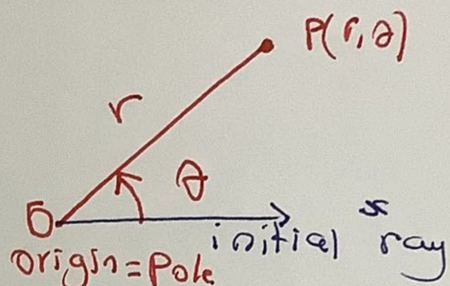
11.3 Polar Coordinates

we will study the Polar Coordinates and its relation to cartesian Coordinats

To define Polar coordinates, we first fix an origin O (called the Pole) and an initial ray from O



Polar coordinates pair (r, θ)

r = gives the directed distance from O to P

θ = gives the directed angle from the initial ray to ray OP

$\boxed{\text{Polar Coordinates}}$

$P(r, \theta)$

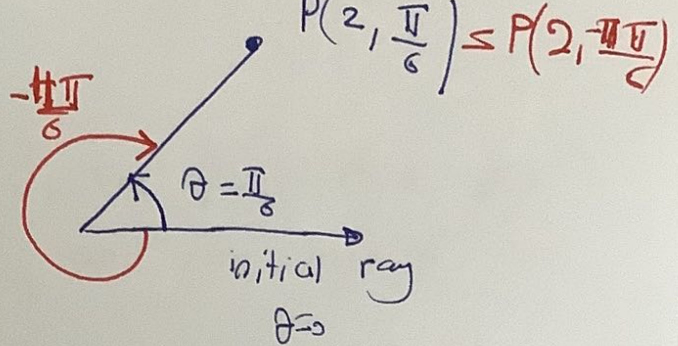
θ: * Directed angle from initial ray to OP

r: * directed distance from O to P

θ is +ve if its measured counterclockwise

θ is −ve if its measured clockwise

Example : Find all the Polar coordinates of the Point $P\left(2, \frac{\pi}{6}\right)$

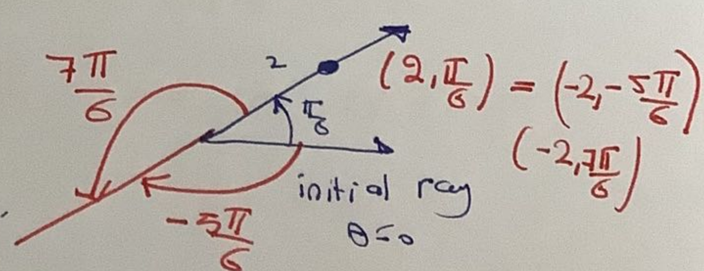


If $r = 2$

$$\theta = \frac{\pi}{6} , \frac{\pi}{6} \pm 2\pi , \frac{\pi}{6} \pm 4\pi , \frac{\pi}{6} \pm 6\pi , \ldots$$

If $r = -2$

$$\theta = -\frac{5\pi}{6} , -\frac{5\pi}{6} + 2\pi , -\frac{5\pi}{6} + 4\pi , \ldots$$

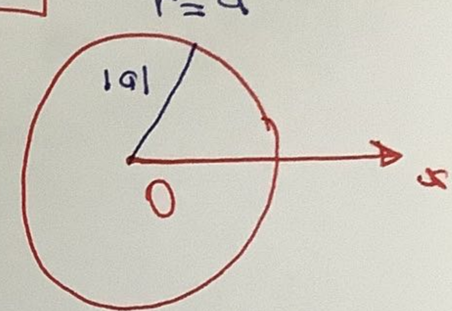


$\left(2, \frac{\pi}{6} + 2n\pi\right)$, $n = 0, \pm 1, \pm 2, \ldots$

$\left(-2, -\frac{5\pi}{6} + 2n\pi\right)$, $n = 0, \pm 1, \pm 2, \ldots$

Polar Coordinates and Graphs

* If we hold $\boxed{r}$ fixed at a constant value $\boxed{r = a} \neq 0$ the point $\boxed{P(r, \theta)}$ will lie $\boxed{|a|}$ units from the origin $\boxed{O}$. As $\boxed{\theta}$ varies over any interval of length $\boxed{2\pi}$, P traces a $\boxed{\text{circle}}$ of $\boxed{\text{radius } |a|}$ centered at $\boxed{O}$

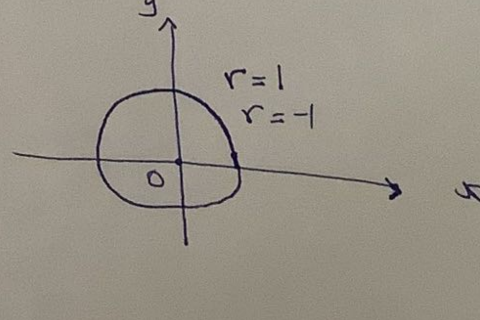


* If we hold $\boxed{\theta}$ fixed at a constant value $\boxed{\theta = \theta_0}$ and let $\boxed{r}$ between $-\infty$ and ∞, the point $P(r, \theta)$ traces the $\boxed{\text{line}}$ through O that makes an angle measure θ_0 with the initial ray

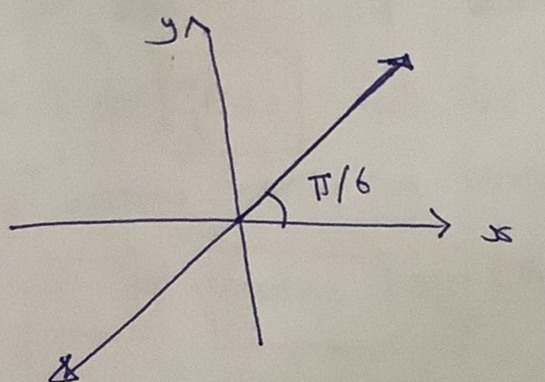
Equation	Graph
$r = a$	Circle of radius $|a|$ centered at O
$\theta = \theta_0$	Line through O making an angle θ_0 with the initial ray

Example:

a) $r = 1$ and $r = -1$

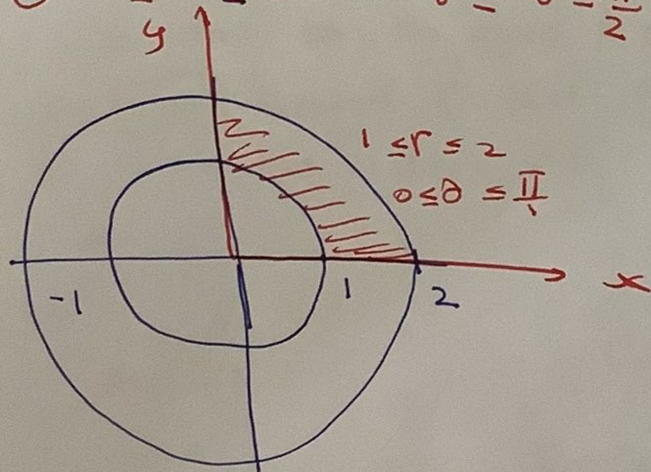


Ex $\theta = \frac{\pi}{6}$, $\theta = \frac{7\pi}{6}$, $\theta = -\frac{5\pi}{6}$

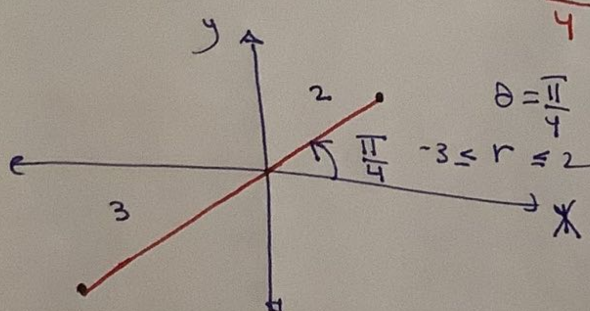


Example

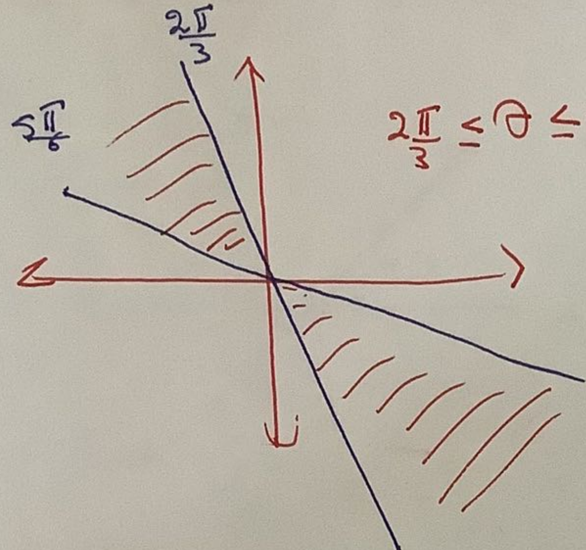
(a) $1 \le r \le 2$ and $0 \le \theta \le \frac{\pi}{2}$



(b) $-3 \le r \le 2$, $\theta = \frac{\pi}{4}$



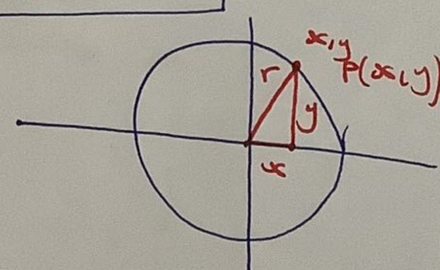
(c) $\frac{2\pi}{3} \le \theta \le \frac{5\pi}{6}$ No restriction on r



$\frac{2\pi}{3} \le \theta \le \frac{5\pi}{6}$

Relating Polar and Cartesian Coordinates

$\boxed{x = r\cos\theta}$, $\boxed{y = r\sin\theta}$, $\boxed{r^2 = x^2 + y^2}$, $\boxed{\tan\theta = \frac{y}{x}}$



Example:

$r\cos\theta = 2$, $\boxed{x = 2}$

Example:

$r^2\cos\theta \cdot \sin\theta = 4$

$r\cos\theta \quad r\sin\theta = 4$

$xy = 4 \longrightarrow \boxed{y = \frac{4}{x}}$

Ex Find a polar equation for the circle

$$x^2+(y-3)^2=9$$

$$x^2+y^2-6y+9=9$$

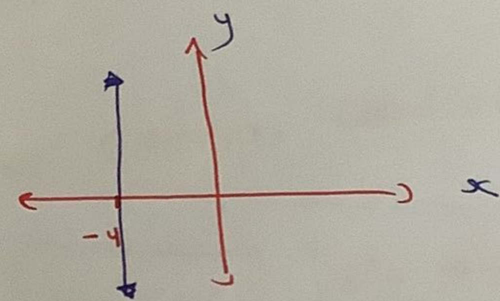
$$x^2+y^2-6y=0$$

$$r^2-6r\sin\theta=0$$

$r=0$ or $r=6\sin\theta$

Ex (a) $r\cos\theta=-4$

$x=-4$



(b) $r^2=4r\cos\theta$

$$x^2+y^2-4x=0$$

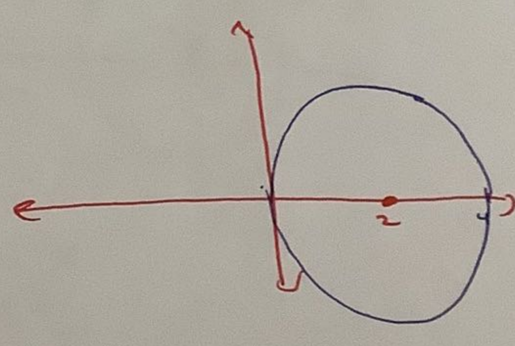
$$x^2-4x+y^2=0$$

$$x^2-4x+4+y^2=4$$

$$(x-2)^2+y^2=4$$

circle

$(2,0)$, radius $=2$



Example:

$r^2\cos^2\theta - r^2\sin^2\theta =$

$(r\cos\theta)^2 - (r\sin\theta)^2 = x^2 - y^2 = 1$

Example:

$r = 1 + 2r\cos\theta$

$\cancel{r = 1 + 2x}$

$\sqrt{x^2+y^2} = 1 + 2x$

$x^2 + y^2 = 1 + 4x + 4x^2$

$3x^2 - y^2 + 4x + 1 = 0$

Example:

$r = 1 - \cos\theta$

$r^2 = r - r\cos\theta$

$x^2 + y^2 = \sqrt{x^2+y^2} - x$

$x^2 + y^2 + x = \sqrt{x^2+y^2}$

$(x^2 + y^2 + x)^2 = x^2 + y^2$

$x^4 + y^4 + 2x^3 + x^2 + 2x^2y^2 + 2xy^2 = x^2 + y^2$

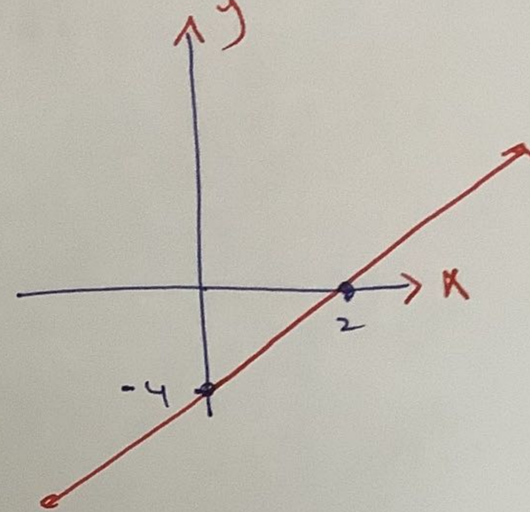
$x^4 + y^4 + 2x^2y^2 + 2x^3 + 2xy^2 - y^2 = 0$

$\boxed{Ex}$ $r = \dfrac{4}{2\cos\theta - \sin\theta}$

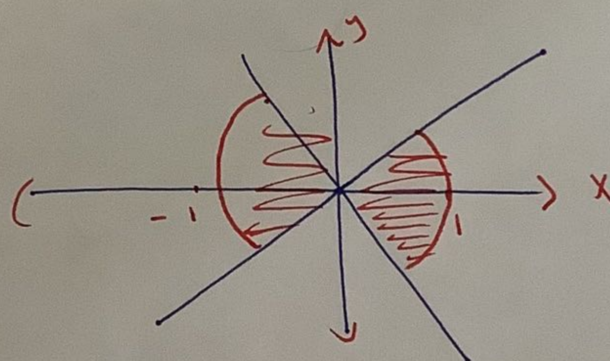
$2r\cos\theta - r\sin\theta = 4$

$2x - y = 4$

$y = 2x - 4$



$\boxed{Ex\,24}$ $-\dfrac{\pi}{4} \le \theta \le \dfrac{\pi}{4}$ & $-1 \le r \le 1$

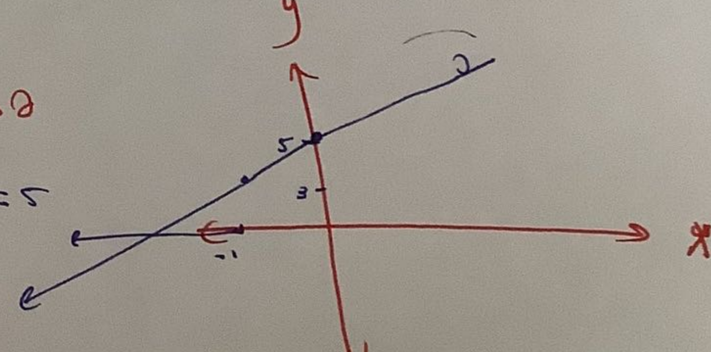


(37) $r = \dfrac{5}{\sin\theta - 2\cos\theta}$

$r\sin\theta - 2r\cos\theta = 5$

$y - 2x = 5$

$y = 2x + 5$



line
slope $= 2$
y-int $= 5$

60 $xy = 2$

$r\cos\theta \; r\sin\theta = 2$

$r^2 \cos\theta \sin\theta = 2$ 6 $\sin 2\theta = 2\sin\theta\cos\theta$

$\boxed{r^2 \sin 2\theta = 4}$