CHAPTER
10
AUTOCORRELATION: WHAT HAPPENS IF ERROR TERMS ARE CORRELATED?

QUESTIONS

10.1. (***a***) The correlation between the current value of the error with its own past value(s).

(***b***) The correlation between the current value of the error with its immediate past value.

(***c***) The correlation between observations over space rather than over time.

Note: Some authors use the term serial correlation for correlation observed in time series data [i.e., in the sense defined in (*a*)] and autocorrelation for correlation observed in cross-section data [in the sense defined in (*c*)].

10.2. Although in general an AR(m) scheme can be used, the AR(1) scheme has been found to be quite useful in many time series analysis. With the AR(1) scheme, many properties of the OLS estimators can be easily established.

10.3. The consequences are: (1) The OLS estimators are unbiased, but are not efficient. (2) The conventionally estimated standard errors of OLS estimators are biased. (3) As a result, the conventionally computed *t* and *F* tests are unreliable, the conventional estimator of σ^2 is biased, and the conventionally computed R^2 may not represent the true R^2.

10.4. The method of generalized difference equation will produce BLUE estimators, provided the first-order autocorrelation parameter, ρ, is known or can be estimated. Also, remember to transform the first observation on the dependent and explanatory variables *a la* Prais-Winsten if the sample size is small.

10.5. These methods are:

(1) The first difference method, where it is assumed that $\rho = 1$

(2) ρ estimated from the Durbin-Watson *d* as: $\rho \approx 1 - d/2$

(3) ρ estimated from the regression $e_t = \hat{\rho}\, e_{t-1} + v_t$

(4) The Cochrane-Orcutt iterative procedure

(5) The Cochrane-Orcutt two-step method

(6) Durbin's two-step method

(7) Hildreth-Lu search procedure

(8) Maximum Likelihood method.

10.6. (1) *The graphical method*: There are no particular assumptions made. We simply plot the residuals from an OLS regression chronologically or plot the current residuals on the residuals in the previous time period, if the AR(1) scheme is assumed.

(2) *The Durbin-Watson test*: This test is based on several assumptions, such as (*i*) an intercept term is included in the model; (*ii*) X variables are non-stochastic (fixed in repeated sampling); (*iii*) AR(1) autoregressive scheme; (*iv*) no lagged values of the dependent variable are included as explanatory variables.

(3) *The runs test*: This is a non-parametric test.

10.7. On the Durbin-Watson *d* test's assumptions, see part (2) of Question 10.6. One drawback of the method is that if the computed *d* value lies in the uncertain zone, no definite decision can be made about the presence of (first-order) autocorrelation.

10.8. (***a***) *False*. The OLS estimators, although inefficient, are unbiased.

(***b***) *True*. Use the Durbin *h* test here.

(***c***) *True*. Except for autocorrelation, we are still retaining the other assumptions of the CLRM.

(***d***) *False*. It assumes that $\rho = +1$. If ρ is -1, we regress the two-period moving average of *Y* on the two-period moving averages of the *X* variables.

(***e***) *True*. Because the dependent variables in the two models are not the same, the two models cannot be directly compared.

10.9. In small samples, if the first observation is omitted from the transformed regression, the resulting estimators can be inefficient.

PROBLEMS

10.10. The answers are in the last column of the following table:

Sample size	Number of explanatory variables	Durbin-Watson d	Evidence of autocorrelation
25	2	0.83	Yes (positive autocorrelation)
30	5	1.24	Uncertain
50	8	1.98	No autocorrelation
60	6	3.72	Negative autocorrelation
200	20	1.61	Uncertain

10.11. The Swed-Eisenhart results are in the last column of the following table:

Sample size	Number of +	Number of −	Number of runs	Autocorrelation (?)
18	11	7	2	Evidence of autocorrelation
30	15	15	24	Evidence of autocorrelation
38	20	18	6	Evidence of autocorrelation
15	8	7	4	Evidence of autocorrelation
10	5	5	1	Evidence of autocorrelation

10.12. (***a***) The estimated d value is 0.6394. The 5% critical d values are 0.971 and 1.331. Since 0.6394 < 0.971, there is evidence of positive (first-order) autocorrelation.

(***b***) $\hat{\rho} \approx 1 - \frac{d}{2} = 0.6803$

(***c***) <u>*Dropping the first observation*</u>, we get:

$$(1)\quad \hat{Y}_t^{\,*} = -1.1230 \; + \; 23.3274\,(1 / X_t^{\,*})$$

$$t = (-0.6210) \quad (3.2700) \qquad\qquad r^2 = 0.5430$$

The residuals from this regression, when subjected to the runs test, gave the number of runs as 4, 5 positive and 6 negative residuals.

<u>*Retaining the first observation*</u>, we obtain:

$$(2)\ \hat{Y}_t^{\,*} = -1.8148 + 27.0485\,(1 / X_t^{\,*})$$

$$t = (-0.9793) \quad (3.8169) \qquad\qquad r^2 = 0.5930$$

In the residuals from this regression there were 5 runs, 6 positive and 6 negative residuals.

(***d***) Based on the runs test, neither regression (1) nor regression (2) seem to have autocorrelation.

Note 1: For *X*, the transformation is $\frac{1}{X_t *} = \frac{1}{X_t} - 0.6803\left(\frac{1}{X}\right)_{t-1}$, given the original format of the independent variable. The intercept in the transformed regressions was entered as $(1-\rho)$.

Note 2: The Prais-Winsten transformation is sensitive to the sample size.

10.13. (***a***) For n = 16 and k' = 1, the 5% critical d values are 1.106 and 1.371. Since the computed d of 0.8252 is less than d_L, there is evidence of positive autocorrelation in the data for Model A. For n = 16 and k' = 2, the 5% critical d values are 0.982 and 1.539. Since the computed d of 1.82 falls between 1.539 (d_U) and 2.461 (4 – d_U), we can conclude that there is no evidence of (first-order) positive autocorrelation in Model B.

(***b***) As this example shows, the Durbin-Watson d can be an indication of a specification error rather than pure auto-correlation.

(***c***) Although popularly used as a test of first-order autocorrelation, the d statistic can also be used to test for specification errors.

10.14. $\hat{Y}_t = -117.8014 + 0.2608\,X_t - 0.629\,X_{t-1} + 0.6562\,Y_{t-1}$

$$t = (-1.8796) \quad (2.6219) \quad (-1.4210) \quad (2.8096) \qquad R^2 = 0.9547$$

The estimated ρ is therefore 0.6562.

The results of the second stage regression with transformed *X* and *Y* are:

$$\hat{Y}_t^{\,*} = -120.3288 + 0.1790\ X_t^{\,*}$$

$$t = (-1.2383) \quad (4.2936) \qquad\qquad r^2 = 0.5060$$

Note: The first observation is included in the analysis *a la* Prais-Winsten. The intercept in the transformed regression has been entered as (1- ρ).

10.15. (***a***) For n = 25 and k' = 2, the 5% critical d values are 1.206 and 1.550. Since the computed d value of 0.8755 is below 1.206, there is evidence of positive (first-order) autocorrelation.

(***b***) Since the Durbin-Watson d test is inappropriate in this case, we cannot trust the computed d value. Perhaps a runs test could be done if the original data were available.

(***c***) Since in the presence of autocorrelation the conventionally estimated standard errors are biased, it is quite possible that in the original regression these standard errors were underestimated. As a result, the t ratios could be over-estimated. The transformed regression shows this clearly.

(***d***) See the answer given in (*b*).

Note: The Durbin-Watson d test assumes an AR(1) scheme. The Durbin two-step procedure implicitly assumes an AR(2) scheme (Why?).

10.16. (***a***) Using the d value given in the problem, we obtain an estimate of ρ as $\left(1-\frac{1.8624}{2}\right)$ = 0.0688. Using this value in the h statistic, we obtain:

$$h \approx (0.0688)\sqrt{\frac{17}{1-17(0.0403)}} = 0.5055.$$

Obviously, this h value is not statistically significant, suggesting that perhaps there is no autocorrelation in the data. But keep in mind that our sample size is rather small. Therefore, the preceding conclusion must be accepted cautiously.

(***b***) In autoregressive models like the one in the present example, the d value is generally around 2, which is the d value expected if there is no autocorrelation in the data. Therefore, there is a built-in bias against finding autocorrelation in such models on the basis of the d test.

10.17. (***a***) $\hat{Y}_t = -2015.2 + 0.7723\,X_t$

t = (-6.58) (19.52) $\qquad r^2 = 0.9380$; $\quad d = 0.4285$

(***b***) For n = 27 and k' = 1, the 5% critical d values are 1.089 and 1.233. Since the computed d value of 0.4285 is less than 1.089, there is evidence of positive autocorrelation.

(***c***) $\hat{\rho} = (1 - d/2) = (1 - 0.4285/2) = 0.7858$

(***d***) *Dropping the first observation*:

$$\hat{Y}_t^* = --617.9 + 0.8624\, X_t^*$$

$$t = (-3.01) \quad (8.47) \qquad r^2 = 0.749; \quad d = 0.9118$$

Retaining the first observation:

$$\hat{Y}_t^* = -642.1 + 0.867\, X_t^*$$

$$t = (-3.13) \quad (8.48) \qquad r^2 = 0.742; \quad d = 0.9248$$

(***e***) $\hat{e}_t = 0.7689\, e_{t-1}$

$t = (6.26)$

Note: There is no intercept in this model (Why?). Therefore, $\hat{\rho} = 0.6156$.

Dropping the first observation:

$$\hat{Y}_t^* = -927.8 + 0.8166\, X_t^*$$

$$t = (-4.42) \; (12.76) \qquad r^2 = 0.872; \quad d = 0.7931$$

Keeping the first observation:

$$\hat{Y}_t^* = -958.5 + 0.823\, X_t^*$$

$$t = (-4.70) \; (13.10) \qquad r^2 = 0.873; \quad d = 0.8209$$

(***f***) *First difference transformation* (i.e., $\hat{\rho} = 1$):

$$\Delta\hat{Y}_t = 0.8684\Delta X_t$$

$$t = (4.75) \qquad d = 0.9315$$

Note: In the transformed regressions, the intercept was entered as $(1-\rho)$.

(***g***) The striking result is that in all the transformations given above, whether one includes the first observation or not, there is a difference compared to the original regression. It is left to the reader to assess the results using the Runs Test discussed in Appendix 10A.

10.18. $(Y_t - \rho Y_{t-1}) = B_1(1-\rho) + B_2(X_{2t} - \rho X_{2t-1}) + ... + B_4(X_{4t} - \rho X_{4t-1}) + v_t$

10.19. Expanding (10.5), we obtain:

$$d = \frac{\sum e_t^2 + \sum e_{t-1}^2 - 2\sum e_t e_{t-1}}{\sum e_t^2} = 2(1-\hat{\rho}),$$

using the fact that:

$$\sum e_t^2 \approx \sum e_{t-1}^2 \text{ and } \hat{\rho} = \sum e_t e_{t-1} / \sum e_t^2 .$$

10.20. Dividing both numerator and denominator by n^2, we get:

$$\hat{\rho} = \frac{[(1 - d/2) + k^2/n^2]}{1 - \frac{k^2}{n^2}}$$

As n tends to infinity, the preceding expression reduces to (1- d / 2).

10.21. At the 5% level, if you routinely apply the Durbin-Watson d test, Model 1 exhibits positive autocorrelation, for the estimated d value lies below the lower critical d value of 1.288 (d_L = 1.288). If you consider model 2, the observed d value of 0.3411 lies below d_L = 1.245, suggesting that there is positive correlation in the error term. For Model 3, the estimated d of 1.611 lies above d_U = 1.474, indicating that this model does not suffer from (first-order) autocorrelation.

The conclusion that we draw from this exercise is that if you estimate a mis-specified model, the observed d value may be more an indication of model specification errors than pure autocorrelation.

10.22. Assign this as a classroom exercise.

10.23. Assign this also as a classroom exercise.