Basic Concepts of Costs

There are at least three different concepts of costs encountered in economics: opportunity cost, accounting cost, and economic cost. For economists, the most general of these is opportunity cost.

Opportunity cost: The cost of a good as measured by the alternative uses that are forgone by producing the good.

Accounting cost: The concept that inputs cost what was paid for them.

Economic cost: The amount required to keep an input in its present use; the amount that it would be worth in its next best alternative use.

To see how the economic definition of cost might be applied in practice and how it differs from accounting ideas, let's look at the economic costs of three inputs: labor, capital, and the services of entrepreneurs (owners)

Labor Costs

Economists and accountants view labor costs in much the same way. To the accountant, firms' spending on wages and salaries is a current expense and therefore is a cost of production. Economists regard wage payments as an explicit cost: labor services (worker-hours) are purchased at some hourly wage rate (which we denote by w), and we presume that this rate is the amount that workers would earn in their next best alternative employment.

Wage rate (w): The cost of hiring one worker for one hour.

Capital Costs

Rental rate (v): The cost of hiring one machine for one hour.

The Two-Input Case

We will make two simplifying assumptions about the costs of inputs a firm use.

First, we can assume, as before, that there are only two inputs: labor (L, measured in labor-hours) and capital (K, measured in machine-hours). Entrepreneurial services are assumed to be included in capital input.

A second assumption we make is that inputs are hired in perfectly competitive markets. Firms can buy (or sell) all the labor or capital services they want at the prevailing rental rates (w and v). In graphic terms, the supply curve for these resources that the firm faces is horizontal at the prevailing input prices.

Economic Profits and Cost Minimization

Given these simplifying assumptions, total costs for the firm during a period are:

$$Total\ costs\ =\ TC\ =\ wL\ +\ vK$$

where, L and K represent input usage during the period.

If the firm produces only one output, its total revenues are given by the price of its product (P) times its total output [$q\ =\ f(K, L)$, where $f(K, L)$ is the firm's production function].

Economic profits (π) are then the difference between total revenues and total economic costs:

$$Economic\ profits\ (\pi) = total\ revenues - total\ costs$$

$$\pi\ =\ Pq\ -\ wL\ -\ vK$$

Economic profits: The difference between a firm's total revenues and its total economic costs.

Cost-Minimizing Input Choice

To minimize the cost of producing q_1, a firm should choose that point on the q_1 isoquant that has the lowest cost. That is, it should explore all feasible input combinations to find the cheapest one. This will require the firm to choose that input combination for which the marginal rate of technical substitution (RTS) of L for K is equal to the ratio of the inputs' costs, w/v.

The Isocost Line

isocost line: Graph showing all possible combinations of labor and capital that can be purchased for a given total cost.

To see what an isocost line looks like, recall that the total cost TC of producing any particular output is given by the sum of the firm's labor cost wL and its capital cost vK:

$$TC\ =\ wL +\ vK$$

If we rewrite the total cost equation as an equation for a straight line, we get

$$K\ =\ \frac{TC}{v} - \frac{w}{v}\ L$$

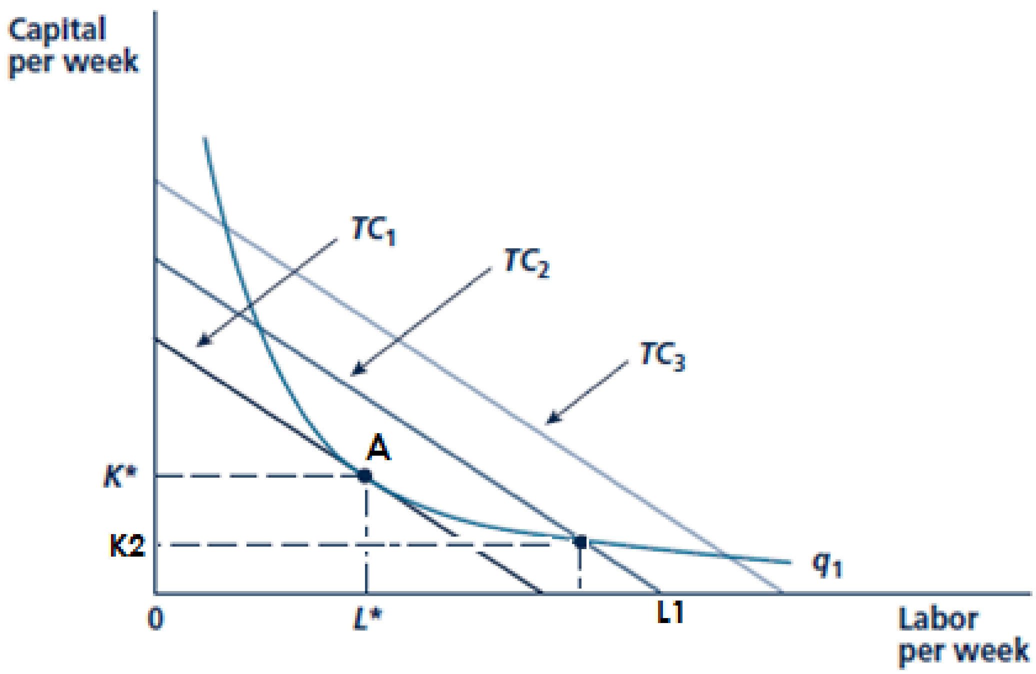
It follows that the isocost line has a slope of $\frac{\Delta K}{\Delta L} = -\ (w/v)$, which is the ratio of the wage rate to the rental cost of capital.

The slope of isocost tells us that if the firm gave up a unit of labor to buy w/r units of capital at a cost or r dollar per unit, its total cost of production would remain the same. For example, if the wage rate were $10 and the rental cost of capital $5, the firm could replace one unit of labor with two units of capital with no change in total cost.

Choosing Inputs

Suppose we wish to produce at an output level q_1. How can we do so at minimum cost?

Output q_1 can be achieved with the expenditure of TC_1, either by using K* units of capital and L* units of labor or by using K_3 units of capital and L_3 units of labor. But TC_2 is not the minimum cost. The same output q_1 can be produced more cheaply, at a cost of TC_1, by using K* units of capital and L* units of labor. In fact, isocost line TC_1 is the lowest isocost line that allows output q_1 to be produced. The point of tangency of the isoquant q_1 and the isocost line TC_1 at point A gives us the cost-minimizing choice of inputs, L* and K*, which can be read directly from the diagram.



At this point, the slopes of the isoquant and the isocost line are just equal.

The slopes of the isoquant = MRTS $= \frac{MPL}{MPK}$

The slopes of the isocost line = $\frac{w}{r}$

When a firm minimizes the cost of producing a particular output, the following condition holds

$$\frac{MPL}{MPK} = \frac{w}{r} \quad \text{or} \quad \frac{MPL}{w} = \frac{MPK}{r}$$

$\frac{MPL}{w}$ Ξ Is the additional output that results from spending an additional dollar for labor.

$\frac{MPK}{r}$ Ξ Is the additional output that results from spending an additional dollar for labor.

Example
A widget manufacturer has a production function of the form q = 4KL. If the wage rate (w) is $8 and the rental rate on capital (r) is $20, what cost minimization combination of K and L will the manufacturer employ to produce 160 units of output? What is the total cost at that output level?

When a firm minimizes the cost of producing a particular output, the following condition holds:

$$\frac{MPL}{MPK} = \frac{w}{r} \Rightarrow \frac{4K}{4L} = \frac{8}{20} \Rightarrow \frac{K}{L} = \frac{2}{5} \Rightarrow 5K = 2L \quad \text{.......... (1)}$$

$$q = 4KL \Rightarrow 160 = 4KL \Rightarrow 40 = KL \Rightarrow K = \frac{40}{L} \quad \text{.......... (2)}$$

Solve (1) and (2):

$$\frac{2}{5} L = \frac{40}{L} \Rightarrow 2L^2 = 200 \Rightarrow L^2 = 100 \Rightarrow L = 10$$

$$K = \frac{40}{L} = \frac{40}{10} = 4$$

$$Tc = wL + vK = (8x10) + (20x4) = 160$$

<u>or:</u>

$$q = 4KL \Rightarrow 160 = 4KL \Rightarrow 40 = KL \Rightarrow K = \frac{40}{L}$$

L	K	TC = 8 L + 20 K
1	40	808
2	20	416
3	13.4	292
4	10	232
5	8	200
6	6.67	181.4
7	5.71	170.2
8	5	164
9	4.45	161
10	4	160 **(min)**
11	3.63	160.6
12	3.34	162.8

Example 2:

A widget manufacturer has a production function of the form $q = 2L^2 K$. If the wage rate (w) is \$2 and the rental rate on capital (r) is \$5.

1. What cost minimization combination of K and L will the manufacturer employs to produce 400 units of output? What is the total cost at that output level?

   When a firm minimizes the cost of producing a particular output, the following condition holds:

   $$\frac{\text{MPL}}{\text{MPK}} = \frac{\text{w}}{\text{r}}$$

   $$\text{MPL} = \frac{\partial q}{\partial L} = 4LK$$

   $$\text{MPK} = \frac{\partial q}{\partial K} = 2L^2$$

   $$\frac{\text{MPL}}{\text{MPK}} = \frac{\text{w}}{\text{r}} \Rightarrow \frac{4\text{LK}}{2L^2} = \frac{2}{5} \Rightarrow \frac{2\text{K}}{\text{L}} = \frac{2}{5} \Rightarrow 10K = 2L \Rightarrow K = \frac{2}{10} L \quad \ldots\ldots\ldots (1)$$

   $$q = 2L^2 K \Rightarrow 400 = 2L^2 K \Rightarrow 200 = L^2 K \Rightarrow \text{K} = \frac{200}{\text{L}^2} \quad \ldots\ldots\ldots (2)$$

   Solve (1) and (2):

   $$\frac{2}{10}\text{L} = \frac{200}{\text{L}^2} \Rightarrow 2L^3 = 2000 \Rightarrow L^3 = 1{,}000 \Rightarrow L = 10$$

   $$K = \frac{2}{10} L = \frac{2}{10}(10) = 2$$

   $$Tc = wL + vK = (2x10) + (5x2) = 30$$

Example

A firm producing good A has a production function of the form $q = 2K + L$. If the wage rate (w) is $1 and the rental rate on capital (r) is $1, what cost minimization combination of K and L will the manufacturer employ to produce 40 units of output?

$$\frac{MPL}{MPK} = \frac{w}{r}$$

$MPL = 1 \quad MPK = 2$

$\frac{1}{2} \neq 1$ ⇨ condition fail

$q = 2K + L \rightarrow 40 = 2K + L \rightarrow 2K = 40 - L \rightarrow K = 20 - \frac{1}{2} L$

L	K	TC = L + K
0	20	20 (min)
40	0	40

To produce 40 units of output: L = 0 and K = 20 with total cost 20.

If $\frac{MPL}{w} > \frac{MPK}{r}$ the firm does not minimize the cost. To reduce the cost of producing its current output level it should employing more labor and less capital

If $\frac{MPL}{w} < \frac{MPK}{r}$ the firm does not minimize the cost. To reduce the cost of producing its current output level it should employing more capital and less labor

Example:

A firm employs 100 workers, each at $8 per hour, and 50 units of capital, each at $10 per hour. The marginal product of labor is 3 and the marginal product of capital is 5. Is the firm minimizing the cost? If not, what it do to reduce the cost of producing its current output level?

To minimize cost: $\frac{MPL}{w} = \frac{MPK}{r}$

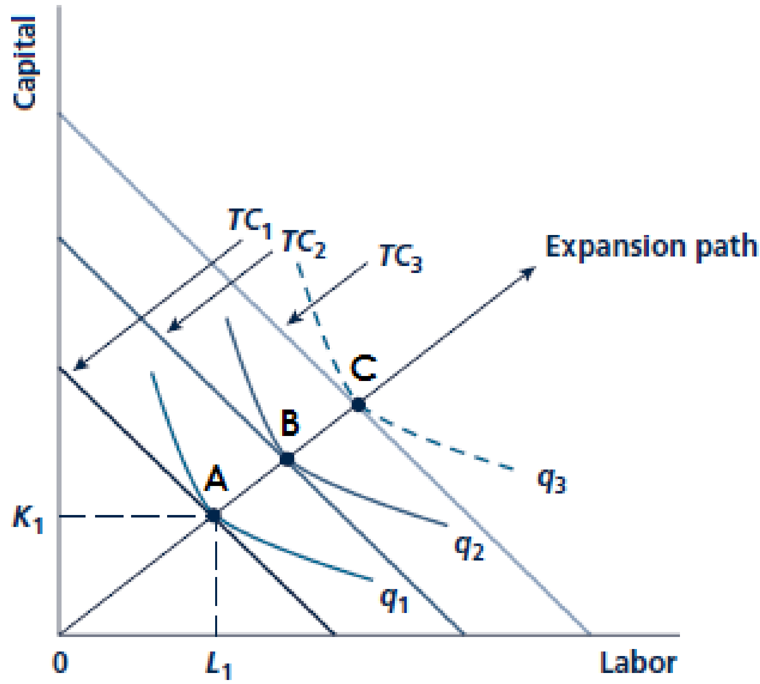
$$\frac{3}{5} < \frac{8}{10}$$

The firm does not minimize the cost. To reduce the cost of producing its current output level it should employing more capital and less labor

The Firm's Expansion Path

The firm's expansion path is the set of cost-minimizing input combinations a firm will choose to produce various levels of output (when the prices of inputs are held constant).

The expansion path (from the origin through points A, B, and C) illustrates the lowest-cost combinations of labor and capital that can be used to produce each level of output



Example

A widget manufacturer has a production function of the form $q = 4KL$. If the wage rate (w) is \$8 and the rental rate on capital (r) is \$20.

2. What cost minimization combination of K and L will the manufacturer employs to produce 160 units of output? What is the total cost at that output level?

   When a firm minimizes the cost of producing a particular output, the following condition holds:

   $$\frac{MPL}{MPK} = \frac{w}{r} \Rightarrow \frac{4K}{4L} = \frac{8}{20} \Rightarrow \frac{K}{L} = \frac{2}{5} \Rightarrow 5K = 2L \text{ .... (1)}$$

   $$q = 4KL \Rightarrow 160 = 4KL \Rightarrow 40 = KL \Rightarrow K = \frac{40}{L} \text{ .... (2)}$$

   Solve (1) and (2):

   $$\frac{2}{5}L = \frac{40}{L} \Rightarrow 2L^2 = 200 \Rightarrow L^2 = 100 \Rightarrow L = 10$$

$K = \frac{40}{L} = \frac{40}{10} = 4$

$Tc = wL + vK =$ (8x10) + (20x4) = 160

3. If the firm wants to increase output level to 360. What cost minimization combination of K and L will the manufacturer employs? What is the total cost at that output level?

$\frac{MPL}{MPK} = \frac{w}{r}$ ⇨ $\frac{4K}{4L} = \frac{8}{20}$ ⇨ $\frac{K}{L} = \frac{2}{5}$ ⇨ 5K = 2L .... (1)

q = 4KL ⇨ 360 = 4KL ⇨ 90 = KL ⇨ $K = \frac{90}{L}$ .... (2)

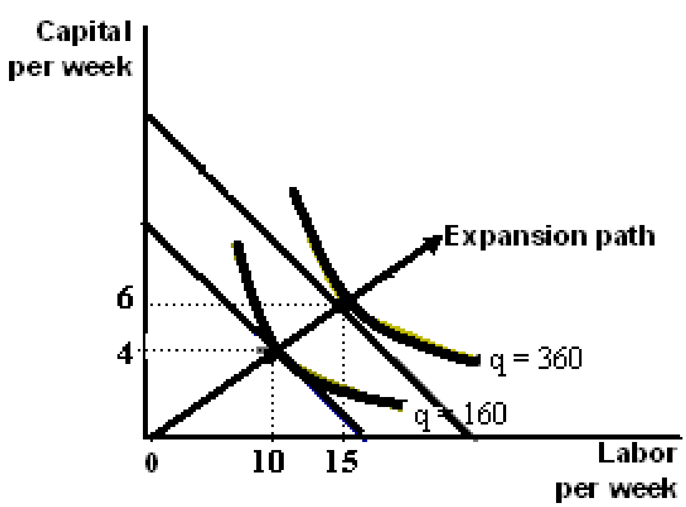
Solve (1) and (2):

$\frac{2}{5}L = \frac{90}{L}$ ⇨ $2L^2 = 450$ ⇨ $L^2 = 225$ ⇨ L = 15

$K = \frac{90}{L} = \frac{90}{15} = 6$

Tc = wL + vK = (8x15) + (20x6) = 240

4. Illustrate your results graphically with a representative isoquant and isocost line consistent with the answers you derived in parts (1) and (2) above and then draw the firm expansion path

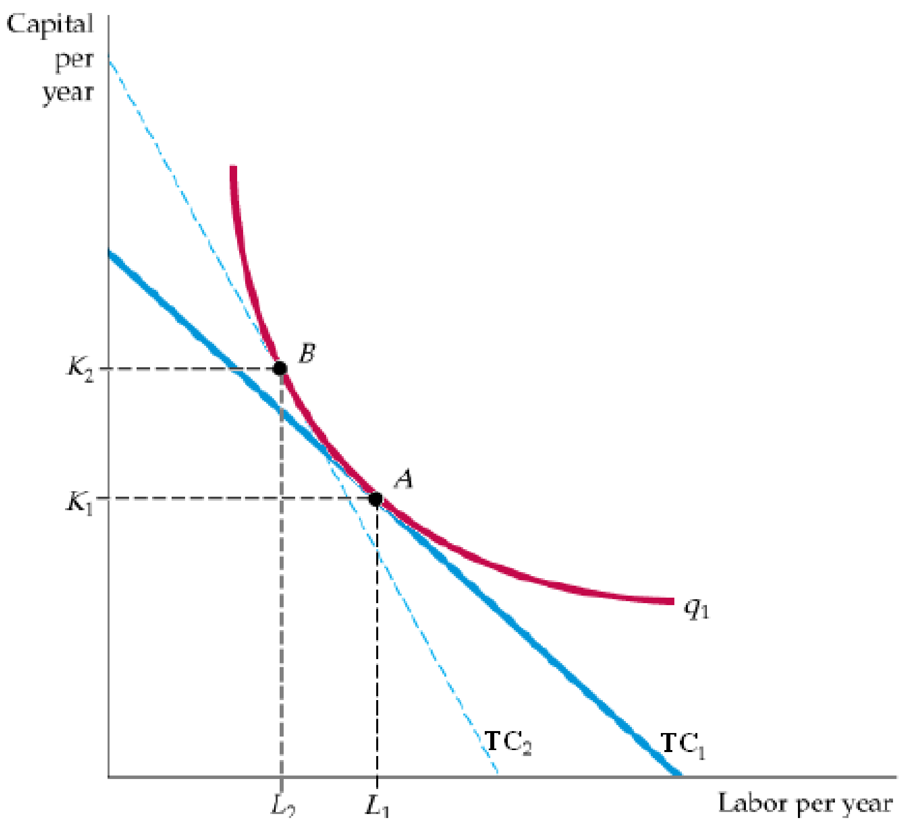


Input substitution with an input price changes:

When the expenditure on all inputs increase, the slope of the isocost line does not change because the prices of the inputs have not changed, but the intercept, will increase (isocost line shift to the right).

Suppose that the price of one of the input, such as labor were to increase. In that case the slope of the isocost line (w/r) would increase and the isocost line would become steeper.

Facing an isocost curve TC_1, the firm produces output q_1 at point A using L_1 units of labor and K_1 units of capital.
When the price of labor increases, the isocost curves become steeper. Output q_1 is now produced at point B on isocost curve TC_2 by using L_2 units of labor and K_2 units of capital. The firm has responded to the higher price of labor by substituting capital for labor in the production process.



Example:
Nike's total cost of producing sport shoes is given by $TC = wL + vK$, where w is the price of labor (= $1) per unit and r is the rental price of capital (= $2) per unit. Nike just won an order to supply q pairs of sport shoes and Nike's production function is of the form q = $2L^2 K$.

1. What is Nike's cost minimization choice of inputs (capital and labor) in order to produce 32 pairs of sport shoes?

   When a firm minimizes the cost of producing a particular output, the following condition holds:

   $$\frac{MPL}{MPK} = \frac{w}{r} \Rightarrow \frac{4LK}{2L^2} = \frac{1}{2} \Rightarrow \frac{2K}{L} = \frac{1}{2} \Rightarrow K = ¼L \quad \text{..........} (1)$$

   $$q = 2L^2 K \Rightarrow 32 = 2L^2 K \Rightarrow K = \frac{32}{2L^2} \quad \text{..........} (2)$$

   Solve (1) and (2):

   $$¼L = \frac{32}{2L^2} \Rightarrow ½L^3 = 32 \Rightarrow L^3 = 64 \Rightarrow L = 4$$

   K = ½L = ½ x 4 =2

2. Suppose that the price of capital increases to $3 per unit. If Nike continues to produce 32 pairs of sport shoes. What cost minimization choice of inputs capital and labor should the firm used.

   $$\frac{MPL}{MPK} = \frac{w}{r} \Rightarrow \frac{4LK}{2L^2} = \frac{1}{3} \Rightarrow \frac{2K}{L} = \frac{1}{3} \Rightarrow K = \frac{L}{6} \quad \text{..........} (1)$$

   $$q = 2L^2 K \Rightarrow 32 = 2L^2 K \Rightarrow K = \frac{32}{2L^2} \quad \text{..........} (2)$$

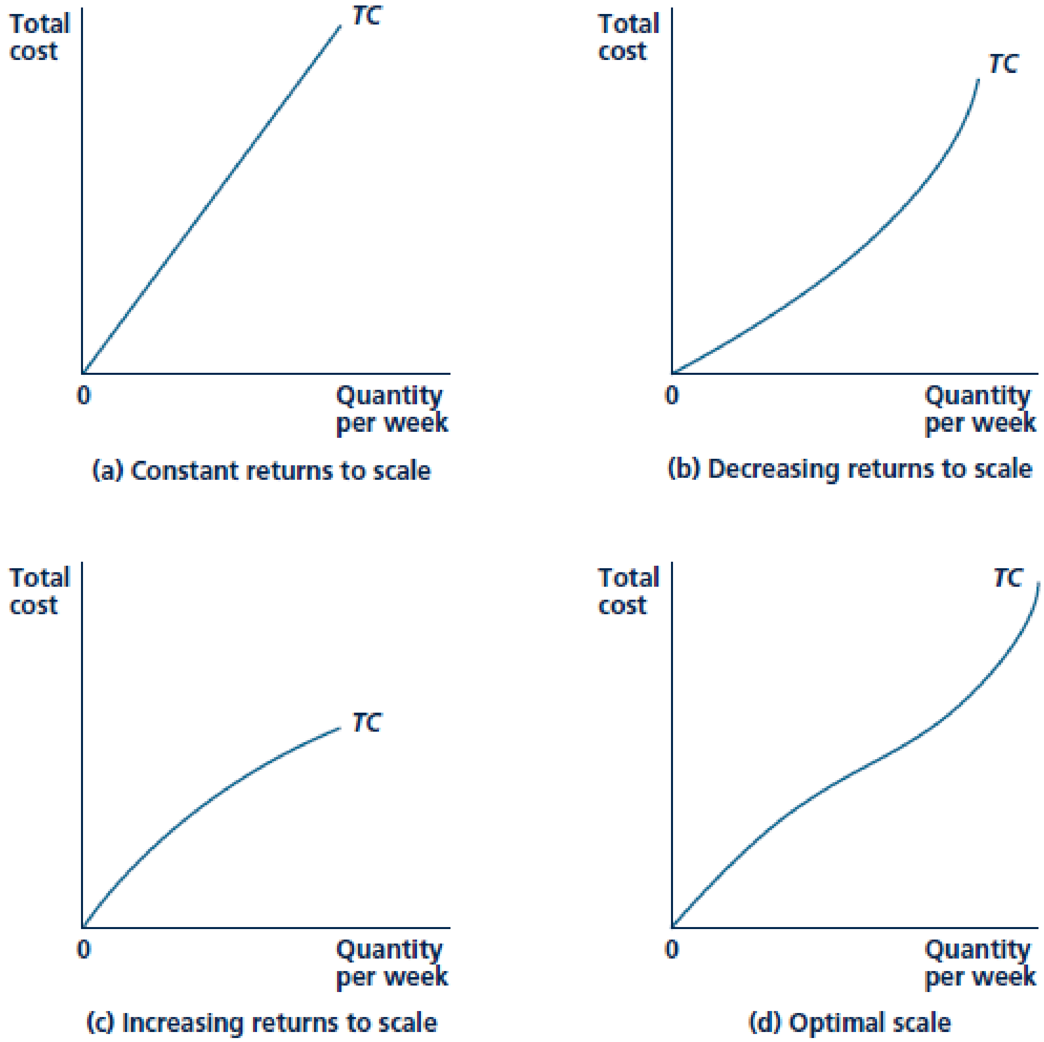
   Solve (1) and (2):

   $$\frac{L}{6} = \frac{32}{2L^2} \Rightarrow 2L^3 = 192 \Rightarrow L^3 = 96 \Rightarrow L = 4.5 \quad ; \; K = \frac{L}{6} = \frac{4.5}{6} = 0.75$$

Cost Curves

In Panel a, output and required input use is proportional which means doubling of output requires doubling of inputs. This is the case when the production function exhibits constant returns to scale.

Panels b and c reflect the cases of decreasing and increasing returns to scale, respectively. With decreasing returns to scale the cost curve is convex, while the it is concave with increasing returns to scale.



Average Costs:

Average cost is total cost divided by output; a common measure of cost per unit.

Average cost = AC = $\frac{TC}{q}$

Marginal Cost:

The additional cost of producing one more unit of output.

If the cost of producing 24 units is $98 and the cost of producing 25 units is $100, the marginal cost of the 25th unit is $2.

MC = $\frac{\Delta\, TC}{\Delta\, q} = \frac{dTC}{dq}$

Marginal costs are reflected by the slope of the total cost curve.

Example

The total cost curve of a firm is $TC = 0.2\, q^2 + 5q + 200$. What is the average total cost and marginal cost to produce 100 units of output?

Average cost = AC = $\frac{TC}{q} = \frac{0.2q^2 + 5q + 200}{q} = 0.2q + 5 + \frac{200}{q}$

AC = $0.2(100) + 5 + \frac{200}{100}$ = 27

MC = $\frac{dTC}{dq} = 0.4q + 5$

MC = $0.4(100) + 5$ = 45

The Relationship Between the Short Run and the Long Run Cost:

The short run is the period of time in which a firm must consider some inputs to be absolutely fixed in making its decisions.

The long run is the period of time in which a firm may consider all of its inputs to be variable in making its decisions.

Holding Capital Input Constant

For the following, the capital input is assumed to be held constant at a level of $K1$, so that, with only two inputs, labor is the only input the firm can vary.

Short Run Total Cost $(STC) = w\, L + v\, K1$

Fixed Cost and Variable cost

Some costs vary with output, while others remain unchanged as long as the firm is producing any output at all.

Total cost (TC): Total economic cost of production, consisting of fixed and variable costs.

Fixed cost (FC): Cost that does not vary with the level of output and that can be eliminated only by shutting down.

Variable cost (VC): Cost that varies as output varies.

Fixed cost does not vary with the level of output—it must be paid even if there is no output. The only way that a firm can eliminate its fixed costs is by shutting down.

Total cost (TC) = Fixed cost (FC) + Variable cost (VC)

Example:

Suppose a firm's short run cost curves were found to be: $TC = Q^2 + 4Q + 5$, where Q is output. What are the firm's FC, VC and TC when the firm producing 10 units of output?

$$FC = 5$$

$$VC = Q^2 + 4Q = 102 + 4(10) = 100 + 40 = 140$$

$$TC = FC + VC = 5 + 140 = 145$$

Average Total Cost:

Average total cost (ATC): Firms total cost dividing by its level of output

ATC = $\frac{\text{TC}}{\text{q}}$

Average fixed cost (AFC): Firms fixed cost dividing by its level of output

AFC = $\frac{\text{FC}}{\text{q}}$

Average variable cost (AVC): Firms variable cost dividing by its level of output

AVC = $\frac{\text{VC}}{\text{q}}$

$$TC = VC + FC \quad \Rightarrow ATC = AVC + AFC$$

Example:

The total cost curve of a firm is $TC = 0.2\,q^2 + 5q + 200$. What are the AFC, AVC, and ATC to produce 100 units of output?

AFC = $\frac{FC}{q} = \frac{200}{100}$ = 2 AVC = $\frac{VC}{q} = \frac{0.2\,q^2 + 5q}{q}$ = $0.2\,q + 5$ = 0.2(100) + 5 = 20 + 5 = 25

ATC = AFC + AVC = 2 + 25 = 27

Example

If average total cost rises from $10 to $30 as total production rises from 100 to 300 units. What is marginal cost?

ATC = $\frac{TC}{q}$ = 10 when q = 100 ⇨ 10 = $\frac{TC}{100}$ ⇨ TC = 1000

ATC = $\frac{TC}{q}$ = 30 when q = 300 ⇨ 30 = $\frac{TC}{300}$ ⇨ TC = 9000

MC = $\frac{\Delta TC}{\Delta q} = \frac{(9000-1000)}{(300-100)} = \frac{8000}{200}$ = 40

Example

A firm producing hockey sticks has a production function given by: q = $2\sqrt{KL}$. In the short run, the firm's amount of capital is fixed at K = 100. The rental rate for K is r = $1, and the wage rate for L is w = $4.

a. Calculate the firm's short run total cost function.

STC = w L + v K_1 = 4L + 100

q = $2\sqrt{KL}$ ⇨ q = $2\sqrt{100L}$ ⇨ q = $20\sqrt{L}$

q^2 = 400L ⇨ L = $\frac{q2}{400}$

STC = 4L + 100 = 4($\frac{q2}{400}$) + 100

⇨ STC = $\frac{q2}{100}$ + 100

b. What are the STC, SAC, and SMC for the firm if it produces 25 hockey sticks?

STC = $\frac{q2}{100}$ + 100

SATC = $\frac{STC}{q} = \frac{q}{100} + \frac{100}{q}$

SMC = $\frac{\partial STC}{\partial q} = \frac{q}{50}$

When q = 25

STC = $\frac{q2}{100} + 100 = \frac{(25)2}{100} + 100 = 106.25$

SATC = $\frac{STC}{q} = \frac{q}{100} + \frac{100}{q} = \frac{25}{100} + \frac{100}{25} = 4.5$

SMC = $\frac{\partial STC}{\partial q} = \frac{q}{50} = \frac{25}{50}$ = ½

Example

Output for a simple production function is given by: q = LK + 3L. The price of capital is \$20 per unit and capital is fixed at 5 units in the short run. The price of labor is \$4 per unit. How many labors would the firm employ in order to producing 80 units of output?

STC = w L + v K_1 = 4L + 100

q = LK + 3L ⇨ 80 = 5L + 3L ⇨ 80 = 8L ⇨ L = $\frac{80}{8}$ = 10

STC = 4L + 100 = 4x10 + 100 = 140

Shifts in Cost Curves:

Any change in input prices and technological innovations, will affect the shape and position of the firm's cost curves.

Changes in Input Prices

A change in the price of an input will tilt the firm's total cost lines. For example, a rise in wage rates will cause firms to use more capital (to the extent allowed by the technology).

Generally, all cost curves will shift upward with the extent of the shift depending upon how important labor is in production and how successful the firm is in substituting other inputs for labor.

Increase in input price → increase total cost → shift cost curves to the right.

Decrease in input price → decrease total cost → shift cost curves to the left.

Technological Innovation

Technological improvements would shift isoquants toward the origin enabling firms to produce the same level of output with less of all inputs → shift the costs curve to the left.