

Math 1351

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Good luck for all



Chapter 0

(Section 1)

Sets :- مجموعات

Definition Sets is a collection of well-defined objects.

Examples :-

A = The set of all female students in this lecture

B = The set of all odd numbers between 2 and 8. $\Rightarrow \{3, 5, 7\}$

C = The set of Natural numbers.

D = The set of all the digits of the number 2620.

digits $\rightarrow 2620$
 $\rightarrow \{2, 6, 0\}$

* There are ~~two~~ two ways to write a set :-

1- By listing its elements :-

A = $\{Nour, Abeer, \dots, Aya\}$

B = $\{3, 5, 7\}$

C = $\{1, 2, 3, 4, 5, \dots, \infty\}$

D = $\{2, 6, 0\} = \{0, 6, 2\}$

غير مهم ترتيب في العناصر
المهم ذكر العناصر الموجودة وعدم تكرار العناصر أكثر من مرة.

(Limits)

* There are two ways to write a set :-

2- By describing its elements :- *طريقة بالوصف*

$A = \{ X : X \text{ is a F student in this Lec} \}$ *يتم تمييز المجموعة بأي رمز*

$B = \{ X : X \text{ is odd and } 2 < X < 8 \}$
 $= \{ 3, 5, 7 \}$ *استخدام الوصف بأي لغة المنهج توصيل للمجموعة*

$C = \{ X : X \text{ is a natural number} \}$

$D = \{ X : X \text{ is digits of } 2620 \}$
 $= \{ 2, 6, 2, 0 \}$ *يمكن كتابة هذه الرقم اعلم انوية كونوا موجودين الارقام*

* To indicate that an element belong (or dose not belong) to a set we use the symbols, \in and \notin respectively.

~~$5 \in B$~~
 $5 \in B$
 $5 \notin D$

Definitions:- *تعريف*

1 \rightarrow A set is finite if all its elements can be listed.

Ex:- $A = \{ 1, 3, 5 \}$ Finite. $C = \{ 1, 2, 3, \dots, 9999 \}$ Finite.

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Definitions:-

2 → A set is Infinite if all of its elements can not be listed.

Ex:- $A = \{x : x \text{ is odd number}\}$ Infinite → لأنه يوجد أعداد لا نهائية من الأعداد الفردية

$B = \{x : x \text{ is } 1 < x < 3\}$ Infinite → لأنه يوجد أعداد طبيعية كثيرة احتمال x أو x يكون 5 أو 6 أو 7 أو 8 أو 9 أو 10 أو 11 أو 12 أو 13 أو 14 أو 15 أو 16 أو 17 أو 18 أو 19 أو 20 أو 21 أو 22 أو 23 أو 24 أو 25 أو 26 أو 27 أو 28 أو 29 أو 30 أو 31 أو 32 أو 33 أو 34 أو 35 أو 36 أو 37 أو 38 أو 39 أو 40 أو 41 أو 42 أو 43 أو 44 أو 45 أو 46 أو 47 أو 48 أو 49 أو 50 أو 51 أو 52 أو 53 أو 54 أو 55 أو 56 أو 57 أو 58 أو 59 أو 60 أو 61 أو 62 أو 63 أو 64 أو 65 أو 66 أو 67 أو 68 أو 69 أو 70 أو 71 أو 72 أو 73 أو 74 أو 75 أو 76 أو 77 أو 78 أو 79 أو 80 أو 81 أو 82 أو 83 أو 84 أو 85 أو 86 أو 87 أو 88 أو 89 أو 90 أو 91 أو 92 أو 93 أو 94 أو 95 أو 96 أو 97 أو 98 أو 99 أو 100 أو 101 أو 102 أو 103 أو 104 أو 105 أو 106 أو 107 أو 108 أو 109 أو 110 أو 111 أو 112 أو 113 أو 114 أو 115 أو 116 أو 117 أو 118 أو 119 أو 120 أو 121 أو 122 أو 123 أو 124 أو 125 أو 126 أو 127 أو 128 أو 129 أو 130 أو 131 أو 132 أو 133 أو 134 أو 135 أو 136 أو 137 أو 138 أو 139 أو 140 أو 141 أو 142 أو 143 أو 144 أو 145 أو 146 أو 147 أو 148 أو 149 أو 150 أو 151 أو 152 أو 153 أو 154 أو 155 أو 156 أو 157 أو 158 أو 159 أو 160 أو 161 أو 162 أو 163 أو 164 أو 165 أو 166 أو 167 أو 168 أو 169 أو 170 أو 171 أو 172 أو 173 أو 174 أو 175 أو 176 أو 177 أو 178 أو 179 أو 180 أو 181 أو 182 أو 183 أو 184 أو 185 أو 186 أو 187 أو 188 أو 189 أو 190 أو 191 أو 192 أو 193 أو 194 أو 195 أو 196 أو 197 أو 198 أو 199 أو 200

3 → A set is empty (Null) if it has no elements.

Ex:- $A = \{\} = \emptyset$ فاني يعني مجموعة فارغة

$B = \{x : x \text{ is a chinese student in this lectures}\} = \{\} = \emptyset$
 لأننا حسب الطلاب داخل المحاضرة لا نرى طالباً صينياً لذلك المجموعة فارغة

Question

Indicate whether the following sets are Finite or Infinite and write them in a second way.

1- $\{x : x \text{ is a natural number less than } 8\}$ Finite → من أقل الأعداد الطبيعية محددة

$\{1, 2, 3, 4, 5, 6, 7\}$ - الطريقة الثانية

2- $\{10, 12, 14, \dots\}$ Infinite → غير محددة

$\{x : x \geq 10\}$ و $\{x : x \text{ is even}\}$
 $\{x : x > 8\}$ و $\{x \text{ is even}\}$

مجموعة ← even ← مجموعة زوجية

اللاقات بين المجموعات :- Relations between sets :-

1 → Two sets are equal if they contain the same elements. ←

Ex: If $A = \{1, 2, a, b\}$ and $B = \{a, b, 1, 2\}$ then $A = B$

2 → Two sets are disjoint if they have no common elements.

Ex: If $A = \{1, 2, 3, 9\}$ and $B = \{1/2, 4, 8\}$

→ A and B are disjoint

أما إذا كان يوجد عدد واحد مشترك في مجموعتين disjoint فلا disjoint

3 → The set A is a subset of the set B (denoted $A \subseteq B$) if every element of A is an element of B.

Ex: $A = \{5, 9, 7, 1\}$, $B = \{x : x \text{ is an odd number}\}$, $C = \{1, 2, 3\}$

$A \subseteq B$ → كل عناصر A موجودون في B / $C \not\subseteq B$ / $A \not\subseteq C$

مجموعة جزئية من مجموعة جزئية / مجموعة غير جزئية

Notes: - Every is a subset of its self ($A \subseteq A, B \subseteq B, C \subseteq C$)

The empty (Null) set is a subset of any set ($\emptyset \subseteq A, \emptyset \subseteq B, \emptyset \subseteq C$)

عنى

مع وطرح الأعداد الكسرية

أشكال

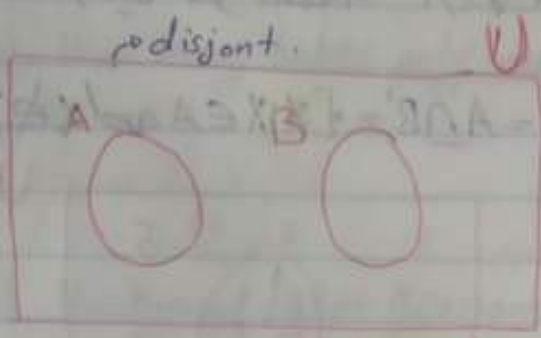
Sets Operations and Venn diagrams

نفسياً أي مجموعة جزئية من مجموعة أكبر من مجموعة U ^{مجموعة}

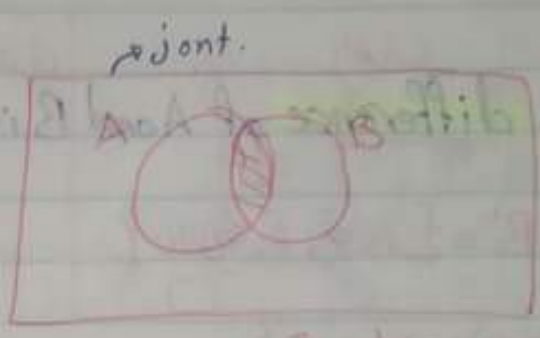
By:- assumption, each set is subset of a largset called the Universal set. This set is represented by a rectangle in Venn diagrams. While the subsets are represented by circles.

مجموعات الكسرية

Ex:- If A and B are sets, than the Venn diagram that represents them is



غير متقاطعة



متقاطعة

* Now do the following example to explain the set operations.

Ex:- If $U = \{x : x \in \mathbb{N} \text{ and } x < 11\}$, $A = \{2, 4, 7, 9\}$, $B = \{1, 3, 5\}$

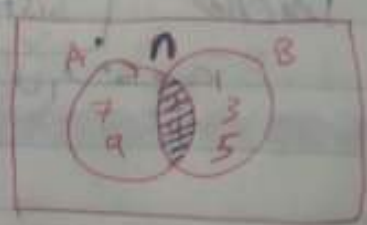
$\hookrightarrow = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$B = \{x : x \text{ is a Natural number less than } 6\}$. Then.

$\hookrightarrow = \{1, 2, 3, 4, 5\}$

* The intersection of A and B is $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

$$A \cap B = \{2, 4\}$$



* The union of A and B is $A \cup B = \{x : x \in A \text{ or } x \in B \text{ (or both)}\}$

$$A \cup B = \{2, 4, 7, 9, 1, 3, 5\}$$

* عدم تكرار الأرقام المتكررة في المجموعة



أي مجموعة X غير متفرقة
ناصر A أو غير متفرقة
ناصر B أو الجاهل

يعتمد على العجل
←

* The complement of A is $A' = \{x : x \in U \text{ and } x \notin A\}$.

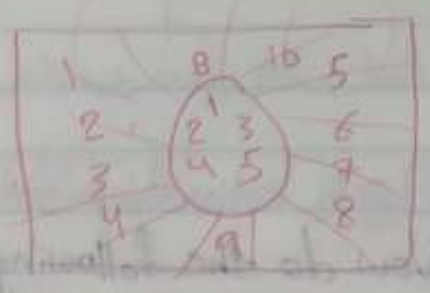


ملاحظة: كل شيء خارج A ينتمي لـ A' ويكون داخل U
A نفسه هو متممة A' ← A

* The difference of A and B is $A - B = A \cap B' = \{x : x \in A \text{ and } x \notin B\}$.

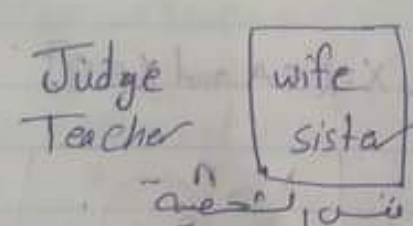
$B' = \{6, 7, 8, 9, 10\}$

$A \cap B' = \{7, 9\}$



* $B - A = \{1, 3, 5\} = B \cap A'$

عبارة عن العناصر الموجودة في B غير موجودة في A



لفظ لغوي السؤال :-
يوجد لنا قاطن وزوجته
ويوجد لنا استاذ وامه
ذبحوا انك معلم وطلبوا
3 وجبات من الطعام و
كل واحد منهم وجبة كاملة

الحل: الزوجية القاطن هي نفسها ام الاستاذ
ايضا الاستاذ



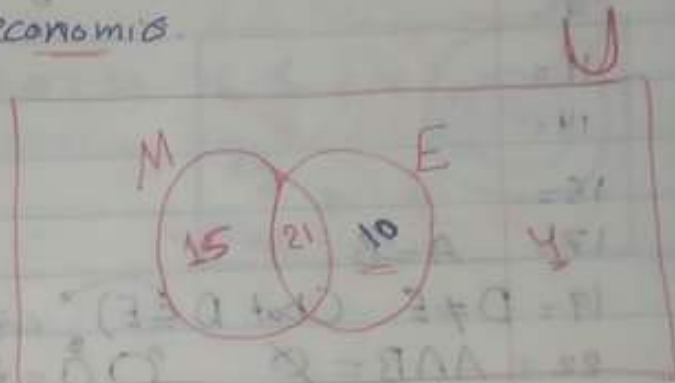
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Application example:-

مثال تطبيقي

Records at a small college show the following about the enrollments of 50 first-year students in math and Economics.

- 36 take math.
- 21 take math and economics.
- 4 do not take neither math nor economics.



a) - How many takes only math?

$36 - 21 = 15$
 $15 + 21 = 36$

b) - How many takes economics?

$10 + 21 = 31$

c) - How many takes only economics?

10

15 take math only
 21 take math and economics
 10 take economics only
 4 do not take either

$31 = 21 + 10 = \text{economics}$

... (A ∪ B)' = A' ∩ B'

- $A \cup \emptyset = A$
- $A \cap \emptyset = \emptyset$
- $A \cap A' = \emptyset$
- $A + A' = \text{Universal set}$
- $A = B = A \cap B'$
- $(A')' = A$
- $(A \cup B)' = A' \cap B'$
- $(A \cap B)' = A' \cup B'$

→ $U' = \emptyset$

ing

Outline

11 ج *

12 = $\{1, 2, 3, 4\}$

3 = $\{x \mid x \text{ is natural number greater than } 5\}$

4 = $3 \neq \emptyset$

6 = $\{7, 8, 9\}$

8 = $\{x \mid x \text{ is non natural number greater than } 6\}$

9 =

11 =

14 =

15 =

17 = $A = B$

19 = $D \neq E$ (but $D \subseteq E$)

22 = $A \cap B = \emptyset$

25 = $A \cap B = \emptyset \rightarrow A = \emptyset, B = \{x, y, a, b\}$

26 = $A \cap B = \{3\}$

28 = $A \cup B = \{a, e, i, o, u, b, c, d\}$

29 = $A \cup B = \{1, 2, 3, 4\} = B$

31 = $A' = U - A = \{4, 6, 9, 10\}$

35 = $(A \cup B)' = A \cup B = \{1, 3, 5, 8, 7, 2, 4, 10\}$

36 = $(A \cup B)' = A \cup B - U = \{6, 9\}$

40 = $A \cap (B' \cup C') = B' = \{1, 2, 5, 6, 7, 9\}, C' = \{1, 3, 5, 7, 9\}$

$B' \cup C' = \{1, 2, 5, 6, 7, 9, 3\}$

$A \cap (B' \cup C') = \{1, 2, 3, 5, 7\}$

43 = $A - B = \{1, 7\}$

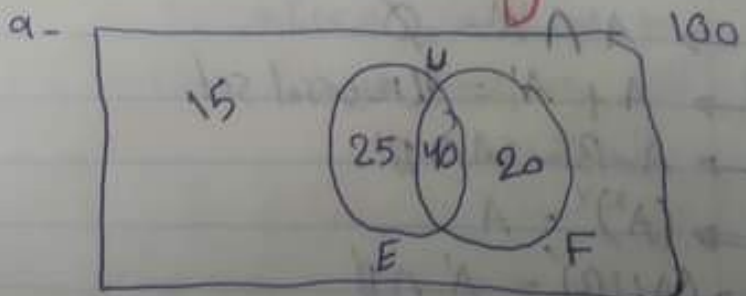
46 = $A - B = \{1, 2, 3, 4, 5\}$

51 =

$b = 40$

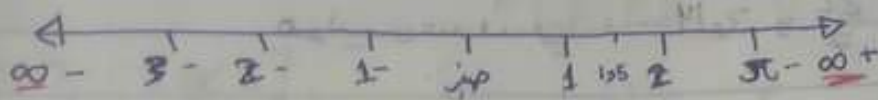
$c = 85$

$d = 25$

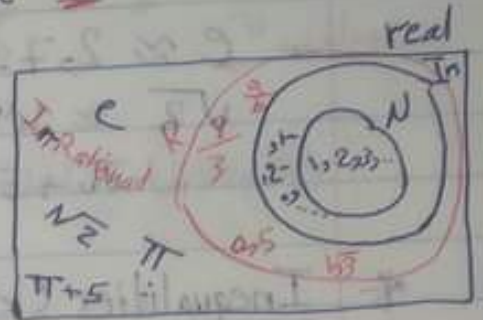


Section 0.2

The set of the real numbers can be represented by the real number line where each point on the line represents a real number.



* Subsets of the set of real numbers :-
 \subseteq + جزء



1. Natural = الأعداد الطبيعية
 $= \{1, 2, 3, 4, \dots, \infty\}$
Natural \subseteq Real

2. Integers = الأعداد الصحيحة (في الأعداد الطبيعية أو صفرية أو سلبية)
 $= \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
Integers \subseteq real number
Natural \subseteq Integers

3. Rational number = الأعداد النسبية
 $= \{x : x = \frac{a}{b}, a, b \text{ are } \underline{\text{Integer}}$ and $b \neq 0\}$.

Ex:- $\frac{8}{3}, \frac{-2}{7}, 4 \rightarrow \frac{4}{1}, \frac{-11}{1}, 0.5 = \frac{1}{2}, \frac{4}{3}$ (دوري)

Any number decimal representations that either ends or repeats is rational number :-
 ما دة صفة - أي عدد مكتوب بالأعداد أو -

تكرار نعتبره الأعداد النسبية عند : هو عدد نسبي
 الأعداد المشهورة هو عدد نسبي
 $\frac{13,5678}{10000}$

$2.0\overline{67} \rightarrow$ هو عدد نسبي

S. Q. 10

4. Irrational = غير نسبي
 $\mathbb{I} = \{x \mid x \in \mathbb{R} \text{ and } x \notin \text{rational}\}$

Ex: $\pi \approx \frac{22}{7}$ و 3.14 ... هو ليس عدداً نسبياً

Ex: $e \approx 2.7$...

$\sqrt{2} \approx 1.4$...
 $\pi + 5 \rightarrow$ Irrational



* Inequalities ($<$, $>$)



$a < b$ / $b > a$
 اقل / أكبر

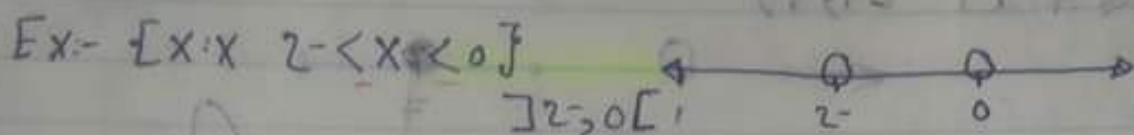
Ex: $3 > 5$, $3 < 5$, $-2 < 0$, $-2 > -4.5$, $2.3 > 2.2222$

$\frac{9}{5} > \frac{11}{8}$

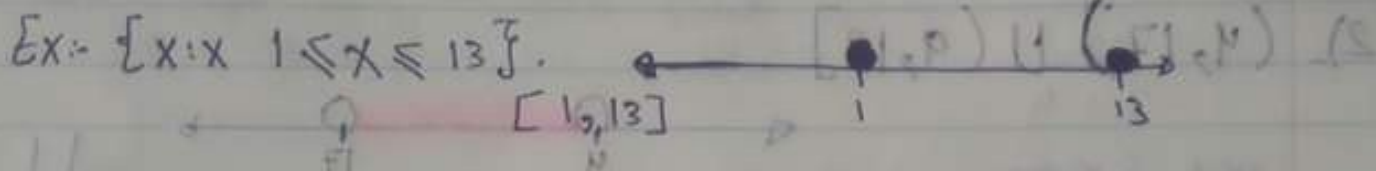
$1.8 > 1.375$

* Intervals : الفترات

1- open intervals :- الفترة المفتوحة (الحدود الأطراف داخلية)
 $\rightarrow = \{x : a < x < b\}$ $a, b \rightarrow$ فرضاً أي عدد حقيقي (Real numbers)



2- closed interval :- الفترة المغلقة (الحدود الأطراف داخلية)
 $\rightarrow = \{x : x a \leq x \leq b\}$



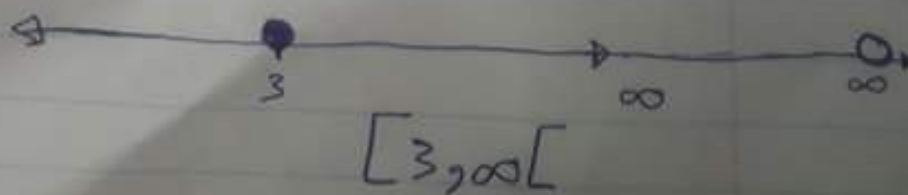
3- half open interval :- الفترة التي يكون فيها فترة مغلقة أو مفتوحة (في طرف داخل و طرف ليس داخل)
 $\rightarrow = \{x : x a \leq x < b\} / \{x : x a < x \leq b\}$



Q :- write the types of the following intervals and express it in a second and third way.

$+\infty > x \geq 3$

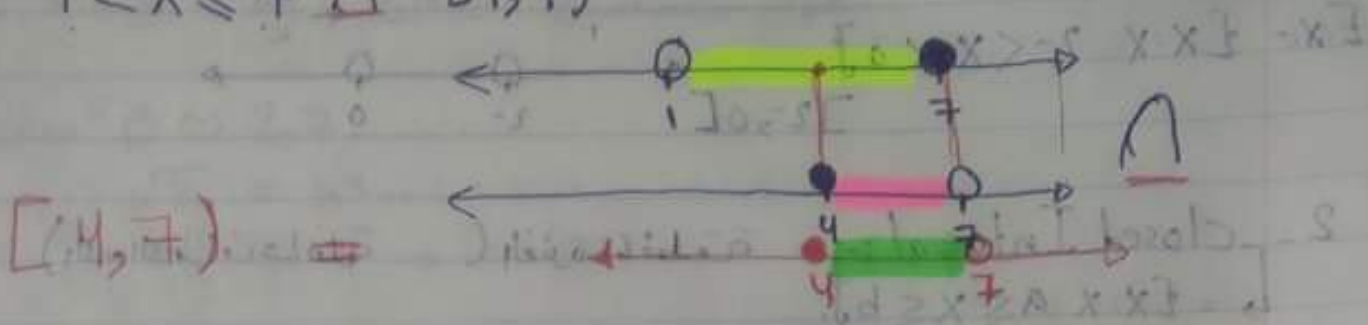
types :- half open



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Q:- perform the following operations, and write your answer in interval notation:-

1). $1 < X \leq 7 \cap [4, 7)$



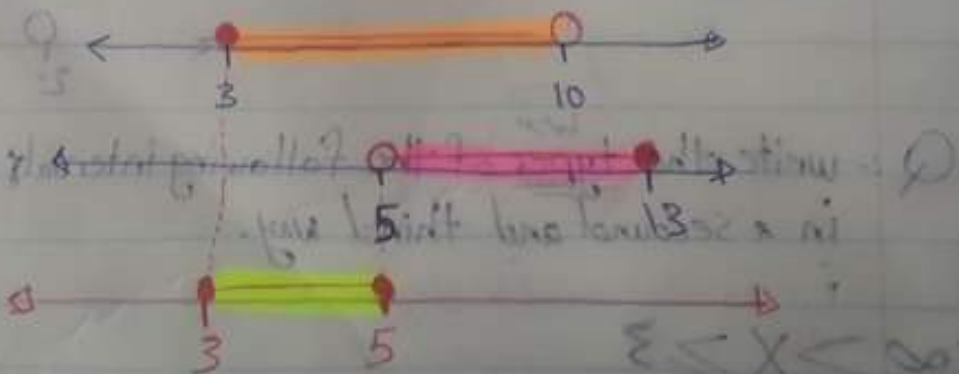
2). $(4, 17) \cup (9, 18]$



$(4, 18]$

Q:- $[3, 10) = [5, 13]$

$= [3, 5]$



Absolute value and operations with real number

القيمة المطلقة

$$|a| = \begin{cases} a & a \geq 0 \\ -a & a < 0 \end{cases}$$

$$|1-4| = 4$$

$$|16| = 16$$

Ex: $3+5=8$ $1+2=3$

$$3-5=-2$$

$$(4) \times -2 = -8$$

$$-4 \times -2 = 8$$

Q: $2+(8 \div 2) =$ الأولوية دائماً للقسمة

$$2+4=6$$

والضرب

الأولوية في العمليات

* () parentheses للاقواس

* powers للقوة

* \times or \div from left to right الضرب أو القسمة من اليسار لليمين

* $-$ or $+$ from left to right الجمع أو الطرح من اليسار لليمين

Done

homework :-

Q:- $(-3)^2 + |2+5|$...

$$2(1-2) + 6 + 3(2)$$

$$1 = 11 - 1$$

$$21 = 18 + 1$$

$$= \frac{9 + | -3 |}{2 \times (-1) + 6 + 3(2)} = \frac{9 + 3}{2 \times (-1) + 6 + 3(2)}$$

$$= \frac{12}{-2 + 2 \times 2}$$

$$= \frac{12}{-2 + 4} = \frac{12}{2} = \boxed{6}$$

$$\begin{array}{l} 0 < 0 \\ 0 > 0 \end{array} \quad \left| \begin{array}{l} a \\ a \end{array} \right| = |a|$$

Simplification

$$x = 5 \quad \text{بعض الأعداد العينية} \quad y = 3 + x - 1$$

$$z = 2 - x$$

$$y = 5 - x - 1$$

$$x + y + z = (5 + 5) + 2 - 1 = 11$$

$$y = 4 + x$$

... ..

... ..

... ..

... ..

... ..

...

Section 0.3 Integral Exponents

= Consider the expression (15), (16), (17), (18) \bullet (19) \bullet (20)

* In general, If a is a real number and n is positive integer, then $a^n = a \cdot a \cdot a \dots a$ (n factors), where n is the exponent & a is the base.

Ex: $2^4 = 16$ $(-2)^4 = 16$
 $-2^4 = -16$ $(-3)^2 = -27$
 ↳ 2 factors

* Rules of exponents

For any real numbers a and b and any integers m and n .

1. $a^m \cdot a^n = a^{m+n}$ Ex: $3^4 \cdot 3^{-2} = 3^{4+(-2)} = 3^2$ $x^1 \cdot x^2 = x^{1+2} = x^3$
2. $\frac{a^m}{a^n} = a^{m-n}$ ($a \neq 0$) Ex: $\frac{4^7}{4^4} = 4^{7-4} = 4^3$ $\frac{y^2}{y^{-1}} = y^{2-(-1)} = y^3$
3. $(ab)^m = a^m \cdot b^m$ Ex: $(3 \cdot 2)^2 = 3^2 \cdot 2^2 = 9 \cdot 4 = 36$ $(xy)^5 = x^5 \cdot y^5$
4. $(a^m)^n = a^{m \cdot n}$ Ex: $(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64$ $(x^2)^3 = x^{2 \cdot 3} = x^6$
5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $b \neq 0$ Ex: $\left(\frac{-2}{5}\right)^3 = \frac{(-2)^3}{(5)^3} = \frac{-8}{125}$ $\left(\frac{x^2}{2^{-1}}\right)^3 = \frac{(x^2)^3}{(2^{-1})^3}$
6. $a^0 = 1$ $a \neq 0$ Ex: $45^0 = 1$ $(-0.126)^0 = 1$ $10^1 = 10^0 = 10$

april

Section 2
Integral Exponents

*

7- $a^{-n} = \frac{1}{a^n}$ ($a \neq 0$). (Ex: $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ and $(-4)^{-3} = \frac{1}{-4^3} = \frac{1}{-64}$)

8- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ ($a, b \neq 0$) (Ex: $\left(\frac{x^2}{3}\right)^{-2} = \left(\frac{3}{x^2}\right)^2 = \frac{3^2}{x^4} = \frac{9}{x^4}$)

عند تحويل الأس إلى موجب في الأس فقط اقلب مقام على البسط.

* Rules of Exponents

for numerical numbers and for algebraic numbers

1- $a^m \cdot a^n = a^{m+n}$ (Ex: $x^2 \cdot x^3 = x^{2+3} = x^5$)

2- $\frac{a^m}{a^n} = a^{m-n}$ ($a \neq 0$) (Ex: $\frac{x^5}{x^2} = x^{5-2} = x^3$)

3- $(a^m)^n = a^{m \cdot n}$ (Ex: $(x^2)^3 = x^{2 \cdot 3} = x^6$)

4- $(a^m)^n = a^{m \cdot n}$ (Ex: $(x^2)^3 = x^{2 \cdot 3} = x^6$)

5- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ ($b \neq 0$) (Ex: $\left(\frac{x^2}{3}\right)^3 = \frac{x^{2 \cdot 3}}{3^3} = \frac{x^6}{27}$)

6- $a^0 = 1$ ($a \neq 0$) (Ex: $x^0 = 1$)

Section 0.4

Radicals and Rational Exponents

do $V = 2 \times 2 \times 2 = 2^3 = 8$

Now suppose we know that $v=8$ and we want the root.
(we want b such that $b^3=8$)

This can be expressed as $b^3 = 8 \Rightarrow b = 2$

In general, ~~the~~ The n^{th} root of a is $\sqrt[n]{a} = b$ only if $b^n = a$ subject to conditions page 22 where n is called the index and a is called the radicand.

Ex: 1) $\sqrt{4} = 2$ $\sqrt[3]{64} = 4$ $\sqrt[3]{-27} = -3$

$\sqrt{-9}$ = Not real number

* Definition: - تعريف

for a positive integer $n = a^{\frac{1}{n}} = \sqrt[n]{a}$

Ex: $9^{\frac{1}{2}} = \sqrt{9} = 3$ $16^{\frac{1}{4}} = \sqrt[4]{16} = 2$ $(-25)^{\frac{1}{2}} = \sqrt{-25}$ = not real number

Definition: - تعريف هذا أسهل من التعريف الأول (more general).

for a positive integer $n = a^{\frac{m}{n}} = \sqrt[n]{a^m} / a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m = (\sqrt[n]{a})^m$

Q:- write the following in radical form and simplify :-

$16^{\frac{3}{4}} = (\sqrt[4]{16})^3 = 2^3 = 8$ or $\sqrt[4]{16^3} = \sqrt[4]{4096} = 8$

$y^{\frac{-3}{2}} = (\sqrt{y})^{-3}$ or $\sqrt{y^{-3}} = \sqrt{\frac{1}{y^3}}$ $(6m)^{\frac{2}{3}} = \sqrt[3]{(6m)^2}$ $\sqrt[3]{6^2 m^2} = \sqrt[3]{36m^2}$

Q-1- write the following without radical signs :-

$$\sqrt{x^3} = x^{\frac{3}{2}} \quad \frac{1}{\sqrt[3]{b^2}} = \frac{1}{b^{\frac{2}{3}}} \quad \sqrt[3]{(ab)^3} = ab^{\frac{3}{3}} = ab$$

* Rules for radicals :- if $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real then so

1. $\sqrt[n]{a^n} = a \Rightarrow (a^n)^{\frac{1}{n}} = a^{\frac{n}{n}} = a^1 = a$

Ex :- 1) $\sqrt[4]{7^4} = 7$ 2) $\sqrt{(-2)^2} = 2$

2. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$

Ex :- $\sqrt[3]{2} \sqrt[3]{4} = \sqrt[3]{2 \cdot 4} = \sqrt[3]{8} = 2$

3. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \quad b \neq 0$

Ex :- $\frac{\sqrt[3]{54}}{\sqrt[3]{2}} = \sqrt[3]{\frac{54}{2}} = \sqrt[3]{27} = 3$

$\sqrt[4]{16} \neq \sqrt[4]{8}$

Q- write the following in radical form and simplify :-

$8 = \sqrt[3]{8^3} = \sqrt[3]{512}$

$\sqrt[3]{\frac{1}{8}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$

Simplifying Radicals :-

→ A radical is considered simplified if the power of its radicand's factors are each less than the index. قوة الجذر

Ex:- $\sqrt[3]{5^2} = \sqrt[3]{25}$ بالمثل للعوامل الأولية

$25 = 5 \cdot 5$ صحة بدني أعرف إذا صيغ أولاً، نضعها في القوة
factor إذا كانت الأس أصغر من قوة الجذر يكون بسيط مثل -

Simplified

$$3 > 2$$

أبسط هو صيغ في أسبسط صورة

$$\sqrt{14} = 14 = 7 \cdot 2 = \sqrt{7 \cdot 2} \therefore \text{simplified}$$

Ex:- $\sqrt[3]{xy^2} \rightarrow x \cdot y \cdot y = \sqrt[3]{x^1 y^2}$ simplified.

Ex:- $\sqrt[3]{16} = \sqrt[3]{2^4} = (2^4)^{\frac{1}{3}}$ simplify the following.

$$16 = 2 \cdot 8 \quad (2^4)^{\frac{1}{3}} = 2^{\frac{4}{3}} = 2^{\frac{1+\frac{1}{3}}{3}} = 2^{\frac{1}{3}} \cdot 2^{\frac{1}{3}} = 2 \cdot \sqrt[3]{2}$$

$2 \cdot 4$
 $2 \cdot 2$

$$\sqrt{10x^3} = \sqrt{2 \cdot 5x^3} = (2 \cdot 5 \cdot x^3)^{\frac{1}{2}} = 2^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} \cdot (x^3)^{\frac{1}{2}}$$

$10 = 2 \cdot 5$

$x^{\frac{3}{2}} = x^{\frac{2}{2} + \frac{1}{2}} = x^{\frac{2}{2}} \cdot x^{\frac{1}{2}}$

$$\sqrt{2} \cdot \sqrt{5} \cdot x \cdot \sqrt{x} = x \sqrt{2 \cdot 5 \cdot x}$$

$$\begin{aligned} \text{Ex:- } \sqrt{8x^5y^3z^2} &= \sqrt{2^3x^5y^3z^2} = 2x^2yz\sqrt{2xy^2z^0} \\ &= 2x^2yz\sqrt{2xy} \end{aligned}$$

outline 0.4.

Simplifying Radicals

15. $\sqrt[4]{m^2 n^5} = \sqrt[4]{m^2} \cdot \sqrt[4]{n^5}$

breaks it = $\frac{2}{4} \sqrt[4]{m^2} \cdot \frac{5}{4} \sqrt[4]{n^5}$ if radical is raised in index A factors are each less than the index

20. $-X^{-5} = -\sqrt[3]{X^{-5}} = -\sqrt[3]{\frac{1}{X^5}} = -\sqrt[3]{\frac{1}{X^5}}$

21. $y^{\frac{1}{4}} y^{\frac{1}{2}} = y^{\frac{1}{4} + \frac{2}{4}} = y^{\frac{3}{4}} = \sqrt[4]{y^3} \Rightarrow (\sqrt[4]{y})^3$

29. $y^{-\frac{5}{2}} = y^{-\frac{5}{2} + \frac{2}{2}} = y^{-\frac{5}{2} + \frac{2}{2}} = y^{-\frac{3}{2}} = \frac{1}{y^{\frac{3}{2}}} = \frac{1}{\sqrt{y^3}}$

34. $(X^{-\frac{2}{3}})^{-\frac{2}{5}} = X^{-\frac{2}{3} \cdot -\frac{2}{5}} = X^{\frac{4}{15}}$

36. Simplify $\sqrt[3]{-64x^6y^3}$

$\sqrt[3]{-64x^6y^3} = \sqrt[3]{-64} \cdot \sqrt[3]{x^6} \cdot \sqrt[3]{y^3} = -4x^2y$

$\sqrt[3]{-64} \cdot \sqrt[3]{x^6} \cdot \sqrt[3]{y^3} = -4 \cdot x^2 \cdot y = -4x^2y$

40. $\sqrt{32x^5y} = \sqrt{16 \cdot 2 \cdot x^2 \cdot x \cdot y} = \sqrt{16} \cdot \sqrt{2} \cdot \sqrt{x^2} \cdot \sqrt{x} \cdot \sqrt{y} = 4x\sqrt{2xy}$

$\sqrt[3]{8x^3y^3} = \sqrt[3]{8} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{y^3} = 2xy$

Section 0.5

Operation with Algebraic Expressions:

1) Algebraic expressions are additions, subtractions, multiplications, divisions or roots of one or more real numbers or letters called variables that represents real numbers.

- Ex: 1) $3x^2$ 2) $2y - 6xz$ 3) $3x - 2\sqrt{y}$

→ Term is a product of real number (coefficient) and one or more variables to powers.

- Ex: 1) $3x^2$ 2) $5xy^2z$ 3) x^2 4) $3\sqrt{x}$ 5) $6x^8$ (constant)
- coefficient = 3, 5, 1, 3, 6

→ The degree of a term is the sum of its variables powers.

- Ex: 1) $3x^2 = 2$ 2) $5xy^2z = 3$ 3) $x^2 = 2$ 4) $3\sqrt{x} = 3 \times \frac{1}{2} = \frac{3}{2}$ 5) $6x^8 = 8$

→ A polynomial is the sum of a finite number of terms with nonnegative integer powers on the variables.

- Ex: 1) $x^2 - 3y + \frac{1}{2}x + xy^2$ - polynomial 2) $x + \frac{2}{x}y + 2 \Rightarrow x + 2y^2 + 2$ - polynomial
- 3) $x^2 + \sqrt{x}$ - not polynomial 4) $y^2 - 2\sqrt{x} = y^2 - 2x^{\frac{1}{2}}$ - not polynomial

→ The degree of a polynomial is the degree of the term with the highest degree.

- leading term: $4x^3$
- leading coefficient: 4

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Ex: 1) $X^2 - 3y + \frac{1}{2}X + X^2y$

degree = 7

درجه بالاتر

Leading coefficient = 1

الرقم ای با أعلى درجة الحد

Leading term = X^2y

الحد ای بیه أعلى درجة (القائم)

2) $X^4 + 2y^3 + 2$

→ degree = 4

Leading coefficient = 1

Leading term = X^4

* Polynomial with Three term is called trinomial =

$2X^5 + X^3 - 2$

$xy - y^2 + X$

* Polynomial with two term is called binomial =

$X^5 + X^2$

$2xy + 3$

$-X^3 - y^2$

* Polynomial with one term is called monomial =

$3X^2$

xy^3

$4y^2zX$

Q:- if a polynomial contains only one variable X , then it is a polynomial in X .

Ex: 1) $X^3 - X + 6$

$3X^7 - X^2 + X - 1$

degree:- 3

Leading term:- X^3

Leading coefficient:- 1

7

$3X^7$

3

[Signature]

Operations :- العمليات الحسابية

Combining polynomials :- (المجموع والطرح والقسمة والضرب)
 نحدد المجموع ونطرح الحدود المتشابهة - هي الحدود التي عندها نفس المتغيرات بنفس القوة.

Ex :- a) $(4xy + 3x) + (5xy - 2x)$ ← مخرج المجموع

$$= 4xy + 5xy + 3x - 2x$$

$$= 9xy + x$$

b) $(3x^2 + 4xy + 5y^2 + 1) - (6x^2 - 2xy + 4)$ ← مثال على الطرح

$$\frac{3x^2 + 4xy + 5y^2 + 1}{3x^2 - 6x^2 + 4xy + 2xy + 5y^2 + 1 - 4} - \frac{6x^2 - 2xy + 4}{-4}$$

$$= -3x^2 + 6xy + 5y^2 - 3$$

Multiplication :- ضرب المبركات الجبرية كل طرف في العبارة الجبرية
 الأولى مع الطرف في العبارة الجبرية الثانية

Ex :- $(4x^2 + 3xy + 4x) \times (2x - 3y)$

$$= (4x^2 \cdot 2x + 4x^2 \cdot -3y) + (3xy \cdot 2x + 3xy \cdot -3y) + (4x \cdot 2x + 4x \cdot -3y)$$

$$= 8x^3 - 12x^2y + 6x^2y - 9xy^2 + 8x^2 - 12xy$$

$$= 8x^3 - 6x^2y - 9xy^2 + 8x^2 - 12xy$$

Ex :- $(x+4) \times (x+2)$

$$= x \cdot x + x \cdot 2 + 4x + 4 \cdot 2$$

$$= x^2 + 2x + 4x + 8$$

$$= x^2 + 6x + 8$$

* Special products :- $\vec{a} \cdot \vec{b} = a \cdot b \cdot \cos \theta$

1. $(X+a)^2 = X^2 + 2Xa + a^2$

2. $(X-a)^2 = X^2 - 2Xa + a^2$

3. $(X+a)(X-a) = X^2 - a^2 \Rightarrow (X-a)^2$

4. $(X+a)^3 = X^3 + 3aX^2 + 3a^2X + a^3$

5. $(X-a)^3 = X^3 - 3aX^2 + 3a^2X - a^3$

1. $(X+2)^2 = X^2 + 2 \cdot 2 \cdot X + 2^2 =$
 $= X^2 + 4X + 4$

2. $(y-3)^3 = y^3 - 3 \cdot 3 \cdot y^2 + 3 \cdot 3^2 \cdot y - 3^3$
 $= y^3 - 9y^2 + 27y - 27$
 $= y^3 - 9y^2 + 27y - 27$

Ans

* Division: القسمة

* تستخدم القسمة الطويلة عندما يكون لدينا متباينين بنفس المتغير
 * إما إذا كان عددي أكثر من متغيراً يستخدم القسمة الطويلة

مثال: $15x^2y^3 \div 3xy^5 = \frac{15x^2y^3}{3xy^5}$
 $= 5x^{1-2}y^{-2} = \frac{5x}{y^2}$ ← * تحتاج على طريقة سكين 0.3

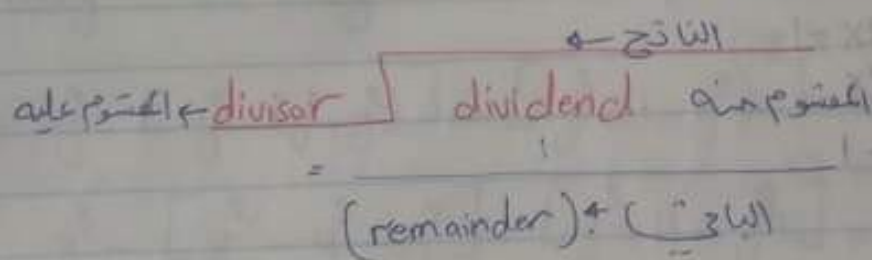
* أما القسمة الطويلة:

Ex:-

$$(4x^3 - 13x - 22) \div (x - 3) = 4x^2 + 12x + 23 + \frac{47}{(x-3)}$$

dividend المتقسم منه

divisor المتقسم عليه



→

$$\begin{array}{r}
 \text{الناتج} \leftarrow 4x^2 + 12x + 23 \\
 (x-3) \overline{) 4x^3 - 13x - 22} \\
 \underline{+ 12x^2} \quad - 4x^2 - 12x^2 \\
 \quad \quad \quad - 12x^2 - 13x - 22 \\
 \quad \quad \quad \underline{+ 23x - 36x} \\
 \quad \quad \quad \quad \quad - 23x - 22 \\
 \quad \quad \quad \quad \quad \underline{+ 23x - 69} \\
 \quad \quad \quad \quad \quad \quad \quad \quad 47 \leftarrow \text{الباقى}
 \end{array}$$

الكل = $4x^2 + 12x + 23 + \frac{47}{(x-3)}$

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$$\text{Ex: } (3x^2 - 2x + 4) \div (x+1) = 3x - 5 + \frac{9}{(x+1)}$$

$$\begin{array}{r} 3x - 5 \\ x+1 \overline{) 3x^2 - 2x + 4} \\ \underline{- 3x^2 + 3x} \\ -5x + 4 \\ \underline{-5x + 5} \\ 9 \end{array}$$

$$\text{Ex: } (4x^3 + 2x - 1) \div (x^2 + 2) = 4x + \frac{-6x - 1}{(x^2 + 2)}$$

$$\Rightarrow \begin{array}{r} 4x \\ x^2 + 2 \overline{) 4x^3 + 2x - 1} \\ \underline{- 4x^3 + 8x} \\ -6x - 1 \end{array}$$

outline:-

1). $10 - 3x - x^2$

④. $2x^5 + 7x^2y^3 - 5y^6 + 0$

a. degree = 2

$x = 6$

b. leading coefficient = -1

= -5

c. constant term = 10

= 0

d. number of variables = 1

(two variables (several variables).

⑨. $10xy - 4(xy)^2$

$10xy - 4(x^2 - 2xy + y^2)$

$10xy - 4x^2 + 8xy - 4y^2$

$18xy - 4x^2 - 4y^2$

⑫. $y^3 - [y^2 - (y^3 + y^2)] - [y^3 + (1 - y^2)]$

$y^3 - [y^2 - y^3 - y^2] - [y^3 + 1 - y^2]$

$y^3 - [y^2 - y^3 - y^2] - [y^3 + 1 - y^2]$
 $\rightarrow y^3 + y^3 - y^2 - 1 + y^2$

$y^3 - 1 + y^2$

⑬. $(5x^3) \cdot (7x^2)$

$5x^3 \cdot 7x^2$

$\rightarrow 35x^{3+2}$

$\rightarrow 35x^5$

⑭. $(4x + 3)^2$

$= 16x^2 + 24x + 9$

outline:-

$$(40) \left(\frac{2}{3} + x\right) \left(\frac{2}{3} - x\right) \Rightarrow (a+b)(a-b) = a^2 - b^2$$

$$\left(\frac{2}{3}\right)^2 (x)^2 = \frac{4}{9} - x^2$$

$$(49) (18m^2n + 6m^3n + 12m^4n^2) \div 6m^2n$$

$$\frac{18m^2n + 6m^3n + 12m^4n^2}{6m^2n} = \frac{3 \cdot 18m^2n}{6m^2n} + \frac{6m^3n}{6m^2n} + \frac{2 \cdot 12m^4n^2}{6m^2n} = 3 + m + 2m^2n$$

$$(53) (x+1)^3 = (x+1)^2(x+1)$$

$$\Rightarrow (x^2 + 2x + 1)(x+1)$$

$$x^2 \cdot x + x^2 \cdot 1 + 2x \cdot x + 2x \cdot 1 + 1 \cdot x + 1 \cdot 1$$

$$x^3 + x^2 + 2x^2 + 2x + x + 1$$

$$x^3 + x^2 + 2x^2 + 3x + 1$$

$$\rightarrow x^3 + 3x^2 + 3x + 1$$

$$(59) (x^4 + 3x^3 - x + 1) \div (x^2 + 1)$$

$$\begin{array}{r} x^2 + 1 \overline{) x^4 + 3x^3 - x + 1} \\ \underline{-x^4 + x^2} \\ 3x^3 - x^2 - x + 1 \\ \underline{-3x^3 + 3x} \\ -x^2 - 4x + 1 \\ \underline{-x^2 + x} \\ -4x + 1 \\ \underline{-(-4x - 2)} \\ 3 \end{array} \Rightarrow x^2 + 3x - 1 + \left(\frac{-4x - 2}{x^2 + 1}\right)$$

$$-4x - 2$$

Section 0.6

* If all the terms of a polynomial has a common factor then it can be factored out.
 (العوامل المشتركة) (أخراج العوامل المشتركة)

Ex: =

$$2x^5 + 2x^1 + 2 = 2(x^5 + x + 1) \rightarrow \text{4 terms} \rightarrow \text{factor out common factor from all terms}$$

$$2x^5 + 6x^1 + 8 = 2(x^5 + 3x + 4) \rightarrow \text{3 terms} \rightarrow \text{factor out common factor from all terms}$$

$$y^6x^2 + 2xy^2 + 2y = y(y^5x^2 + 2xy^2 + 2) \rightarrow \text{3 terms} \rightarrow \text{factor out common factor from all terms}$$

$$-3x^7 + 6x^2 = 3x^2(-x^5 + 2) \rightarrow \text{Factored completely} \rightarrow \text{3 terms} \rightarrow \text{factor out common factor from all terms}$$

* factoring trinomials (العوامل المشتركة بين الحدود)

Since $(x+a)(x+b) = x^2 + bx + ax + ab = x^2 + (a+b)x + ab$ therefore certain trinomials in x can be factored to $(x+a)(x+b)$

Ex: $x^2 + 5x + 6 = (x+2)(x+3)$ (بأخذ عددين ضربهم 6 و مجموعهم 5)
 $x^2 + 5x + 6$ (تأكد) $2+3=5$ / $2 \times 3 = 6$

$$x^2 - 7x + 6 = (x-1)(x-6) \quad \begin{matrix} -1 \times -6 = 6 \\ -1 + -6 = -7 \end{matrix}$$

$$y^2 - 10y + 25 = (y-5)(y-5) \quad \begin{matrix} -5 \times -5 = 25 \\ -5 + -5 = -10 \end{matrix}$$

Amal

Ex:-

اعيد كتابة البنية
صحيح

$$x^2 - 8 - 2x =$$

$$x^2 - 2x - 8 = (x - 4)(x + 2)$$

$$-4 \times 2 = -8$$

$$-4 + 2 = -2$$

باستخدام شكل
حد مربع

$$9x^2 + 6x - 3 =$$

$$3^2x^2 + 6x - 3 =$$

$$(3x)^2 + 2 \times 3x - 3 =$$

$$z = 3x$$

$$z^2 + 2z - 3 = (z + 3)(z - 1)$$

$$(3x + 3)(3x - 1)$$

$$3x - 1 = -3$$

$$3x + 3 = 2$$

$$x^2 - 16 = (x - 4)(x + 4)$$

$$-4 \times 4 = -16$$

$$-4 + 4 = 0$$

* Special Factorizations :-

الفرق بين

الفرق بين مربعين

Difference of two squares :- $x^2 - a^2 = (x + a)(x - a)$

Ex:- $x^2 - 16 = (x - 4)(x + 4)$

$$25x^2 - 36y^2 =$$

$$(5x)^2 - (6y)^2 = (5x - 6y)(5x + 6y)$$

Difference of two cubes :- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Ex:- $x^3 - 64 = (x - 4)(x^2 + 4x + 16)$

$$27 - 8x^3 = (3 - 2x)(9 + 6x + 4x^2)$$

* Sum of two cubes so $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$.

Ex: $x^3 + 64 = (x^3 + 4^3) = (x+4)(x^2 - 4x + 16)$

$27 + 8x^3 = (3^3 + (2x)^3) = (3+2x)(9 - 6x + 4x^2)$

outline:-

(4) $12y^3z + 4yz^2 - 8y^2z^3$ $12 = (4 \times 3) \quad | \quad 4 = (4) \quad | \quad 8 = (4) \cdot 2$

$4yz(3y^2 + z - 2yz^2)$

(9) $x^2 + 8x + 12 = (x+2)(x+6) \rightarrow$ معادلة تربيعية

(10) $x^2 - 2x - 8 = (x-4)(x+2)$

(17) $49a^2 - 144b^2$ * $x^2 - y^2 = (x+y)(x-y)$
 $7^2a^2 - 12^2b^2 = (7a)^2 - (12b)^2$ * $(ab)^n = a^n \cdot b^n$
 $(7a + 12b)(7a - 12b)$

a(19) $9x^2 + 21x - 8$

$3^2x^2 + 7 \cdot 3 \cdot x - 8$

$\rightarrow (3x)^2 + 7(3x) - 8 \Rightarrow (3x - 1)(3x + 8)$

(21) $4x^2 - x = x(4x - 1)$ Common factor.

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Section 0.7

Algebraic Fractions :- (صلاوات الجبرية الكسور)

Algebraic Fractions are :- $\frac{\text{algebraic expression}}{\text{algebraic expression}}$ \rightarrow $\frac{\text{numerator}}{\text{denominator}}$
المقسوم عليه المقسوم

له صيغتين عند دمج كسرين مختلفين

$$\text{Ex: } \frac{6}{8} = \frac{3x^2 - 14x + 8}{x^2 - 16} = \frac{ab}{c}$$

\rightarrow In this section, we will learn how to multiply, divide, add and subtract algebraic fractions. (الضرب والقسمة والجمع والطرح)

Simplifying fractions into simplify, factor the denominator and numerator and divide them by the common factors

$$\text{Ex: } \frac{6}{9} = \frac{3 \cancel{x}}{3 \cancel{x} \cdot 3} = \frac{2}{3}$$

قرب Products of Fractions :- to multiply, multiply the numerators together and denominator together.

$$\text{Ex: } \frac{4}{5} \times \frac{10}{12} = \frac{40 \div 10}{60 \div 10} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

$$\frac{-4x+8}{3x+6} \times \frac{2x+4}{4x+12} =$$

$$\frac{4(-x+2)}{3(x+2)} \cdot \frac{2(x+2)}{4(x+3)} = \frac{(-x+2) \cdot 2}{3 \cdot (x+3)} = \frac{-2x+4}{3x+9}$$

محمد

المقسوم عليه
Quotients of fractions:- to divide, multiply the dividend by the reciprocal of the divisor.

$$\text{Ex: } \frac{4}{7} \div \frac{5}{21} = \frac{4}{7} \times \frac{21}{5} = \frac{12}{5}$$

$$\text{Ex: } \frac{a^2b}{c} \div \frac{ab}{c^2} = \frac{a^2b}{c} \times \frac{c^2}{ab} = \frac{ac}{1} = ac.$$

$$\text{Ex: } \frac{6x^2-6}{x^2+3x+2} \div \frac{x-1}{x^2+4x+4} =$$

$$\frac{6x^2-6}{x^2+3x+2} \times \frac{x^2+4x+4}{x-1} =$$

$$\frac{6(x^2-1)}{(x+1)(x+2)} \cdot \frac{(x+2)(x+2)}{(x-1)} =$$

$$\frac{6(x-1)(x+1)}{(x+1)(x+2)} \cdot \frac{(x+2)(x+2)}{(x-1)} = 6(x+2) = 6x+12.$$

د. م. م.

* Combining Fractions: To add or subtract fractions find a common denominator or subtract the numerator.

$$Ex: \frac{3x+3}{12a^2} + \frac{3ax}{21a} = \frac{24}{96} + \frac{84}{96} = \frac{108}{96} = \frac{54}{48} = \frac{27}{24} = \frac{27 \div 3}{24 \div 3} = \frac{9}{8}$$

* Least Common Denominator: (LCD) = (unlike) $\frac{1}{2} + \frac{1}{3}$ etc

$$Ex: \frac{3(4x)}{12(12)} + \frac{3(1)}{8(8)} =$$

$12 \div 2 = 6, 12 \div 3 = 2$
 $8 \div 2 = 4, 8 \div 2 = 4$
 (common denominator) LCD = $2 \cdot 3 \cdot 2 = 24$
 $24 \cdot \frac{1}{24} = \frac{1}{1}$
 distinct factors

$$Ex: \frac{x \cdot 3x}{x^2 a^2} + \frac{4 \cdot a}{a \cdot a}$$

$a^2 = a \cdot a$ distinct factors
 $ax = a \cdot x$
 LCD = $a^2 \cdot x$

$$= \frac{3x^2 + 4a}{a^2 x}$$

$$Ex: \frac{y^2 + y - 3}{(y+1)(y-5)^2} - \frac{(y-2)(y-5)}{(y-4)(y-5)(y-5)}$$

$(y-5)^2 = y^2 - 10y + 25 = (y-5)(y-5)$
 distinct factors = $(y-5)(y+1)$
 LCD = $(y-5)^2(y+1)$

$$= \frac{(y^2 + y - 3) - (y-2)(y-5)}{(y-5)^2(y+1)}$$

$$= \frac{(y^2 - 3y - 3) - (y^2 + 7y + 10)}{(y-5)^2(y+1)} = \frac{5y - 13}{(y-5)^2(y+1)}$$

* find the least common denominator :- for :-

$$\frac{1}{x^3-x}, \frac{x+1}{x^2}, \frac{x}{y+1}$$

distinct factors -

$$x, (x-1), (x+1), (y+1)$$

$$\text{LCD} := x^2, (x-1), (x+1), (y+1)$$

$$x^3 - x = x(x^2 - 1) = (x-1)(x+1)$$

$$x^2 = x \cdot x$$

$$y+1 = y+1$$

sent

Chapter 1

(Section 1.1) وحدة

Solutions of linear Equations and

Inequalities in one variable.

معادلة

عبارة

متساوية

An equation is a statement that two algebraic expressions are equal.

(المعادلة: عبارتين جبريتين بينهما (إشارة مساواة).

Ex: 1. $x + x = -6 \Rightarrow x = -7$

2. $x^2 + x = 2x + 6 \rightarrow$ معادلات

3. $xy = 6$

4. $6(3-x) = 5 + \frac{x-1}{2}$

The values of the variables that makes the statements true are called solutions of the equations.

* Solving linear equations:-

Ex:- $x + x = -6 \Rightarrow x = -6 + -1 = -7 \Rightarrow -7 + 1 = -6$

Solutions

Ex. $4x - 7 = 8x + 2 = -7 = 4x + 2 = \frac{-9}{4} = \frac{4x}{4} = x = \frac{-9}{4}$ Solution

$4 \cdot \frac{-9}{4} - 7 = \frac{-9}{4} + 2$

(✓) $-16 = -16$ نعم متساويتان

Ex:- $x + 8 = 8(x + 1)$

$x + 8 = 8x + 8$

$x = 8x$
 $-x -x$

$\frac{0}{7} = \frac{7x}{7} = x = 0$ (solution).

Chapter 1

Ex: $\frac{2x}{3} - 1 = \frac{x-2}{2}$

$$6 \cdot \left(\frac{2x}{3} - 1 \right) = 6 \cdot \left(\frac{x-2}{2} \right)$$

$$\frac{2x}{3} \cdot 6 - 6 \cdot 1 = \frac{6 \cdot x - 6 \cdot 2}{2}$$

$$4x - 6 = 3x - 6$$

$$x - 6 = -6$$

$$x = 0$$

Ex: $\frac{2x-2}{x-3} = 4 + \frac{5}{x-3}$

$$(x-3) \cdot \left(\frac{2x-2}{x-3} \right) = (x-3) \cdot \left(4 + \frac{5}{x-3} \right)$$

$$\cancel{x-3} \cdot \frac{2x-2}{\cancel{x-3}} = x-3 \cdot 4 + \frac{5}{\cancel{x-3}} \cdot (x-3)$$

$$2x-2 = 4x-12+5$$

$$2x-2 = 4x-7$$

$$-2 = 2x - 7$$

$$\frac{2x}{2} = \frac{5}{2} = x = \frac{5}{2}$$

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$$\text{Ex: } \frac{2x-1}{x-3} = 4 + \frac{5}{x-3}$$

$$\frac{(x-3) \cdot 2x-1}{x-3} = (x-3) \cdot 4 + \frac{5}{x-3} \cdot (x-3)$$

$$2x-1 = 4x-12+5$$

$$\begin{array}{r} 2x-1 = 4x-7 \\ -2x \quad -2x \end{array}$$

$$\begin{array}{r} -1 = 2x-7 \\ +7 \quad +7 \end{array}$$

$$\frac{6}{2} = \frac{2x}{2} \Rightarrow x = \frac{3}{1} = \text{No solution.}$$

$$\text{Ex: } \frac{3x}{2x+10} = 1 + \frac{1}{x+5}$$

$$(2(x+5)) \cdot \frac{3x}{2x+10} = 2(x+5) \cdot 1 + \frac{1}{x+5} \cdot (2(x+5))$$

$$3x = 2x+10+2$$

$$\begin{array}{r} 3x = 2x+12 \\ -2x \quad -2x \end{array}$$

$$x = 12$$

$$\text{Ex: } \frac{2x+5}{-2x} = -3 + \frac{2x}{2x} \Rightarrow 5 \neq -3 \text{ always false } \therefore \text{No solution.}$$

$$\text{Ex: } 2x+6 = 2(3+x)$$

$$\begin{array}{r} 2x+6 = 6+2x \\ -2x \quad -2x \end{array} \Rightarrow 6 = 6 \text{ always true } \therefore \text{Solution or real number}$$

[Handwritten signature]

* Solving linear equations of two variables for one variable :-

$$\text{Ex - 1) } 4x + 3y = 12 \quad (\text{Solve the following equations for } x)$$

$$3y = 12 - 4x$$

$$y = \frac{12 - 4x}{3}$$

$$\text{Ex - 2) } (9x + \frac{3}{2}y = 11) \times 2$$

$$= 18x + 3y = 22$$

$$3y = 22 - 18x$$

$$y = \frac{22 - 18x}{3}$$

* Linear Inequalities :-

Linear Inequality :- is a statement that one algebraic expression is greater or less than another algebraic expression.

البيان الجبري الذي أكبر أو أصغر من الآخر.
البيان الجبري الذي أكبر أو أصغر من الآخر.

Ex 20

$$1) 3x + \frac{2}{x} \leq \frac{2}{-2}$$

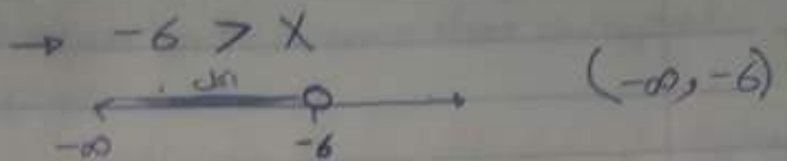
$$\frac{3x}{3} \leq \frac{0}{3} \Rightarrow x \leq 0$$



Ex:-

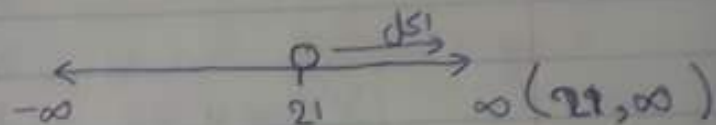
$$2) \quad \frac{2x-1}{-2x} > \frac{3x+5}{-2x}$$

$$\frac{-1}{-5} > \frac{x+5}{5} \rightarrow -6 > x$$



$$3) \quad \frac{17-x}{-17} < -4$$

$$\frac{x}{+1} < \frac{-21}{+1}$$



$$x > 21$$

قلنا الإشارة لأنه عندما نضرب بالـ 1 أو ما

نقسم على الـ 1 نغير الإشارة مثل

$$3 > 2$$

$$\hookrightarrow -1 \times 3 < -1 \times 2$$

$$-3 < -2$$

$$4) \quad \frac{3x}{4} - \frac{1}{6} \geq \frac{x - 2(x-1)}{3}$$

$$4 = 2 \cdot 2$$

$$6 = 3 \cdot 2$$

$$3 = 3$$

$$\frac{3}{12} \cdot \frac{3x}{4} - \frac{2}{12} \cdot \frac{1}{6} \geq \frac{12 \cdot x - 4}{12} \cdot \frac{2(x-1)}{3}$$

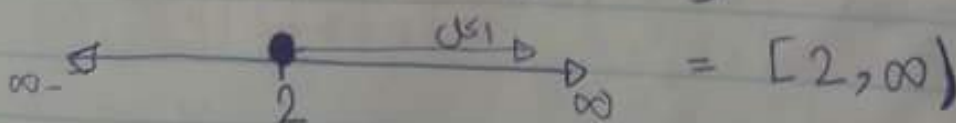
$$LCD: 2^2 \cdot 3$$

$$9x - 2 \geq 12x - 8(x-1)$$

$$9x - 2 \geq 12x - 8x + 8$$

$$\frac{9x-2}{-4x} \geq \frac{4x+8}{-4x}$$

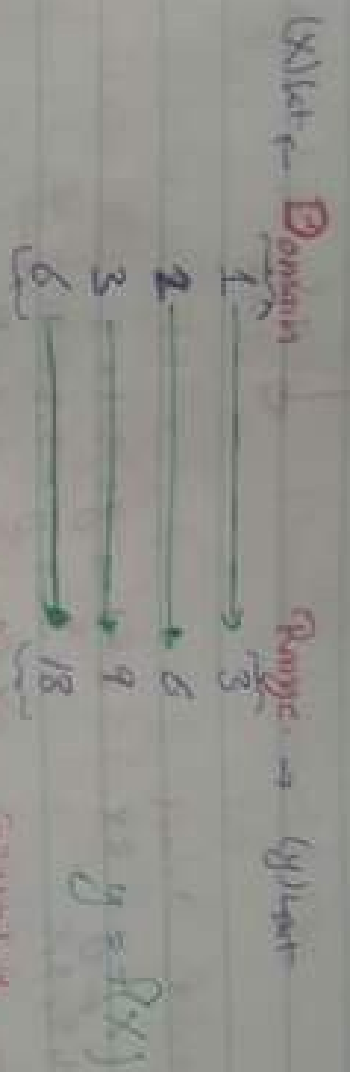
$$5x - 2 \geq 8 \Rightarrow \frac{5x}{5} \geq \frac{10}{5} \Rightarrow x \geq 2$$



(Section 1.2)
 Functions 80

Function is relation between two sets (domain and range) such that for each that for each element in the domain there corresponds only one element of the range.

with each given that set



A function may be defined by a set of ordered pairs in a table, a graph, or an equation.

1. Set of ordered pairs:

- $f = \{ (1, 3), (2, 6), (3, 9), (6, 18) \}$

Domain = $\{ 1, 2, 3, 6 \}$
 Range = $\{ 3, 6, 9, 18 \}$

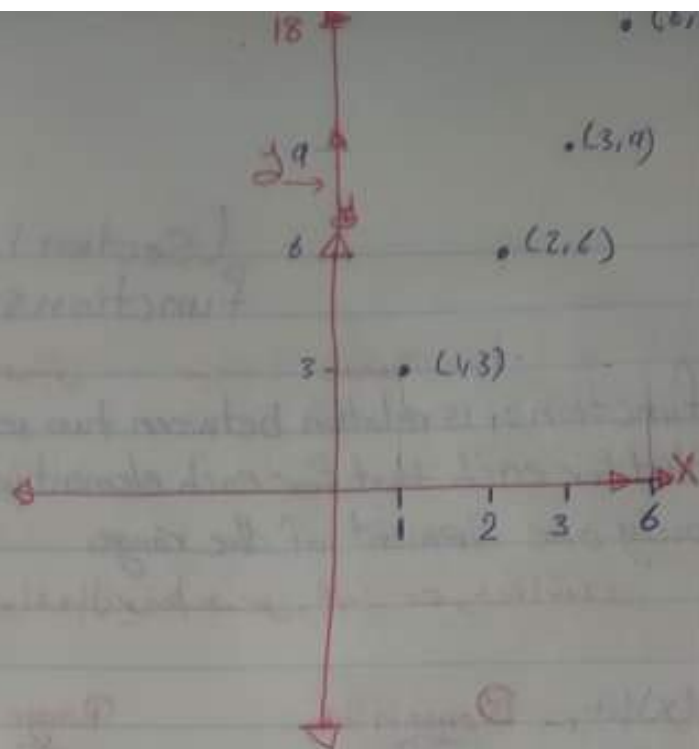
Ex: $f = \{ (1, 3), (2, 7) \}$ is a function (Not a function)

2. Table:

x	1	2	3	6
y	3	6	9	18

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3. Graph :- الرسم البياني



4. equation :- المعادلة

$$y = 3x \quad x = 1, 2, 3, 6$$

$$x = 1 \rightarrow y = 3 \cdot 1 = 3$$

$$x = 3 \rightarrow y = 3 \cdot 3 = 9$$

$$x = 6 \rightarrow y = 3 \cdot 6 = 18$$

$$x = 4 \rightarrow y = 3 \cdot 4 = 12$$

Evaluating functions: Finding the range (y , output, dependent variable) for a specified domain (x , input, independent variable).

Ex: evaluate the function above at $x=2$ and at $x=6$

$$\text{at } x=2 \rightarrow y=6 \quad y=3 \cdot 2=6$$

$$\text{at } x=6 \rightarrow y=18 \quad y=3 \cdot 6=18$$

Ex: evaluate the function $y = \frac{2-x}{x+4}$ at $x=6$ and $x \neq -4$

$$\text{at } x=6 \Rightarrow y = \frac{2-6}{6+4} = \frac{-4}{10} = -\frac{2}{5}$$

not in the domain

$$\text{at } x=-4 \rightarrow y = \frac{2-(-4)}{-4+4} = \frac{6}{0} \rightarrow \text{undefined (division by zero)}$$

Domain and Range.

If the domain is not specified then it is the set of all real number inputs that results in real number outputs the range is the set of all outputs obtained from evaluating all the domain.

Ex: $y = f(x) = x^2$ Domain is all real number $(-\infty, \infty)$.

$y = 1 + \frac{1}{x-2}$ Domain $(-\infty, \infty) \setminus \{2\}$ $\frac{x-2}{x-2} = \frac{0}{0}$
 $x = \frac{2}{1} = 2$

$f(x) = \frac{3}{2x - \frac{1}{3}}$ $3(2x - \frac{1}{3}) = 0 \Rightarrow 3$

$6x - 1 = 0 \rightarrow \frac{6x}{6} = \frac{1}{6} = x = \frac{1}{6}$
 Domain $(-\infty, \infty) - \{\frac{1}{6}\}$

$f(x) = \sqrt{4-x}$ $4-x \geq 0$
 $-x \geq -4$

Domain $(-\infty, 4]$ $-x \geq -4 \Rightarrow x \leq 4$

operation with functions: $(f+g)(x) = f(x) + g(x)$

$(\frac{f}{g})(x) = \frac{f(x)}{g(x)}$ $g(x) \neq 0$

Ex: if $f(x) = 3x+3$ and $g(x) = 8-2x$ $h(x) = x^2-1$ then.

1) $(f+g)(x) = f(x) + g(x)$
 2) $(f+g)(2) = 3x+3+8-2x = 2+11 = 13 \neq x+11$
 3) $(h-g)(x) = h(x) - g(x)$
 $(h-g)(2) = (2^2-1) - (8-2 \cdot 2) = 3 - 4 = -1$
 $(h-g)(x) = x^2-1 - (8-2x) = x^2-1-8+2x = x^2+2x-9$

3. $(g \cdot h)(x) = g(x) \cdot h(x)$

$(g \cdot h)(1) = (8-2x) \cdot (x^2-1) = 8x^2 - 8 - 2x^3 + 2x$
 $= -2x^3 + 8x^2 + 2x - 8 \Rightarrow -2 + 8 + 2 - 8 = 0$

$$4) \left(\frac{g}{h}\right)(x) = \frac{g(x)}{h(x)} = \frac{8-2x}{x^2-1}$$

$$5) \left(\frac{f}{h}\right)(x) = \frac{f(x)}{h(x)} = \frac{3x+3}{x^2-1} = \frac{3(x+1)}{(x-1)(x+1)} = \frac{3}{x-1}$$

$$\left(\frac{f}{h}\right)(x) = \frac{3}{x-1} \text{ domain } (-\infty, \infty) - \{1\}$$

$$5.5) \left(\frac{f}{h}\right)(7) = \frac{3}{7-1} = \frac{3}{6} = \frac{1}{2}$$

$f(x)$ evaluate $f(x)$ at $x=3$
 $f(3)$

evaluate $f(x)$ at $x=b$
 $f(b)$

evaluate $f(x)$ at $x=\frac{1}{2}$
 $f\left(\frac{1}{2}\right)$

evaluate $f(x)$ at $x=\infty$
 $f(\infty)$

* Composite functions:- f composed of g written as $(f \circ g)(x) = f(g(x))$

Ex:- if $f(x) = 3x+3$ and $g(x) = 8-2x$ $h(x) = x^2-1$ then:-

$$\begin{aligned} 1) (f \circ g)(x) &= f(g(x)) \\ &= 3(8-2x)+3 \\ &= 24-6x+3 \\ &= -6x+27 \end{aligned}$$

$$\begin{aligned} 2) (h \circ f)(x) &= h(f(x)) \\ &= (3x+3)^2-1 \\ &= 9x^2+18x+9-1 \\ &= 9x^2+18x+8 \end{aligned}$$

$$1.5) (f \circ g)(g(3)) = -6x(3)+27 = -9$$

$$\begin{aligned} 2.5) (h \circ f)(9) &= h(f(9)) \\ &= 9(9)^2+18(9)+8 \\ &= 9+18+8 = 35 \end{aligned}$$

$$\begin{aligned} 6) (f \circ g \circ h)(x) &= f(g(h(x))) \\ &= 3|8-2(x^2-1)|+3 \\ &= 3(8-2x^2+2)+3 \\ &= 24-6x^2+6+3 \\ &= -6x^2+33 \end{aligned}$$

$$* f(x) = \frac{3}{2x-7}$$

$$2x-7=0$$

$$\frac{2x}{2} = \frac{7}{2} \Rightarrow x = \frac{7}{2}$$

$$\text{Domian } (-\infty, \infty) - \left\{ \frac{7}{2} \right\}$$

$$* g(x) = \sqrt[4]{x-3}$$

$$x-3 \geq 0$$

$$x \geq 3$$

$$\text{Domian } [3, \infty)$$

$$* m(x) = \sqrt[4]{x+3}$$

$$x+3 \geq 0$$

$$x \geq -3$$

$$\text{Domian } [-3, \infty)$$

John

Section 1,3 Linear Functions

A linear function is a function of the form $y = f(x) = ax + b$ where a and b are constants and variables.

Ex: $y = f(x) = (2x + 4)$

$a = 2$ $b = 4$

$f(x) = 2x - 5$

$a = 2$ $b = -5$

$4x = 2x - y$

$-2x = -y - 2x$
 $y = 2 - 4x$

$a = -4$ $b = 2$

$y = -x + 4$
 $a = -1$ $b = 4$

$y = f(x) = 3 + 6x$
 $a = 6$ $b = 3$

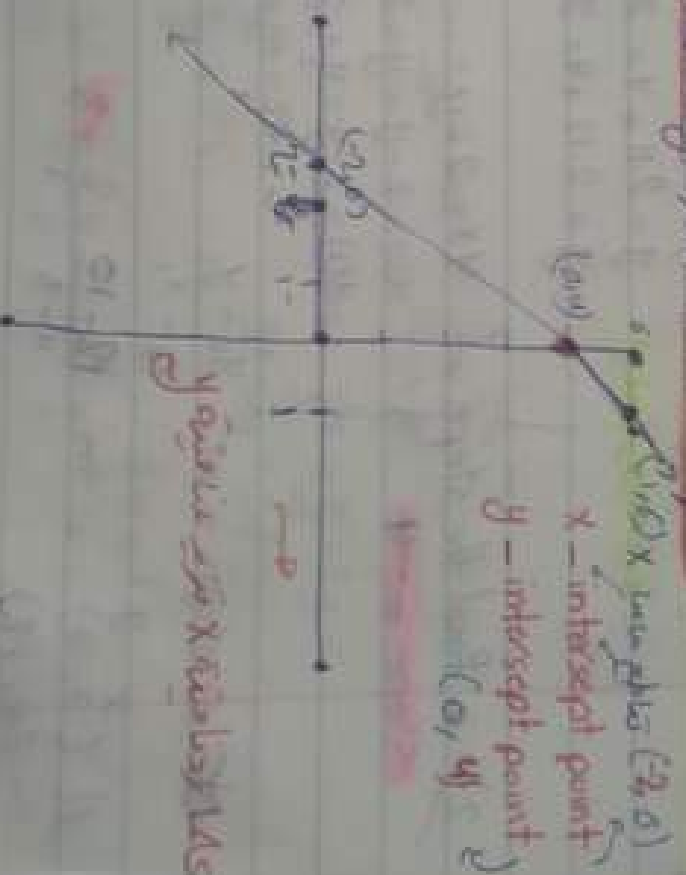
a - slope
 b - y-intercept

In this section we will learn how to graph linear functions, find their slopes, rates of change, and find their equations.

1- graphing linear functions =

$y = 2x + 4$

$y = 2x + 4$	x	y
0		4
1		6
$y = 2x + 4 = 0$	-2	0



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Ex: Graph $4x = 2 - y$

$$4x(0) = 2 - y$$

$$0 = 2 - y$$

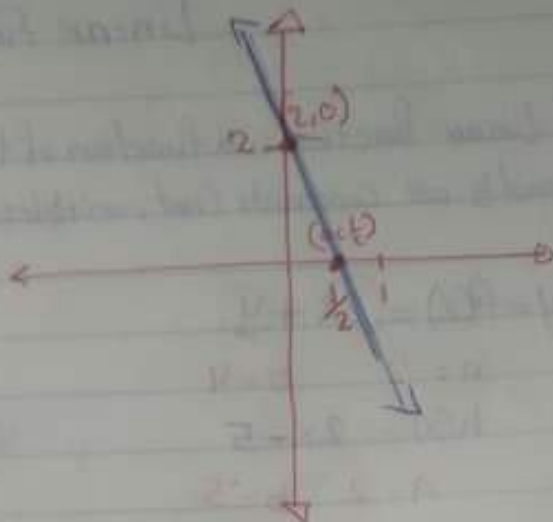
$$y = 2$$

x	y
0	2
$\frac{1}{2}$	0

$$4x = 2 - 0$$

$$\frac{4x}{4} = \frac{2}{4}$$

$$x = \frac{1}{2}$$



* Rate of change (slope) :-

نسبة التغير

كم (y) تتغير عندما نزيد قيمة (x) قيمة زائدة

1) * Find the slope of $y = 2x + 4$

Slope = 2

$$y = 2 \cdot 3 + 4 = 10$$

$$y = 2 \cdot 4 + 4 = 12$$

$$y = 2 \cdot 11 + 4 = 26$$

$$y = 2 \cdot 12 + 4 = 28$$

x	y
3	10
4	12

$$\frac{2}{1} = \frac{2}{1} \rightarrow \frac{2}{1} = \frac{2}{1}$$

x	y
11	26
12	28

$$\frac{2}{1} = \frac{2}{1} \rightarrow \frac{2}{1} = \frac{2}{1}$$

2) * Find the slope of $4x = 2 - y$

Slope = -4

$$4(0) = 2 - y = y = 2$$

$$4(1) = 2 - y = y = 2 - 4 = -2$$

$$-1 + 2 = -1 + y + 1 = -2 = y$$

x	y
0	2
1	-2
2	-6

$$\frac{-6 - 2}{2 - 0} = \frac{-8}{2} = -4$$

$$\frac{-2 - 2}{1 - 0} = \frac{-4}{1} = -4$$

$$\frac{-6 - 2}{2 - 0} = \frac{-8}{2} = -4$$

* or slope = $m = \frac{y_2 - y_1}{x_2 - x_1}$

1) $\frac{(3, 10)}{(4, 12)} = m = \frac{12 - 10}{4 - 3} = \frac{2}{1} = 2$

2) $\frac{(0, 2)}{(2, -6)} = \frac{-6 - 2}{2 - 0} = \frac{-8}{2} = -4$

* or slope:- 1) $y = 2x + 4$

$$y = ax + b$$

a = slope

b = y-intercept

$$a = 2$$

$$\text{slope} = 2$$

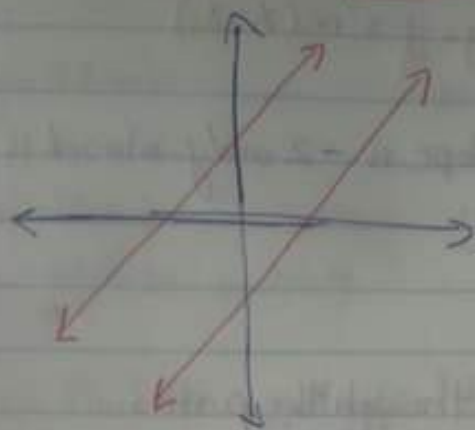
2) $4x = 2 - y$

$$y = -4x + 2$$

$$a = -4$$

$$\text{Slope} = -4$$

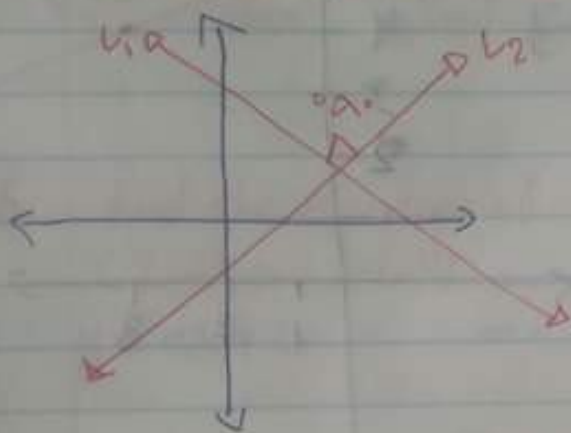
* Two Lines are parallel if their slope are equal



متوازيين
إذا كان نفس (slope) للخطين ما يتقاطعا
حتى لو اشواك النهاية

$$m_1 = m_2$$

* Two lines are perpendicular if their slope product of their slopes is negative one



$$m_1 \times m_2 = -1$$

متعامدين
إذا كان حاصل ضرب (slope) = -1 تكون
الخطين متعامدين

Ex:- are the lines $3x = 2 - 2y$ and $y = \frac{2}{3}x + 107$

$m_1 \neq m_2$ - Parallel, perpendicular, or neither

$$m_2 = \frac{2}{3}$$

$$m_1 = \frac{-3}{2}$$

$$m_2 \cdot m_1 = \frac{2}{3} \cdot \frac{-3}{2} = -1$$

$$\text{slope} = 3x = 2 - 2y$$

$$\frac{3x - 2}{-2} = \frac{-2y}{-2}$$

$$y = \frac{-3}{2}x + 1$$

abd

* Writing equations of lines so $y = ax + b$
 $y - y_1 = m(x - x_1)$
 slope \rightarrow m , y-intercept \rightarrow b

Ex:- Find the equations of lines whose ~~equation~~ slope is -2 and y-intercept is 7 .

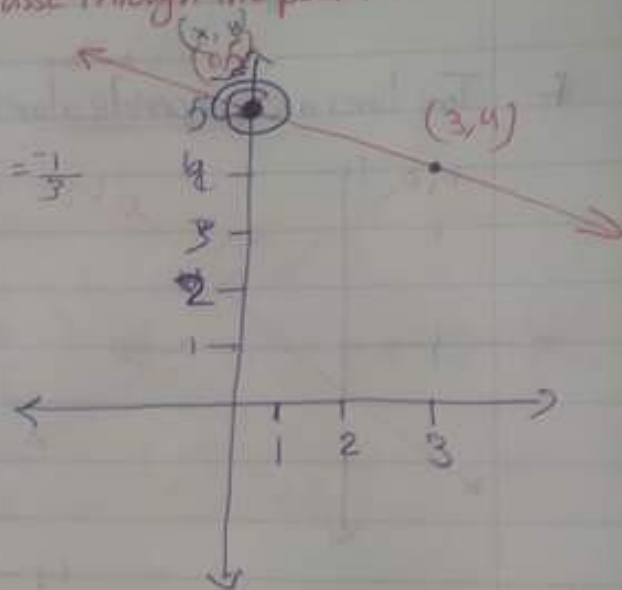
equation $\Rightarrow y = -2x + 7$

Ex:- Find the equations of the line that passes through the points $(3, 4)$ and $(0, 5)$.

$y = ax + b$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 4}{0 - 3} = -\frac{1}{3}$
 slope \rightarrow m

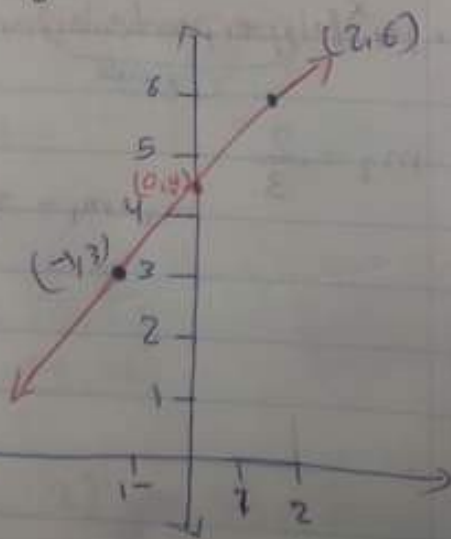
$y = -\frac{1}{3}x + 5$



Ex:- Find the equation of the line that passes through the points $(-1, 3)$ and $(2, 6)$

$y = ax + b$
 slope \rightarrow a , y-intercept \rightarrow b

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 3}{2 - (-1)} = \frac{3}{3} = 1$ slope



$y = x + b$

$6 = 2 + b$
 $-2 \quad -x$

∴ the equation $y = x + 4$

$b = 4$

Ex: Find the equations of the line that passes through $(4,0)$ and $(1,3)$

$$y = ax + b$$

$$m = \frac{3-0}{1-4} = \frac{3}{-3} = \boxed{-1}$$

$$y = -1x + b \quad \text{Equation: } y = -x + 4$$

$$\begin{array}{l} 0 = -4 + b \\ +4 \quad +4 \end{array} \rightarrow b = 4$$

Ex: Find the equations of the line that passes through the point $(2,1)$ and

parallel to $y = 3x + 117$.

$$y = ax + b$$

slope a y-intercept b

$$y = 3x + b \quad \text{equation } y = 3x + -5$$

$$1 = 3(2) + b$$

$$\begin{array}{l} 1 = 6 + b \\ -6 \quad -6 \end{array} \rightarrow b = -5$$

Ex: Find the equations of the line that passes through the points $(2,4)$ and $(-1,4)$

$$y = ax + b$$

$$m = \frac{4-4}{-1-2} = \frac{0}{-3} = 0$$

horizontal
(افقي)

$$y = 0(x) + b$$

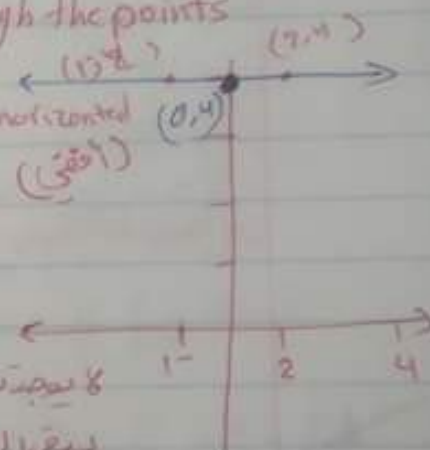
$$y = 4$$

$$4 = 2(0) + b$$

$$\underline{4 = b}$$

$$\therefore \text{the equations } \rightarrow y = 4$$

المعادلة افقية
يعني ان كل
النقاط على
الخط لها نفس
القيمة لـ y



Ex: Find the equation of the line that passes through $(-1,2)$ and $(-1,4)$

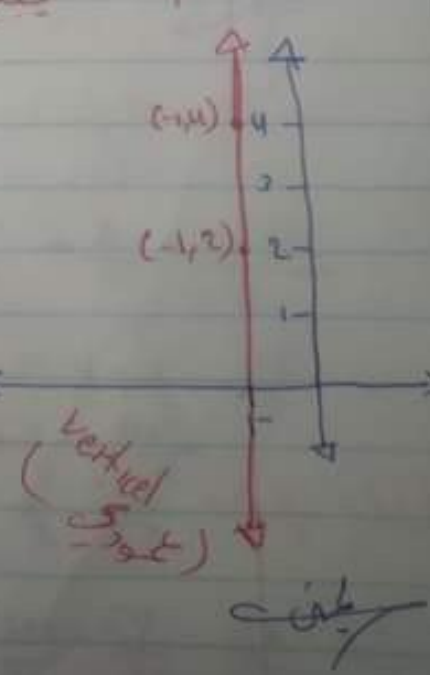
$$y = ax + b$$

$$m = \frac{4-2}{-1-(-1)} = \frac{2}{0} = \text{undefined}$$

$$\frac{2}{0}$$

ليس هناك اقتران
الغير (H) مرتبوع غير
ممكن

$$\therefore \text{equation is } x = -1$$



Section 1.5 Solutions of systems of linear Equations

A solution of the linear equation with two variables (x and y) is the pair (x, y) that will make the equation true.

Ex:- For the equation $4x + 3y = 11$ $(0, \frac{11}{3})$ $(\frac{11}{4}, 0)$ $(\frac{2}{3}, 3)$ are all solutions.

$$4 \cdot 0 + 3 \cdot \frac{11}{3} = 11 \quad (\checkmark)$$

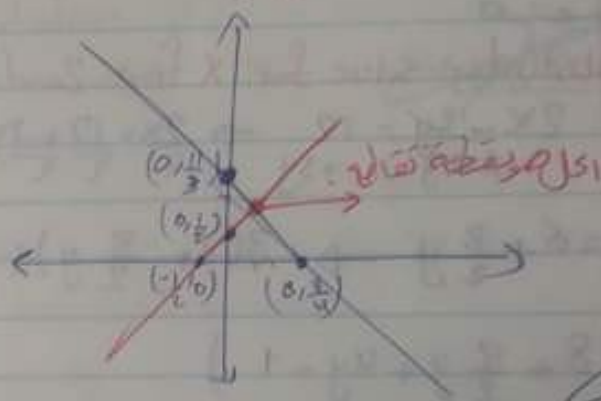
Suppose we want a solution that satisfies more than one equation (system of equations). This solution is called a simultaneous solution and it could be found using three methods.

Ex:- Find a simultaneous solution for the following system of equations.

$$4x + 3y = 11$$

$$2x - 5y = -1$$

x	y
0	$\frac{11}{3}$
$\frac{11}{4}$	0



1- Graphical methods.

2. Substitution :-

$$4x + 3y = 11 \rightarrow \text{Solve for } y \rightarrow 3y = 11 - 4x \rightarrow \frac{3y}{3} = \frac{11 - 4x}{3}$$

$$2x - 5y = -1$$

$$2x - 5\left(\frac{11 - 4x}{3}\right) = -1$$

$$y = \frac{11 - 4x}{3}$$

$$2x - \frac{55}{3} + \frac{20x}{3} = -1 \quad \times 3$$

$$y = \frac{11}{3} - \frac{4x}{3} = \frac{3}{3} = 1$$

$$6x - 55 + 20x = -3 \rightarrow 26x - 55 = -3 \rightarrow 26x = 52 \rightarrow x = 2$$

$$(2, 1)$$

3. elimination \Rightarrow

$$\begin{aligned} 4x + 3y &= 11 \\ 2 \cdot (2x - 5y) &= -1 \cdot 2 \end{aligned}$$

$$\begin{aligned} 4x + 3y &= 11 \\ -4x - 10y &= -2 \end{aligned}$$

$$\frac{13y}{-7} = \frac{13}{-7} \Rightarrow y = 1 \rightarrow 4x + 3 \cdot 1 = 11$$

$$4x + 3 = 11$$

$$\rightarrow -3 \quad \frac{4x}{4} = \frac{8}{4} \quad x = 2$$

~~$$4x + 3y = 11$$~~

~~$$-4x - 10y = -2$$~~

$$(2, 1)$$

Ex:

1) substitution solve for x from 2nd

$$\begin{aligned} 2x - 3y &= 12 \\ +3y &+3y \end{aligned} \rightarrow \frac{2x}{2} = \frac{12+3y}{2}$$

$$3x + 4y = 1 \rightarrow$$

$$2x - 3y = 12$$

$$x = 6 + \frac{3}{2}y \rightarrow 3(6 + \frac{3}{2}y) + 4y = 1$$

$$2(18 + \frac{9}{2}y + 4y = 1)$$

$$36 + 9y + 8y = 2$$

$$\begin{aligned} 36 + 17y &= 2 \\ -36 &-36 \end{aligned}$$

$$\frac{17y}{17} = \frac{-34}{17} \rightarrow y = -2$$

$$(x, -2)$$

$$3x + 4 \cdot (-2) = 1$$

$$\begin{aligned} 3x - 8 &= 1 \\ +8 &+8 \end{aligned}$$

$$\frac{3x}{3} = \frac{9}{3}$$

$$\boxed{x = 3}$$

Ex: elimination

$$\begin{aligned} 3 \cdot (3x + 4y = 1) &\rightarrow 9x + 12y = 3 \\ 4 \cdot (2x - 3y = 12) &\rightarrow 8x - 12y = 48 \\ \hline 17x &= 51 \rightarrow \boxed{x = 3} \end{aligned}$$

$(3, -2)$

$$\begin{aligned} 3 \cdot 3 + 4y &= 1 \\ 9 + 4y &= 1 \rightarrow \frac{4y}{4} = \frac{-8}{4} \rightarrow y = \boxed{-2} \end{aligned}$$

* Inconsistent system: Not they have the same slope.

$$\begin{aligned} \text{Ex: } 2 \cdot (4x + 3y) &= (4) \cdot 2 \\ 8x + 6y &= 18 \end{aligned}$$

$$\begin{aligned} 8x + 6y &= 8 \\ -8x + 6y &= 18 \end{aligned}$$

$$\boxed{0 = 10} \quad \therefore \text{No solution}$$

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Chapter 2

Sections 2.1

Quadratic Equations

A quadratic equation in one variable has a general form of

$$ax^2 + bx + c = 0 \quad (a \neq 0)$$

Ex: - 1) $x^2 - 3x + 2 = 0$
 $a = 1, b = -3, c = 2$

2) $3x^2 + 6x + 12 = 0$
 $a = 3, b = 6, c = 12$

3) $4x^2 - 9 = 0$
 $a = 4, b = 0, c = -9$

4) $7x^2 - 4 = x^2 - 10x$
 $= 7x^2 + 4 - 7x^2 + 10x$
 $-1 \cdot (7x^2 + 10x + 4) = (0) \cdot 15$
 $a = 7, b = 10, c = 4$

Solving quadratic equations:-

Using Quadratic Formula.

Once the equation is in the general form ($ax^2 + bx + c = 0$) then the solution is can be found using the quadratic formula.

$$x = \frac{-b \pm \sqrt{\text{discriminant}}}{2a} \quad \text{where discriminant} = b^2 - 4ac$$

Not so

discriminant < 0 No real solution.

discriminant $= 0$ one solution $x = \frac{-b}{2a}$

discriminant > 0 Two solution

Solve Ex 1-

1.8 method

Chapter 2

1) $x^2 - 3x + 2 = 0$

$a = 1, b = -3, c = 2$

$$x = \frac{-b \pm \sqrt{\text{disc}}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{1}}{2 \cdot 1} \Rightarrow x = \frac{3 \pm 1}{2}$$

disc = $b^2 - 4ac$

$(-3)^2 - 4 \cdot 1 \cdot 2$

$9 - 8 = 1$

or $x = \frac{3+1}{2} = \frac{4}{2} = 2$

$x = \frac{3-1}{2} = \frac{2}{2} = 1$

2) $3x^2 + 6x + 12 = 0$

$a = 3, b = 6, c = 12$

disc = $(6)^2 - 4 \cdot 3 \cdot 12$

$= 36 - 144 < 0$

$= -108 < 0 \therefore$ No real solution.

3) $4x^2 - 9 = 0$

$a = 4, b = 0, c = -9$

disc = $0^2 - 4 \cdot 4 \cdot -9$

disc = $0 + 144 = 144$

$$x = \frac{-0 \pm \sqrt{144}}{2 \cdot 4} = \frac{-0 \pm \sqrt{144}}{8}$$

or $x = \frac{\pm 12}{8} \Rightarrow \frac{\pm 12}{8} = \frac{3}{2}$

$\frac{-12}{8} = \frac{-3}{2}$

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Using Factoring

If the quadratic equation can be factored, then we can use the Zero property.

if $ab=0$ then either $a=0$ or $b=0$ or both.

Ex. Solve the following :-

$$1) - x^2 - 3x + 2 = 0$$

$$(x-1)(x+2) = 0 \rightarrow \begin{array}{l} x-1=0 \\ x=1 \end{array} \quad / \quad \begin{array}{l} x+2=0 \\ x=-2 \end{array}$$

$$x=1 \text{ or } x=-2.$$

$$2) 4x^2 - 9 = 0$$

$$(2x)^2 - (3)^2 = 0$$

$$(2x-3)(2x+3) = 0 \rightarrow \begin{array}{l} 2x-3=0 \\ 2x = \frac{3}{2} \\ x = \frac{3}{4} \end{array} \quad \rightarrow \quad \begin{array}{l} 2x+3=0 \\ 2x = -3 \\ x = -\frac{3}{2} \end{array}$$

$$x = \frac{3}{4} \text{ or } x = -\frac{3}{2}$$

Done

Section 2.2.

Quadratic Functions

A quadratic function has a general form of $y = f(x) = ax^2 + bx + c$ where $a \neq 0$

Ex:-

1. $f(x) = x^2 + 2x - 3$

$y = a = 1, b = 2, c = -3$

2) $2y + 24 = 2x + 2x^2 - 24 \rightarrow \frac{2y}{2} = \frac{2x}{2} + \frac{2x^2}{2} - \frac{24}{2}$

$y = x^2 + x - 12$
 $a = 1, b = 1, c = -12$

3) $y = (x+2)^2 - 3 \rightarrow y = x^2 + 4x + 4 - 3$

$y = x^2 + 4x + 1$
 $a = 1, b = 4, c = 1$

The graph of the quadratic function is a parabola open upward U (if $a > 0$) or downward \cap (if $a < 0$). :- الرسم القطع المكافئ

1- $a > 0 \rightarrow U$ 2. $a < 0 \rightarrow \cap$ Vertex = $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right)$

Ex:-

1- $f(x) = x^2 + 2x - 3 \rightarrow$ Graph

$y = x^2 + 2x - 3$

$a = 1, b = 2, c = -3$

Vertex = $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right)$

$\frac{-b}{2a} = \frac{-2}{2 \cdot 1} = \boxed{-1}$

$f(-1) = (-1)^2 + 2(-1) - 3$

$f(-1) = 1 - 2 - 3$

$f(-1) = \boxed{-4}$

(x, y)

⇒ Ex: axis

$a > 0 \rightarrow U$ (parabola opening upwards)

x-intercept (Zero of the function)

$$y = x^2 + 2x - 3$$

$$0 = x^2 + 2x - 3$$

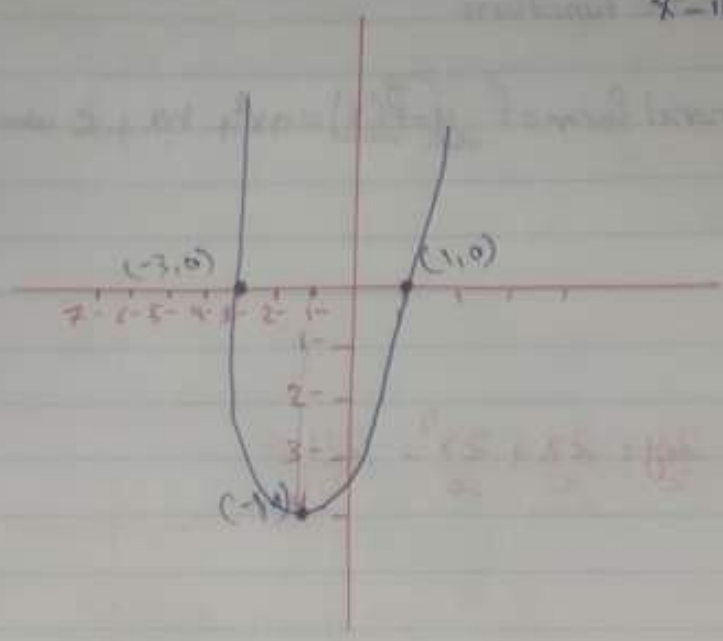
$$(x + 3)(x - 1) = 0$$

$$x = -3 \text{ or } x = 1$$

y-intercept

$$y = (0)^2 + 2(0) - 3$$

$$y = -3$$



2) Ex: $2y + 24 = 2x + 2x^2$

$$y = x^2 + x - 12$$

$$a = 1, b = 1, c = -12$$

$$\text{Vertex} = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right) \rightarrow \left(-\frac{1}{2}, -12,25 \right)$$

$$\frac{-1}{2 \cdot 1} = -\frac{1}{2}$$

$$f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 12$$

$$f\left(-\frac{1}{2}\right) = \frac{1}{4} + \frac{-1}{2} - 12$$

$$\frac{1}{4} + \frac{-2}{4} - \frac{48}{4}$$

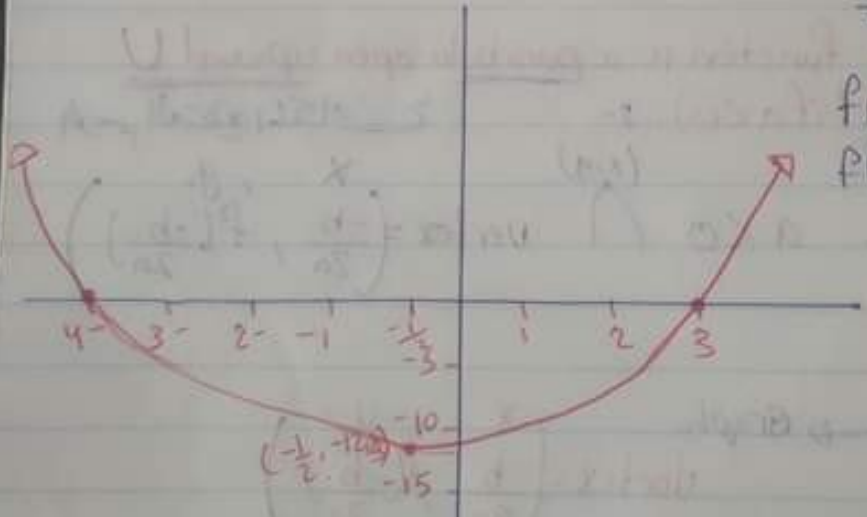
$$= \frac{-49}{4} = -12,25$$

Zero

$$0 = x^2 + x - 12$$

$$0 = (x - 3)(x + 4)$$

$$x = +3, x = -4$$



What is the pointed Value $-12,25$ (Minimum)

optimal point = $\left(-\frac{1}{2}, -12,25\right)$.
is the optimal point a min or Max
= minimum point.

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Vertex = optimal point.

$a < 0$
Maximum
Point

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right) \right)$$

Minimum
Point

$a > 0$

$f\left(-\frac{b}{2a}\right)$ optimal value.

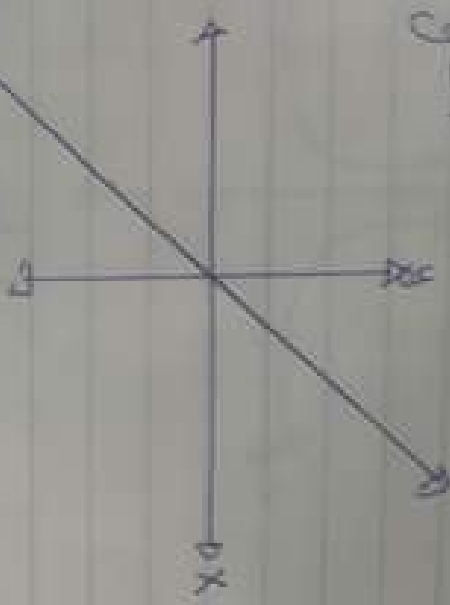
opt

Sactions 2.14

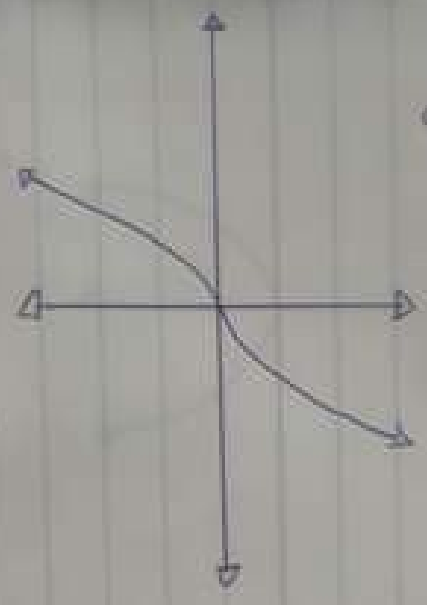
Special Functions and Their Graphs.

Graphs of all cubic polynomials $y = ax^3 + bx^2 + cx + d$ (where $a \neq 0$)

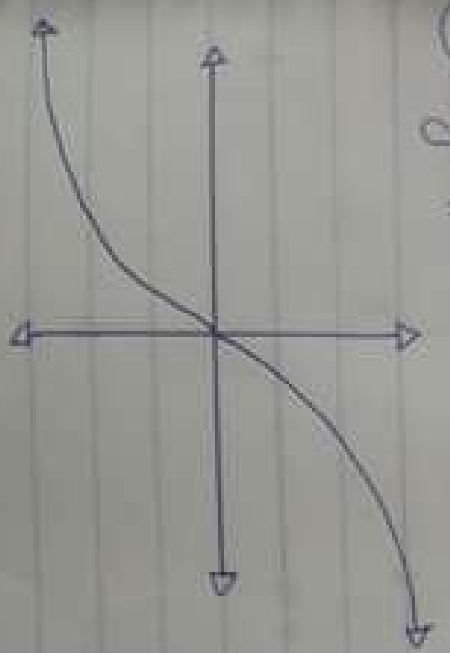
① $y = ax + b$
 $y = x$



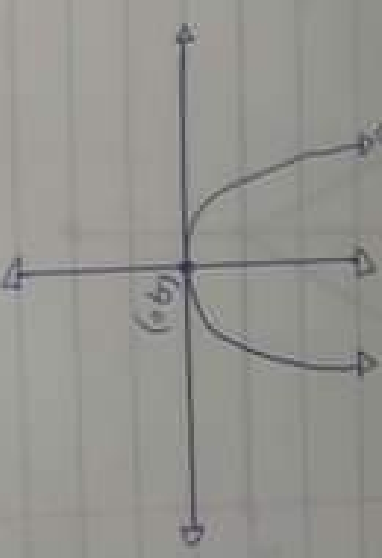
② $y = x^3$



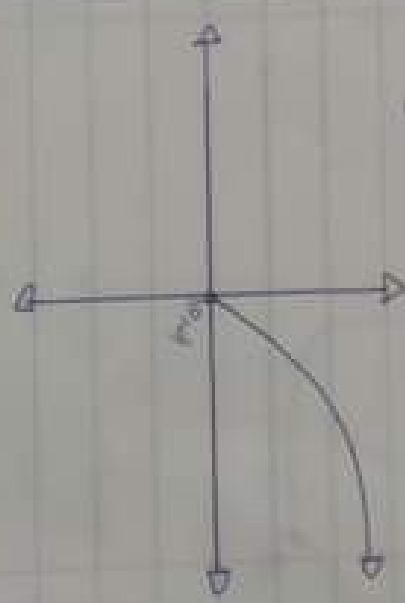
③ $y = x^{\frac{1}{2}} = \sqrt{x}$



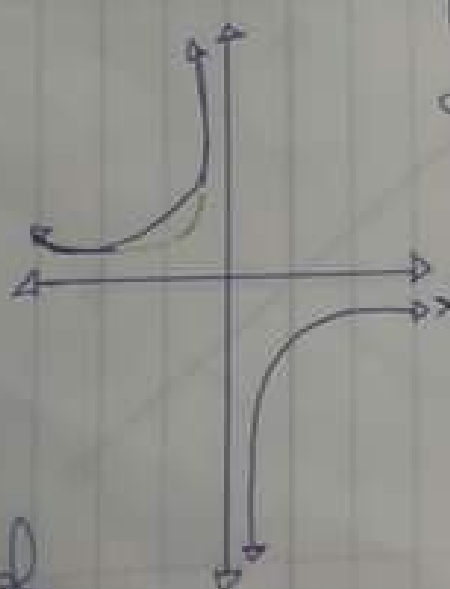
④ $y = x^2$



⑤ $y = x^{\frac{1}{2}} = \sqrt{x}$



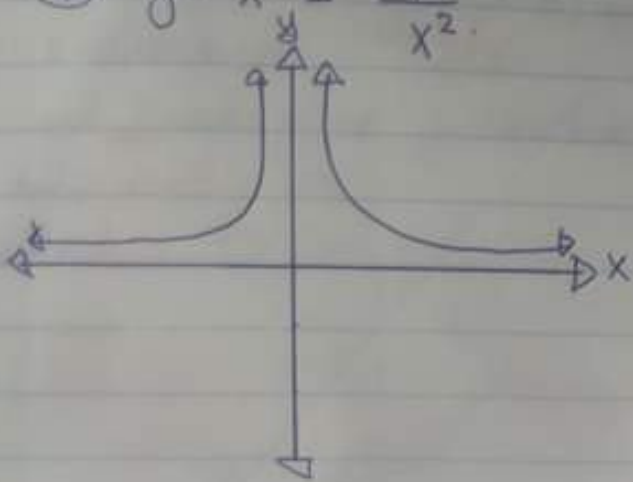
⑥ $y = x^{-1} = \frac{1}{x}$



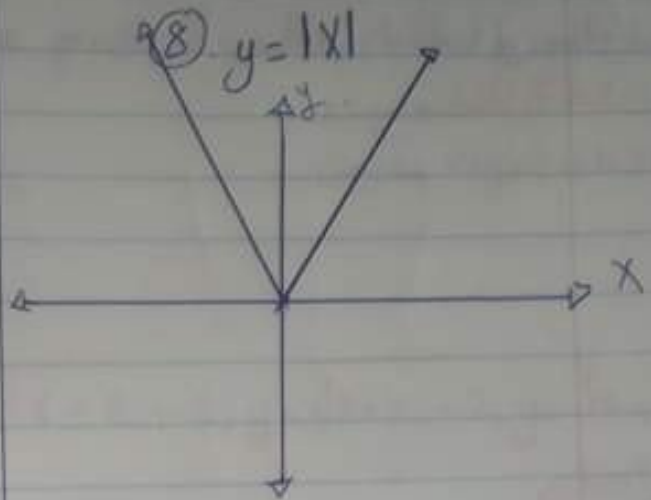
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Sections 2.4.

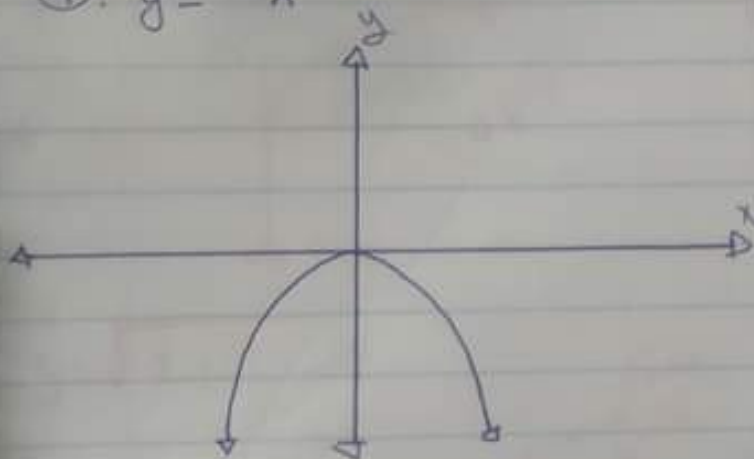
⑦. $y = x^{-2} = \frac{1}{x^2}$



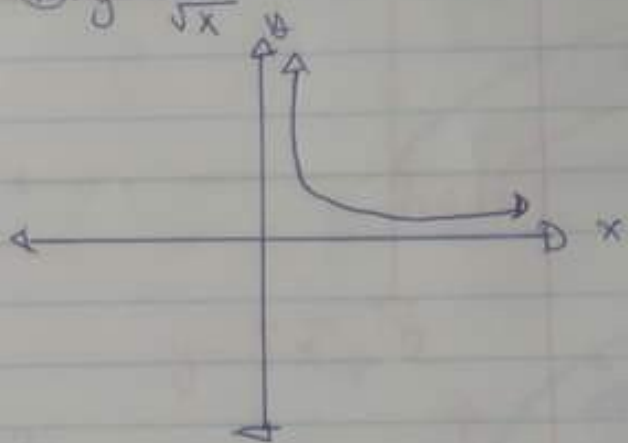
⑧. $y = |x|$



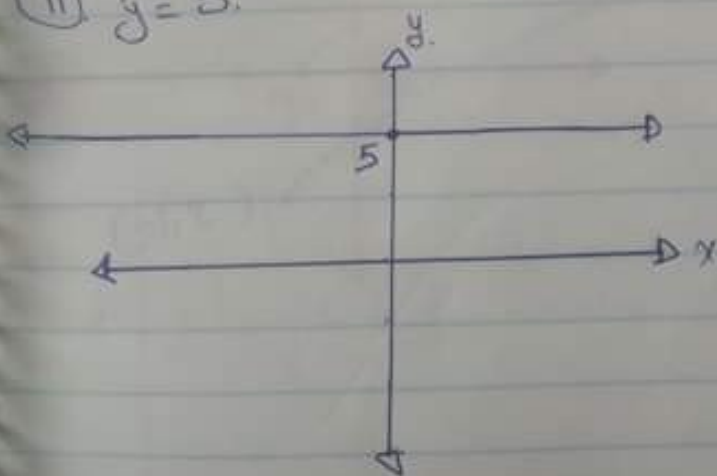
⑨. $y = -x^2$



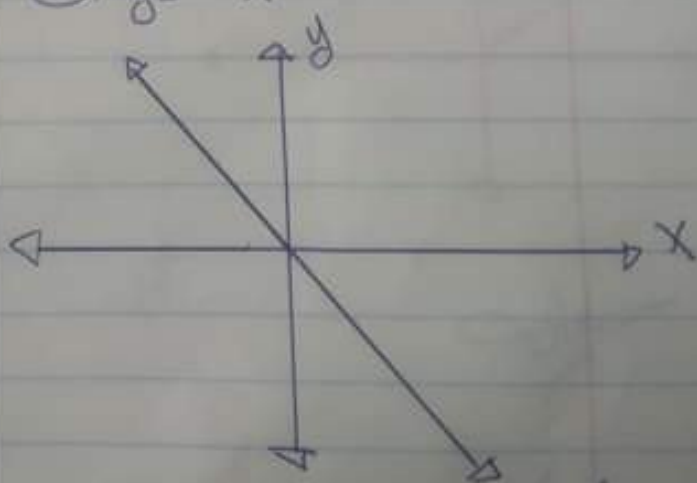
⑩. $y = \frac{1}{\sqrt{x}}$



⑪. $y = 5$



⑫. $y = -x$



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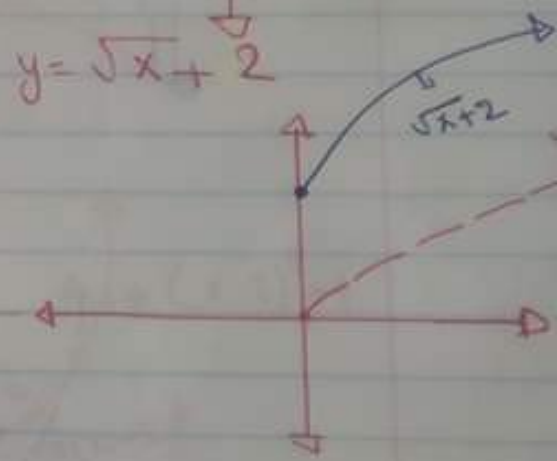
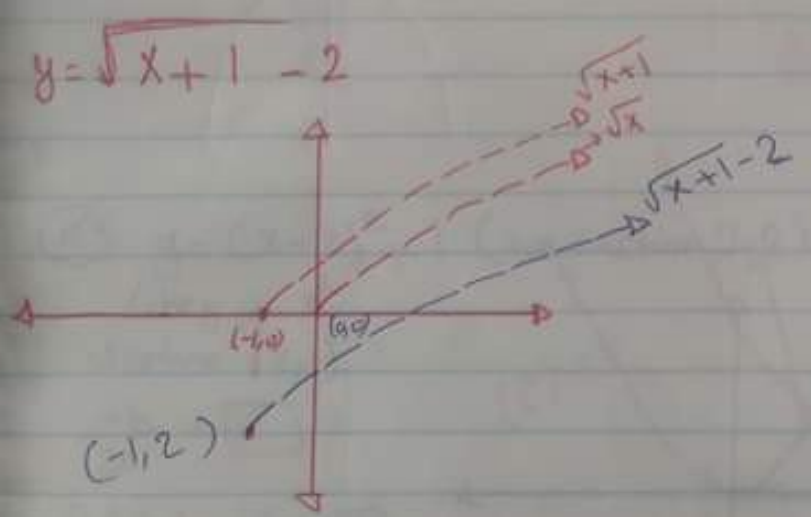
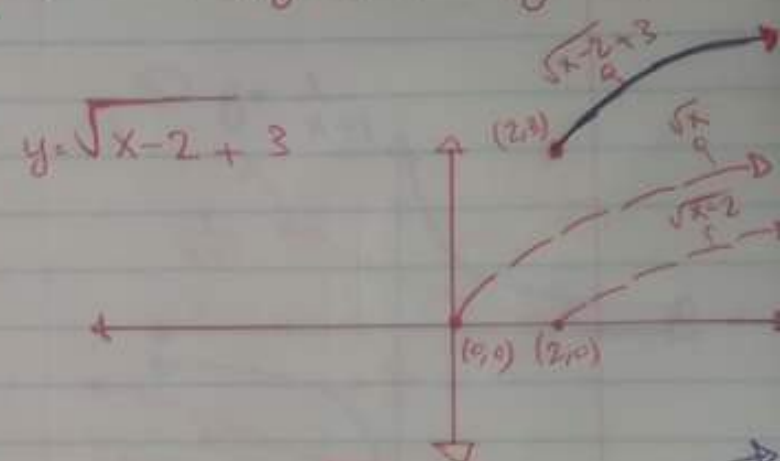
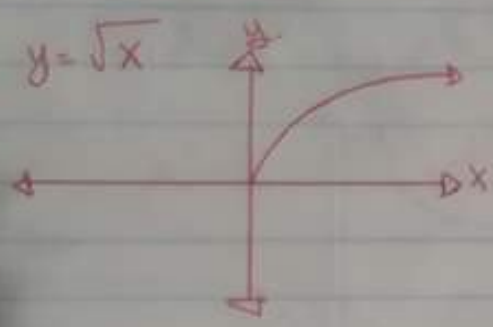
* Shifts of Graphs 80

The graph of $y = f(x-h) + k$ is the graph of $y = f(x)$ shifted h units in the x -direction and k units in the y -direction.

↑ up if $k > 0$
 ↓ down if $k < 0$

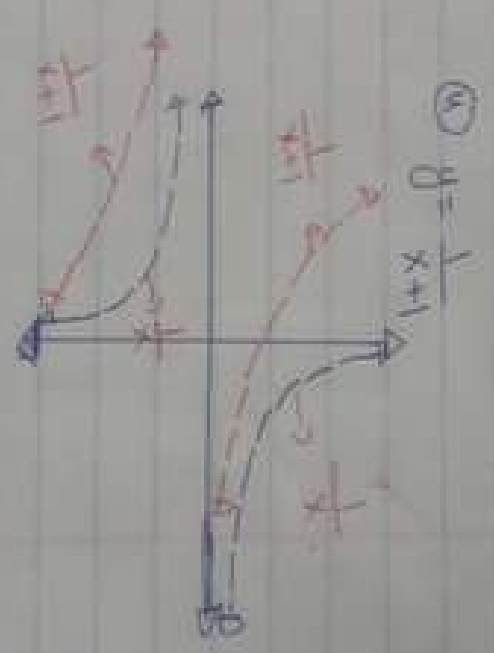
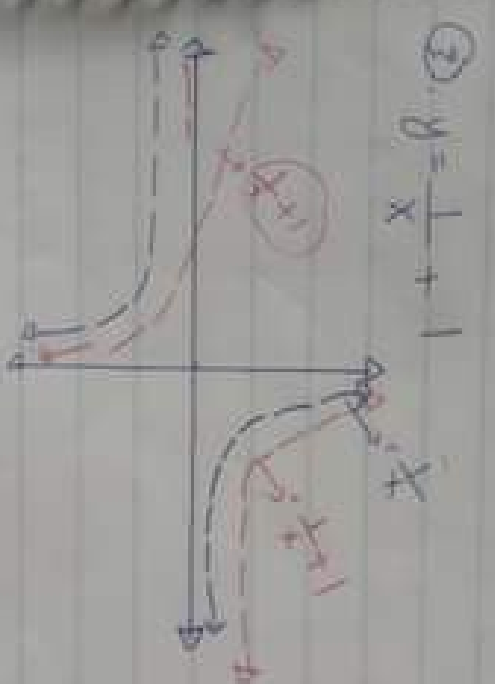
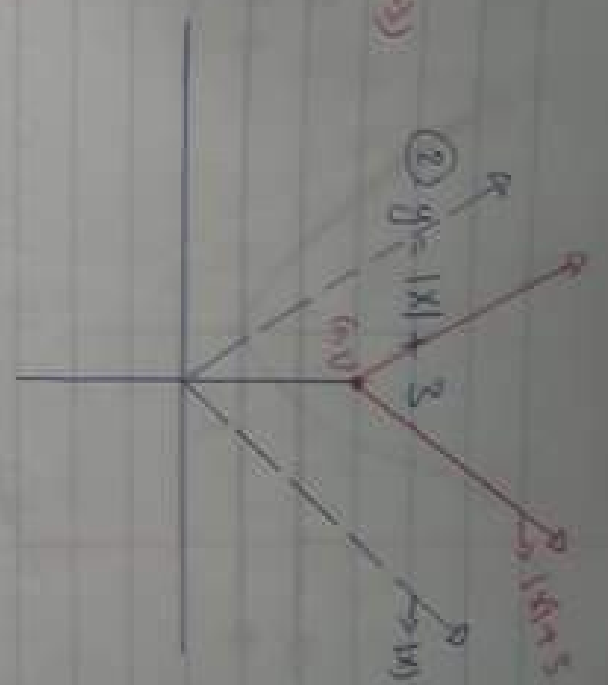
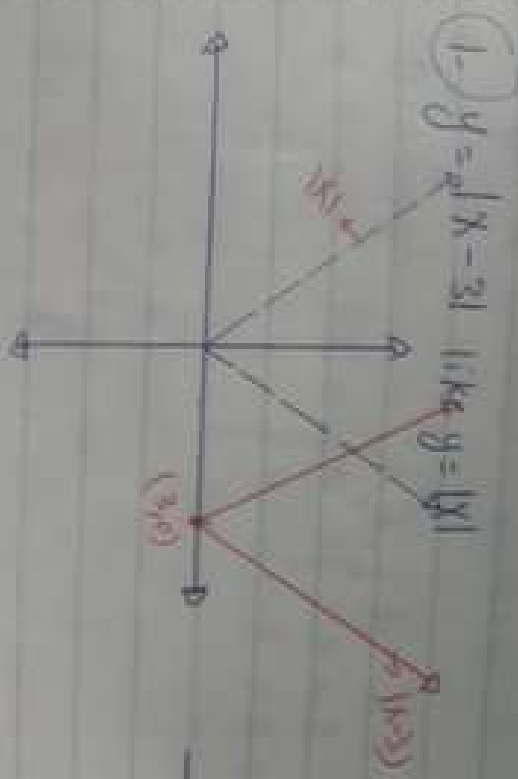
← Left if $h < 0$
 → Right if $h > 0$

Ex:- Graph $y = \sqrt{x}$ than graph $y = \sqrt{x-2} + 3$, $y = \sqrt{x+1} - 2$, $y = \sqrt{x} + 2$ and $y = \sqrt{x+1}$

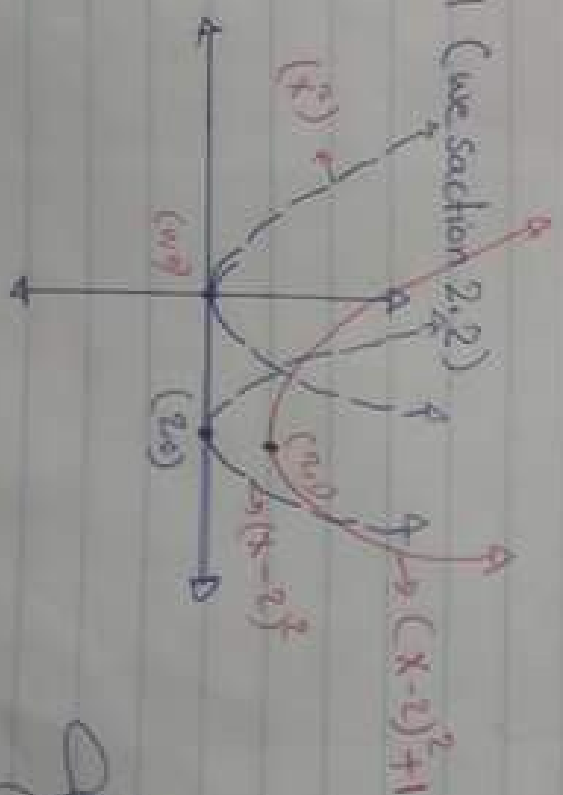


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Ex: graph



⑤ $y = (x-2)^2 + 1$ (we section 2.2)
 Like $y = x^2$
 Vertex (2, 1)
 $\frac{dy}{dx} = 2x$
 $f'(2) = f''(2) = 2$

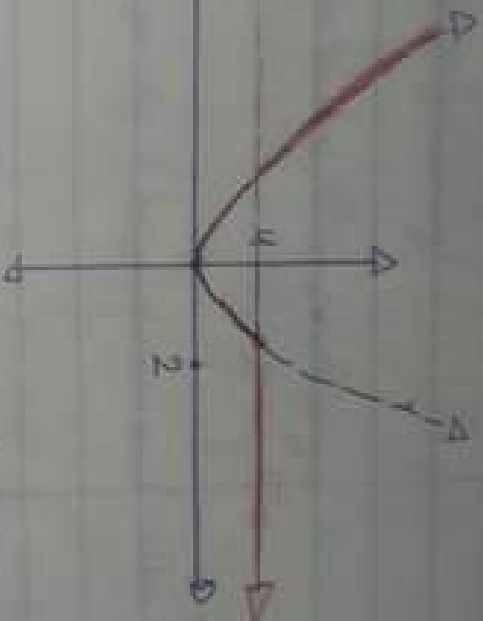


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Piecewise Defined Functions

Ex-Graph

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ 4 & \text{if } x > 2 \end{cases}$$



done

Chapter 4

Section 4.1

Inequalities and linear programming

- * Suppose that the daily profit, a Carpenter makes, from producing and selling chairs and tables are 10\$ per chairs and 15\$ per table.

How much profit would the carpenter makes per day if he produces and sells 4 chairs per day? $10 \times 4 = 40\$$

How much profit would the carpenter makes per day, if he produces and sells 2 chairs and 2 table per day? $10 \times 2 + 15 \times 2 = 50\$$

In general, if we let x = number of chairs produced and sold each day
 y = number of table produced and sold each day

Then the total daily profit = $p = 10x + 15y$

- * What value of x and y will maximize the daily profit?

- * Suppose that each chair requires $1m^2$ of wood, each table requires $2m^2$ of woods and the total amount of wood available each day is $14m^2$. This will impose the following constraint on x and y . $\Rightarrow 1x + 2y \leq 14$

- * Suppose that each chair requires 2 hours of labour, each table requires 1 hour of labor, and the total daily work time 10 hours. This will impose the following constraints on x and y . $\Rightarrow 2x + 1y \leq 10$

Now what value of x and y will maximize the daily profit? 4.2

This kind of problem where you need to optimize a function called the Object function = $p = 10x + 15y$

Subject to a system of linear inequalities called constraints $\begin{cases} x + 2y \leq 14 \\ 2x + y \leq 10 \\ x \geq 0 \\ y \geq 0 \end{cases}$ is called linear programming problem

* To solve linear programming problems, we need to know how to solve linear inequalities in two variables. (Section 4.1)

1.1 \Rightarrow radical $\rightarrow x + y \geq 15$ $\Rightarrow x \geq 15 - y$

Examples - Graph the solution of: $4x - 2y < 6$

$4x - 2y = 6$

$4x - 2y = 6$

$2y = 4x - 6$

$y = 2x - 3 \Rightarrow (0, -3)$

$y = 0 \Rightarrow 4x - 2(0) = 6$

$x = \frac{6}{4} = \frac{3}{2}$

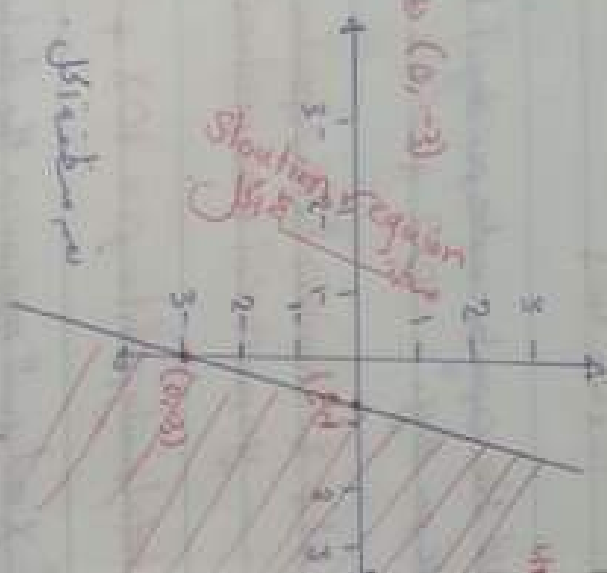
$(\frac{3}{2}, 0)$

test point $(0, 0)$

$4(0) - 2(0) < 6$

$0 < 6$ (✓)

Shaded region



$4x - 2y < 6$

$2x - y < 3$

$2x < 3 + y$

$x < \frac{3+y}{2}$

graph

Ex - Graph the solution of: $2x + y \geq 4$ *feasible region, constraint*

$2x + y = 4$

$2x + y = 4$

$2(0) + 0 = 4$

$0 = 4$

$0 \neq 4$

$0 > 4$

$0 < 4$

test point $(3, 0)$

$2(3) + 0 \geq 4$

$6 \geq 4$ (✓)



Examples:- Graph the solution of $3x - 2y \geq 4$ and $x + y \geq 3$

1) $3x - 2y = 4$

$3 \cdot 0 - 2y = 4$

$-2y = \frac{4}{-2} \Rightarrow (0, 2)$

$3x - 2 \cdot 0 = 4$

$\frac{3x}{3} = \frac{4}{3} \Rightarrow (\frac{4}{3}, 0)$

test point $(0, 0)$

$3 \cdot 0 - 2 \cdot 0 \geq 4$

$0 \geq 4 \rightarrow$ X

2. $x + y = 3$

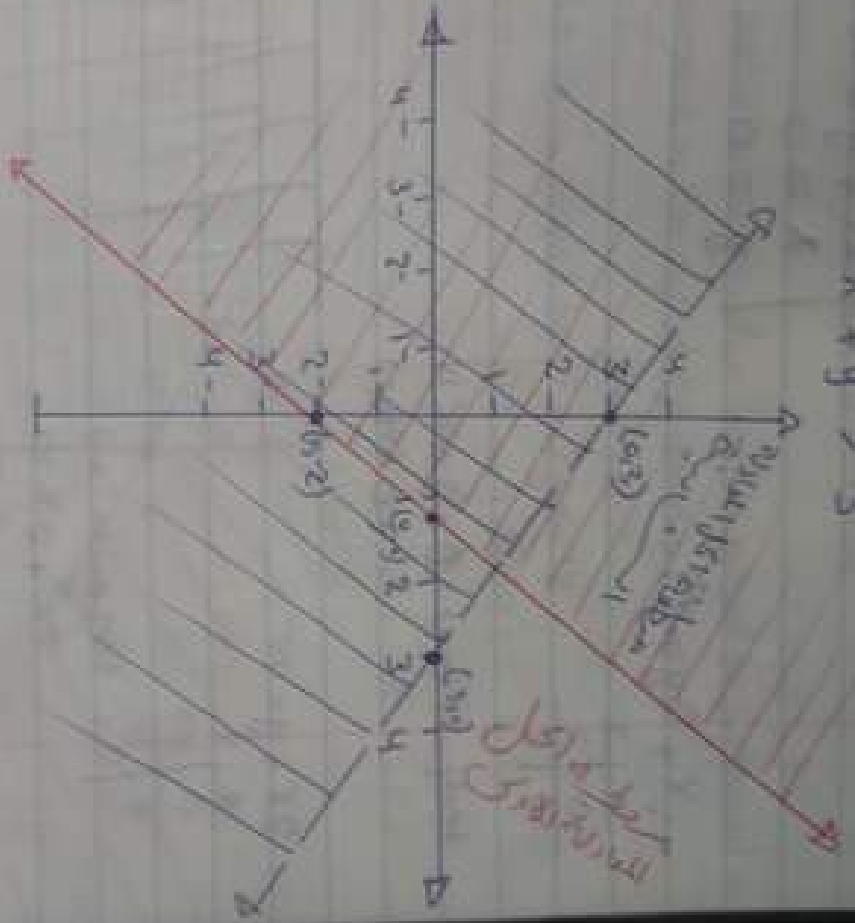
$0 + y = 3 \Rightarrow (0, 3)$

$x + 0 = 3 \Rightarrow (3, 0)$

test point $(0, 0)$

$0 + 0 > 3$

$0 > 3$ X



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Examples :- Graph the solution of

$$\begin{cases} x + 2y \leq 14 \\ 2x + y \leq 10 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

$$x + 2y = 14$$

x	y
0	7
14	0

test point (0,0) = $0 + 2 \cdot 0 \leq 14$
 $0 \leq 14$ ✓

$$2x + y = 10$$

x	y
0	10
5	0

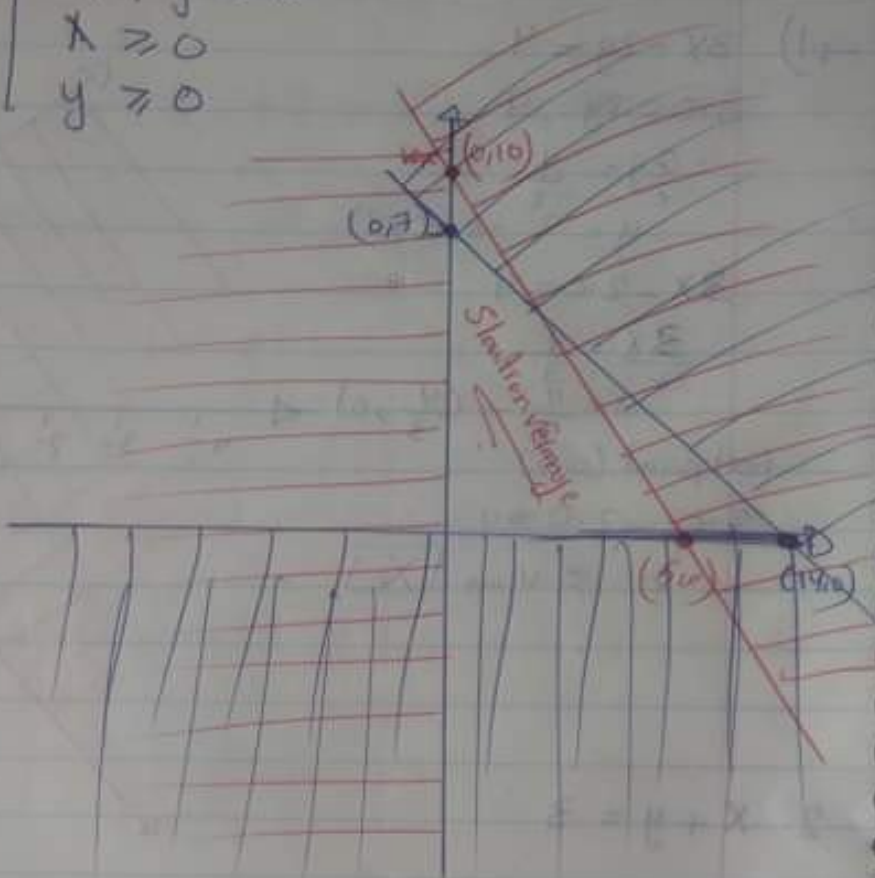
test point = (0,0)
 $2 \cdot 0 + 0 \leq 10$
 $0 \leq 10$ ✓

$$x \geq 0$$

الاعداد الموجبة x

$$y \geq 0$$

الاعداد الموجبة y



Section 4.2

Linear programming: Graphical Methods

* A problem where we need to optimize an objective function subject to constraint is called a linear programming problem such as the carpenter problem:-

Maximize $P = 10x + 15y$ → subject to

$x + 2y = 14$

$x + 2y \leq 14$

$2x + y \leq 10$

$x \geq 0$

$y \geq 0$

x	y
0	7
14	0

test point (0,0)

$x + 2 \cdot 0 \leq 14$

$0 \leq 14$ ✓

$2x + y = 10$

x	y
0	10
5	0

test point (0,0)

$2 \cdot 0 + 0 \leq 10$

$0 \leq 10$ ✓

$x = 0 \rightarrow (0, 7)$

$x = y \rightarrow (0, 10)$

← wie stbla

$(x + 2y = 14) \cdot 2$

$2x + y = 10$

$2x + 4y = 28$

$-2x + y = 10$

$\frac{3y = 18}{3} \Rightarrow y = 6$

$x + 2 \cdot 6 = 14 \Rightarrow x = 2$

objekt (bla)

Corners

$(0, 0)$

$(0, 7)$

$(5, 0)$

$(2, 6)$

$P = 10x + 15y$

$P = 10 \cdot 0 + 15 \cdot 7$

$P = 10 \cdot 5 + 15 \cdot 0$

$P = 10 \cdot 2 + 15 \cdot 6$

$P = 10 \cdot 0 + 15 \cdot 0$

$P = 105$

$= 50$

$P = 20 + 90$

$P = 0$

$P = 110$

Maximize point is 110 \$ when $\Rightarrow x = 2, y = 6$

Examples to Maximize $90x + 120y$ Subject to

$$\begin{cases} 2x + 3y \leq 240 \\ x + 3y \leq 150 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

① $2x + 3y = 240$

x	y	total point (x,y)
0	80	
120	0	

$\Rightarrow 2 \cdot 0 + 3 \cdot 0 \leq 240$
 $0 \leq 240 (✓)$

② $x + 3y = 150$

x	y	total point (x,y)
0	50	
150	0	

$\Rightarrow 0 + 3 \cdot 0 \leq 150$
 $0 \leq 150 (✓)$

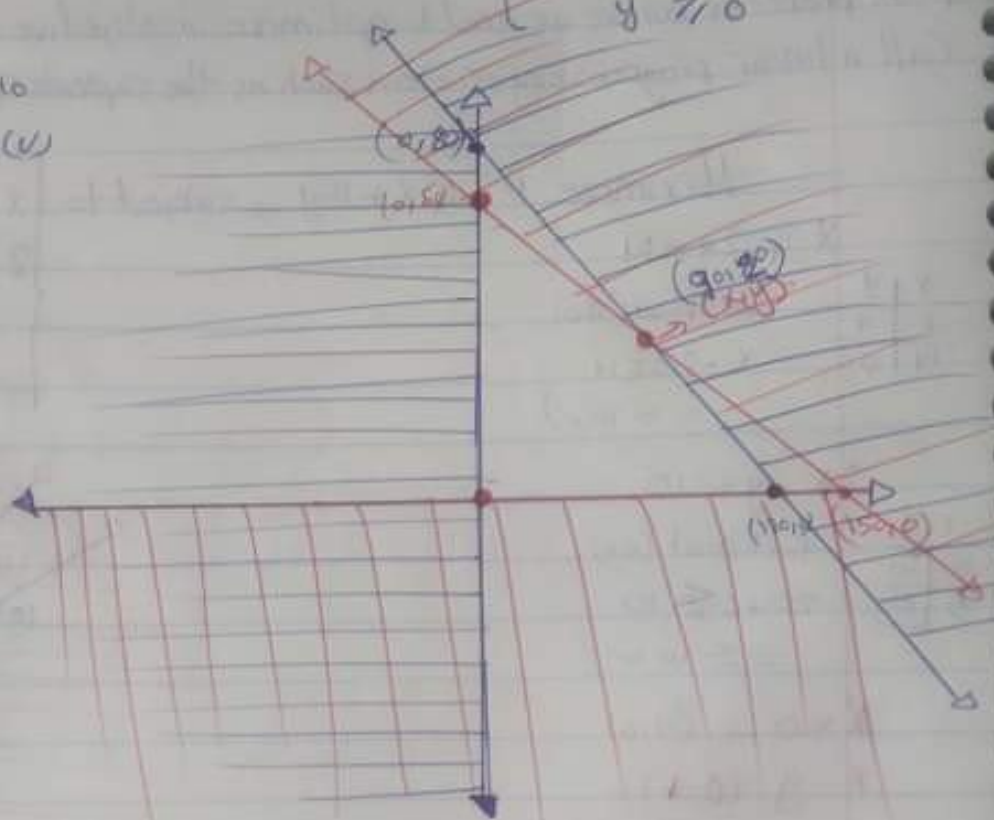
$$\begin{aligned} 2x + 3y &= 240 \\ x + 3y &= 150 \quad - \\ \hline x &= 90 \end{aligned}$$

$$\begin{aligned} 90 + 3y &= 150 \\ -90 & \quad -90 \\ \hline 3y &= 60 \\ y &= 20 \end{aligned}$$

Cornes :- $(0,0)$
 $P = 90 \cdot 0 + 120 \cdot 0$
 $P = 0$

$(120,0)$	$(0,50)$	$(90,20)$
$P = 120 \cdot 90 + 120 \cdot 0$	$P = 90 \cdot 0 + 50 \cdot 120$	$P = 90 \cdot 90 + 120 \cdot 20$
$P = 10800$	$P = 6000$	$8100 + 2400$
		10500

Maximize is 10800 when $x=120/y=0$



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* In each problem 5-8, Graph and find Maximum and minimums.

6) $P = 5x + 8y$

$4x + 2y = 140$

$x + y = 50$

$\frac{3x}{3} = \frac{90}{3} \rightarrow x = 30$
 $y = 20$

$x + 2y = 80$

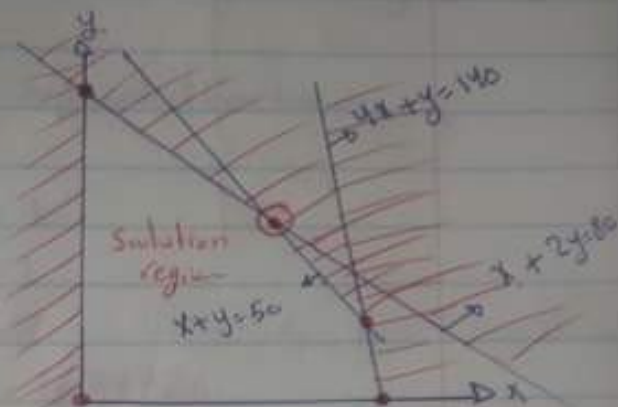
$0 + 2y = 80$

$y = 40$

$4x + y = 140$

$\frac{4x + 0}{4} = \frac{140}{4}$

$x = 35$



Corner: $(0,0)$

$P = 5 \cdot 0 + 8 \cdot 0$

$P = 0$

$(0,40)$

$P = 5 \cdot 0 + 8 \cdot 40$

$P = 320$

$(30,20)$

$P = 30 \cdot 5 + 20 \cdot 8$

$P = 150 + 160$

$P = 310$

$(35,0)$

$P = 35 \cdot 5 + 8 \cdot 0$

$P = 175$

$x + 2y = 80$

$x + y = 50$

$y = 30 \rightarrow x = 20$

$(20,30)$

$P = 20 \cdot 5 + 30 \cdot 8$

$P = 100 + 240$

$P = 340$

Maximum is 340 when $x = 20$, $y = 30$

Minimum is 0 when $x = 0$, $y = 0$

Chapter 5

Section 5.1

Exponential Functions

* Consider a microorganism that doubles every hour. If we start with 1 organism and let y = number of organisms, that at

time	y
0 hours	1
1 hour	$1 \cdot 2 = 2$
2 hours	$1 \cdot 2 \cdot 2 = 4$
3 hours	$2 \cdot 2 \cdot 2 = 8$
x hours	$2 \cdot 2 \cdot 2 \cdot \dots \cdot x$ times

exponential functions:

$$y = 2^x$$

$x = 2 \Rightarrow y = 2^2 = 4$
 $x = 0 \Rightarrow y = 2^0 = 1$
 $x = 3 \Rightarrow y = 2^3 = 8$
 $x = 25 \Rightarrow y = 2^{25} = 18,446,744,073,709,551,616$

In general if $a > 0$, and $a \neq 1$, then $f(x) = c(a)^x$ is an exponential function.

- Ex - $y = 4^x$ $y = c(a)^x$ $y = 5c(10)^x$ $y = 6c(4)^x$ $y = (2/9)^x$
- (growth) $c = 1/a = 1/4$ $c = 1/a = 0.1$ (decay) $c = 5/a = 0.1$ (growth) $c = 6/a = 1.5$ (growth) $c = 1/a = 9/2$ (decay)

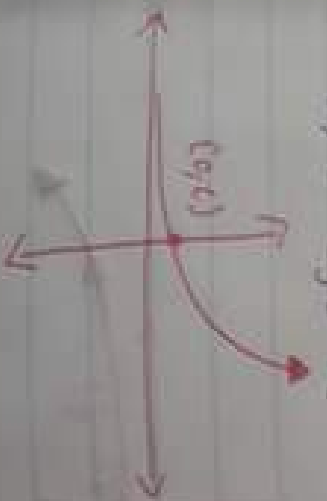
For $c > 0 \Rightarrow$

- $c(a)^x$ is a growth $a > 1$
- $c(a)^x$ is a decay $0 < a < 1$

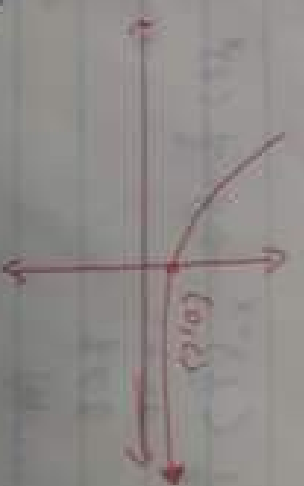
Graphing Exponential Functions:

If $c > 0$ then the graph of $f(x) = c(a)^x$ is

$a > 1$ (growth)



$a < 1$ (decay)

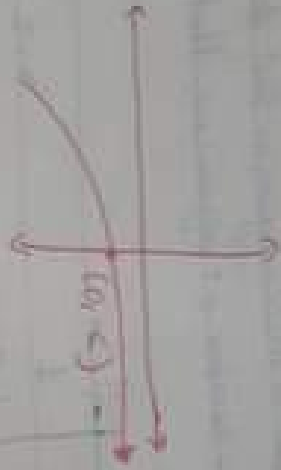


* if $C < 0$ then the graph is the same but reflected about the x -axis.

$a > 1$



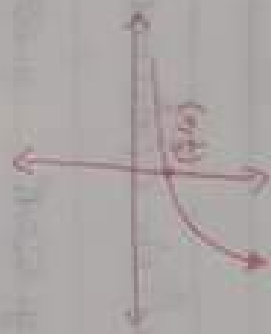
$a < 1$



Example :- graph the following :-

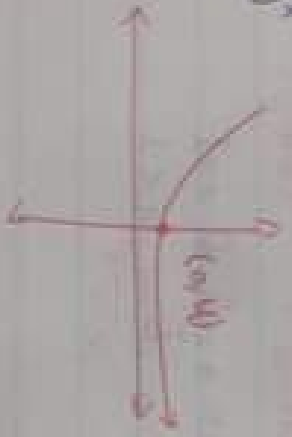
1) $y = \frac{1}{6} 6^x$

$C = \frac{1}{6}$ $a = 6$
 $\frac{1}{6} > 0$ $6 > 1$
 (growth)



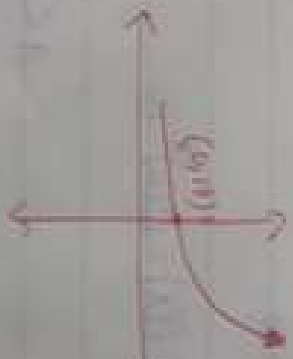
2) $y = 4\left(\frac{3}{4}\right)^x \Rightarrow y = 4\left(\frac{3}{4}\right)^x$

$C = 4$ $a = \frac{3}{4}$
 $4 > 0$ $\frac{3}{4} < 1$
 (decay)



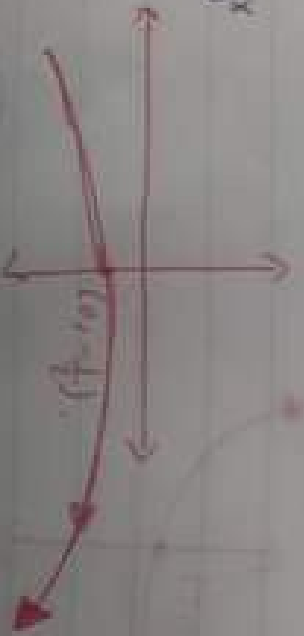
3) $f(x) = 17(10)^x$

$C = 17$ $a = 10 > 1$
 $17 > 0$ $10 > 1$
 (growth)



4) $y = -\frac{1}{2}\left(\frac{1}{2}\right)^x \Rightarrow y = -\frac{1}{2}\left(\frac{1}{2}\right)^x$

$C = -\frac{1}{2}$ $a = \frac{1}{2}$
 $-\frac{1}{2} < 0$ $\frac{1}{2} < 1$
 (growth)



[Signature]

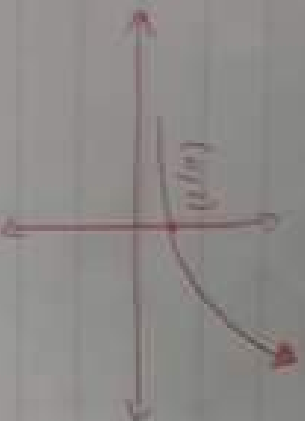
* A special exponential function is the base $e \approx 2.7$ function. Talk about it and graph it.

$$f(x) = e^x$$

$$e = 1 / a = 2.7$$

$$1.70 \quad / \quad 2.7 > 1$$

(growth)



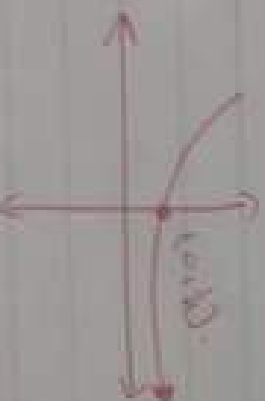
Questions: indicate whether the following is a growth or decay function from their graphs.

$$y = \frac{1}{9} \left(\frac{\pi}{e} \right)^{-x} \rightarrow y = \frac{1}{9} \left(\frac{e}{\pi} \right)^x$$

$$c = \frac{1}{9}$$

$$\frac{1}{9} > 0$$

$$a = \frac{e}{\pi} < 1$$



graph:

$$y = \pi \cdot 6^x$$

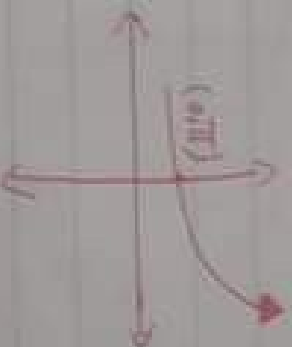
$$c = \pi$$

$$\pi > 0$$

$$a = 6$$

$$6 > 1$$

(growth)

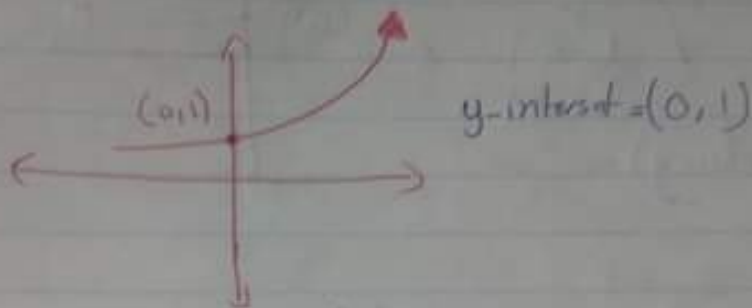


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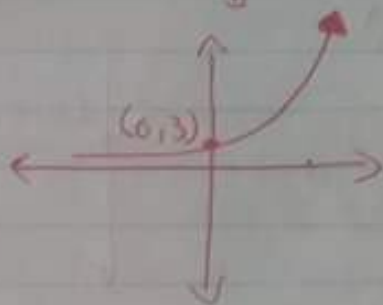
outline 5.1

the Graph: $f(x) = c(a)^x$ and is: the function growth or decay function.

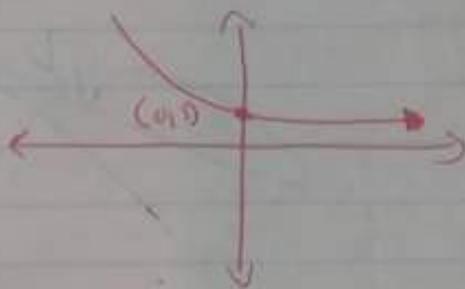
6.) $y = 8^x$
 $c = 1 \quad | \quad a = 8$
 $1 > 0 \quad | \quad 8 > 1$
 (growth)



8.) $y = 3(2)^x$
 $c = 3 \quad | \quad a = 2$
 $3 > 1 \quad | \quad 2 > 1$
 (growth.)



10.) $y = \left(\frac{2}{3}\right)^x$
 $c = 1 \quad | \quad a = \frac{2}{3}$
 $1 > 0 \quad | \quad \frac{2}{3} < 1$
 (decay).



12.) $y = 3^{x-1} \rightarrow y = \frac{3^x}{3} = \frac{1}{3}(3^x)$

$c = \frac{1}{3} \quad | \quad a = 3$

$\frac{1}{3} > 0 \quad | \quad 3 > 1$
 (growth)



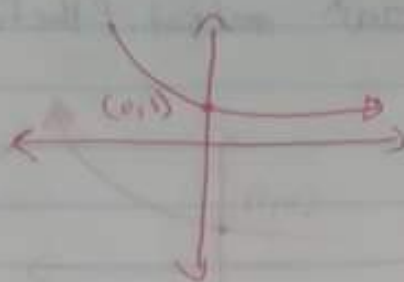
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$$14.) y = 3^{-x} \rightarrow y = \left(\frac{1}{3}\right)^x$$

$$C=1 \quad / \quad a = \frac{1}{3}$$

$$1 > 0 \quad / \quad \frac{1}{3} < 1$$

(decay)

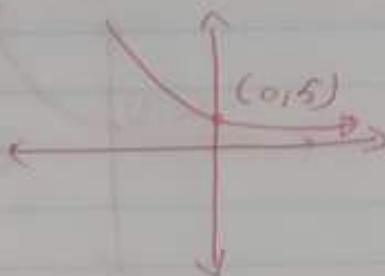


$$16.) y = 5e^{-x} \rightarrow y = 5\left(\frac{1}{e}\right)^x$$

$$C=5 \quad / \quad a = \frac{1}{e}$$

$$5 > 0 \quad / \quad \frac{1}{e} < 1$$

(decay)



~~int~~

The End... ♡