91/100

Birzeit University

Department of Mathematics

Second Hour Exam Math 132. Summer 2013

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Q#1 (68%)circle the correct answer.

1) The series $\sum_{1}^{\infty}\frac{1}{\sqrt{n(n+1)(n+2)}}$ $\frac{1}{\sqrt{n^3}}$

$n^2+2n+n+2$

$\sqrt{n^3+3n^2+3n}$

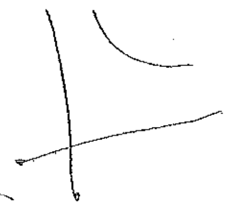
$n^{3/2}\sqrt{n^3(1+\frac{3}{n}+\frac{3}{n^2})}$

a) Converges by Limit comparison test with $\sum_{1}^{\infty}\frac{1}{\sqrt{n^3}}$ div

b) Converges by direct comparison test with $\sum_{1}^{\infty}\frac{1}{\sqrt[5]{n^4}}$ conv

c) Converges by direct comparison test with $\sum_{1}^{\infty}\frac{1}{\sqrt[4]{n^3}}$ conv $\frac{1}{n^{3/4}}$

d) Diverges by the ratio test



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$(n^2+n)(n+2)$

$n^3+2n^2+n^2+2n$

n^3+3n^2+2n

2) The series $\sum_{1}^{\infty}(\log_2 x)^n$ converges if

a) $x\in(e^{-1},e)$

b) $x\in(\frac{1}{2},2)$

c) $x\in(-\frac{1}{2},\frac{1}{2})$

d) $x\in(-1,1)$

$\frac{\ln|x|}{\ln 2}<1$

$-1<\frac{\ln x}{\ln 2}<1$

$-\ln 2<\ln x<\ln 2$

$\frac{1}{2}<x<2$

$\frac{n+1}{(n+1)^2+1}\cdot\frac{(n^2+1)}{n}$

3.The series $\sum_{n=1}^{\infty}(1+\frac{1}{n})^n$ $e^n\neq 0$

a) diverges by nth term test.

b) diverges by nth root test

c) converges by nth root test

d) converges by nth term test

$\frac{n^{1/n}}{n^{3/n}+1^{1/n}}$

$\frac{n^2}{n^2+1}$

$\frac{-1}{2}+\frac{-2}{5}+\frac{-3}{10}+\frac{-4}{17}+\frac{-5}{26}$

$\lim \frac{n+\frac{1}{n}}{n}=1$ no conclusion

$n^x \,?\, x^n$

$\frac{n^{2/n}}{e} = \frac{1}{e}$ $\quad \frac{1}{2.3} < 1$

4. The series $\sum_{1}^{\infty} \frac{n^2}{e^n}$

$\frac{(n+1)^2}{e^{n+1}} \cdot \frac{e^n}{n^2}$ $\quad \frac{2n}{e^n} \Rightarrow \frac{2}{e^n} \Rightarrow$

a) Converges By n^{th} term Test ✗
b) Converges By Ratio Test
c) Diverges By Integral Test
d) Diverges by Alternating Series theorem ✗

$\frac{(n+1)^2}{e^{n+1}} \cdot \frac{e^n}{n^2}$ $\quad \frac{e^n}{e^{n+1}} \times \frac{(n+1)^2}{n^2} = \frac{n^2+2n+1}{n^2 e}$

5. Consider $\sum_{1}^{\infty} a_n$ Where $a_n \geq 0$ Then

a) If $\lim_{x\to\infty} a_n = 0$ then $\sum_{1}^{\infty} a_n$ converges. $\frac{1}{n}$ $\quad \frac{1}{e}$

b) If $\sum_{1}^{\infty} a_n$ diverges then $\lim_{x\to\infty} a_n \neq 0$

c) If $\lim_{x\to\infty} a_n \neq 0$ then $\sum_{1}^{\infty} a_n$ diverges

d) If $\sum_{1}^{\infty} a_n$ converges then $\lim_{x\to\infty} a_n \neq 0$ $=0$

$\int_1^{\infty} e^{-n} x^2$

$\lim_{a\to\infty}$

$\frac{3^n}{2^n}$ $\quad \frac{3^n}{2^n+5^n} < \frac{3^n}{5^n} < \frac{3^n}{7^n}$

6) The series $\sum_{0}^{\infty} \frac{3^n}{2^n + 5^n}$

a) Converges by direct comparison test with $\sum_{1}^{\infty} \frac{3^n}{7^n}$ conv.

b) Converges by direct comparison test with $\sum_{1}^{\infty} \frac{3^n}{4^n}$ conv.

c) Converges by direct comparison test with $\sum_{1}^{\infty} \frac{1}{2^n}$

d) Diverges

$\frac{\sin}{\cos}$

$u = \cos$
$du = -\sin x \, dx$

7. $\int_{0}^{\frac{\pi}{2}} \tan x dx =$ $\quad \lim_{a\to\pi/2} \int_0^a \tan x$

$-\int \frac{\sin}{u} \frac{du}{-\sin x}$

$\ln|\sec a| - \cancel{\ln|\sec 0|}$

$-\ln u \Rightarrow -\ln\cos$
$\ln|\sec x|$

a) 0
b) -1
c) ∞
d) -∞

8) The series $\sum_{2}^{\infty}\frac{(n+1)\ln n}{\sqrt{n}}$

$a)$ Converges by the integral test

$b)$ Converges by direct comparison test with $\sum_{2}^{\infty}\frac{1}{\sqrt[5]{n^4}}$

$c)$ Diverges by the nth term test

$d)$ Diverges by the ratio test

9. Which of the following sequences diverges?

(a) $\{\frac{n}{1+\ln n}\}$

(b) c) $\{n^2/e^n\}$

(d) $\{(-1)^{n+1}/n\}$

(c) $\{\sqrt[n]{10n}\}$

10. Which of the following series converges conditionally?

(a) 3 - 1 + 1/9 - 1/27 + ...

(b) $\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}-\frac{1}{\sqrt{5}}+\dots$

(c) $1/2^2 - 1/3^2 + 1/4^2 - \dots$

(d) $\frac{1}{1\times 2}-\frac{1}{2\times 3}+\frac{1}{3\times 4}-\frac{1}{4\times 5}+\dots$

11) The Integral $\int_{2}^{\infty}\frac{dx}{\sqrt[3]{x}(\sqrt{x}-1)}$

$a)$ Converges by limit comparison test with $\int_{2}^{\infty}\frac{dx}{\sqrt[3]{x}}$

$b)$ Converges by limit comparison test with $\int_{2}^{\infty}\frac{dx}{\sqrt{x}}$

$c)$ Diverge by direct comparison test with $\int_{2}^{\infty}\frac{dx}{\sqrt[3]{x^2}}$

$d)$ Diverge by direct comparison test with $\int_{2}^{\infty}\frac{dx}{\sqrt[6]{x^5}}$

$$\frac{\cancel{n^2}\left(\frac{2}{n}-\frac{1}{n^2}\right)}{\cancel{n^2}(n-1)^2}$$

12. The series $\sum_{n=2}^{\infty}\frac{2n-1}{n^2(n-1)^2}$

0

a) 1

[illegible]1

c) $\frac{1}{4}$

d) 2

13. The Series $\sum_{1}^{\infty}\frac{2^n-4}{3^n}$

$\left(\frac{2}{3}\right)^n - \frac{4}{3^n}$

a) Converges to 1

b) Converges to $\frac{3}{4}$

$\frac{\frac{2}{3}}{1-\frac{2}{3}} - 4\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)$

c) Converges to 0

d) Converges to $\frac{3}{2}$

$\left(\frac{2}{\cancel{3}}\cdot\frac{\cancel{3}}{1}\right) - 4\left(\frac{1}{\cancel{3}}\cdot\frac{\cancel{3}}{2}\right)$ $\quad 2-2 \Rightarrow 0$

14. Assuming its convergence,find the limit of the following recursively sequence, $a_1 = 8$ and $a_{n+1} = \sqrt{a_n + 8} - 2$

a) 1

$a_1 = 8,\ 2,$

b) -4

c) -2

$\sqrt{10}-2,\ \sqrt{\sqrt{10}-6}$

d) 8

$\frac{\ln(n)}{n^3} \Rightarrow \frac{\frac{1}{n}}{3n^2} \Rightarrow \frac{1}{n}\cdot\frac{1}{3n^2} \Rightarrow \frac{1}{3n^3} \Rightarrow 0$

15. $\sum_{1}^{\infty}(-1)^n\frac{\ln(n)}{n^3}$

$\frac{1}{n^3}$ $\quad \ln(n) \Rightarrow \infty$

a) Is ageometric series

b) Converges conditionaly

c) Converges abslutely

d) Diverges

$\frac{7}{24} + \frac{1}{4} = \frac{9}{12} - \frac{2}{12}$

$\frac{7}{12}$

16) The Series $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{6}+\cdots$

a) Convrges to L Where $\frac{7}{12} \le L \le \frac{3}{4}$

b) Convrges to L Where $\frac{3}{4} \le L \le \frac{11}{12}$

c) Convrges to L Where $\frac{1}{4} \le L \le \frac{1}{2}$

d) Diverges

$\boxed{\begin{aligned} u &= \tan^{-1} x \\ du &= \frac{dx}{1+x^2} \end{aligned}}$

$\frac{\sin}{\cos}$

17. If $\{s_n\} = \{(-1)^n(\frac{n+1}{n})\}$,then

$\frac{1+\frac{1}{n}}{1} \Rightarrow 1$

(a) $\{s_n\}$ diverges

(b) $\{s_n\}$ converges to zero

(c) $\{s_n\}$ converges to e^{-1}

(d) $\{s_n\}$ converges to 1

1.(15%) Test for Convergence

a) $\int_0^\infty \frac{\tan^{-1} x}{1+x^2} dx$

$u = \tan^{-1} x$
$du = \frac{1}{1+x^2} dx$

$$\lim_{a\to\infty} \int_0^a \frac{\tan^{-1} x}{1+x^2} dx = \lim_{a\to\infty} \int_0^a \frac{u \; du \cancel{(1+x^2)}}{\cancel{1+x^2}} = \lim_{a\to\infty} \int_0 u\,du$$

$$= \lim_{a\to\infty} \frac{u^2}{2}$$

$$= \lim_{a\to\infty} \frac{(\tan^{-1} x)^2}{2}\Big|_0^a = \frac{(\tan^{-1} a)^2}{2} - 0$$

$$= \frac{\left(\frac{\pi}{2}\right)^2}{2} = \frac{\frac{\pi^2}{4}}{2}$$

$$= \frac{\pi^2}{4} \cdot \frac{1}{2} = \boxed{\frac{\pi^2}{8}}$$

$\boxed{\text{Converges}}$

$\frac{\cancel{n}(\frac{1}{n})}{\cancel{n}(\frac{2}{n}+1)}$

0

) $\int_0^\infty \frac{dx}{\sqrt{x^6+1}}$

$$\int_0^\infty \frac{dx}{\sqrt{x^6(1+\frac{1}{x^6})}} = \int_0^\infty \frac{dx}{x^3\sqrt{(1+\frac{1}{x^6})}} = \boxed{\int_0^1 \frac{dx}{x^3(\sqrt{1+\frac{1}{x^6}})} + \int_1^\infty \frac{dx}{x^3\sqrt{1+\frac{1}{x^6}}}}$$

$$\frac{1}{x^3\sqrt{1+\frac{1}{x^6}}} \div \frac{1}{x^3} \Rightarrow \frac{\cancel{x^3}}{\cancel{x^3}\sqrt{1+\frac{1}{x^6}}} = \frac{1}{\sqrt{1+\frac{1}{x^6}}}$$

$$\lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x^6}}} = 1$$

so by the Limit comparison test with $\frac{1}{x^3}$ both converge or both diverge $\Rightarrow$ $\int_0^1 \frac{dx}{x^3}$ $\Rightarrow$ diverge - P - Test

so $\boxed{\text{diverge}}$

$\sum_1^\infty (\frac{1}{2+n})^n$

by applying the root test $\sqrt[n]{\left(\frac{1}{2+n}\right)^n} = \frac{1}{2+n}$

$$\lim_{n\to\infty} \frac{1}{2+n} = \frac{\cancel{n}(\frac{1}{n})}{\cancel{n}(\frac{2}{n}+1)} = \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{2}{n}+1} = \frac{0}{1}$$

$\rho = 0 < 1$ so the series converges by the root test

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$$\frac{(n+1)\,x^{\cancel{n+1}}}{4^{\cancel{n+1}}((n+1)^2+1)} \qquad \frac{4^{\cancel{n}}(\;)}{n\cancel{x^n}} \qquad \frac{x\,\cancel{(n+1)}\,(n^2+1)}{n\,4\,(n^2+1)^{\cancel{2}}+4n}$$

$$\frac{x n^2 + x}{4n^2+4n+4n}$$

3. (17%) Consider the power series $\sum_{1}^{\infty} \frac{nx^n}{4^n(n^2+1)}$ Find

a) Interval and radius of convergence

b)For what values of x does the series converges

1) absolutely   2) coditionally   3) diverge

by the ratio test :-

$$\frac{(n+1)\,(x)^{\cancel{n+1}}}{(4)^{\cancel{n+1}}((n+1)^2+1)} \cdot \frac{\cancel{4^n}(n^2+1)}{n\,\cancel{x^n}} = |x|\,\frac{(n+1)\,(n^2+1)}{n4\,(n+1)^2+4n}$$

$$\lim_{n\to\infty} \frac{|x|\,(n+1)\,(n^2+1)}{4n\,(n+1)^2+4n} = \frac{|x|}{4} = \rho$$

$$\frac{|x|}{4} < 1 \Rightarrow -1 < \frac{|x|}{4} < 1 \Rightarrow -4 < |x| < 4$$

$\boxed{\text{Radius} = 4}$ Converges abs $\text{when } x = (-4, 4]$

div whe $x = (-\infty, -4)\ (4, \infty)$

Test for intervals begining and endings

when $x = 4$

$$\sum_{1}^{\infty} \frac{n\,\cancel{4^n}}{\cancel{4^n}(n^2+1)} = \frac{n}{n^2+1}$$

L.C.T with $\frac{1}{n} \Rightarrow \frac{n}{n^2+1} \cdot n = \frac{n^2}{n^2+1} \Rightarrow \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$

So both converg or both diverge

and $\frac{1}{n}$ diverges $\Rightarrow$ p-Test

so the series diverges when $x = 4$

When $x = -4 \Rightarrow \sum_{1}^{\infty} \frac{-n\,\cancel{4^n}}{\cancel{4^n}(n^2+1)}$

$= \frac{-n}{n^2+1}$ by the root test

$$\lim_{n\to\infty} \frac{-n^{\frac{1}{n}}}{(n^2)^{\frac{1}{n}}+1^{\frac{1}{n}}} = \frac{-\cancel{1}}{\cancel{1}+1}$$

$= -\frac{1}{2}$ So converges by the root test when $\boxed{x=-4}$

because $\rho < 1$

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