in a foreign country.

$$\text{Budget line: } P_D * D + P_F * F = I \rightarrow 100 D + 400 F = 4000 \text{ ----- (1)}$$

$$\text{To maximize utility: } \frac{MUD}{MUF} = \frac{P_D}{P_F}$$

$$MUD = \frac{dU(D,F)}{dD} = 10F$$

$$MUF = \frac{dU(D,F)}{dF} = 10D$$

$$\frac{10F}{10D} = \frac{100}{400} \rightarrow \frac{F}{D} = \frac{1}{4} \text{ (ضرب تبادلي ينتج)} \rightarrow D = 4F \text{ ----- (2)}$$

Solve equations (1) and (2):

$$100(4F) + 400F = 4000 \rightarrow 400F + 400F = 4000 \rightarrow 800F = 4000 \rightarrow \underline{F = 5 \text{ days}}$$

$$\text{From equation (2): } D = 4F \rightarrow \underline{D = 4*5 = 20 \text{ days}}$$

Example (3):

Taleen consumes only grapefruits and grapes. Her utility function is $U(X, Y) = XY - 2Y$, where X is the number of grapefruits consumed and Y is the number of grapes consumed. Fanny's income is 48, and the prices of grapefruits and grapes are 1 and 3, respectively. How many grapefruits will she consume?

$$\text{Budget line: } P_X * X + P_Y * Y = I \rightarrow X + 3 Y = 48 \text{ ----- (1)}$$

$$\text{To maximize utility: } \frac{MUX}{MUY} = \frac{P_X}{P_Y}$$

$$MUX = \frac{dU(X,Y)}{dX} = Y$$

$$MUY = \frac{dU(D,F)}{dF} = X - 2$$

$$\frac{Y}{X-2} = \frac{1}{3} \text{ (ضرب تبادلي ينتج)} \quad 3Y = X - 2 \text{ ----- (2)}$$

Solve equations (1) and (2): بتعويض المعادلة 2 في المعادلة 1 ينتج

يتم تعويض محل 3Y في المعادلة رقم 1 : X - 2

$$X + (X - 2) = 48 \rightarrow X + X - 2 = 48 \rightarrow 2X = 50 \rightarrow \underline{X = 25}$$

بتعويض قيمة X في المعادلة رقم 2 ينتج:

$$\text{From equation (2): } 3Y = X - 2 \rightarrow 3Y = 25 - 2 = 23 \rightarrow Y = 23/2 = 11.5$$

Example (4):

Cindy consumes good X and good Y. Candy has utility function $U(X, Y) = 2XY$. His income is \$12, the price of good X is \$2 and the price of good Y is \$1.

a. Write the Cindy budget constraint

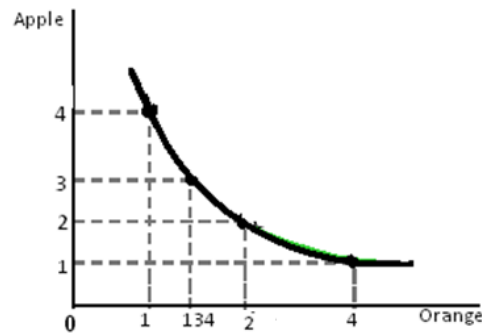
$$\text{Budget line: } P_X * X + P_Y * Y = I \rightarrow 2X + Y = 12$$

b. Graph the $U(X, Y) = 8$ indifference curve

$$\text{When } U = 8 \rightarrow 8 = 2XY \rightarrow XY = 4 \rightarrow Y = \frac{4}{X}$$

يتم اختيار 4 قيم ل X موجبة (أي اربع قيم) وبعدها يتم حساب قيم Y

X	Y
1	4/1 = 4
2	4/2 = 2
3	4/3 = 1.33
4	4/4 = 1



c. How much good X and good Y should he buy to maximize utility?

$$\text{Budget line: } P_X * X + P_Y * Y = I \rightarrow 2X + Y = 12 \quad \text{--- (1)}$$

12 -----

$$\text{To maximize utility: } \frac{MUX}{MUY} = \frac{P_X}{P_Y}$$

$$MUX = \frac{dU(X, Y)}{dX} = 2Y$$

$$MUY = \frac{dU(D, F)}{dF} = 2X$$

$$\frac{2Y}{2X} = \frac{2}{1} \quad (\text{ضرب تبادلي ينتج}) \quad 2Y = 4X \quad \text{-----} (2)$$

Solve equations (1) and (2): بتعويض المعادلة 2 في المعادلة 1 ينتج

يتم تعويض محل 2Y في المعادلة رقم 1 : 4X

$$2X + 4X = 12 \rightarrow 6X = 12 \rightarrow X = 2$$

بتعويض قيمة X في المعادلة رقم 2 ينتج:

$$\text{From equation (2): } 2Y = 4X \rightarrow 2Y = 4(2) \rightarrow 2Y = 8 \rightarrow Y = 4$$

When utility maximization conditions not Satisfy

- If the equation (MU per dollar of good X = MU per dollar of good Y) is not fulfilled (غير متحقق), then some reallocation of the consumer's expenditures between good X and good Y will increase the consumer's total utility.

$$\text{If } \frac{MUX}{P_X} > \frac{MUY}{P_Y} \quad \text{or} \quad \frac{MUX}{MUY} > \frac{P_X}{P_Y}$$

The last dollar spent on good X provide more utility than the last dollar spent on good Y. so the consumer can increase total utility by purchasing (consuming) more of good X and less of good Y.

$$\text{If } \frac{MUX}{P_X} < \frac{MUY}{P_Y} \quad \text{or} \quad \frac{MUX}{MUY} < \frac{P_X}{P_Y}$$

- The last dollar spent on good Y provide more utility than the last dollar spent on good X. so the consumer can increase total utility by purchasing (consuming) more of good Y and less of good X.

Example:

Muhanad divides her consumption between orange juice (O) and bagels (B). Orange juice costs \$1 per glass and bagels cost \$3each. Muhanad consumes positive amounts of both goods, and has chosen consumption quantities where his marginal utility of orange juice is 10 utils per glass, his marginal utility of bagels is 25 utils per bagel, and he spends all her income. Is Muhanad maximizing utility? If no, what he do to increase his utility?

$$\text{To max utility: } \frac{MUO}{MUB} = \frac{P_O}{P_B} \quad \Leftrightarrow \quad \frac{10}{25} \neq \frac{1}{3} \quad \Leftrightarrow \quad \text{Muhanad does not maximizing utility}$$

$$\text{But } \frac{MUO}{MUB} > \frac{P_O}{P_B} \quad \text{Since, } \frac{10}{25} > \frac{1}{3}$$

⇨ The last dollar spent on orange juice provides more utility than the last dollar spent on bagels. So the consumer can increase total utility by purchasing (consuming) more of orange juice and less of bagels.

Utility maximization for perfect substitutes goods case:

لمعرفة اذا كانت الحالة تمثل حالة السلع البديلة من خلال شكل منحنى المنفعة : $U(X, Y) = aX + bY$

Example:

Ms. Caffeine enjoy coffee (C) and tea (T) according to the function $U(C, T) = 3C + 4T$. If coffee and tea cost \$3 each and Ms. Caffeine has \$12 to spend on these products, how much coffee (C) and tea (T) should she buy to maximize her utility?

$$\text{Budget line: } P_C * C + P_T * T = I \quad \Rightarrow \quad 3C + 3T = 12 \text{ ----- (1)}$$

$$\text{To maximum utility: } \frac{MUC}{MUT} = \frac{P_C}{P_T}$$

$$MUC = \frac{dU(C, T)}{dC} = 3$$

$$MUT = \frac{dU(C, T)}{dT} = 4$$

$$\frac{MUC}{MUT} = \frac{P_C}{P_T} \quad \Rightarrow \quad \frac{3}{4} \neq \frac{3}{3} \quad \Rightarrow \quad \text{Condition fail}$$

عند تطبيق شرط تعظيم المنفعة وصلنا لنتيجة تناقض $\frac{3}{4} \neq \frac{3}{3}$ وبالتالي لا نستطيع حل هذا المثال باستخدام هذه الطريقة

لحل هذه الحالة يتم الاعتماد على الطريقة الثانية وهي تعويض ارقام

$$\text{Budget line: } P_C * C + P_T * T = I \quad \Rightarrow \quad 3C + 3T = 12$$

نستخدم فقط قيمتين : من معادلة خط الميزانية يتم تعويض في المره الاولى قيمة C صفر وبالتالي تكون قيمة $T = 4$.

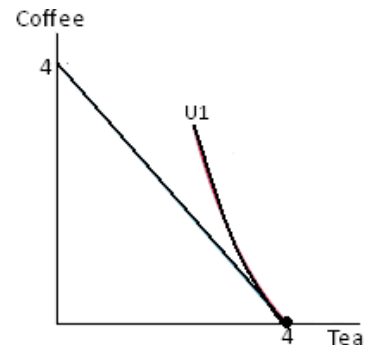
$$3(0) + 3T = 12 \rightarrow T = 12/3 = 4$$

من معادلة خط الميزانية يتم تعويض في المره الثانية قيمة T صفر وبالتالي تكون قيمة $C = 4$.

وبعدها يتم تعويض قيمة T وقيمة C في منحنى المنفعة $U(T, C) = 3T + 3C$ كما في الجدول في الاسفل واعلى قيمة ل U بين القيمتين هي نقطة تعظيم المنفعة.

C	T	$U(C, T) = 3C + 4T$
0	4	$3*0 + 4*4 = 16 \Rightarrow \text{max}$
4	0	$3*4 + 4*0 = 12$

Ms. Caffeine should consume 4 units of tea and gets 16 utility



Utility maximization for perfect complement goods case:

لمعرفة اذا كانت الحالة تمثل حالة السلع المكملة من خلال شكل منحنى المنفعة : $U(X, Y) = \min (X, Y)$

Example:

A consumer consume two goods movies (M) and popcorn (C) according to the utility function : $U(M, C) = \min (2M, C)$, the price of movies ticket is \$10 and the price of popcorn is \$2.5 and consumer budget is \$30. how much movies and popcorn should she buy to maximize her utility?

Where “Min” means that utility is given by the smaller of the two terms in parentheses. If, for example, this person attends a movie but buys no popcorn, utility is zero. If he or she attends a movie and buys three bags of popcorn, utility is 2—the extra bag of popcorn does not raise utility.

$$\text{Budget line: } P_M * M + P_C * C = I \quad \Leftrightarrow \quad 10 M + 2.5C = 30 \text{ ----- (1)}$$

$U(M, C) = \min (2M, C)$: this person should only consume bundles for which $C = 2M$ (that is, two bags of popcorn for each movie).

$$U(M, C) = \min (2M, C) \rightarrow C = 2M \text{ ----- (2)}$$

Solve the 2 equations: بتعويض المعادلة رقم 2 بالمعادلة رقم 1 ينتج

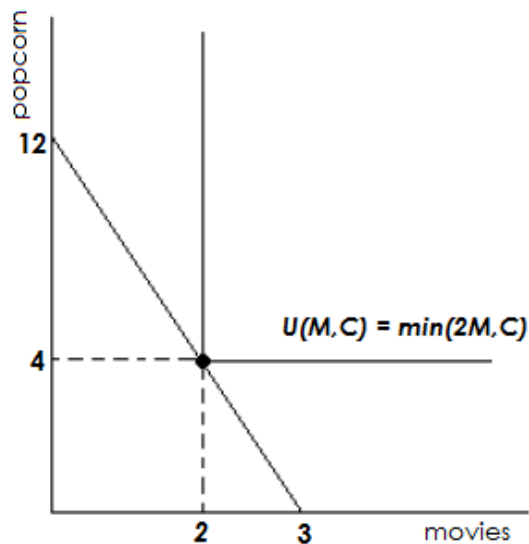
$$10 M + 2.5C = 30 \text{ ----- (1)}$$

$$C = 2M \text{ ----- (2)}$$

$$10M + 2.5(2M) = 30 \rightarrow 10M + 5M = 30 \rightarrow 15M = 30 \rightarrow M = \frac{30}{15} = 2$$

$$C = 2M = 2 * 2 = 4$$

$$U(M, C) = \min (2M, C) = \min (2 * 2, 4) = \min (4, 4) = 4$$



Example:

Shireen has a monthly income of \$20 that she allocates among two goods: Meat (M) and Chickens (C). Suppose also that her utility function is given by the equation $U(M,C) = 2M + C$

- Suppose meat cost \$2 per pound and chickens cost \$4 per pound. Derive her budget line equation and draw her budget constraint putting (M) on the vertical axes (y-axes).
- Find her marginal rate of substitution.
- what combination of meat and chickens should she buy to maximize her utility?
- What is maximum utility will Shireen realized?

Solution:

a. Budget line: $P_M * M + P_C * C = I \Rightarrow 2M + 4C = 20$

b. $MRS = \frac{MUM}{MUC}$

$$U(M,C) = 2M + C$$

$$MUM = \frac{dU(M,C)}{dM} = 2$$

$$MUC = \frac{dU(M,C)}{dC} = 1$$

$$MRS = \frac{MUM}{MUC} = \frac{2}{1} = 2$$

(المستهلك مستعد ان يضحي (يقلل الاستهلاك) وحدتين من السلعة سي مقابل حصوله على سلعة اضافية من السلعة أم

c. $U(M,C) = 2M + C$

هذا شكل منحى المنفعة ل السلع البديلة وبالتالي طريقة الحل هي:

Budget line: $P_M * M + P_C * C = I \Rightarrow 2M + 4C = 20$

نستخدم فقط قيمتين : من معادلة خط الميزانية يتم تعويض في المره الاولى قيمة M صفر وبالتالي تكون قيمة $C = 4$.

$$2(0) + 4C = 20 \rightarrow C = 20/4 = 5$$

من معادلة خط الميزانية يتم تعويض في المره الثانية قيمة C صفر وبالتالي تكون قيمة $M = 10$.

وبعدها يتم تعويض قيمة M وقيمة C في المنحى المنفعة $U(M,C) = 2M + C$
كما في الجدول في الاسفل واعلى قيمة ل U بين القيمتين هي نقطة تعظيم المنفعة.

M	C	$U(M,C) = 2M + C$
0	5	$2(0) + 5 = 5$
10	0	$2(10) + 0 = 20 \Rightarrow \max$

Shireen should consume 10 units of meat and 0 unit of Chicken to get maximize utility.