**Solutions to selected problems for Chapter Three:**

6. The null hypothesis is that the true (but unknown) value of beta is equal to one, against a one-sided alternative that it is greater than one

H0 : *β* = 1

H1 : *β* > 1

The test statistic is given by



We want to compare this with a value from the *t*-table with *T*–2 degrees of freedom, where *T* is the sample size, and here *T*–2 =60. We want a value with 5% all in one tail since we are doing a one-sided test. The critical *t-*value from the *t*-table is 1.671



The value of the test statistic is in the rejection region and hence we can reject the null hypothesis. We have statistically significant evidence that this security has a beta greater than one, i.e., it is significantly more risky than the market as a whole.

7. We want to use a two-sided test to test the null hypothesis that shares in Chris Mining are completely unrelated to movements in the market as a whole. In other words, the value of beta in the regression model would be zero so that whatever happens to the value of the market proxy, Chris Mining would be completely unaffected by it.

The null and alternative hypotheses are therefore

H0 : *β* = 0

H1 : *β* ≠ 0

The test statistic has the same format as before, and is given by



We want to find a value from the *t*-tables for a variable with 38–2=36 degrees of freedom, and we want to look up the value that puts 2.5% of the distribution in each tail since we are doing a two-sided test and we want to have a 5% size of test overall



The critical *t*-value is therefore ±2.03.

Since the test statistic is not within the rejection region, we do not reject the null hypothesis. We therefore conclude that we have no statistically significant evidence that Chris Mining has any systematic risk. In other words, we have no evidence that changes in the company’s value are driven by movements in the market.

8. A confidence interval for beta is given by the formula



Confidence intervals are almost invariably two-sided, unless we are told otherwise (which we are not here), so we want to look up the values which put 2.5% in the upper tail and 0.5% in the upper tail for the 95% and 99% confidence intervals, respectively. The 0.5% critical values are given as follows for a *t*-distribution with *T*–2=38–2=36 degrees of freedom



The confidence interval in each case is thus given by

(0.214±0.186\*2.03) for a 95% confidence interval, which solves to (–0.164,0.592)

and

(0.214±0.186\*2.72) for a 99% confidence interval, which solves to (–0.292,0.720)

There are a couple of points worth noting.

First, one intuitive interpretation of an X% confidence interval is that we are X% sure that the true value of the population parameter lies within the interval. So we are 95% sure that the true value of beta lies within the interval (–0.164,0.592) and we are 99% sure that the true population value of beta lies within (–0.292,0.720). Thus in order to be more sure that we have the true value of beta contained in the interval, i.e., as we move from 95% to 99% confidence, the interval must become wider.

The second point to note is that we can test an infinite number of hypotheses about beta once we have formed the interval. For example, we would not reject the null hypothesis contained in the last question (i.e., that beta = 0), since that value of beta lies within the 95% and 99% confidence intervals. Would we reject or not reject a null hypothesis that the true value of beta was 0.6? At the 5% level, we should have enough evidence against the null hypothesis to reject it, since 0.6 is not contained within the 95% confidence interval. But at the 1% level, we would no longer have sufficient evidence to reject the null hypothesis, since 0.6 is now contained within the interval. Therefore we should always if possible conduct some sort of sensitivity analysis to see if our conclusions are altered by (sensible) changes in the level of significance used.