**Answers to selected problems for Chapter 4**

2. (a) H0 : *β*3 = 2

We could use an *F*- or a *t*- test for this one since it is a single hypothesis involving only one coefficient. We would probably in practice use a *t*-test since it is computationally simpler and we only have to estimate one regression. There is one restriction.

(b) H0 : *β*3 + *β*4 = 1

Since this involves more than one coefficient, we should use an *F*-test. There is one restriction.

(c) H0 : *β*3 + *β*4 = 1 and *β*5 = 1

Since we are testing more than one hypothesis simultaneously, we would use an *F*-test. There are 2 restrictions.

(d) H0 : *β*2 =0 and *β*3 = 0 and *β*4 = 0 and *β*5 = 0

As for (c), we are testing multiple hypotheses so we cannot use a t-test. We have 4 restrictions.

(e) H0 : *β*2*β*3 = 1

Although there is only one restriction, it is a multiplicative restriction. We therefore cannot use a *t*-test or an *F*-test to test it. In fact we cannot test it at all using the methodology that has been examined in this chapter.

3. THE regression *F*-statistic would be given by the test statistic associated with hypothesis (iv) above. We are always interested in testing this hypothesis since it tests whether all of the coefficients in the regression (except the constant) are jointly insignificant. If they are, then we have a completely useless regression, where none of the variables that we have said influence *y* actually do. So we would need to go back to the drawing board!

The alternative hypothesis is

H1 : *β*2 ≠ 0 or *β*3 ≠ 0 or *β*4 ≠ 0 or *β*5 ≠ 0

Note the form of the alternative hypothesis: ‘or’ indicates that only one of the components of the null hypothesis would have to be rejected for us to reject the null hypothesis as a whole.

5. The null hypothesis is H0 : *β*3 + *β*4 = 1 and *β*5 = 1

The first step is to impose this on the regression model

*yt = β*1 *+ β*2*x*2*t + β*3*x*3*t + β*4*x*4*t + β*5*x*5*t + ut* subject to *β*3 + *β*4 = 1 and *β*5 = 1.

We can rewrite the first part of the restriction as *β*4 = 1 - *β*3

Then rewrite the regression with the restriction imposed

*yt = β*1 *+ β*2*x*2*t + β*3*x*3*t* + (1*-β*3)*x*4*t + x*5*t + ut*

which can be re-written

*yt = β*1 *+ β*2*x*2*t + β*3*x*3*t + x*4*t - β*3*x*4*t + x*5*t + ut*

and rearranging

*(yt – x*4*t – x*5*t ) = β*1 *+ β*2*x*2*t + β*3*x*3*t - β*3*x*4*t + ut*

(*yt – x*4*t – x*5*t*) *= β*1 *+ β*2*x*2*t + β*3(*x*3*t –x*4*t*)*+ ut*

Now create two new variables, call them *Pt* and *Qt*

*pt* = (*yt - x*3*t - x*4*t*)

*qt* = (*x*2*t -x*3*t*)

We can then run the linear regression

*pt* *= β*1 *+ β*2*x*2*t + β*3*qt+ ut* ,

which constitutes the restricted regression model.

The test statistic is calculated as ((RRSS–URSS)/URSS)\*(*T*–*k*)/*m*

In this case, *m*=2, *T*=96, *k*=5 so the test statistic = 5.704. Compare this to an *F*-distribution with (2,91) degrees of freedom, which is approximately 3.10. Hence we reject the null hypothesis that the restrictions are valid. We cannot impose these restrictions on the data without a substantial increase in the residual sum of squares.

6. *ri =* 0.080 *+* 0.801*Si +*  0.321*MBi +* 0.164*PEi –* 0.084*BETAi*

(0.064) (0.147) (0.136) (0.420) (0.120)

*1.25 5.45 2.36 0.390 –0.700*

The *t*-ratios are given in the final row above, and are in *italics*. They are calculated by dividing the coefficient estimate by its standard error. The relevant value from the t-tables is for a two-sided test with 5% rejection overall. *T*–*k* = 195; *tcrit* = 1.97. The null hypothesis is rejected at the 5% level if the absolute value of the test statistic is greater than the critical value. We would conclude based on this evidence that only firm size and market to book value have a significant effect on stock returns.

If a stock’s beta increases from 1 to 1.2, then we would expect the return on the stock to FALL by (1.2–1)\*0.084 = 0.0168 = 1.68%.

This is not the sign we would have expected on beta, since beta would be expected to be positively related to return, since investors would require higher returns as compensation for bearing higher market risk.

We would thus consider deleting the price/earnings and beta variables from the regression since these are not significant in the regression - i.e., they are not helping much to explain variations in *y*. We would not delete the constant term from the regression even though it is insignificant since there are good statistical reasons for its inclusion.

8. A researcher estimates the following two econometric models





(a) The value of *R*2 will almost always be higher for the second model since it has another variable added to the regression. The value of *R*2 would only be identical for the two models in the very, very unlikely event that the estimated coefficient on the *x*4*t* variable was exactly zero. Otherwise, the *R*2 must be higher for the second model than the first.

(b) The value of the adjusted *R*2 could fall as we add another variable. The reason for this is that the adjusted version of *R*2 has a correction for the loss of degrees of freedom associated with adding another regressor into a regression. This implies a penalty term, so that the value of the adjusted *R*2 will only rise if the increase in this penalty is more than outweighed by the rise in the value of *R*2.