*Ch11: Parametric Equations And Polar Coordinates.

- Sec 11.1: Parametrizations of plane curves.

- If x and y are given as functions

$$x = f(t) \quad , \quad y = g(t)$$

over an iterval, then the set of points is a parametric curves. The equations are parametric equations.

- Exercises: page 616

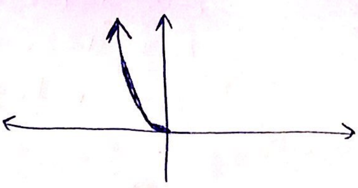
$\boxed{2}$: Identify the particle's path by finding a Cartesian equation.

$$x = -\sqrt{t} \quad , \quad y = t \quad , \quad t \geq 0$$

$\longrightarrow$ $y = t = x^2$; $x \leq 0$. since $t \geq 0$

$\sqrt{t} \geq 0$

$x = -\sqrt{t} \leq 0$



$\boxed{6}$ $x = \cos(\pi - t)$, $y = \sin(\pi - t)$; $0 \leq t \leq \pi$

$\rightarrow$ $\cos^2(\pi - t) + \sin^2(\pi - t) = 1$

$x^2 + y^2 = 1$ $\rightsquigarrow$ $0 \leq t \leq \pi$

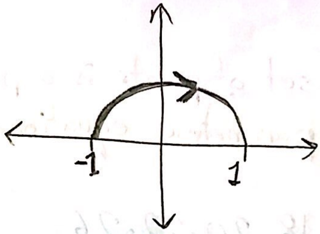
«circle»

$\therefore \cos(\pi - 0) = -1$

$\cos(\pi - \pi) = 1$

$\rightarrow -1 \leq x \leq 1$

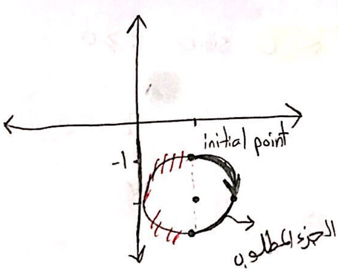
$0 \leq \sin(\pi - t) \leq 1$



$\boxed{10}$ $x = 1 + \sin t$, $y = \cos t - 2$; $0 \leq t \leq \pi$

$\rightarrow$ $\sin t = x - 1$, $\cos t = y + 2$

$\therefore$ $(x-1)^2 + (y+2)^2 = 1$ circle with center $(1, -2)$ and radius 1



$t = 0$: $x = 1$, $y = -1$

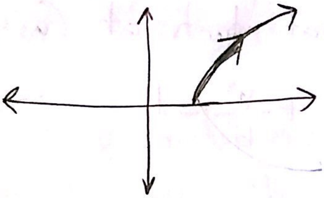
$t = \frac{\pi}{2}$: $x = 2$, $y = -2$

$t = \pi$: $x = 1$, $y = -3$

$\boxed{14}$ $x=\sqrt{t+1}$, $y=\sqrt{t}$; $t \geq 0$.

$\longrightarrow \quad y = \sqrt{t} = \sqrt{x^2-1}$; $x \geq 1$
$y \geq 0$

$\therefore \quad x^2 - y^2 = 1$ hyperbola.



$\boxed{15}$ $x = \sec^2 t - 1$, $y = \tan t$; $-\frac{\pi}{2} < t < \frac{\pi}{2}$

$\longrightarrow \quad \sec^2 t = 1 + \tan^2 t$

$x+1 = 1 + y^2$

$\therefore \quad y^2 = x$

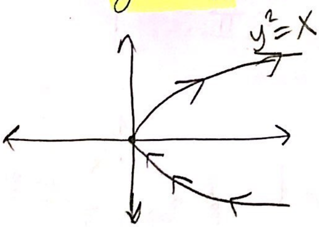
$t = -\frac{\pi}{2}$; $x = \frac{1}{\sec^2 \frac{-\pi}{2}^+} = -\infty$

$y = \tan \frac{-\pi}{2}^+ = -\infty$

$t = 0$; $x = 0$, $y = 0$

$t = \frac{\pi}{2}$; $x = \frac{1}{\sec^2 \frac{\pi}{2}^-} = \infty$

$y = \tan \frac{\pi}{2}^- = \infty$

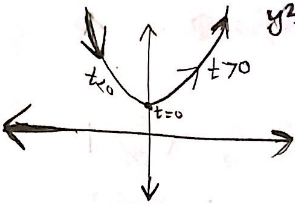


$\boxed{18}$ $x = 2\sinh t$, $y = 2\cosh t$; $-\infty < t < \infty$

$\longrightarrow \cosh^2 t - \sinh^2 t = 1$

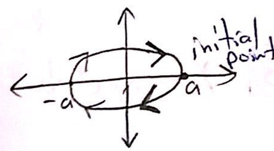
$\frac{y^2}{4} - \frac{x^2}{4} = 1 \quad \therefore \quad y^2 - x^2 = 4$

$-\infty < x < \infty$
$0 < y < \infty$



$\boxed{20}$ Find parametric equations and interval for the motion of a particle that starts at $(a, 0)$ and traces the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

a) once clockwise.



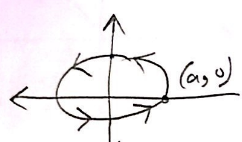
$x = a\sin t$, $y = b\cos t$

$t=\frac{\pi}{2}$: $(a, 0)$

حتى نكون قد اكتمل الشكل يجب أن ندور دورة كاملة. $\frac{\pi}{2}+2\pi$

$\therefore$ $\frac{\pi}{2} \leq t \leq \frac{5\pi}{2}$.

b) once counterclockwise.



نلاحظ أنه عند استخدام هذا parametrization فإن فترة t تكون معكوسة لذا ~~...~~ نعدّل على parametrization ونضرب y بالسالب.

$x = a\sin t$, $y = b\cos t$

$t: \frac{\pi}{2} \longrightarrow 0 \longrightarrow -\frac{\pi}{2} \rightarrow -\pi \rightarrow -\frac{3\pi}{2}$

تغيير parametrization

$x = a\cos t$, $y = b\sin t$

$0 \leq t \leq 2\pi$

c) twice clockwise.

فقط نضاعف الفترة:

$x = a\sin t$, $y = b\cos t$; $\frac{\pi}{2} \leq t \leq \frac{9\pi}{2}$

d) twice counterclockwise.

$x = a\cos t$, $y = b\sin t$; $0 \leq t \leq 4\pi$

22 find a parametrization for the curve:-
the line segment with endpoints $(-1,3)$ and $(3,-2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-3}{3--1} = \frac{-5}{4}$$

$$y - 3 = m(x - -1)$$

$\rightarrow y - 3 = \frac{-5}{4}(x+1)$. cartesian.

parametric equation:-

$x = t$,
$y = 3 - \frac{5}{4}(t+1)$; $-1 \leq t \leq 3$

ويجوز أيضاً اعتبار $x+1 = t$

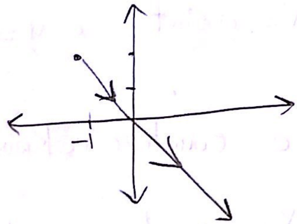
$\rightarrow x = t - 1$ $\quad -1 \leq x \leq 3$
$y = 3 - \frac{5}{4}t$ $\quad -1 \leq t-1 \leq 3$
$\quad 0 \leq t \leq 4$

وأيضاً: $\frac{-5}{4}(x+1) = t$

⋮

يوجد عدد لا نهائي

26 the ray with initial point $(-1,2)$ that passes through the point $(0,0)$.



$\rightarrow m = \frac{0-2}{0+1} = -2$

$y-0 = -2(x-0)$

$\therefore y = -2x \quad ; \quad x \geq -1$

let $x = t$, $y = -2t$; $t \geq -1$

- Sec 11.2 : Calculus with parametric Curves.

* If $x = f(t)$, $y = g(t)$. then:-

① $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$; $\frac{dx}{dt} \neq 0$

② $\frac{d^2 y}{dx^2} = \frac{dy'/dt}{dx/dt}$

③ If f' and g' are continuous and is traced exactly once as t increases from $t=a$ to $t=b$, then the length of C is : $L = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2}\, dt$

④ Area of surface of revolution: * غير مطلوب

→ about the x-axis:

$S = \int_a^b 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ X

→ about the y-axis:

$S = \int_a^b 2\pi x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$ X

- Exercises: page 625

$\boxed{7}$ Find an equation for the line tangent to the curve at the point, and find $\frac{d^2y}{dx^2}$ at this point.

$$X = \sec t \quad , \quad y = \tan t \quad ; \quad t = \frac{\pi}{6}$$

$$\rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sec^2 t}{\sec t \tan t} = \frac{\sec t}{\tan t} = \frac{1}{\sin t} = \csc t$$

$$\therefore m = \left.\frac{dy}{dx}\right|_{t=\frac{\pi}{6}} = \boxed{2}$$

the equation is: $y - y_0 = m(X - X_0)$; $X_0 = \sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}$

$y - \frac{1}{\sqrt{3}} = 2\left(X - \frac{2}{\sqrt{3}}\right)$; $y_0 = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$$\longrightarrow y = 2X - \frac{3}{\sqrt{3}} = 2X - \sqrt{3}$$

$$\rightarrow \frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt} = \left.\frac{-\csc t \cot t}{\sec t \tan t}\right|_{t=\frac{\pi}{6}} = \frac{(-2)(\sqrt{3})}{\left(\frac{2}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)} = -3\sqrt{3}$$

$\boxed{14}$ $X = t + e^t \quad , \quad y = 1 - e^t \quad ; \quad t = 0$.

$$\longrightarrow \left.\frac{dy}{dx}\right|_{t=0} = \left.\frac{dy/dt}{dx/dt}\right|_{t=0} = \left.\frac{-e^t}{1+e^t}\right|_{t=0} = \frac{-1}{2}.$$

the equation is: $y - y_0 = m(X - X_0)$; $X_0 = 1$

$y - 1 = \frac{-1}{2}(X - 1)$; $y_0 = 1$

$$\longrightarrow \boxed{y = -\frac{1}{2}X - \frac{1}{2}}$$

$$\longrightarrow \frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt} = \frac{\left[(1+e^t)(-e^t)-(-e^t)(e^t)\right]/\left[1+e^t\right]^2}{1+e^t}$$

$$= \frac{-e^t - e^{2t} + e^{2t}}{(1+e^t)^3}$$

$$= \frac{-e^t}{(1+e^t)^3}\Bigg|_{t=0} = \frac{-1}{8}$$

$\boxed{20}$ find the slope of the curve :-

$t = \ln(x-t)$, $y = te^t$; $t=0$.

↳ $x = e^t + t$

$$\longrightarrow m = \frac{dy}{dx}\Bigg|_{t=0} = \frac{dy/dt}{dx/dt}\Bigg|_{t=0} = \frac{t \cdot e^t + e^t(1)}{e^t + 1}\Bigg|_{t=0} = \frac{1}{2}$$

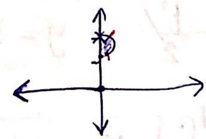
$\boxed{22}$ Find the area enclosed by the y-axis and the curve:

$x = t - t^2$, $y = 1 + e^{-t}$

$t - t^2 = 0$

$\rightarrow t = 0, 1$

$$\longrightarrow \text{Area} = \int_a^b y\,dx \quad \times$$



$$\therefore \text{Area} = \int_a^b x\,dy$$

$$= \int_0^1 (t-t^2)(-e^{-t})\,dt = \int_0^1 (t^2-t)e^{-t}\,dt$$

$$\begin{array}{ll} t^2-t & e^{-t} \\ 2t-1 & \xrightarrow{+} -e^{-t} \\ 2 & \xrightarrow{-} e^{-t} \\ 0 & \xrightarrow{+} -e^{-t} \end{array} \qquad = (t^2-t)e^{t}-(2t-1)e^{t}+2e^{t}$$

$$\therefore \text{Area} = (t-t^2)e^{-t}-(2t-1)e^{-t}-2e^{-t}\Big|_0^1$$

$$= \left(-e^{-1}-2e^{-1}\right)-\left(1-2\right)$$

$$= -3e^{-1}+1$$

25 Find the lengths of the curve :-

$X=\cos t \; , \; y=t+\sin t \; , \; 0\leqslant t\leqslant \pi$

$$\longrightarrow L=\int_0^{\pi}\sqrt{(-\sin t)^2+(1+\cos t)^2}\,dt=\int_0^{\pi}\sqrt{2+2\cos t}\;dt$$

$$=\sqrt{2}\int_0^{\pi}\sqrt{1+2\cos^2\tfrac{t}{2}-1}\,dt=\sqrt{2}\int_0^{\pi}\sqrt{2}\,\left|\cos\tfrac{t}{2}\right|dt=2\left[\int_0^{\frac{\pi}{2}}\cos\tfrac{t}{2}\,dt+\int_{\frac{\pi}{2}}^{\pi}-\cos\tfrac{t}{2}\,dt\right]=4$$

27 $X=\dfrac{t^2}{2} \; , \; y=\dfrac{(2t+1)^{\frac{3}{2}}}{3} \; ; \; 0\leqslant t\leqslant 4$

$$L=\int_0^{4}\sqrt{t^2+2t+1}\;dt \qquad \frac{dx}{dt}=\frac{2t}{2}=t$$

$$=\int_0^{4}\sqrt{(t+1)^2}\;dt \qquad \frac{dy}{dt}=\frac{3}{2}\,\frac{(2t+1)^{\frac{1}{2}}}{3}\cdot 2 =\sqrt{2t+1}$$

$$= \int_0^4 |t+1|\, dt$$

$$= \frac{t^2}{2} + t \Big|_0^4$$

$$= (8+4) - 0$$

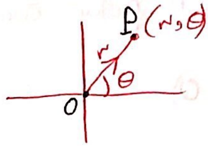
$$= \boxed{12}$$

- Sec 11.3 : Polar Coordinates الإحداثيات القطبية

* Polar Coordinate: $P(r,\theta)$

r: Directed distance from O to P

θ: Directed ray



* Polar Equations and graphs:-

1. $r=a$; circle of radius $|a|$ centerd at O.
2. $\theta=\theta_0$; line through O making an angle θ_0 with the initial ray.

* Equations relating Polar and Cartesian Coordinates.

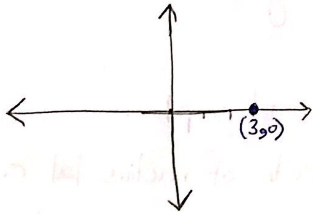
1. $x=r\cos\theta$
2. $y=r\sin\theta$
3. $r^2=x^2+y^2$
4. $\tan\theta=\frac{y}{x}$.

- Exercises: page 630

1 Which polar coordinate pairs label the same point:

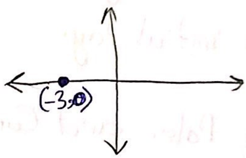
a) $(3, 0)$

$r = 3$, $\theta = 0$



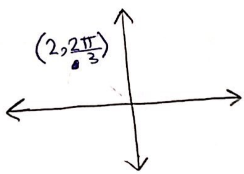
b) $(-3, 0)$

$r = -3$, $\theta = 0$



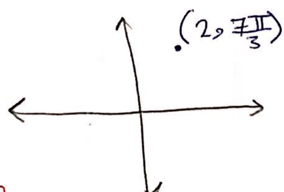
c) $\left(2, \frac{2\pi}{3}\right)$

$r = 2$, $\theta = \frac{2\pi}{3}$



d) $\left(2, \frac{7\pi}{3}\right)$.

$r = 2$, $\theta = \frac{7\pi}{3}$



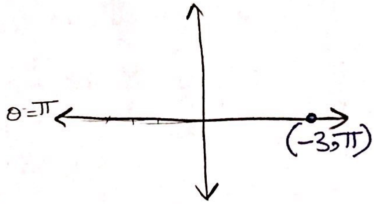
$\theta = \frac{7\pi}{3} \equiv \frac{\pi}{3}$.

θ تجاوزت الدورة الكاملة أي
2π ← لمعرفة الزاوية التي تكافئها
نطرح 2π. $= \frac{\pi}{3}$

e) $(-3, \pi)$

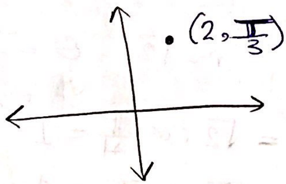
$r = -3 \ , \ \theta = \pi$

عندما تكون r بالسالب
أي أنها تكون معكس
الاتجاه .



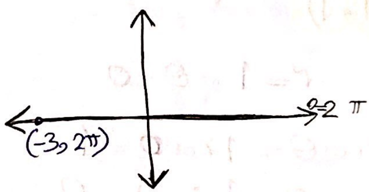
f) $(2, \frac{\pi}{3})$

$r = 2 \ , \ \theta = \frac{\pi}{3}$



g) $(-3, 2\pi)$

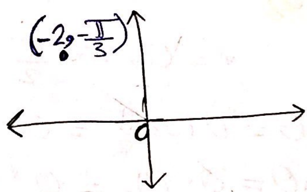
$r = -3 \ , \ \theta = 2\pi$



h) $(-2, -\frac{\pi}{3})$

$r = -2 \ , \ \theta = -\frac{\pi}{3}$

بما أن r=-2 ، نعكس
الاتجاه ، أي في
الربع الذي يقابلها .



$a \equiv e$,

$b \equiv g$

$c \equiv h$

$d \equiv f$

6 Find the cartesian coordinates of the following points:-

a) $(\sqrt{2}, \frac{\pi}{4})$ $r = \sqrt{2}$, $\theta = \frac{\pi}{4}$

$\longrightarrow X = r\cos\theta = \sqrt{2}\cos\frac{\pi}{4} = 1$

$y = r\sin\theta = \sqrt{2}\sin\frac{\pi}{4} = 1$

$\therefore (1, 1)$

b) $(1, 0)$ $r = 1$, $\theta = 0$

$\longrightarrow X = r\cos\theta = 1\cos 0 = 1$

$y = r\sin\theta = 1\sin 0 = 0$

$\therefore (1, 0)$

c) $(0, \frac{\pi}{2})$ $r = 0$, $\theta = \frac{\pi}{2}$

$\longrightarrow X = r\cos\theta = 0\cos\frac{\pi}{2} = 0$

$y = r\sin\theta = 0\sin\frac{\pi}{2} = 0$

$\therefore (0, 0)$

d) $\left(-\sqrt{2}, \frac{\pi}{4}\right)$ $\quad r = -\sqrt{2}, \; \theta = \frac{\pi}{4}$

$\longrightarrow x = r\cos\theta = -\sqrt{2}\cos\frac{\pi}{4} = -1$

$y = r\sin\theta = -\sqrt{2}\sin\frac{\pi}{4} = -1$

$\therefore (-1, -1)$

e) $\left(-3, \frac{5\pi}{6}\right)$ $\quad r = -3, \; \theta = \frac{5\pi}{6}$

$\longrightarrow x = r\cos\theta = -3\cos\left(\frac{5\pi}{6}\right) = -3\left(-\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{2}$

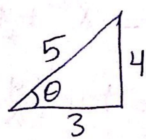
$y = r\sin\theta = -3\sin\left(\frac{5\pi}{6}\right) = -3\left(\frac{1}{2}\right) = \frac{-3}{2}$

$\therefore \left(\frac{3\sqrt{3}}{2}, \frac{-3}{2}\right)$

f) $\left(5, \tan^{-1}\left(\frac{4}{3}\right)\right)$ $\quad r = 5, \; \theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\longrightarrow x = r\cos\theta = 5\cos\left(\tan^{-1}\frac{4}{3}\right)$

$= 5\left(\frac{3}{5}\right) = 3$



$y = r\sin\theta = 5\sin\left(\tan^{-1}\frac{4}{3}\right) = 5\left(\frac{4}{5}\right) = 4$

$\therefore (3, 4)$

h) $\left(2\sqrt{3}, \frac{2\pi}{3}\right)$ $\quad r = 2\sqrt{3}, \; \theta = \frac{2\pi}{3}$

$\longrightarrow x = r\cos\theta = 2\sqrt{3}\cos\left(\frac{2\pi}{3}\right) = 2\sqrt{3}\left(\frac{-1}{2}\right) = -\sqrt{3}$

$y = r\sin\theta = 2\sqrt{3}\sin\left(\frac{2\pi}{3}\right) = 2\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = 3$

$\therefore (-\sqrt{3}, 3)$

7 Find the polar coordinates, $0 \leq \theta \leq 2\pi$ and $r \geq 0$ of the following points given in Cartesian coordinates:-

a) $(1, 1)$ $\quad x = 1, \quad y = 1$ } النقطة في الربع الأول ∴ نختار الزاوية في الربع 1

$\longrightarrow r^2 = x^2 + y^2 = 2 \longrightarrow r = \sqrt{2}$ } نأخذ فقط الموجب لأن $r \geq 0$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1} 1 = \frac{\pi}{4}$$

$\therefore \left(\sqrt{2}, \frac{\pi}{4}\right)$.

b) $(-3, 0)$ $\quad x = -3, \; y = 0$ } النقطة في الربع الثالث نختار الزاوية في الربع 3

$\longrightarrow r^2 = x^2 + y^2 = 9 \longrightarrow r = 3$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1} 0 = 0 + \pi = \pi$$

$\therefore (3, \pi)$

c) $(\sqrt{3}, -1)$ $\quad x = \sqrt{3}, \; y = -1$ } النقطة في الربع الرابع نختار الزاوية في الربع الرابع

$\longrightarrow r^2 = x^2 + y^2 = (\sqrt{3})^2 + (-1)^2 = 4 \longrightarrow r = 2$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

$\therefore \left(2, \frac{11\pi}{6}\right)$

d) $(-3,4)$ $\quad x=-3,\ y=4$ النقطة في الربع الثاني فنأخذ الزاوية في الربع الثاني.

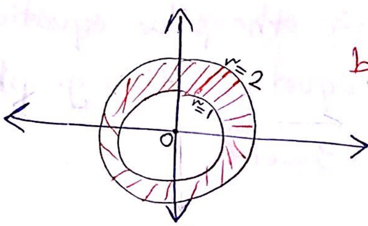
$\rightarrow r^2 = x^2+y^2 = 25 \rightarrow r=5$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-4}{3}\right) \cong 180-53 \cong 126.7 = \frac{126}{180}\pi$$

$\left(5, \frac{126}{180}\pi\right)$

14 Graph the sets of points whose polar coordinates satisfy the equations and inequalities:-

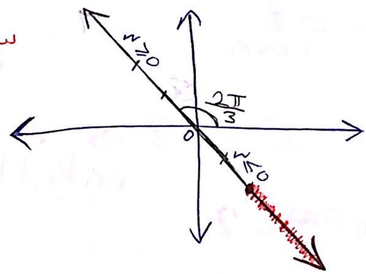
$1 \leq r \leq 2$



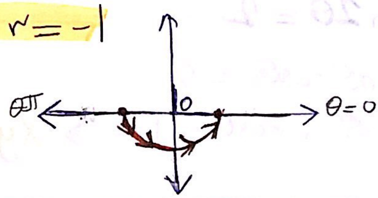
لا يوجد شرط على θ.

16 $\theta = \frac{2\pi}{3}$, $r \leq -2$

نبدأ برسم θ.



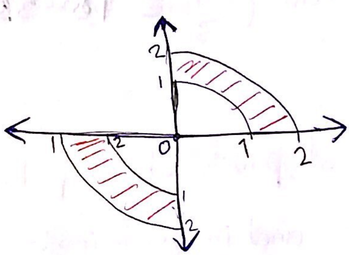
22 $0 \leq \theta \leq \pi$, $r=-1$



26 $0 \leq \theta \leq \frac{\pi}{2}, \ 1 \leq |r| \leq 2$

$1 \leq r \leq 2$

$-2 \leq r \leq -1$



32 Replace the polar equations with equivalent cartesian equations, then graph:-

$r = -3 \sec\theta$

$\longrightarrow r = -3 \sec\theta$

$r = \frac{-3}{\cos\theta}$

$; \ r\cos\theta = -3 \longrightarrow x = -3$

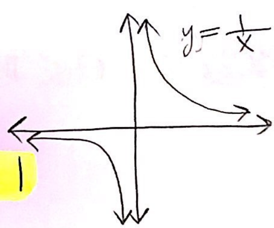
vertical line

38 $r^2 \sin 2\theta = 2$

$\longrightarrow r^2 \sin 2\theta = 2$

$r^2(2\sin\theta\cos\theta) = 2$

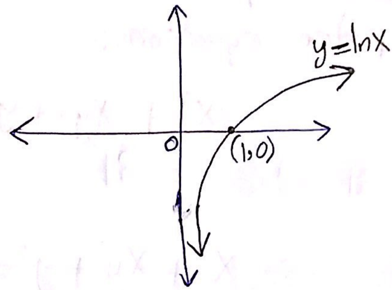
$r\sin\theta \cdot r\cos\theta = 1 \longrightarrow xy = 1$



42 $r \sin\theta = \ln r + \ln\cos\theta$

$\rightarrow r\sin\theta = \ln(r \cdot \cos\theta)$

$$y = \ln x$$



52 $r \sin\left(\frac{2\pi}{3} - \theta\right) = 5$

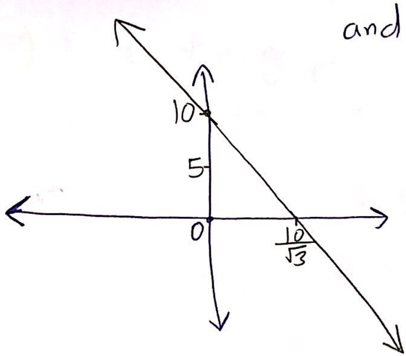
$\rightarrow r\sin\left(\frac{2\pi}{3} - \theta\right) = 5$

$r\left[\sin\frac{2\pi}{3}\cos\theta - \sin\theta\cos\frac{2\pi}{3}\right] = 5$

$r\left[\frac{\sqrt{3}}{2}\cos\theta - \frac{-1}{2}\sin\theta\right] = 5$

$\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 5$ « line with x-intercept: $\frac{10}{\sqrt{3}}$ and y-intercept: 10 »

$\rightarrow \boxed{\sqrt{3}\,x + y = 10}$



62 Replace the cartesian equations with equivalent polar equations:-

$$x^2+xy+y^2=1$$

$r\cos\theta = x$ نستبدل

$r\sin\theta = y$

$\rightarrow x^2+xy+y^2=1$

$$r^2\cos^2\theta + r^2\cos\theta\sin\theta + r^2\sin^2\theta = 1$$

$$r^2\left(\cos^2\theta+\sin^2\theta+\cos\theta\sin\theta\right)=1$$

$$r^2\left(1+\cos\theta\sin\theta\right)=1$$

$$\therefore\; r^2=\frac{1}{1+\cos\theta\sin\theta}$$

Sec 11.4 : Graphing in Polar Coordinates:

* Symmetry Tests for Polar Graphs:-

1) Symmetry about the x-axis:

If the point (r, θ) lies on the graph, then $(r, -\theta)$ or $(-r, \pi - \theta)$ lies on the graph.

← اذا كانت النقاط لا تقع لا نستطيع الحكم.

2) Symmetry about the y-axis:-

If the point (r, θ) lies on the graph, then $(r, \pi - \theta)$ or $(-r, -\theta)$ lies on the graph.

← في حال كانت النقاط المذكورة لا تقع لا نستطيع الحكم.

3) Symmetry about the origin:

If the point (r, θ) lies on the graph, then $(-r, \theta)$ or $(r, \theta + \pi)$ lies on the graph. في حال كانت النقاط المذكورة لا تقع لا نستطيع الحكم.

- Notes:-

- If the curve is symmetric about the x-axis and y-axis, then it is symmetric about the origin.

العكس غير صحيح.

← بشكل عام اذا كان متماثل حول أي اثنتين فإنه يكون متماثل حول الثالث

-Notes:

1. $\cos(-\theta) = \cos\theta$
2. $\cos(\pi - \theta) = -\cos\theta$
3. $\cos(\pi + \theta) = -\cos\theta$
4. $\sin(-\theta) = -\sin\theta$
5. $\sin(\pi - \theta) = \sin\theta$
6. $\sin(\pi + \theta) = -\sin\theta$
7. $\cos(\frac{\pi}{2} - \theta) = \sin\theta$
8. $\sin(\frac{\pi}{2} - \theta) = \cos\theta$
9. $\sin(\frac{\pi}{2} + \theta) = \cos\theta$
10. $\sin(\frac{\pi}{2} + \theta) = -\sin\theta$

-Exercises: page 634

1 Identify the symmetry, then sketch:-

$r = 1 + \cos\theta$.

1) Symmetry about the x-axis:

we check the points $(r, -\theta)$ or $(-r, \pi - \theta)$:-

$(r, -\theta)$: $r \overset{?}{=} 1 + \cos(-\theta)$

$r \overset{?}{=} 1 + \cos\theta$ ✓

$\therefore$ the curve is symmetric about the x-axis.

2) Symmetry about the y-axis:

we check the points $(r, \pi - \theta)$ or $(-r, -\theta)$

$(r, \pi - \theta)$: $r \overset{?}{=} 1 + \cos(\pi - \theta)$

$r \overset{?}{=} 1 - \cos\theta$ ✗ we can't tell.

$(-r, -\theta)$: $\quad -r \stackrel{?}{=} 1+\cos(-\theta)$

$-r \stackrel{?}{=} 1+\cos\theta \quad \times \qquad$ can't tell.

3) Symmetry about the origin:

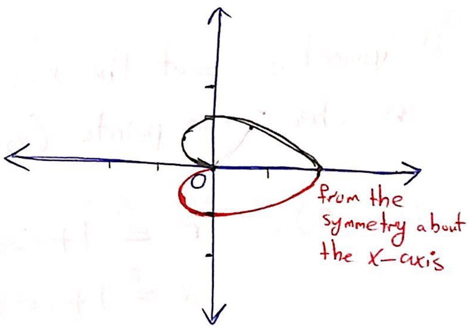
We check the points $(-r, \theta)$ or $(r, \pi+\theta)$.

$(-r, \theta)$: $\quad -r \stackrel{?}{=} 1+\cos\theta \quad \times \qquad$ can't tell.

$(r, \pi+\theta)$: $\quad r \stackrel{?}{=} 1+\cos(\pi+\theta)$

$r \stackrel{?}{=} 1-\cos\theta \quad \times \qquad$ can't tell.

θ	$r = 1+\cos\theta$
0	2
$\frac{\pi}{3}$	$\frac{3}{2}$
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	$\frac{1}{2}$
π	0



$\boxed{6}$ $r = 1 + 2\sin\theta$

1) Symmetry about the x-axis:-

$(r, -\theta)$: $r \overset{?}{=} 1 + 2\sin(-\theta)$

$r \overset{?}{=} 1 - 2\sin\theta$ ✗ can't tell

$(-r, \pi - \theta)$: $-r \overset{?}{=} 1 + 2\sin(\pi - \theta)$

$-r \overset{?}{=} 1 + 2\sin\theta$ ✗ can't tell.

2) Symmetry about the y-axis:

$(r, \pi - \theta)$: $r \overset{?}{=} 1 + 2\sin(\pi - \theta)$

$r \overset{?}{=} 1 + 2\sin\theta$ ✓

$\therefore$ the curve is symmetric about the y-axis.

3) Symmetry about the origin:-

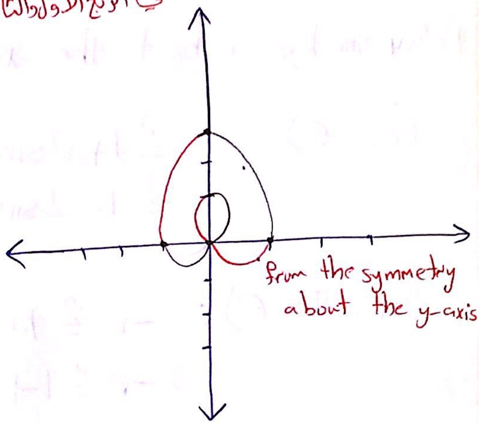
$(-r, \theta)$: $-r \overset{?}{=} 1 + 2\sin\theta$ ✗ can't tell.

$(r, \pi + \theta)$: $r \overset{?}{=} 1 + 2\sin(\pi + \theta)$

$r \overset{?}{=} 1 - 2\sin\theta$ ✗ can't tell.

لأنه متماثل حول y-axis بناخذ زوايا في الربع الأول والثالث

θ	$r = 1 + 2\sin\theta$
0	1
$\frac{\pi}{6}$	2
$\frac{\pi}{2}$	3
π	1
$\frac{7\pi}{6}$	0
$\frac{3\pi}{2}$	-1



$\boxed{8}$ $r = \cos\left(\frac{\theta}{2}\right)$

1) Symmetry about the x-axis:-

$(r, -\theta)$: $r \overset{?}{=} \cos\left(\frac{-\theta}{2}\right)$

$r \overset{?}{=} \cos\left(\frac{\theta}{2}\right)$ ✓

∴ the curve is symmetric about the x-axis.

2) Symmetry about the y-axis:

$(-r, -\theta)$: $-r \overset{?}{=} \cos\left(\frac{-\theta}{2}\right)$

$-r \overset{?}{=} \cos\left(\frac{\theta}{2}\right)$ ✗ can't tell

$(r, \pi-\theta)$: $r \overset{?}{=} \cos\left(\frac{\pi-\theta}{2}\right) = \sin\left(\frac{\theta}{2}\right)$ ✗ can't tell

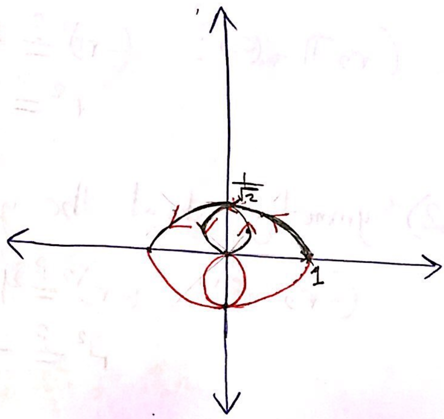
3) Symmetry about the origin:-

$(-r, \theta)$: $\quad -r \overset{?}{=} \cos\left(\frac{\theta}{2}\right)$ ✗. can't tell.

$(r, \pi + \theta)$: $\quad r \overset{?}{=} \cos\left(\frac{\pi}{2} + \theta\right) = -\sin\left(\frac{\theta}{2}\right)$ ✗. can't tell

θ	$r = \cos\left(\frac{\theta}{2}\right)$
0	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2} = 0.86$
$\frac{\pi}{2}$	$\frac{1}{\sqrt{2}} = 0.7$
$\frac{2\pi}{3}$	$\frac{1}{2}$
π	0
$\frac{4\pi}{3}$	$-\frac{1}{2}$
$\frac{3\pi}{2}$	$-\frac{1}{\sqrt{2}} = 0.7$
$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2} = -0.8$
2π	-1

لأن معامل
$\theta \leftrightarrow \frac{1}{2}$
نأخذ كمان
دورة



from the graph
→ the curve is symmetric about the x-axis, y-axis and the origin.

14 What symmetries do these curves have?

$r^2 = 4\sin 2\theta$

1) Symmetry about the x-axis:-

$(r, -\theta)$: $r^2 \overset{?}{=} 4\sin(-2\theta)$

$r^2 \overset{?}{=} -4\sin(2\theta)$ ✗ can't tell

$(-r, \pi-\theta)$: $(-r)^2 \overset{?}{=} 4\sin(2\pi-2\theta)$

$r^2 \overset{?}{=} -4\sin(2\theta)$ ✗ can't tell

2) Symmetry about the y-axis:-

$(-r, -\theta)$: $(-r)^2 \overset{?}{=} 4\sin(-2\theta)$

$r^2 \overset{?}{=} -4\sin(2\theta)$ ✗ can't tell.

$(r, \pi-\theta)$: $r^2 \overset{?}{=} 4\sin(2\pi-2\theta)$

$r^2 \overset{?}{=} -4\sin(2\theta)$ ✗ can't tell

3) Symmetry about the origin:-

$(-r, \theta)$: $(-r)^2 \overset{?}{=} 4\sin 2\theta$

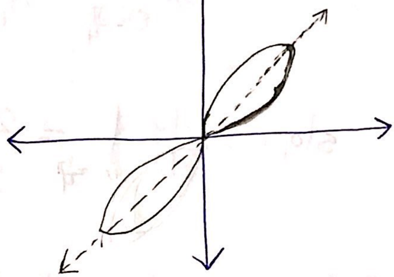
$r^2 = 4\sin 2\theta$ ✓

$\therefore$ the curve is symmetric about the origin.

θ	$r^2 = 4\sin(2\theta)$	r
0	0	0
$\frac{\pi}{6}$	$4\frac{\sqrt{3}}{2} = 3.46$	∓ 1.86
$\frac{\pi}{4}$	4	∓ 2
$\frac{\pi}{2}$	0	0
$\frac{5\pi}{6}$	$-\frac{4\sqrt{3}}{2} = -3.64$	✗
$\frac{3\pi}{4}$	-4	
π	0	

لأن معامل 2θ نأخذ من $\frac{\pi}{2} \to 0$

$-r \to$ هي نفسها الي سنحصل عليها من Symmetry حول نقطة الأصل.



$\boxed{19}$ Find the slopes of the curve at the given points then sketch the curve.

$$r = \sin(2\theta) \quad ; \quad \theta = \mp\frac{\pi}{4}, \mp\frac{3\pi}{4}.$$

$$\to \text{slope} = \left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = \left.\frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}\right|_{\theta=\frac{\pi}{4}}$$

$$= \left.\frac{2\cos(2\theta)\sin\theta + \sin(2\theta)\cos\theta}{2\cos(2\theta)\cos\theta - \sin(2\theta)\sin\theta}\right|_{\theta=\frac{\pi}{4}}$$

$$= \frac{2(0)\left(\frac{1}{\sqrt{2}}\right) + (1)\left(\frac{1}{\sqrt{2}}\right)}{2(0)\left(\frac{1}{\sqrt{2}}\right) - (1)\left(\frac{1}{\sqrt{2}}\right)} = \boxed{-1}$$

$$\text{slope} = \left.\frac{dy}{dx}\right|_{\theta=\frac{-\pi}{4}} = \frac{2(0)\left(\frac{-1}{\sqrt{2}}\right)+(-1)\left(\frac{1}{\sqrt{2}}\right)}{2(0)\left(\frac{1}{\sqrt{2}}\right)-(-1)\left(\frac{-1}{\sqrt{2}}\right)} = \boxed{1}$$

$$\text{slope} = \left.\frac{dy}{dx}\right|_{\theta=\frac{3\pi}{4}} = \frac{2(0)\left(\frac{1}{\sqrt{2}}\right)+(-1)\left(\frac{-1}{\sqrt{2}}\right)}{2(0)\left(\frac{-1}{\sqrt{2}}\right)-(-1)\left(\frac{1}{\sqrt{2}}\right)} = \boxed{1}$$

$$\text{slope} = \left.\frac{dy}{dx}\right|_{\theta=\frac{-3\pi}{4}} = \frac{2(0)\left(\frac{-1}{\sqrt{2}}\right)+(1)\left(\frac{-1}{\sqrt{2}}\right)}{2(0)\left(\frac{-1}{\sqrt{2}}\right)-(1)\left(\frac{-1}{\sqrt{2}}\right)} = \boxed{-1}$$

1) Symmetry about the x-axis:-

$(r, -\theta)$: $\quad r \stackrel{?}{=} \sin(-2\theta)$

$\quad r \stackrel{?}{=} -\sin(2\theta)$ ✗ can't tell.

$(-r, \pi-\theta)$: $\quad r \stackrel{?}{=} \sin(2\pi-2\theta)$

$\quad -r \stackrel{?}{=} -\sin(2\theta)$ ✓ $\therefore$ r is symmetric about the x-axis

2) Symmetry about the y-axis:-

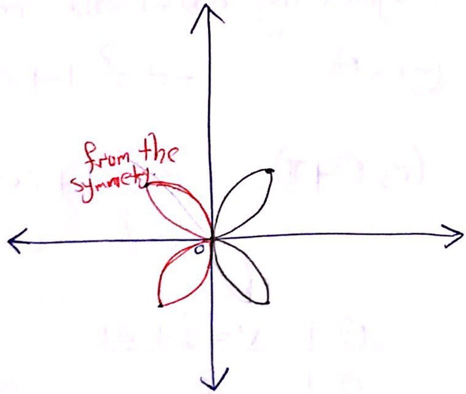
$(-r, -\theta)$: $\quad -r \stackrel{?}{=} \sin(-2\theta)$

$\quad -r \stackrel{?}{=} -\sin(2\theta)$ ✓

$\therefore$ the curve is symmetric about the y-axis

from 1) and 2) $\longrightarrow$ the curve is symmetric about the origin.

θ	$r=\sin(2\theta)$
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	-1
π	0



21 Graph:- a) $r=\frac{1}{2}+\cos\theta$

1) Symmetry about the x-axis:

$(r, -\theta)$: $\quad r \overset{?}{=} \frac{1}{2}+\cos(-\theta)$

$r \overset{?}{=} \frac{1}{2}+\cos\theta$ ✓

$\therefore$ the curve is symmetric about the x-axis.

2) Symmetry about the y-axis:-

$(-r, -\theta)$: $\quad -r \overset{?}{=} \frac{1}{2}+\cos(-\theta)$

$-r \overset{?}{=} \frac{1}{2}-\cos\theta$ ✗. can't tell.

$(r, \pi-\theta)$: $\quad r \overset{?}{=} \frac{1}{2}+\cos(\pi-\theta)$

$r \overset{?}{=} \frac{1}{2}-\cos\theta$ ✗. can't tell

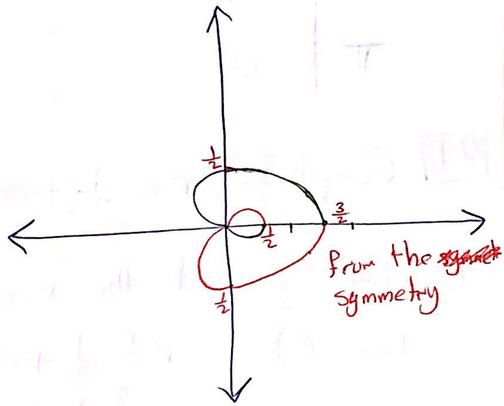
3) Symmetry about the origin:-

$(-r, \theta)$: $\quad -r \overset{?}{=} 1+\cos\theta$ ✗ can't tell.

$(r, \theta+\pi)$: $\quad r \overset{?}{=} 1+\cos(\theta+\pi)$

$r \overset{?}{=} -\cos\theta$ ✗ can't tell.

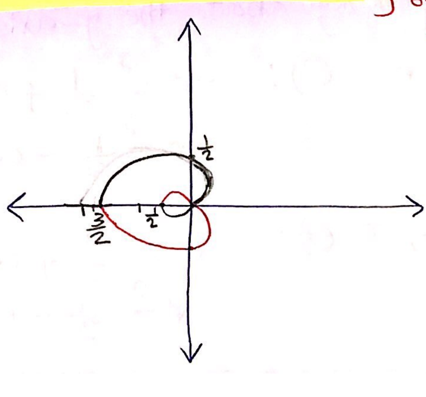
θ	$r=\frac{1}{2}+\cos\theta$
0	$\frac{3}{2}$
$\frac{\pi}{3}$	1
$\frac{\pi}{2}$	$\frac{1}{2}$
$\frac{2\pi}{3}$	0
π	$-\frac{1}{2}$



b) $r=\frac{1}{2}-\cos\theta$ } → $r = 1-\sin\theta$ السؤال

the curve is symmetric about the x-axis } نفحصه كما الفرع السابق

θ	$r=\frac{1}{2}-\cos\theta$
0	$-\frac{1}{2}$
$\frac{\pi}{3}$	0
$\frac{\pi}{2}$	$\frac{1}{2}$
$\frac{2\pi}{3}$	1
π	$\frac{3}{2}$



28 sketch the region defined by:-

$0 \leqslant r^2 \leqslant \cos\theta$

$\longrightarrow r^2 = \cos\theta$

1) symmetry about the x-axis:-

$(r, -\theta)$: $r^2 \overset{?}{=} \cos(-\theta)$

$r^2 \overset{?}{=} \cos\theta$ ✓

$\therefore$ the curve is symmetric about the x-axis.

2) Symmety about the y-axis:-

$(r, \pi-\theta)$: $r^2 = \cos(\pi-\theta)$

$r^2 \overset{?}{=} -\cos\theta$ ✗. can't tell

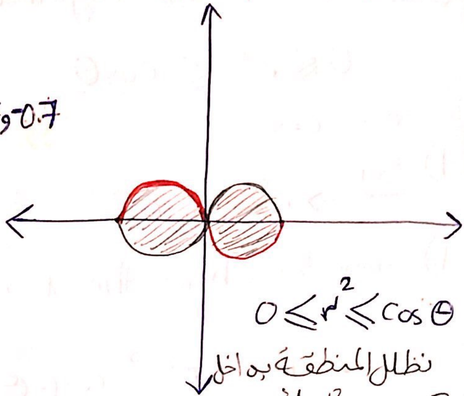
$(-r, -\theta)$: $(-r)^2 \overset{?}{=} \cos(-\theta)$

$r^2 \overset{?}{=} \cos\theta$ ✓

$\therefore$ the curve is symmetric about the y-axis.

3) the curve is symmetric about the origin.

θ	$r^2 = \cos\theta$	r
0	1	$1, -1$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} = 0.7, -0.7$
$\frac{\pi}{2}$	0	0
$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\times$
π	-1	$\times$



$0 \leq r^2 \leq \cos\theta$

نظلل المنطقة بداخل

$r^2 = \cos\theta$ الـ

$0 \leq r^2 \leq \cos\theta$