12/20

جامعة بيرزيت
BIRZEIT UNIVERSITY

MATHEMATICS DEPARTMENT
MATH132 - THIRD EXAM
SUMMER 2013/2014

- Name [redacted]
- Number [redacted]

- (For Question 1) Fill your answers in the tables below:

| Page 1 | |
|---|---|
| 1 | b |
| 2 | d |
| 3 | d |
| 4 | a |
| 5 | c |

| Page 2 | |
|---|---|
| 6 | a |
| 7 | d |
| 8 | d |
| 9 | a |
| 10 | b |

| Page 3 | |
|---|---|
| 11 | b |
| 12 | a |

- Instructions:

1. No Calculators.
2. Mobiles Off.
3. BZU ID On Your Desk.
4. No Cheating At All.
5. Time Limit: 60 Minutes.

5

Question 1. *(12 points)* Circle the best answer.

Root test $= \sum$

1. The series $\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-2}\right)^n$

$\lim_{n\to\infty} \left(\frac{4n+3}{3n-2}\right)$

$\boxed{\frac{4}{3} > 1}$

a) Converges by root test
b) Diverges by root test
c) Converges by integral test
d) Diverges by alternating series test

2. If we approximate $e^x$ by $1+x+\frac{x^2}{2!}$, then the error in estimating $e^{-1}$ is

$\boxed{x=-1}$

$\boxed{a=0}$

$n=2$

$e^x = 1+x+\frac{x^2}{2!}$ $\quad e^x = \frac{x^n}{n!}$

a) less than $\frac{1}{2}$
b) less than $\frac{1}{2e}$
c) less than $\frac{1}{6}$
d) less than $\frac{1}{e}$

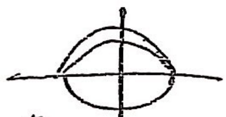
$\left|\frac{f^{n+1}(c)(x-a)^{n+1}}{(n+1)!}\right| < \frac{M(x-a)^{n+1}}{(n+1)!}$ $\quad a < c < x$

$0 < c < -1$

$e^c < e^{-1}$

$\left|\frac{e^c (x)^3}{3!}\right| < \frac{e^{-1}(x)^3}{3!}$

3. The radius of convergence of the series $\sum_{n=0}^{\infty} (n+1)!\,(x-4)^n$ is



a) $R=0$
b) $R=1$
c) $R=4$
d) $R=\infty$

$= \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

4. The series $\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$

$-1 + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}$

Ratio Test

$\lim_{n\to\infty} \left|\frac{(n+1+1)!\,(x-4)^{n+1}}{(n+1)!\,(x-4)^n}\right|$

a) Converges absolutely
b) Converges conditionally
c) Diverges by alternating series test
d) Diverges by nth term test

5. $1+\pi+\frac{\pi^2}{2!}+\frac{\pi^3}{3!}+\cdots=$

$\lim_{n\to\infty} \left|\frac{(n+2)\cancel{(n+1)!}\,\cancel{(x-4)^n}(x-4)}{\cancel{(n+1)!}\,\cancel{(x-4)^n}}\right|$

a) 0
b) $-1$
c) $e^\pi$
d) None of the above

$|x-4|$

$\frac{1}{\sqrt{n}}$ $-a_n = \frac{1}{\sqrt{n}}$ ~~...~~ $u_n > 0$ decrecing

$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

$\cos 0 = 1$

$-\sin = 0$

$-\cos 0 = -1$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$-1 + x - \frac{x^3}{3!}$

6. The series $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n^5}}$

a) Converges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^5}}$

b) Converges by direct comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^5}}$

c) Converges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{n^3}$

d) Diverges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt[3]{n}}$

$\lim_{n\to\infty} 1 = 1$

$1 \neq 0$ diverges

7. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges conditionally if

a) $0 < p < 1$

b) $0 \le p < 1$

c) $0 < p \le 1$

d) $0 \le p \le 1$

$\lim \frac{1}{n^p}$ $p \le 1$

$\lim_{n\to\infty} \dfrac{\frac{\ln n}{\sqrt{n^5}}}{\frac{1}{\sqrt{n^5}}}$

8. The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$

a) Converges by integral test

b) Diverges by integral test

c) Converges by nth term test

d) None of the above

$\lim_{n\to\infty} \frac{\ln n}{\sqrt{n^5}} = \infty$

$\frac{1}{n^{5/2}}$ $\frac{5}{2} > 1$ Converges

9. The binomial series of $\frac{1}{\sqrt{1+x}}$ is

a) $1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \cdots$

b) $1 + \frac{x}{2} - \frac{3x^2}{8} - \frac{5x^3}{16} + \cdots$

c) $1 - \frac{x}{2} - \frac{3x^2}{8} - \frac{5x^3}{16} - \cdots$

d) $1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{x^3}{16} + \cdots$

$(1+x)^{-\frac{1}{2}}$ $m = -\frac{1}{2}$

$1 + \sum_{n=1}^{\infty} \binom{m}{n} x^n$

$-\frac{1}{2}\left(-\frac{1}{2}-1\right)$

10. The Maclaurin series generated by $x \sin x^2$ is

a) $x^3 + \frac{x^7}{3!} - \frac{x^{11}}{5!} + \cdots$

b) $x^3 - \frac{x^7}{3!} + \frac{x^{11}}{5!} - \cdots$

c) $x + \frac{x^5}{2!} - \frac{x^9}{4!} + \cdots$

d) $x - \frac{x^5}{2!} + \frac{x^9}{4!} - \cdots$

$\lim_{n\to\infty} \frac{1}{\ln n} \Rightarrow 0$

$\frac{3}{4}x^2$

$\frac{3}{8}x^2$

11. The Taylor polynomial of order 3 generated by $f(x) = e^{2x}$ about $a = 0$ is

a) $P_3(x) = 1 + 2x + x^2$

b) $P_3(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3$

c) $P_3(x) = 1 + x + 2x^2 + \frac{4}{3}x^3$

d) $P_3(x) = 1 + x + x^2$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{2x} = \frac{(2x)^n}{n!}$$

$$1 + 2x + \frac{4x^2}{2}$$

$$1 + 2x + 2x^2 + \frac{48x^3}{36}$$

12. The series $\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$

a) Converges by ratio test

b) Diverges by ratio test

c) Converges to 4

d) Converges by root test

$$\frac{(2n+1)!}{(n+1)!(n+1)!} \cdot \frac{n!n!}{2n!}$$

$$\frac{1}{n^2} = \boxed{0} < 1 \qquad \lim_{n \to \infty} \frac{(2n+1)}{(n+1)(n+1)}$$

$$x - \frac{x^3}{3!} + \frac{x^5}{5!}$$

Question 2. *(4 points)* Given that $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$.

$$\cancel{1 - \frac{x^2}{2!} + \frac{x^4}{4!}}$$

(a) Find the Maclaurin series of $\cos x^3$.

$$\cos x^3 = \sum_{n=0}^{\infty} \frac{(-1)^n (x^3)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!}$$

2

(b) Use part (a) to estimate $\int_0^1 \cos x^3 \, dx$ with error less than 0.01

$$\int_0^1 \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!} = \int_0^1 1 + -\frac{x^6}{2!} + \frac{x^{12}}{4!} - \frac{x^{18}}{6!}$$

1

$$= x - \frac{x^7}{7(2!)} + \frac{x^{13}}{13(4!)} - \frac{x^{19}}{19(6!)} \Bigg]_0^1$$

Question 3. *(2 points)* Express $\frac{1}{(1+x)^2}$ as a power series ~~and~~ find its radius of convergence.

( Hint: $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots$ )

$\frac{d}{dx}\left(\frac{1}{1+x}\right) = 1 - x + x^2 - x^3 + \cdots$

$\frac{-1}{(1+x)^2} = -1 + 2x - 3x^2$

$\frac{-1}{1+x^2} = \sum_{n=1}^{\infty} (-1)^n (nx)^{n-1} \cdots$

$\frac{1}{1+x^2} = \sum_{n=1}^{\infty} (nx)^{n-1}$

by Ratio Test

$\lim_{n\to\infty} \left|\frac{((n+1)x)^n}{(nx)^{n-1}}\right| \Rightarrow \left|\frac{(n+1)^n x^n}{n^{n-1} x^{n-1}}\right|$

$|x| \lim_{n\to\infty} \frac{(n+1)^n}{n^{n-1}}$

Question 4. *(2 points)* Use series to find $\lim_{x\to 0} \frac{\sin x}{e^{-x} - 1}$

$\sin x = \sum_{n=0}^{\infty} x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \Rightarrow e^{-x} = \frac{(-1)^n (x^n)}{n!}$

$= 1 - x + \frac{x^2}{2!}$

$\lim_{x\to 0} \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)}{\left(\cancel{1} - x + \frac{x^2}{2!} - \cancel{1}\right)}$

BONUS. *(2 points)* Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{x^n n!}{(1)(3)(5)\dots(2n-1)}$.

$$\lim_{x \to 0} \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)}{\left(x + \frac{x^2}{2!} + \frac{x^3}{3!}\right)}$$

$$\lim_{x \to 0} \frac{x\left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots\right)}{x\left(-1 + \frac{x}{2!} - \frac{x^2}{3!} - \cdots\right)} = -1$$

# Birzeit University
## Mathematics Department
## Math 132
## Final Exam
## First Summer Semester 2012/2013

Student Name: ............................................ Student Number: ..........................

Time: 150 minutes There are 4 questions in 10 pages

**Question 1.** (60%) Circle the most correct answer:

(1) The volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $x = 1$, and the $x$-axis, about the $y$-axis, is:

(a) $\frac{3\pi}{5}$

(b) $\frac{\pi}{5}$

(c) $\frac{2\pi}{5}$

(d) $\frac{4\pi}{5}$

(2) $\sum_{n=2}^{\infty}(0.5)^{-n} =$

(a) 2

(b) 1

(c) $\frac{1}{2}$

(d) None of the above

(3) $\int_0^{\frac{\pi}{2}} \tan x \, dx =$

(a) 0

(b) $-1$

(c) $\infty$

(d) $-\infty$

(4) If $y$ is the solution of the differential equation $\frac{dy}{dx} = 3x^2y + y$, $y(1) = e$, then $y(-1) =$

(a) $-1$

(b) $-3$

(c) $e^{-1}$

(d) $e^{-3}$

(10)

(5) $\int_{1}^{4} \frac{3^{\sqrt{x}}}{2\sqrt{x}}\,dx =$

(a) $\frac{6}{\ln 3}$

(b) $\frac{3}{\ln 3}$

(c) $\frac{78}{\ln 3}$

(d) $\frac{9}{\ln 3}$

(6) The volume of the solid whose base is the region enclosed between the curves $y = x^2$ and $y = x$, and whose cross sections perpendicular to the $x$-axis are equilateral triangles of height 4, is:

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{6}$

(d) $\frac{1}{4}$

(7) If $a_n = n3^{\frac{1}{n}}$, $n \in \mathbb{N}$, then $\lim\limits_{n\to\infty} a_n =$

(a) 1

(b) 0

(c) $\infty$

(d) $\ln 3$

(8) $\sum\limits_{n=2}^{\infty} \frac{2n-1}{n^2(n-1)^2} =$

(a) $-1$

(b) 1

(c) $\frac{1}{4}$

(d) 2

(9) Assuming its convergence, find the limit of the following recursively defined sequence, $a_1 = 8$, $a_{n+1} = \sqrt{a_n + 8} - 2$:

(a) 1

(b) $-4$

(c) $-2$

(d) 8

(10) $\int e^{\sqrt{2x+1}}\,dx =$

(a) $2\sqrt{2x+1}\,e^{\sqrt{2x+1}} + C$

(b) $\dfrac{e^{\sqrt{2x+1}}}{2\sqrt{2x+1}} + C$

(c) $\sqrt{2x+1}\,e^{\sqrt{2x+1}} - e^{\sqrt{2x+1}} + C$

(d) $\sqrt{2x+1}\,e^{\sqrt{2x+1}} - \sqrt{2x+1} + C$

(11) If $\tanh x = \dfrac{1}{2}$, $x < 0$, then $\text{sech}x =$

(a) $\dfrac{\sqrt{5}}{2}$

(b) $\dfrac{-\sqrt{5}}{2}$

(c) $\dfrac{\sqrt{3}}{2}$

(d) $\dfrac{-\sqrt{3}}{2}$

(12) Which one of the following functions is the fastest growing as $x \to \infty$:

(a) $e^{\frac{x}{2}}$

(b) $\ln(\ln x)$

(c) $3^x$

(d) $4 + 2^x$

(13) The series $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{5^n + 2^n}$:

(a) Converges by direct comparison with $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{4^n}$

(b) Converges by direct comparison with $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^n}$

(c) Converges by direct comparison with $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{7^n}$

(d) Converges by summing its terms as a geometric series

(14) The series $\displaystyle\sum_{n=2}^{\infty} \frac{(n+1)\ln n}{\sqrt{n}}$:

(a) Converges by the integral test

(b) Converges by direct comparison with $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2}$

(c) Diverges by the ratio test

(d) Diverges by the $n$th-term test

(15) If $a_n = \left(1 - \frac{2}{n}\right)^{\frac{n}{2}}$, $n \in \mathbb{N}$, then $\lim\limits_{n\to+\infty} a_n =$

(a) $e^{-2}$

(b) $e^{-1}$

(c) $e^{-4}$

(d) $e^{\frac{-1}{2}}$

(16) The series $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n(n+2)(n+3)}}$:

(a) Converges by direct comparison with $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt[5]{n^4}}$

(b) Converges by limit comparison with $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$

(c) Converges by direct comparison with $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$

(d) Diverges by the ratio test

(17) $i^{215} =$

(a) $i$

(b) $-i$

(c) $1$

(d) $-1$

(18) The integral $\int\limits_{2}^{\infty} \frac{dx}{\sqrt[3]{x}\,(\sqrt{x}-1)}$:

(a) Converges by limit comparison with $\int\limits_{2}^{\infty} \frac{dx}{\sqrt[3]{x}}$

(b) Converges by limit comparison with $\int\limits_{2}^{\infty} \frac{dx}{\sqrt{x}}$

(c) Diverges by direct comparison with $\int\limits_{2}^{\infty} \frac{dx}{\sqrt[3]{x^2}}$

(d) Diverges by direct comparison with $\int\limits_{2}^{\infty} \frac{dx}{\sqrt[6]{x^5}}$

(19) The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{e^n(x-1)^n}{n^2 3^n}$ is:

(a) $\frac{3}{e}+1$
(b) $\frac{e}{3}+1$
(c) $\frac{3}{e}$
(d) $\frac{e}{3}$

(20) $\int_0^1 x^2 \ln x \, dx =$

(a) $\frac{-1}{4}$
(b) $\frac{-1}{9}$
(c) $\infty$
(d) $-\infty$

(21) The series $\sum_{n=1}^{\infty} \frac{(-1)^n(\sqrt{n}+n)}{\sqrt{n^5+1}}$:

(a) Converges absolutely
(b) Converges conditionally
(c) Diverges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$
(d) Diverges by the $n$th-term test

(22) $\int_1^{\sqrt{3}} \frac{dx}{x\sqrt{x^2+1}} =$

(a) $\ln\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right)$
(b) $\ln\left(\frac{\sqrt{2}}{\sqrt{3}+1}\right)$
(c) $\ln\left(\frac{\sqrt{2}+1}{\sqrt{3}}\right)$
(d) $\ln\left(\frac{\sqrt{3}}{\sqrt{2}+1}\right)$

(27)

(23) $\displaystyle\int_{-\frac{\pi}{3}}^{\frac{\pi}{6}} \sec^2 x \tan^2 x \, dx =$

(a) $\dfrac{-26}{9\sqrt{3}}$
(b) $\dfrac{28}{9\sqrt{3}}$
(c) $\dfrac{-13}{3\sqrt{3}}$
(d) $\dfrac{20}{3\sqrt{3}}$

(24) A partial fraction for the function $f(x) = \dfrac{3x+1}{x^3-8}$ is:

(a) $\dfrac{A}{x-2} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2}$
(b) $\dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{C}{x+2}$
(c) $\dfrac{A}{x-2} + \dfrac{Bx+C}{x^2-2x+4}$
(d) $\dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+2x+4}$

(25) The series $\displaystyle\sum_{n=2}^{\infty} \left(\frac{n}{n^2-1}\right)^{n^2}$:

(a) Converges by summing its terms as a telescoping series
(b) Converges by the $n$th-term test
(c) Converges by the root test
(d) Diverges by direct comparison with $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt[n]{n}}$

(26) $\dfrac{4-i}{1+i} =$

(a) $\dfrac{3}{2} - \dfrac{5}{2}i$
(b) $\dfrac{3}{2} + \dfrac{5}{2}i$
(c) $\dfrac{5}{2} - \dfrac{3}{2}i$
(d) $\dfrac{5}{2} + \dfrac{3}{2}i$

(27) The series $\sum_{n=3}^{\infty} \frac{\sqrt{\ln n}}{n^{1.1}}$:

(a) Converges by limit comparison with $\sum_{n=3}^{\infty} \frac{1}{n^{1.05}}$

(b) Converges by limit comparison with $\sum_{n=3}^{\infty} \frac{1}{n^{1.1}}$

(c) Converges by direct comparison with $\sum_{n=3}^{\infty} \frac{1}{n^{0.95}}$

(d) Diverges by the ratio test

(28) If $x = \ln(\sec t + \tan t)$, $y = t \sec t$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$, then $\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{4}} =$

(a) $\frac{\pi}{2} + 1$

(b) $\frac{\pi}{2\sqrt{2}} + \frac{1}{\sqrt{2}}$

(c) $\frac{\pi}{4} + 1$

(d) None of the above

(29) The series $\sum_{n=1}^{\infty} (-1)^n \left(\sqrt{n+1} - \sqrt{n-1}\right)$:

(a) Converges absolutely

(b) Converges conditionally

(c) Diverges by the $n$th-term test

(d) Diverges by direct comparison with $\sum_{n=1}^{\infty} (-1)^n \sqrt{n}$

(30) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)2^{n+1}} =$

(a) $\ln\left(\frac{2}{3}\right)$

(b) $\ln\left(\frac{1}{3}\right)$

(c) $\ln\left(\frac{3}{2}\right)$

(d) $\ln\left(\frac{3}{4}\right)$

Question 2. (15%) (a) Use the binomial series to find out the first four nonzero terms of the Maclaurin series of $(1+x)^{\frac{2}{3}}$, $-1 < x < 1$.

(b) (1) Find the Taylor series of $f(x) = \tan^{-1}(3x^2)$, about $a = 0$, and specify its interval of convergence.

(2) Use the above series to estimate the value of $\tan^{-1}\left(\frac{1}{3}\right)$ with an error of magnitude less than 0.001.

Question 3. (13%) (a) Find the length of the parametric curve:

$$x = t, \quad y = \frac{t^2}{2}, \quad 0 \le t \le 1.$$

(b) Sketch the parametric curve defined by the equations:

$$x = 3\cos t, \quad y = 2\sin t, \quad -\frac{\pi}{2} \le t \le \pi.$$

**Question 4.** (12%) (a) Find the four forth roots of $-81$.

(b) Solve the equation: $2|z-1-i| = |z+\overline{z}-2|$.

Final (+ U) Shadi Key



BIRZEIT UNIVERSITY

MATH 132 Final exam 09/10

NO:

Name (............................ ...................)

INSTRUCTORS : RASEM + SHADI

**PROBLEM 1:10 MARKS**

**Find the four fourth roots of** $-8+8\sqrt{3}\,i$

(2)

The modulus $= \sqrt{(-8)^2+(8\sqrt{3})^2} = 16$ (2)

64

$\cos\theta = \frac{-8}{16} = -\frac{1}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{-8}{16}\right) = 120 = \boxed{\frac{2\pi}{3}}$ (2)

$w_k = 16^{1/4}\left[\cos\left(\frac{120+2k180}{4}\right) + i\sin\left(\frac{120+2k180}{4}\right)\right]$ $k=0,1,2,3$

$w_0 = 2\left[\cos(30) + i\sin(30)\right] = 2\left[\frac{\sqrt{3}}{2}+\frac{1}{2}i\right] = \sqrt{3}+i$ (1)

$w_1 = 2\left[\cos(120) + i\sin(120)\right] = 2\left[-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right] = -1+i\sqrt{3}$ (1)

$w_2 = 2\left[\cos(210) + i\sin(210)\right] = 2\left[-\frac{\sqrt{3}}{2} - i\frac{1}{2}\right] = -\sqrt{3}-i$ (1)

$w_3 = 2\left[\cos 300 + i\sin 300\right] = 2\left[\frac{1}{2} - i\frac{\sqrt{3}}{2}\right] = 1-i\sqrt{3}$ (1)

$w_0 = \sqrt{3}+i$
$w_1 = -1+i\sqrt{3}$
$w_2 = -\sqrt{3}-i$
$w_3 = 1-i\sqrt{3}$

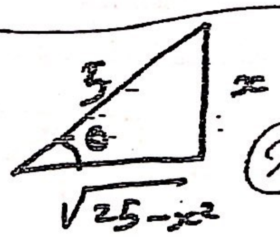
**PROBLEM 2:10 MARKS EVALUATE**



$$\int \frac{1}{x^2\sqrt{25-x^2}}\,dx$$

$x = 5\sin\theta$ 2

$dx = 5\cos\theta\, d\theta$




2

$$\int \frac{5\cos\theta\, d\theta}{25\sin^2\theta\sqrt{25-25\sin^2\theta}}$$ 2

$$\int \frac{5\cos\theta\, d\theta}{25\sin^2\theta\,(5\cos\theta)}$$ 2

$$\frac{1}{25}\int \csc^2\theta\, d\theta$$

$$= -\frac{1}{25}\cot(\theta) + c$$

$$= -\frac{1}{25}\frac{\sqrt{25-x^2}}{x} + c$$ 2

AV1505614535

**PROBLEM 3(10 MARKS):COSIDER S IS CONSTANT EVALUATE**

$$\int_0^{\infty} t^2 e^{-st}\, dt$$

| | |
|---|---|
| $t^2$ | $e^{-st}$ |
| $2t$ | $-\frac{e^{-st}}{s}$ |
| $2$ | $\frac{e^{-st}}{s^2}$ |
| $0$ | $-\frac{e^{-st}}{s^3}$ |

$$\lim_{B\to\infty}\int_0^B t^2 e^{-st}\, dt$$

$$= \lim_{B\to\infty}\left[-\frac{t^2}{se^{st}} - \frac{2t}{s^2e^{st}} - \frac{2}{s^3e^{st}}\right]_0^B$$

$$= \lim_{B\to\infty} \frac{-B^2}{se^{sB}} - \frac{2B}{s^2e^{sB}} - \frac{2}{s^3e^{sB}} - \left(0-0-\frac{2}{s^3}\right)$$

$$= \frac{2}{s^3}$$

**PROBLEM 4(10 MARKS)DISCUSS THE CONVERGENCE**

$$\sum_{k=0}^{\infty} \frac{k^2}{2^{3k}}(x+4)^k$$

By Ratio test

$$\lim_{k\to\infty} \left| \frac{\frac{(k+1)^2}{2^{3(k+1)}}(x+4)^{k+1}}{\frac{k^2}{2^{3k}}(x+4)^k} \right| < 1$$

$$= \lim \frac{(k+1)^2}{k^2}\left(\frac{1}{2^3}\right)|x+4| < 1$$

$$= \lim \frac{1}{8}|x+4| < 1$$

$$= |x+4| < 8 \longrightarrow \boxed{R=8}$$

$$-8 < x+4 < 8$$

$$\boxed{-12 < x < 4}$$

when $\boxed{x=-12}$ $\sum_{k=0}^{\infty} \frac{k^2}{2^{3k}}(-12+4)^k = \sum \frac{k^2}{(8)^k}(-8)^k = \sum_{k=0}^{\infty}(-1)^k k^2$

$\lim_{k\to\infty} k^2 = \infty$ div.

when $\boxed{x=4}$ $\sum_{k=0}^{\infty} \frac{k^2}{2^{3k}}(4+4)^k = \sum_{k=0}^{\infty} k^2$

$\lim k^2 = \infty$ div

$\therefore$ The interval of conv. $(-12, 4)$

## PROBLEM 5: 60 MARKS

Consider the test form the first column and the result form the second column

1. geometric series
2. p- series
3. telescoping series
4. the nth-term test
5. the integral test
6. alternating series test
7. the direct comparison test
8. the limit comparison test
9. the ratio test
10. the root test

a. converges absolutely
b. converges conditionally
c. diverges

**solve in details then circle the correct answer**

1. radius of conv.

$$\sum_{k=1}^{\infty}\frac{2^k(x-3)^k}{\sqrt{k+3}}=$$

a. $\frac{1}{2}$

b. $\frac{7}{2}$

c. $\frac{19}{6}$

d. $\frac{5}{2}$

$\lim\left|\frac{2^{k+1}(x-3)^{k+1}}{\sqrt{k+4}}\cdot\frac{\sqrt{k+3}}{2^k(x-3)^k}\right|<1$

$=\lim 2|x-3|\frac{\sqrt{k+3}}{\sqrt{k+4}}<1$

$=2|x-3|<1$

$|x-3|<\frac{1}{2}$

$-\frac{1}{2}<x-3<\frac{1}{2}$

$\frac{5}{2}<x<\frac{7}{2}$

$x=\frac{5}{2}$ by conv. by alternating

$x=\frac{7}{2}$ div. by limit cmp. test

2. $$\sum_{k=1}^{\infty}\frac{\sin k}{k^2+1}$$

a. divergent by p- series
b. divergent by geometric series
c. converges conditionally
d. converges absolutely

$a_k = \left|\frac{\sin k}{k^2+1}\right|\le\frac{1}{k^2+1}\le\frac{1}{k^2}$

$\sum\frac{1}{k^2}$ conv. p-ser

$\sum a_k$ conv. by D.C.T.

3. $$\sum_{k=0}^{\infty}\left(\frac{(-1)^k}{\sqrt{k+1}}\right)=$$

a. divergent by the ratio test
b. divergent by the nth-root test
c. converges conditionally
d. converges absolutely

1) Let $a_k=\frac{1}{\sqrt{k+1}}>0$

2) $f(x)=\frac{1}{\sqrt{x+1}}=(x+1)^{-1/2}$

$f'(x)=\frac{-1}{2\sqrt{(x+1)^3}}<0$

decreasing series

3) $\lim a_k=0$

$\sum(-1)^k a_k$ conv. alternating series

Let $b_k=\frac{1}{\sqrt{k}}=\frac{1}{k^{1/2}}$

$\lim\frac{a_k}{b_k}=\lim\frac{\sqrt{k}}{\sqrt{k+1}}=1$

$\sum b_k$ div. p-series

$\Rightarrow\sum a_k$ div. by limit C.T.

4. $\sum_{k=2}^{\infty}\frac{(-1)^k\sqrt{k}}{\ln k}=$

a. divergent by the direct comparison test
b. divergent by the kth-term test
c. converges absolutely
d. converges conditionally

$\text{Lim}\ \frac{\sqrt{k}}{\text{Ln}k}=\text{Lim}\ \frac{\frac{1}{2\sqrt{k}}}{1/k}=\text{Lim}\ \frac{\sqrt{k}}{2}\neq 0$

By k-th term test

the seres div.

5. $\sum_{k=1}^{\infty}\frac{\sqrt{n^2-1}}{n^3+2n^2+5}=$

a. convergent by the integral test
b divergent .by limit comparison test
c. convergent by limit comparison test
d. divergent by by direct comparison test

Let $a_n=\frac{\sqrt{n^2-1}}{n^3+2n^2+5}$

$b_n=\frac{1}{n^2}$

$\text{Lim}\ \frac{a_n}{b_n}=\frac{\sqrt{n^2-1}\ n^2}{n^3+2n^2+5}$

But $\sum b_n=\sum\frac{1}{n^2}$

Conv. p-series

So $\sum a_n$ Conv. Limit C.T.

6. $\sum_{k=0}^{\infty}\frac{(-2)^{3k-1}}{9^k}=$

a. $\frac{-2}{9}$

b. $\frac{9}{7}$

c. $\frac{-9}{7}$

d. $\frac{-9}{34}$

$\sum_{k=0}\frac{(-2)^{-1}\left((-2)^3\right)^k}{9^k}=\sum_{k=0}-\frac{1}{2}\left(\frac{-8}{9}\right)^k$

$=-\frac{1}{2}\left[\frac{1}{1-\left(-\frac{8}{9}\right)}\right]$

$=-\frac{1}{2}\left[\frac{1}{1+\frac{8}{9}}\right]$

$=-\frac{1}{2}\left[\frac{9}{9+8}\right]=\frac{-9}{34}$

7. $\sum_{k=1}^{\infty}\frac{e^{\frac{1}{n}}}{n^2}=$

a. convergent by the integral test
b divergent.by limit comparison test
c. convergent by telescoping series
d. divergent by by direct comparison test

$\int_1^{\infty}\frac{e^{1/x}}{x^2}\,dx$

$u=\frac{1}{x}$ | $x=1\to u=1$, $x\to\infty\to u\to 0$

$du=-\frac{1}{x^2}dx$

$-\int_1^{0}e^u\,du$

$e^{1/x},\frac{1}{x^2}$ Cont., +ve, dec.

$=\int_0^1 e^u\,du=e^u\Big]_0^1$

$=e-1$

8. $\sum_{k=1}^{\infty} \frac{\cos(\frac{k\pi}{6})}{k\sqrt{k}} =$

$a_k \leq \frac{|\cos k\pi|}{k\sqrt{k}} \leq \frac{1}{k^{3/2}} = b_k$

$\Sigma b_k$ conv. p-series $\Rightarrow \Sigma a_k$ Con by D.C.T.

a. divergent by the direct comparison test
b. divergent by the kth-term test
c. converges absolutely
d. converges conditionally

---

9.. $\lim_{x\to 0}(\cos 3x)^{\frac{5}{x}} =$

a. 1
b. 0
c. $\frac{1}{2}$
d. $\infty$

Let $y = (\cos 3x)^{5/x} \Rightarrow \ln y = \frac{5\ln(\cos 3x)}{x}$

$\lim_{x\to 0} \ln y = \lim \frac{5\ln\cos 3x}{x} = \lim \frac{-15\tan 3x}{1} = 0$

$\lim y = \lim e^{\ln y} = e^0 = 1$

---

10. $\lim_{x\to 0}(1\text{-}2x)^{\frac{1}{x}} =$

a. does not exit
b. $e^{e-2}$
c. $e^{-2}$
d. $\frac{2}{e}$

Let $y = (1-2x)^{\frac{1}{x}}$

$\ln y = \frac{\ln(1-2x)}{x}$

$\lim_{x\to 0} \frac{\ln(1-2x)}{x}$

$\lim_{x\to 0} \frac{-2}{1-2x} = -2$

$\Rightarrow \lim(1-2x)^{\frac{1}{x}} = \lim e^{y} = e^{-2}$

---

11. the sequence $a_n = \frac{\ln(2+e^n)}{3n}$

a. converges to $\frac{2}{3}$
b. converges to $\frac{1}{3}$
c. converges to 0
d. divergent sequences

$\lim \frac{\ln(2+e^n)}{3n}$

$\lim \frac{\frac{e^n}{2+e^n}}{3} = \lim \frac{e^n}{6+3e^n}$

$= \lim \frac{e^n}{3e^n} = \frac{1}{3}$

12. $\int \frac{2x^2-3x+2}{x^3+x}$ CAN BE INTERATED BY PARTIAL FRACTION

a. $\frac{A}{X}+\frac{BX+C}{X^2+1}$

b. $\frac{A}{X}+\frac{B}{X^2+1}$

c. $\frac{A}{X}+\frac{C}{X^2+1}+\frac{D}{X^2}$

d. $\frac{A}{X}+\frac{BX^2+C}{X^3}$

13. $\int_0^1 \tan^{-1}(x)dx$

$u=\tan^{-1}x$ $dv=dx$

$du=\frac{1}{1+x^2}dx$ $v=x$

a. 1

b. 0

c. $\frac{\pi}{4}$

d. $\frac{\pi}{4}-\frac{1}{2}\ln 2$

$x\tan^{-1}x-\int\frac{x}{1+x^2}$

$x\tan^{-1}x-\frac{1}{2}\ln(1+x^2)\Big]_0^1$

$\tan^{-1}1-\tan^{-1}(0)-\left[\frac{1}{2}\ln 2-\frac{1}{2}\ln 1\right]$

$=\pi/4-\frac{1}{2}\ln 2$

14. $\int_1^{e^{\frac{\pi}{2}}}\frac{\cos(\ln x)}{x}dx$

$u=\ln x$

$du=\frac{1}{x}dx$

a. diverge

b. 1

c. 0

d. e

$\int_0^{\pi/2}\cos u\,du=\sin u\Big]_0^{\pi/2}=1$

غلط

15. $\int_0^1\frac{x}{1+3x}dx$

a. $\frac{1}{3}-\frac{1}{3}\ln 4$

b. $\frac{1}{3}$

c. $\frac{1}{\sqrt{3}}\tan^{-1}\sqrt{3}$

d. 1/4 $\frac{1}{3}+\frac{1}{3}\ln 2$

$=\frac{1}{3}\int_0^1\frac{3(x)+1}{1+3x}dx-\int_0^1\frac{1}{1+3x}dx$

$=\frac{1}{3}x\Big]_0^1-\frac{1}{3}\ln(1+3x)\Big]_0^1$

$=\frac{1}{3}-\frac{1}{3}\ln 4$

16. $y = e^{\sinh x}$

a. $y' = \cosh x$

b. $y' = \cosh x\, e^{\sinh x}$

c. $y' = \sinh x\, e^{\sinh x}$

d. $y' = e^{\cosh x}$

17. the center of the ellipse

$4x^2 + y^2 - 8x + 4y - 8 = 0$

Is

a. (2,4)

b. (4,2)

c. (1,2)

d. (1,-2)

$4(x^2 - 2x + 1) + (y^2 + 4y + 4) = 8 + 4 + 4$

$4(x-1)^2 + (y+2)^2 = 16$

$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$

18. the equation of the asymptotes for the hyperbola

$4x^2 - 3y^2 + 8x + 16 = 0$

a. $y = \frac{2}{\sqrt{3}}(x+1)$ and $y = -\frac{2}{\sqrt{3}}(x+1)$

b. $y = \frac{2}{\sqrt{3}}(x)$ and $y = -\frac{2}{\sqrt{3}}(x)$

c. $y - 1 = \frac{2}{\sqrt{3}}(x)$ and $y - 1 = -\frac{2}{\sqrt{3}}(x)$

d. $y = (x-1)$ and $y = -(x-1)$

$\frac{y^2}{4} - \frac{(x+1)^2}{3} = 1$

$a = 2$

$b = \sqrt{3}$

19. the focus of the parabola $2y = 1 - 2x - x^2$ is

a. (0,0)

b. $(-1, \frac{1}{2})$

c. (1,-1)

d. $(1, \frac{-1}{2})$

$(x+1)^2 = -2(y-1)$

$(x-h)^2 = 4p(y-k)$

$(h,k) = (-1,1)$

$p = -\frac{1}{2}$

Focus $= (-1, \frac{1}{2})$

19 the focus of the parabola $2y = 1 - x - x^2$ is

a. (0,0)

b. $(-1, \frac{1}{2})$

c. (1,-1)

d. $(1, \frac{-1}{2})$

$x^2 + x - 1 = -2y$

$x^2 + x + \cancel{\frac{1}{4}} = -2y + 1 + \frac{1}{4}$

$(x + \frac{1}{2})^2 = -2(y - \frac{5}{8})$

20. Consider the function $f(x) = \frac{1}{x}$ find the maximum error in using a Taylor polynomial Of order 3 centered at a=1to estimate 1.2

a. $(0.2)^4$

b. $(1.2)^3$

c. 1

d. $(\frac{1}{1.2})^3$

$R_3 = \frac{24(x-1)^4}{4!\, c^5}$ ; $1 \le c \le 1.2$

$|R_3| \le (1 - 1.2)^4 = (.2)^4$

21 From quastion20the infinite series represent

$$f(x) = \frac{1}{x} = \sum_{n=0}^{\infty} (1 - x)^n$$

a. true

b. false

$$f(x) = \frac{1}{1-(1-x)} = \sum_{n=0}^{\infty} (1-x)^n$$

# MATH DEPARTMENT

## MATH 132 TEST THREE

**TIME: 60 Min.** **JANUARY 2008**

**NAME:** George Hannuneh

**NUMBER:** 106 15 15

**SECTION:** 1

Instructor's name: Dr. Rimon Jad'on

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**QUESTION ONE: (MULTIPLE CHOICE) [40 POINTS]**

**CIRCLE THE RIGHT ANSWER:**

28
30
10
14
82

1. Let $f(x)=\sum_{n=0}^{\infty} x^n$. The interval of convergence of the definite integral 0 to x, $\int_0^x f(t)dt$ is

$\frac{x^{n+1}}{n+1}\Big|_0 = \frac{x^{x+1}}{x+1} - x$ ?

$\frac{x^{x+1} - x(x+1)}{x+1}$

$\ln|1+x|$

$-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}$

(A) $x=0$ only
(B) $|x|\le 1$
(C) $-\infty < x < \infty$
(D) ~~$-1 \le x < 1$~~
(E) $-1 < x < 1$

2. The coefficient of $x^4$ in the Maclaurin series for $f(x)=e^{-x/2}$ is

$\frac{1}{e^{\frac{x}{2}}}$ $-\frac{e^{-\frac{x}{2}}}{2}$ $-\frac{1}{2e^{\frac{x}{2}}}$

$\frac{e^{-\frac{x}{2}}}{4}$ $\frac{1}{4e^{\frac{x}{2}}}$

$\frac{(-1)^n \cdot (x)^n}{2^{n+1}\, n!}$

$\frac{1 \cdot x^4}{32 \cdot 4\times3\times2}$

(A) -1/24
(B) 1/24
(C) 1/96
(D) -1/384
(E) 1/384

3. Which of the following series diverges?
(A) $\Sigma 1/n^2$ conv
(B) $\Sigma 1/(n^2+n)$ conv.
(C) $\Sigma n/(n^3+1)$
(D) $\Sigma \frac{n}{\sqrt{(4n^2-1)}}$ $\frac{n}{n\sqrt{4-\frac{1}{n^2}}}$ $\frac{1}{4}$
(E) none of the preceding.

4. For which of the following series does the Ratio Test fail?
(A) $\Sigma 1/n!$
(B) $\Sigma n/2^n$
(C) $1 + 1/2^{3/2} + 1/3^{3/2} + 1/4^{3/2} + \ldots$
(D) $(\ln 2)/2^2 + (\ln 3)/2^3 + (\ln 4)/2^4 + \ldots$
(E) $\Sigma n^n/n!$

$\left(\frac{1}{2}\right)^{\frac{3}{2}}$ $\frac{1}{3^{\frac{3}{2}}} \times 2^{\frac{3}{2}}$ ?

$\left(\frac{2}{3}\right)^{\frac{3}{2}}$ $\left(\frac{3}{4}\right)^{\frac{3}{2}}$

$\frac{\ln 3}{2^3} \cdot \frac{2^2}{\ln 2}$ $\frac{1}{2} \cdot \frac{\ln 4}{\ln 3}$

5. Which of the following alternating series diverges?
(A) $\Sigma (-1)^{n-1}/n$
(B) $\Sigma (-1)^{n+1}(n-1)/(n+1)$
(C) $\Sigma (-1)^{n+1}/\ln(n+1)$
(D) $\sum \frac{(-1)^{n-1}}{\sqrt{n}}$
(E) $\Sigma (-1)^{n-1} n/n^2 + 1$

$2\sqrt{n}$

6. Which of the following series converges conditionally?
(A) $3 - 1 + 1/9 - 1/27 + \ldots$
(B) $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \ldots\ldots\ldots\ldots$
(C) $1/2^2 - 1/3^2 + 1/4^2 - \ldots$
(D) $1 - 1.1 + 1.21 - 1.332 + \ldots$
(E) $1/(1*2) - 1/(2*3) + 1/(3*4) - 1/(4*5) + \ldots$

$\frac{1}{3}$ $\frac{1}{9}$ $\frac{1}{3}$

7. Suppose $f(x)$ is a function with Taylor series converging to $f(x)$ for all $x \in \Re$.

If $f(0) = 2$, $f'(0) = 2$ and $f^n(0) = 3$ for $n \geq 2$ then $f(x)=$

(A) $3e^x + 2x - 1$ 2 5

(B) $e^{3x} + 2x + 1$ 2

(C) $e^{3x} - x + 1$ 2

(D) $3e^x - x - 1$ 2 0 2

(E) $3e^x + 5x + 5$ 8

8. Which of the following series converge?

(I) $\sum_{n=1}^{\infty} \frac{\ln(n^{-3})}{n^{-3}}$ (II) $\sum_{n=1}^{\infty} \frac{\ln 3}{3n}$ (III) $\sum_{n=1}^{\infty} \frac{n}{3^n}$

(A) I only (B) II only (C) III only

(D) None (E) I and III

9. What is the Taylor series for $f(x) = e^x$ about $x = 1$?

$\frac{x^n}{n!}$

(A) $\sum_{n=0}^{\infty} \frac{-(x-1)^n}{n!}$ (B) $\sum_{n=0}^{\infty} \frac{-e(x-1)^n}{n!}$ (C) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!e}$

(D) $\sum_{n=0}^{\infty} \frac{e(x-1)^n}{n!}$ (E) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!}$

$\frac{e(x-1)^n}{n!}$

10. Let $\{a_n\}$ be a sequence of positive real numbers such that

$$\frac{1}{2} \quad \frac{a_{n+1}}{a_n} \leq \frac{n+4}{2n+1} \quad \text{for all } n. \text{ Then } \lim_{n\to\infty} a_n =$$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} \leq \frac{1}{2}$

$\Rightarrow \sum a_n$ converges

$\Rightarrow \lim_{n\to\infty} a_n = 0.$

(A) 0 (B) 1/2 (C) 1

(D) 2 (E) 4

**QUESTION TWO: [30 points]**

**Test the following series for convergence or divergence. State the test you are using, and verify that it applies. Determine whether the convergence is absolute or conditional.**

(a) $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^{100}}$ $\Rightarrow$ Converges by Integral test

$$\int_3^{\infty} \frac{1}{n(\ln n)^{100}} = \frac{(\ln n)^{-99}}{-99}\Big|_3$$

$$= \frac{-1}{99(\ln n)^{99}}\Big|_3^{\infty} = \frac{-1}{\infty} + \frac{1}{99(\ln 3)^{99}} \Rightarrow \text{Integral Converges}$$

(b) $\sum_{n=0}^{\infty} \frac{n^{2n}}{(1+2n^2)^n}$ ⇒ converges by the nth Root test absol

$\lim_{n\to\infty} \sqrt[n]{\frac{n^{2n}}{(1+2n^2)^n}}$

$= \lim_{n\to\infty} \frac{n^2}{1+2n^2} = \boxed{\frac{1}{2}} < 1$ ⇒ convergs

(c) $\sum_{n=1}^{\infty} \frac{n^3-\sqrt{n}+6}{n^4-n-3}$ diverges by limit comparison test

$\lim_{n\to\infty} \frac{n^3-\sqrt{n}+6}{n^4-n-3} \div \frac{1}{n}$

$\lim_{n\to\infty} \frac{n^4-n^{\frac{3}{2}}+6n}{n^4-n-3}$

$= 1$ ⇒ ~~both~~ both diverge orconverg

$\frac{1}{n}$ diverges ( power series with $p=1$)

⇒ both diverg

30

**QUESTION THREE: [14 points]**

Consider the power series $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}}(x+3)^n$

(a) When $x = -4$ does this series converge or diverge?
(b) Determine all values for which the series converges.

$$\lim_{n\to\infty} \sqrt[n]{\frac{(-2)^n}{n^{\frac{1}{2}}}(x+3)^n}$$

$$= \frac{-2}{n^{\frac{1}{2n}}}(x+3) \quad \therefore \quad |x+3| \leq \frac{1}{2} \Rightarrow R = \frac{1}{2}$$

$$-\frac{1}{2} < x+3 < \frac{1}{2}$$
$$-4 < x < -2$$

when $x = -4$

$$\sum \frac{(-2)^n (-1)^n}{\sqrt{n}}$$

$$\sum \frac{2^n}{\sqrt{n}}$$

$$\lim_{n\to\infty} \frac{2^n}{\sqrt{n}} \div \frac{1}{\sqrt{n}}$$

$$\lim_{n\to\infty} 2^n$$
$$= \infty$$

both diverg by L.C.T

when $x = -2$

$$\sum \frac{(-2)^n}{\sqrt{n}}$$

Converges conditionally b A.S.T

$\Rightarrow$ the series converges on the interval $(-4, -2]$

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**QUESTION FOUR: [16 points]**

Consider the integral $\int x\cos(x^3)dx$.

(a) Write down the Maclaurin series for cos(x), $\cos(x^3)$, and $x\cos(x^3)$.

(b) Evaluate $\int_0^1 x\cos(x^3)dx$ as an infinite series.

$\cos(x) = \frac{(-1)^n x^{2n}}{2n!}$

$\cos(x^3) = \frac{(-1)^n x^{6n}}{2n!}$

Maclaurin

$\cos(x^3) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$

$\cos(x^3) = 1 + \frac{x^6}{2!} + \frac{x^{12}}{4!} + \frac{x^{18}}{6!}$

$x\cos(x^3) = x + \frac{x^7}{2!} + \frac{x^{13}}{4!} + \frac{x^{19}}{6!} + \dots$

$\int_0^1 x\cos(x^3) = x + \frac{x^7}{2!} + \cancel{\frac{x^{13}}{4!}} + \cancel{\frac{x^{19}}{6!}} + \cancel{\frac{x^{25}}{8!}} + \cancel{R}$

$\int_0^1 x\cos(x^3) = \frac{2x + x^7}{2}\Big|_0^1$ ~~R ≤ $a_5$~~

$\int_0^1 x\cos(x^3) = \boxed{\cancel{\frac{3}{2}}}$

14