Second

Birzeit University- Mathematics Department
Calculus II-Math 132

Spring 2011/2012

econd Exam [redacted]

ame(Arabic):...[redacted]............ Number:..[redacted]...

ıstructor of Discussion(Arabic):.............[illegible]..... Section:...14...........

'ime: 90 Min. Calculators are not allowed. There are 4 questions in 7 pages.

Question 1.(51%) Circle the correct answer:

1. The sequence $a_n = \left(1+\frac{1}{n}\right)^{-n}$, $n = 1, 2, 3, \ldots$

$(1-\frac{1}{n})(1+\frac{1}{n})$??

$\frac{1}{(1+\frac{1}{n})^n} = \frac{1}{e^{-1}} = e$

$\frac{1}{(1+\frac{1}{n})^n}$ $\frac{1}{e}$ e^{-1}

33
08
02
09
52

(a) Converges to 1.
(b) Converges to e.
(c) Converges to e^{-1}.
(d) Diverges.

2. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

$\left|\frac{(-1)^{n+1}}{\sqrt{n}}\right| = \frac{1}{\sqrt{n}}$ div by P-series

(a) Converges conditionally.
(b) Converges absolutely.
(c) Converges by nth term test.
(d) Diverges.

$\frac{(-1)^{n+1}}{\sqrt{n}} \to u_n$ decreas positive

$\lim \frac{1}{\sqrt{n}} = 0$

$\frac{1}{\sqrt{n}} \to$

3. $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$

$> \sqrt{n}$

$n + \sqrt{n} > n$

(a) Diverges by nth term test.
(b) Converges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{2n}$ div
(c) Diverges by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}}$.
(d) Diverges by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{2n}$ div

$\frac{1}{n+\sqrt{n}} = \frac{1}{\infty+\infty} = \frac{1}{\infty} = 0$

$n+\sqrt{n} > n$

$\frac{1}{n+\sqrt{n}} < \frac{1}{n}$ div

$\frac{n}{1+\sqrt{n}} = 1$

4. $\sum_{n=1}^{\infty} \left(\frac{2n+1}{3n+1}\right)^n$

(a) Converges by nth root test.
(b) Diverges by nth root test.
(c) Converges by alternating series test.
(d) None of the above.

$\frac{2n+1}{3n+1} = \frac{2}{3}$ conv.

second

Birzeit University- Mathematics Department
Calculus II-Math 132

Second Exam | Spring 2011/2012

Name(Arabic):..[redacted]..........

Number:..[redacted]...

Instructor of Discussion(Arabic):............[illegible].....

Section:...14...........

Time: 90 Min. Calculators are not allowed. There are 4 questions in 7 pages.

33
8
2
9
52

Question 1.(51%) Circle the correct answer:

1. The sequence $a_n = (1+\frac{1}{n})^{-n}$, $n = 1, 2, 3, \dots$

$\frac{1}{(1+\frac{1}{n})^n} = \frac{1}{e^{-1}} = e$

   (a) Converges to 1.
   (b) Converges to e.
   (c) Converges to e^{-1}.
   (d) Diverges.

2. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$

$\left|\frac{(-1)^{n+1}}{\sqrt{n}}\right| = \frac{1}{\sqrt{n}}$ div by P- series

$\frac{(-1)^{n+1}}{\sqrt{n}} \rightarrow u_n$ decreas positive

$\lim \frac{1}{\sqrt{n}} = 0$

   (a) Converges conditionally.
   (b) Converges absolutely.
   (c) Converges by nth term test.
   (d) Diverges.

3. $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$

$> \sqrt{n}$

$n + \sqrt{n} > n$

$\frac{1}{n+\sqrt{n}} = \frac{1}{\infty+\infty} = \frac{1}{\infty} = 0$

   (a) Diverges by nth term test.
   (b) Converges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{2n}$.
   (c) Diverges by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}}$.
   (d) Diverges by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{2n}$.

4. $\sum_{n=1}^{\infty} \left(\frac{2n+1}{3n+1}\right)^n$

   (a) Converges by nth root test.
   (b) Diverges by nth root test.
   (c) Converges by alternating series test.
   (d) None of the above.

5. One of the following statements is always true $(-1, 1, -1, 1)$

(a) A bounded sequence always converges. X

(b) A monotonic sequence converges. X ~~monotonic + bounded~~ $a_n = n$

(c) A convergent sequence is monotonic. X $(1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8} \cdots)$

(d) A convergent sequence is bounded.

6. The series $\sum_{n=1}^{\infty} \frac{1}{n^{1/n}}$ $\lim \frac{1}{\sqrt[n]{n}} = \frac{1}{1} = 1$

(a) Converges by nth term test.

(b) Converges by nth root test.

(c) Diverges by nth term test.

(d) Diverges by nth root test.

7. The series $\sum_{n=1}^{\infty} (\log_2 x)^n$ converges if

$\left(\frac{\ln x}{\ln 2}\right)^n = \left|\frac{\ln x}{\ln 2}\right| =$

$-1 < \frac{\ln x}{\ln 2} < 1$

$-\ln 2 < \ln x < \ln 2$

$e^{-\ln 2} < x < e^{\ln 2}$

(a) $x \in (e^{-1}, e)$.

(b) $x \in (\frac{1}{2}, 2)$.

(c) $x \in (-\frac{1}{2}, \frac{1}{2})$.

(d) $x \in (-1, 1)$.

8. One of the following statements is true

(a) If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ converges. X

(b) If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ then $\sum a_n$ and $\sum b_n$ both converge or both diverge. X

(c) The alternating harmonic series diverges. X

(d) If $\lim_{n\to\infty} a_n = 1$ then $\sum_{n=1}^{\infty} a_n$ diverges. ✓ nth term test

9. The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{x^n}{2^n}$ is $\left(\frac{x}{2}\right)^n$ $\left|\frac{x}{2}\right| < 1$

$\sqrt[n]{\left(\frac{x}{2}\right)^n}$

(a) $R = 1$.

(b) $R = 2$.

(c) $R = 0$.

(d) $R = \infty$.

$-1 < \frac{x}{2} < 1$

$-2 < x < 2$

center zero

10. One of the following series converges to 1

(a) $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$. $\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}$ $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ $\frac{\frac{1}{2}}{1-\frac{1}{2}}$

(b) $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$. converge to 2

(c) $\sum_{n=1}^{\infty} \left(\frac{-1}{2}\right)^n$. $\sum_{n=0}^{\infty}\left(\frac{-1}{2}\right)^{n+1}$

(d) $\sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n$. converge to 2

$\frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{2}{2}-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = (2)\left(\frac{1}{2}\right) = 1$

$\frac{1}{\frac{2}{2}-\frac{1}{2}}$ $\frac{1}{\frac{1}{2}} = 2 \cdot \frac{1}{2}$

conv or Div

11. The series $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$ $\lim \ln\left(\frac{n}{n+1}\right)$ $\sum_{1}^{\infty} \ln\left(\frac{n}{n+1}\right)$

(a) Converges to 1.

(b) Converges to $\ln\left(\frac{1}{2}\right)$.

(c) Converges to 0.

(d) Diverges. ←

$\ln(n) - \ln(n+1)$

$\lim \ln(n) - \ln(n+1)$

$(\ln(1) - \ln(2)) + (\ln(2) - \ln(3)) \ldots (\ln(n) - \ln(n)) + (\ln(n) - \ln(n+1))$

$\sum \ln(n+1) = \infty$ Div

12. $1 + \ln 2 + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} + \cdots + \frac{(\ln 2)^n}{n!} + \cdots =$ $\frac{(\ln 2)^n}{n!} = \frac{x^n}{n!}$

(a) $\ln 2$.

(b) $\frac{1}{1-\ln 2}$.

(c) 2

(d) The series diverges.

$\frac{x^n}{n!} = e^x$ $e^{\ln 2} = 2$

$\frac{(\ln 2)^n}{n!}$ $\frac{x^n}{n!} = e^x$ $x<1$ $= e^{\ln 2} = 2$

13. The sequence $a_n = n(2^{1/n} - 1)$, $n = 1, 2, 3, \ldots$

(a) Converges to 0.

(b) Converges to 1.

(c) Converges to $\ln 2$.

(d) Diverges.

$\lim n\sqrt[n]{2} - n$

$n(\sqrt[n]{2} - 1)$ $\lim \frac{\sqrt[n]{2}}{\frac{1}{n}} - \frac{1}{\frac{1}{n}}$

$\lim \frac{\sqrt[n]{2}}{\frac{1}{n}} - \lim \frac{1}{\frac{1}{n}}$

$\lim n(\sqrt[n]{2} - 1)$ $\frac{e^{n\sqrt[n]{2}}}{e^n}$

$x^{\frac{1}{n}}$

$n(\sqrt[n]{2} - 1)$

$\lim m \times \lim 0$

$$\frac{(n+2)!}{(2n+1)!} \cdot \frac{\cancel{2n!}}{\cancel{(n+1)!}}$$

14. The series $\sum_{n=1}^{\infty} \frac{(n+1)!}{(2n)!}$

(a) Converges by ratio test.
(b) Diverges by ratio test.
(c) Diverges by nth term test.
(d) None of the above.

$$\lim \frac{n+2}{2n+1} = \frac{1}{2} < 1$$

$$\frac{(n+2)!}{(2n+2)!} \cdot \frac{(2n)!}{(n+1)!}$$

$$\frac{(n+2)}{(2n+2)(2n+1)} \to 0 < 1 \quad \text{conv}$$

15. The series $\sum_{n=2}^{\infty} (-1)^n \frac{\ln(n)}{n}$

(a) Converges by nth term test.
(b) Diverges by nth term test.
(c) Converges absolutely.
(d) Converges by alternating series test.

$$\frac{\ln n}{n} = \frac{\frac{1}{n}}{1}$$

LCT with $\ln(n)$

div $\frac{\ln(n)}{n} \to 0$

$$\frac{1-\ln(n)}{n^2} \qquad 1-\ln(n) = 0 \qquad n = e \qquad \text{dec}$$

16. The series $\sum_{n=2}^{\infty} \frac{n}{\sqrt{n^4-1}}$

(a) Converges by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^3}$.
(b) Diverges by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$.
(c) Converges by direct comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^3}$.
(d) Diverges by nth term test.

$$\frac{n}{n^2} = \frac{1}{n}$$

$$\frac{n^2}{n^2-1}$$

$$\frac{n}{\sqrt{n^4-1}} > \frac{n}{n^2}$$

17. We can approximate e^{-x} by $1-x$ with error less than 0.02 when

(a) $|x| < 0.4$.
(b) $|x| < 0.01$.
(c) $|x| < 0.02$.
(d) $|x| < 0.2$.

$$|1-x| < 0.02$$

$$e^{-x} = 1 - x + \frac{x^2}{2}$$

$$\text{error} < \frac{x^2}{2}$$

$$\frac{x^2}{2} \le 0.02$$

$$x^2 \le 0.04$$

$$\le 0.2$$

Question 2(18%) Find the radius and interval of convergence of the series

$$\sum_{n=2}^{\infty} \frac{(x-1)^n}{\sqrt{n}\ln n}$$

When does the series converge conditionally, absolutely, diverge?

$$\left| \frac{(X-1)^{n+1}}{\sqrt{n+1}\,\ln(n+1)} \cdot \frac{\sqrt{n}\,\ln n}{\cancel{(X-1)^n}} \right| = \left| (X-1) \cdot \underbrace{\sqrt{\frac{n}{n+1}}}_{\lim = 1} \cdot \frac{\ln n}{\ln(n+1)} \right|$$

$$= |X-1| \lim_{n\to\infty} \left| 1 \cdot \underbrace{[\ln n - n + 1]}_{} \right| = |X-1| \underbrace{\lim_{n\to\infty} |\ln 1|}_{\text{zero}}$$

$|X-1| < 1$

$-1 < X-1 < 1$

2 $\boxed{0 < X < 2}$ 5

at $X = 0 \rightarrow \sum \frac{(-1)^n}{\sqrt{n}\ln n}$, ~~Alternating Series~~ diverge by nth term test

at $X = 2 \rightarrow \sum \frac{(1)^n}{\sqrt{n}\ln n} = \sum \frac{1}{\sqrt{n}\ln n}$

diverge by nth term test

1

radius $= \frac{2+0}{2} = 1$ 1

1

Absolutly converge $x \in (0, 2)$

1

diverge $X \in (-\infty, 0] \cup [2, \infty)$

$\ln n \sqrt{n} < \frac{1}{\ln n}$ $\quad \frac{1}{\infty} = 0$

$> \ln n$

$\ln n \sqrt{n} > \sqrt{n}$

$$\frac{\frac{1}{\ln n \sqrt{n}}}{\frac{1}{\sqrt{n}}}$$

$\frac{1}{\ln n \sqrt{n}} \cdot \sqrt{n}$

$\lim_{n\to\infty} \frac{1}{\ln n} = 0$

$\sqrt{n}$

$\frac{1}{2\sqrt{\ }}$

$\lim \frac{\frac{1}{\sqrt{n}}}{\ln n}$

$\frac{\infty}{\infty}$ $\quad \frac{\frac{1}{\sqrt{n}}}{\ln n} = \frac{2\sqrt{n}}{\frac{1}{n}}$

$= 2\sqrt{n} \cdot \sqrt{n^2}$

$= 2\sqrt{n \cdot n^2}$

$= \lim_{n\to\infty} \sqrt{n^3} = \infty$ div by nth term test

Question 3(15%) Determine whether the series converge or diverge, justify your answer:

(a) $\sum_{n=1}^{\infty} \frac{3^n}{3^n+4^n}$

$$\frac{3}{3^{n+1}+4^{n+1}} \cdot \frac{3^n+4^n}{3^n} = 3\left(\frac{3^n+4^n}{3^{n+1}+4^{n+1}}\right)$$

Ratio

$$= \frac{3^n}{3^{n+1}+4^{n+1}} + \frac{4^n}{3^{n+1}+4^{n+1}}$$

$$= \frac{1}{3+\frac{4^{n+1}}{3^n}} + \frac{1}{\frac{3^{n+1}}{4^n}+4} = \frac{1}{3+\left(\frac{4}{3}\right)^n 4} + \frac{1}{3\left(\frac{3}{4}\right)^n+4}$$

$$\lim_{n\to\infty} \frac{1}{3+\left(\frac{4}{3}\right)^n 4} + \lim \frac{1}{3\left(\frac{3}{4}\right)^n+4} = \frac{1}{3+0} + \frac{1}{0+4} = \frac{4}{12}+\frac{3}{12} = \frac{7}{12} < 1$$ converg

by Ratio test

(b) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$ $\quad \operatorname{Ln} a_n = n^2 \operatorname{Ln}\left(\frac{n}{n+1}\right)$

$$(n+1)^2 \operatorname{Ln}\left(\frac{n+1}{n+2}\right) \cdot \frac{1}{n^2 \operatorname{Ln}\left(\frac{n}{n+1}\right)} = \frac{(\cancel{n^2}+2n+1).\left[\operatorname{Ln}(n+1) - \operatorname{Ln}(n+2)\right]}{\cancel{n^2}\left[(\operatorname{Ln} n) - (\operatorname{Ln} n+1)\right]} = (2n+1)\frac{\operatorname{Ln}\frac{n+1}{n+2}}{\operatorname{Ln}\frac{n}{n+1}}$$

$$= 2n+1\left[\operatorname{Ln}\frac{n+1}{n+2} - \operatorname{Ln}\frac{n}{n+1}\right] = \frac{\operatorname{Ln}\frac{n+1}{n+2}}{\frac{1}{2n+1}} - \frac{\operatorname{Ln}\frac{n}{n+1}}{\frac{1}{2n+1}} = \lim \frac{\infty}{\infty} - \frac{\infty}{\infty}$$

$$= \frac{\frac{n+2}{n+1}\left(\frac{1}{(n+2)^2}\right)}{\frac{1}{2}} - \frac{\frac{n+1}{n}\left(\frac{n+1-n}{(n+1)^2}\right)}{\frac{1}{2}} = \frac{2}{n^2+3n+2} - \frac{2}{n^2+n} = 2$$

$\infty - \infty$, $\frac{-\infty}{e} = c$

إكمال الحل خطأ الصفحة

$a_n = e^2 < 1$

converge by Ratio

(c) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

$$\frac{\frac{1}{n \ln n}}{\frac{1}{n}} = \frac{1}{n \ln n} \cdot n = \frac{1}{\ln n} \quad \lim_{n\to\infty} = \frac{1}{\infty} = \text{zero}$$

$$= \frac{1}{\operatorname{Ln} n^n} = -\operatorname{Ln}(n^n) = -n \operatorname{Ln} n$$

$$= -\frac{\operatorname{Ln} n}{\frac{1}{n}} \overset{\text{L'Hôpital}}{=} \cancel{-}\frac{\frac{1}{n}}{\cancel{-}\frac{1}{n^2}} = \frac{1}{n} \cdot n^2 = n \lim_{n\to\infty} \infty$$

diverge by nth term test

$-(1-x)^{-2}$

Question 4(16%) In this question, you can use $(1-x)^{-1} = \sum_{n=0}^{\infty} x^n$, $|x| < 1$.

(a) Use substitution to show that $\frac{2x}{1+x^2} = \sum_{n=0}^{\infty}(-1)^n 2x^{2n+1}$, $\quad |x| < 1$

$\cancel{2x\left(\frac{-1}{1+x^2}\right) = -2x\sum_{n=0}^{\infty}(-1)^n\, n\, x^{n-1}}$

$f(0) = 2$

$\cancel{\sum_{n=0}^{\infty}(-1)^n 2x\, x^{n-1} = \sum_{n=0}^{\infty}(-1)^n 2x^n}$

$f'(0) = -2x$

4

$(2x)\frac{1}{1-(-x^2)} = 2x\sum_{n=0}^{\infty}(-x^2)^n = 2x\sum_{n=0}^{\infty}(-1)^n(x^{2n})$

$= \sum_{n=0}^{\infty}(-1)^n 2x\cdot x^{2n} = \sum_{n=0}^{\infty}(-1)^n 2x^{2n+1}, \quad |x|<1$

(b) Use (a) to find the Maclaurin series of $\ln(1+x^2)$. $\quad f'(x) = \frac{2x}{1+x^2}$

$\frac{2x}{1+x^2}$ is the first derivative of $\ln(1+x^2)$ ∴

$\frac{2x}{1+x^2} = \sum_{n=0}^{\infty}(-1)^n 2x^{2n+1}$ (by integrat~~ing term by term~~)

$\ln(1+x^2) = \sum_{n=0}^{\infty}(-1)^n\frac{2x^{2n+2}}{2n+2} = \sum_{n=0}^{\infty}(-1)^n\frac{\cancel{2}x^{2n+2}}{\cancel{2}(n+1)} = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+2}}{n+1} + C$

5

(c) Estimate $\ln(2)$ with error less than 0.1.

$\int\frac{1}{1+x} = 1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+x^4$

$\ln x \quad \frac{1}{x}$

$\ln(1+x)$

$\ln x$

$\ln x = \frac{1}{x} = \cancel{-\frac{1}{x^2}+1}$

0

92

Birzeit University- Mathematics Department
Calculus II-Math 132

Second Exam
Name(Arabic):[illegible]
Instructor of Discussion(Arabic):[illegible]

Spring 2012/2013
Number: 1102234
Section: 8:00

Time: 80 Minutes **There are 4 questions in 7 pages.**

Question 1.(54%) Circle the correct answer:

1. The sequence $a_n = \sqrt{n+1} - \sqrt{n}$
   (a) Diverges.
   (b) Converges to 0.
   (c) Converges to 1.
   (d) None.

2. The series $\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$
   (a) Diverges by nth term test.
   (b) Converges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{n^3}$.
   (c) Converges by direct comparison with $\sum_{n=2}^{\infty} \frac{1}{n^3}$.
   (d) Converges by direct comparison with $\sum_{n=2}^{\infty} \frac{1}{n^2}$.

3. The series $\sum_{n=1}^{\infty} \frac{n}{n+1}$
   (a) Converges to 1.
   (b) Converges by ratio test.
   (c) Diverges by ratio test.
   (d) Diverges by nth term test.

4. $\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{2}{3}\right)^n =$
   (a) $\frac{3}{5}$.
   (b) $\frac{2}{3}$.
   (c) $\frac{2}{5}$.
   (d) Diverges by alternating series test.

$\int \frac{}{e^x+e^{-x}}$ $\qquad \frac{1}{e^x(1+\frac{1}{e^{2x}})}$

5. The series $\sum_{n=1}^{\infty}\left(\frac{1}{e^n+e^{-n}}\right)$

(a) Converges to 1. $\qquad e^n+e^{-n} > e^n$

(b) Is a geometric series. $\qquad \frac{1}{e^n+e^{-n}} < \frac{1}{e^n}$ con

(c) Converges by integral test.

(d) Diverges by nth term test. X

6. The series $\sum_{n=0}^{\infty} \frac{3^n}{2^n+5^n}$ $< \frac{3^n}{5^n}$

$e^x = du$

$1+e^{-2x} = 0$

$-2e^{-2x} = du$

(a) Diverges by direct comparison with $\sum_{n=0}^{\infty}\left(\frac{3}{2}\right)^n$. X

(b) Diverges by nth term test.

(c) Converges by direct comparison with $\sum_{n=0}^{\infty}\left(\frac{3}{5}\right)^n$.

(d) Converges by direct comparison with $\sum_{n=0}^{\infty}\left(\frac{3}{2}\right)^n$. X

$\frac{3^n}{2^n+5^n} < \frac{3^n}{5^n}$

7. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ $\qquad \frac{1}{n^p}$ $\qquad \frac{(-1)^{n+1}}{(n+1)^p}\cdot\frac{n^p}{(-1)^n}$ $\qquad \frac{1}{n^p}$ $p>1$

(a) Converges absolutely if $p \geq 1$.

(b) Converges conditionally if $0 < p \leq 1$.

(c) Converges absolutely if $0 < p \leq 1$.

(d) Diverges.

8. The series $\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$

$\frac{(n+1)!}{(2n+2)!}\cdot\frac{(2n)!}{n!}$

(a) Converges by ratio test. $\qquad \frac{n+1}{(2n+1)(2n+2)} \to 0$

(b) Diverges by ratio test. $\qquad \frac{(n+1)}{(2n+2)(2n+1)} = 0 < 1$

(c) Diverges by integral **test**.

(d) Diverges by nth term test.

9. The series $\sum_{n=1}^{\infty} \frac{n^2}{n^n}$

$\lim \frac{n^2}{n^n} = 0$

(a) Diverges by nth term test.

(b) Diverges by nth root test. $\qquad \frac{\sqrt[n]{n^2} \to 1}{n \to \infty} = 0 < 1$

(c) Converges by nth root test.

(d) Converges by alternating series test. X

10. The Maclaurin series generated by the function e^{x^2} is

$f(0) = 1$

(a) $\sum_{n=0}^{\infty} \frac{x^n}{(2n)!}$.

(b) $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$.

(c) $\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$.

(d) $\sum_{n=0}^{\infty} \frac{x^{2n}}{n}$.

$f' = e^{x^2} 2x = 0$

$f'' = (e^{x^2} 2 + 4x^2 e^{x^2}) = 2$

$f''' = 4xe^{x^2} + (8x + 4e^{x^2}) = 0$

$1 + 0 + \frac{2(x)^2}{2!} + \frac{24(x)^3}{3!}$

$1 + \frac{2(x)^2}{2!} + \frac{2}{3} x^3$

$f''' = 4xe^{x^2} + (4x^2 \cdot 2x e^{x^2}$

$(8x^3 e^{x^2} + e^{x^2} 8x)$

$f'''' =$

11. One of the following improper integrals converges

(a) $\int_1^{\infty} \frac{e^x}{x} dx$.

(b) $\int_0^1 \frac{dx}{x}$. div

(c) $\int_0^1 \frac{dx}{\sqrt{x}}$. ✓

(d) $\int_2^{\infty} \frac{dx}{\ln x}$. div

$x > \ln x$

$\frac{1}{x} < \frac{1}{\ln x}$

div

$a_n < b_n$

12. If $\lim_{n \to \infty} \frac{a_n}{b_n} = 0$, $a_n, b_n > 0$ for all n then

(a) $\sum a_n$ and $\sum b_n$ both converge or both diverge.

(b) If $\sum a_n$ converges then $\sum b_n$ converges.

(c) If $\sum b_n$ converges then $\sum a_n$ converges. ✓

(d) If $\sum b_n$ diverges then $\sum a_n$ diverges.

$a_n < b_n$

conv conv

13. The sequence $a_n = (1 + \frac{1}{n})^{-n}$

(a) converges to 1.

(b) converges to e.

(c) converges to $-e$.

(d) converges to e^{-1}.

$\frac{1}{(1+\frac{1}{n})^n}$ $\frac{1}{e}$ e^{-1}

abs cond ??

14. The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{x^n}{n^n}$ is

(a) $R = 1$.

(b) $R = \infty$.

(c) $R = 0$.

(d) $R = e$.

$\left(\frac{|x|^n}{n^n} \right)^{1/n} \to \frac{|x|}{n} = 0$

conv on $(-\infty, \infty)$

$R = \infty$

$\frac{1}{1-x} = 3$

$3 - 3x = 1$

15. $\sum_{n=1}^{\infty} x^n = 2$ if $x =$ $x + x^2 + x^3 = \frac{x}{1-x} = 2$

$2 = \frac{1}{1-x}$ $2 - 2x = x$

$2 - 2x = 1$

(a) $\frac{1}{2}$.
(b) $\frac{2}{3}$.
(c) $\frac{1}{3}$.
(d) $\frac{1}{4}$.

$\frac{x}{1-x} = 1$

$1 - x = x$

$2x = 1 \rightarrow \boxed{x = \frac{1}{2}}$

16. The series $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n+1}}$

(a) Diverges by direct comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$.

(b) Diverges by nth term test.

(c) Converges by limit comparison test with $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$.

(d) Diverges by limit comparison test with $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$.

$\frac{\frac{1}{\sqrt{n+1}}}{\frac{1}{\sqrt{n}}}$ $\frac{\sqrt{n}}{\sqrt{n+1}} = 1$

S_9 or S_8

17. If we approximate the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ using the 8th partial sum s_8 then the error in this approximation

error $< S_9$

(a) is less than $\frac{1}{64}$.
(b) is less than $\frac{1}{81}$.
(c) is greater than $\frac{1}{64}$.
(d) is greater than $\frac{1}{81}$.

$e < S_9$

$|e| < \frac{(-1)^{10}}{81} < \frac{1}{81}$

18. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$

$\left|\frac{1}{n^3+1}\right| < \frac{1}{n^3}$ conv.

(a) Diverges.
(b) Converges absolutely.
(c) Converges conditionally.
(d) Diverges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

48

Question 2(16%) Find the radius and interval of convergence of the following series

$$\sum_{n=0}^{\infty} \frac{nx^n}{4^n(n^2+1)}$$

Then, specify the points at which the series converges conditionally, absolutely.

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(n+1)\,|x|^{\cancel{x}+1}}{4^{\cancel{n}+1}\,((n+1)^2+1)} \cdot \frac{\cancel{4^n}\,(n^2+1)}{n\,\cancel{|x|^n}}$$

$$= \lim_{n\to\infty} \frac{|x|\,(n+1)(n^2+1)}{4\,((n+1)^2+1)\,n} = \frac{|x|}{4}$$

$$\frac{|x|}{4} < 1$$

$$|x| < 4$$

$$-4 < x < 4$$

$\boxed{x=-4} \to \sum_0^{\infty} \frac{n(-4)^n}{4^n(n^2+1)} = \sum_0^{\infty} \frac{n(-1)^n(\cancel{4})^n}{\cancel{4^n}(n^2+1)} = \sum_0^{\infty} \frac{(-1)^n n}{n^2+1}$

$|u_n| = \sum \frac{n}{n^2+1}$

$\lim_{n\to\infty} \frac{\frac{n}{n^2+1}}{\frac{1}{n}} = 1$ div by nth test

$\sum_0^{\infty} \frac{(-1)^n n}{n^2+1}$ conv by A.S.Th

1. $U_{n+1} < U_n$
2. U_n positive
3. $\lim_{n\to\infty} U_n \to 0$

$\boxed{x=4} \to \sum_0^{\infty} \frac{n\,\cancel{4^n}}{\cancel{4^n}(n^2+1)} = \sum^{\infty} \frac{n}{n^2+1}$

$\lim_{n\to\infty} \frac{\frac{n}{n^2+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$ div by L.C.T by compare with $\frac{1}{n}$ which is div by p-test

الرحمن

Question 3(15%) Answer the questions below

(a) Find the sum of the series $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$.

$$= \sum_{2}^{\infty} \frac{2}{(n+1)(n-1)} = \sum_{2}^{\infty} \frac{1}{(n+1)} - \frac{1}{(n-1)}$$

$$= \left(\frac{1}{3} - \frac{1}{1}\right) + \left(\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{1}{5} - \frac{1}{3}\right) + \left(\frac{1}{6} - \frac{1}{4}\right) + \left(\frac{1}{7} - \frac{1}{5}\right) + \frac{1}{(n+1)} - \frac{1}{(n-1)}$$

$$= \lim_{k\to\infty} -1 - \frac{1}{2} + \frac{1}{n+1} = \lim_{k\to\infty} -\frac{3}{2} + \frac{1}{n+1} = -\frac{3}{2}$$

4

(b) Determine whether the integral $\int_1^{\infty} \frac{dx}{e^x+x^2}$ converges or diverges.

$$\int_1^{\infty} \frac{dx}{e^x+x^2} < \frac{1}{x^2}$$

$\int_1^{\infty} \frac{1}{x^2}$ is conv by p-test

so $\int_1^{\infty} \frac{dx}{e^x+x^2}$ is conv by D.C.T with $\int_1^{\infty} \frac{1}{x^2}$

5

(c) Determine whether the integral $\int_0^1 \frac{2dx}{x(x+2)}$ converges or diverges.

$$\frac{2}{x^2+2x} < \int_0^1 \frac{2}{x^2} \quad \text{div.}$$

$$\int_0^1 \frac{2dx}{x^2+2x} = \lim_{a\to 0^+} \int_a^1 \frac{2\,dx}{x^2+2x} = \lim_{a\to 0^+} \int_a^1 \frac{2}{(x)(x+2)}\,dx$$

$$\Rightarrow \int_a^1 \frac{2}{x(x+2)}\,dx \Rightarrow \frac{2}{x(x+2)} = \frac{A}{x} + \frac{B}{(x+2)}$$

$$\frac{1}{x^2} \quad \frac{x^2}{x^2+2x} \to 1 \quad \text{both div}$$

$$2 = A(x+2) + Bx$$

$$x=0 \to \boxed{A=1}, \quad x=-2 \to \boxed{B=-1}$$

$$\Rightarrow \int_a^1 \frac{1}{x} + \frac{-1}{x+2}$$

$$= \ln|x| - \ln|x+2| \Big|_a^1 = \ln 1 - \ln 2 - \ln a + \ln(a+2)$$

خلاف الوردي ←

4

Question 4(15%) Answer the following:

(a) Find the Taylor series generated by the function $f(x) = 2^x$ at $a = 1$.

$$\sum_0^{\infty} \frac{f^n(a)(x-1)}{n!} = f(1) + f'(1)(x-1) + f''(1)\frac{(x-1)^2}{2!} + f'''(1)\frac{(x-1)^3}{3!}$$

$$= 2 + 2\ln 2 (x-1) + 2(\ln 2)^2 \frac{(x-1)^2}{2!} + \frac{2(\ln 2)^3 (x-1)^3}{3!}$$

$$= \sum_0^{\infty} \frac{2(\ln 2)^n (x-1)^n}{n!}$$

$f(x) = 2^x \to f(1) = 2$

$f'(x) = 2^x \ln 2 \to f'(1) = 2\ln 2$

$f''(x) = (\ln 2)^2 2^x \to f''(1) = 2(\ln 2)^2$

$f'''(x) = (\ln 2)^3 2^x \to f'''(1) = (\ln 2)^3 2$

(b) Use the fact that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, $|x| < 1$ to answer the following questions:

(i) Find the Maclaurin series of the function $\frac{1}{1+x^2}$.

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)}$$

$$\sum_0^{\infty} (-x^2)^n = \sum_0^{\infty} (-1)^n x^{2n}$$

(ii) Use (i) to find the Maclaurin series of the function $\tan^{-1} x$.

$$\int \frac{1}{1+x^2} = \tan^{-1} x$$

$$\int \sum_0^{\infty} (-1)^n x^{2n} = \sum_0^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$$

(iii) What is the interval of convergence of the series in (i) and (ii).

series(i) $\to \sum_0^{\infty} (-1)^n x^{2n}$ is div by nth test ~~A.S.T.~~

$\cancel{\lim_{n\to\infty} a_n}$

Conv at $\boxed{x=0}$

is

Series(ii) $\to \sum_0^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$

Conv at $-1 < x < 1$

7

$$\lim_{n\to\infty} \frac{|x|^{2n+3}}{2n+3} \cdot \frac{2n+1}{|x|^{2n+1}} \Rightarrow \lim_{n\to\infty} \frac{|x|^2 (2n+1)}{(2n+3)} \to |x|^2 \quad |x|^2 < 1$$

$(|x| < 1)\; -1 < x < 1$

91/100

Birzeit University

Department of Mathematics

Second Hour Exam Math 132. Summer 2013

Student name: Malek Jamjoum Section Student no.: 1120064 (18)

Q#1 (68%)circle the correct answer.

1) The series $\sum_{1}^{\infty}\frac{1}{\sqrt{n(n+1)(n+2)}}$ $\frac{1}{\sqrt{n^3}}$

a) Converges by Limit comparison test with $\sum_{1}^{\infty}\frac{1}{\sqrt{n^3}}$ div

b) Converges by direct comparison test with $\sum_{1}^{\infty}\frac{1}{\sqrt[5]{n^4}}$ conv

c) Converges by direct comparison test with $\sum_{1}^{\infty}\frac{1}{\sqrt[4]{n^3}}$ conv $\frac{1}{n^{3/4}}$

d) Diverges by the ratio test

$(n^2+n)(n+2)$

$n^3+2n^2+n^2+2n$

n^3+3n^2+2n

$\sqrt{n^3+3n^2+3n}$

60

2) The series $\sum_{1}^{\infty}(\log_2 x)^n$ converges if

a) $x\in(e^{-1},e)$

b) $x\in(\frac{1}{2},2)$

c) $x\in(-\frac{1}{2},\frac{1}{2})$

d) $x\in(-1,1)$

$\frac{\ln|x|}{\ln 2} < 1$

$-1 < \frac{\ln x}{\ln 2} < 1$

$-\ln 2 < \ln x < \ln 2$

$\frac{1}{2} < x < 2$

3.The series $\sum_{n=1}^{\infty}(1+\frac{1}{n})^n$ $e^n \neq 0$

a) diverges by nth term test.

b) diverges by nth root test

c) converges by nth root test

d) converges by nth term test

$\frac{n^2}{n^2+1}$

$\frac{-1}{2}+\frac{-2}{5}+\frac{-3}{10}+\frac{-4}{17}+\frac{-5}{26}$

$\lim \frac{n+1}{n} = 1$ no conclusion

4. The series $\sum_{1}^{\infty} \frac{n^2}{e^n}$

a) Converges By nth term Test

b) Converges By Ratio Test

c) Diverges By Integral Test

d) Diverges by Alternating Series theorem

$$\frac{(n+1)^2}{e^{n+1}} \cdot \frac{e^n}{n^2}$$

$$\frac{2n}{e^n} \Rightarrow \frac{2}{e^n}$$

$$\frac{n^{2/n}}{e} = \frac{1}{e} \quad \frac{1}{2.3} < 1$$

5. Consider $\sum_{1}^{\infty} a_n$ Where $a_n \geq 0$ Then

a) If $\lim_{x\to\infty} a_n = 0$ then $\sum_{1}^{\infty} a_n$ converges.

b) If $\sum_{1}^{\infty} a_n$ diverges then $\lim_{x\to\infty} a_n \neq 0$

c) If $\lim_{x\to\infty} a_n \neq 0$ then $\sum_{1}^{\infty} a_n$ diverges

d) If $\sum_{1}^{\infty} a_n$ converges then $\lim_{x\to\infty} a_n \neq 0$

$$\int_1^{\infty} e^{-n} x^2$$

$\lim_{a\to\infty}$

6) The series $\sum_{0}^{\infty} \frac{3^n}{2^n + 5^n}$

a) Converges by direct comparison test with $\sum_{1}^{\infty} \frac{3^n}{7^n}$

b) Converges by direct comparison test with $\sum_{1}^{\infty} \frac{3^n}{4^n}$

c) Converges by direct comparison test with $\sum_{1}^{\infty} \frac{1}{2^n}$

d) Diverges

$$\frac{3^n}{2^n + 5^n} < \frac{3^n}{5^n} < \frac{3^n}{7^n}$$

7. $\int_0^{\frac{\pi}{2}} \tan x dx =$

a) 0

b) -1

c) ∞

d) $-\infty$

$$\lim_{a\to\pi/2} \int_0^a \tan x$$

$u = \cos$

$du = -\sin x\, dx$

$$-\int \frac{\sin}{u} \frac{du}{\sin x}$$

$$\ln|\sec a| - \cancel{\ln|\sec 0|}$$

$$-\ln u \Rightarrow -\ln\cos$$

8) The series $\sum_{2}^{\infty}\frac{(n+1)\ln n}{\sqrt{n}}$

a) Converges by the integral test

b) Converges by direct comparison test with $\sum_{2}^{\infty}\frac{1}{\sqrt[5]{n^4}}$

c) Diverges by the nth term test

d) Diverges by the ratio test

9. Which of the following sequences diverges?

(a) $\{\frac{n}{1+\ln n}\}$

(b) $\{n^2/e^n\}$

(d) $\{(-1)^{n+1}/n\}$

(c) $\{\sqrt[n]{10n}\}$

10. Which of the following series converges conditionally?

(a) $3 - 1 + 1/9 - 1/27 + \ldots$

(b) $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{5}} + \ldots$

(c) $1/2^2 - 1/3^2 + 1/4^2 - \ldots$

(d) $\frac{1}{1\times 2} - \frac{1}{2\times 3} + \frac{1}{3\times 4} - \frac{1}{4\times 5} + \ldots$

11) The Integral $\int_{2}^{\infty}\frac{dx}{\sqrt[3]{x}(\sqrt{x}-1)}$

a) Converges by limit comparison test with $\int_{2}^{\infty}\frac{dx}{\sqrt[3]{x}}$

b) Converges by limit comparison test with $\int_{2}^{\infty}\frac{dx}{\sqrt{x}}$

c) Diverge by direct comparison test with $\int_{2}^{\infty}\frac{dx}{\sqrt[3]{x^2}}$

d) Diverge by direct comparison test with $\int_{2}^{\infty}\frac{dx}{\sqrt[6]{x^5}}$

12. The series $\sum_{n=2}^{\infty} \frac{2n-1}{n^2(n-1)^2}$

$$\frac{\cancel{n^2}\left(\frac{2}{n}-\frac{1}{n^2}\right)}{\cancel{n^2}(n-1)^2}$$

0

a) 1

[illegible]1

c) $\frac{1}{4}$

d) 2

13. The Series $\sum_{1}^{\infty} \frac{2^n-4}{3^n}$

$$\left(\frac{2}{3}\right)^n - \frac{4}{3^n}$$

$$\frac{\frac{2}{3}}{1-\frac{2}{3}} - 4\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)$$

$$\left(\frac{2}{\cancel{3}} \cdot \frac{\cancel{3}}{1}\right) - 4\left(\frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{2}\right) \qquad 2-2 \Rightarrow 0$$

a) Converges to 1

b) Converges to $\frac{3}{4}$

c) Converges to 0

d) Converges to $\frac{3}{2}$

14. Assuming its convergence,find the limit of the following recursively sequence, $a_1 = 8$ and $a_{n+1} = \sqrt{a_n + 8} - 2$

a) 1

b) -4

c) -2

d) 8

$a_1 = 8,\ 2,$

$\sqrt{10}-2,\ \sqrt{\sqrt{10}-6}$

15. $\sum_{1}^{\infty} (-1)^n \frac{\ln(n)}{n^3}$

$$\frac{\ln(n)}{n^3} \Rightarrow \frac{\frac{1}{n}}{3n^2} \Rightarrow \frac{1}{n}\cdot\frac{1}{3n^2} \Rightarrow \frac{1}{3n^3} \Rightarrow 0$$

$\frac{1}{n^3}$

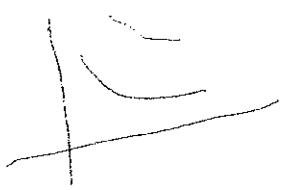
$\ln(n) \Rightarrow \infty$

a) Is ageometric series

b) Converges conditionaly

c) Converges abslutely

d) Diverges



$\frac{2}{24} + \frac{1}{4} = \frac{9}{12} - \frac{2}{12}$

$\frac{7}{12}$

16) The Series $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{6}+\cdots$

a) Convrges to L Where $\frac{7}{12} \leq L \leq \frac{3}{4}$

b) Convrges to L Where $\frac{3}{4} \leq L \leq \frac{11}{12}$

c) Convrges to L Where $\frac{1}{4} \leq L \leq \frac{1}{2}$

d) Diverges

$$\boxed{\begin{aligned} u &= \tan^{-1} x \\ du &= \frac{dx}{1+x^2} \end{aligned}}$$

$\frac{\sin}{\cos}$

17. If $\{s_n\} = \{(-1)^n(\frac{n+1}{n})\}$, then

(a) $\{s_n\}$ diverges
(b) $\{s_n\}$ converges to zero
(c) $\{s_n\}$ converges to e^{-1}
(d) $\{s_n\}$ converges to 1

$\frac{1+\frac{1}{n}}{1} \Rightarrow 1$

1.(15%) Test for Convergence

a) $\int_0^\infty \frac{\tan^{-1} x}{1+x^2} dx$

$u = \tan^{-1} x$
$du = \frac{1}{1+x^2} dx$

$$\lim_{a\to\infty} \int_0^a \frac{\tan^{-1} x}{1+x^2} dx = \lim_{a\to\infty} \int_0^a \frac{u \, du \cancel{(1+x^2)}}{\cancel{1+x^2}} = \lim_{a\to\infty} \int u \, du$$

$$= \lim_{a\to\infty} \frac{u^2}{2}$$

$$= \lim_{a\to\infty} \frac{(\tan^{-1} x)^2}{2}\Bigg|_0^a = \frac{(\tan^{-1} a)^2}{2} - 0$$

$$= \frac{\left(\frac{\pi}{2}\right)^2}{2} = \frac{\frac{\pi^2}{4}}{2}$$

$$= \frac{\pi^2}{4} \cdot \frac{1}{2} = \boxed{\frac{\pi^2}{8}}$$

$\boxed{\text{Converges}}$

$\frac{\cancel{n}(\frac{1}{n})}{\cancel{n}(\frac{2}{n}+1)}$

0

$\int_0^\infty \frac{dx}{\sqrt{x^6+1}}$

$$\int_0^\infty \frac{dx}{\sqrt{x^6(1+\frac{1}{x^6})}} = \int_0^\infty \frac{dx}{x^3\sqrt{(1+\frac{1}{x^6})}} = \boxed{\int_0^1 \frac{dx}{x^3(\sqrt{1+\frac{1}{x^6}})} + \int_1^\infty \frac{dx}{x^3\sqrt{1+\frac{1}{x^6}}}}$$

$$\frac{1}{x^3\sqrt{1+\frac{1}{x^6}}} \div \frac{1}{x^3} \Rightarrow \frac{\cancel{x^3}}{\cancel{x^3}\sqrt{1+\frac{1}{x^6}}} = \frac{1}{\sqrt{1+\frac{1}{x^6}}}$$

$$\lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x^6}}} = 1$$

So by the Limit comparison test with $\frac{1}{x^3}$ both converge or both diverge $\Rightarrow$ $\int_0^1 \frac{dx}{x^3}$ $\Rightarrow$ diverge – P – Test

Conv

So $\boxed{\text{diverge}}$

$\sum_1^\infty (\frac{1}{2+n})^n$

by applying the root test $\sqrt[n]{\left(\frac{1}{2+n}\right)^n} = \frac{1}{2+n}$

$$\lim_{n\to\infty} \frac{1}{2+n} = \frac{\cancel{n}(\frac{1}{n})}{\cancel{n}(\frac{2}{n}+1)} = \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{2}{n}+1} = \frac{0}{1}$$

$\rho = 0 < 1$ so the series converges by the root test

14

$$\frac{(n+1)\,x^{\cancel{(n+1)}}}{4^{\cancel{(n+1)}}\left((n+1)^2+1\right)} \qquad \frac{\cancel{4}\,(\;\;)}{n\,\cancel{x}} \qquad \frac{x\,\cancel{(n+1)}\,(n^2+1)}{n\,4\,(n^2+1)^2+4n}$$

$$\frac{x\,n^2+x}{4n^2+4n+4n}$$

3. (17%) Consider the power series $\sum_{1}^{\infty} \frac{nx^n}{4^n(n^2+1)}$ Find

a) Interval and radius of convergence

b)For what values of x does the series converges

1) absolutely 2) coditionally 3) diverge

by the ratio test :

$$\frac{(n+1)\,(x)^{\cancel{n+1}}}{(4)^{\cancel{n+1}}\left((n+1)^2+1\right)} \cdot \frac{\cancel{4^n}\,(n^2+1)}{n\,\cancel{x^n}} = |x|\,\frac{(n+1)\,(n^2+1)}{n4\,(n+1)^2+4n}$$

$$\lim_{n\to\infty} |x|\,\frac{(n+1)\,(n^2+1)}{4n\,(n+1)^2+4n} = \frac{|x|}{4} = \rho$$

$$\frac{|x|}{4} < 1 \Rightarrow -1 < \frac{|x|}{4} < 1 \Rightarrow -4 < |x| < 4$$

$\boxed{\text{Radius} = 4}$

Converges abs $\overset{\text{when } x=}{}$ $(-4, 4]$

div when $x = (-\infty, -4)\ (4, \infty)$

Test for intervals beginng and endings

when $x = 4$

$$\sum_{1}^{\infty} \frac{n\,\cancel{4^n}}{\cancel{4^n}(n^2+1)} = \frac{n}{n^2+1}$$

L.C.T with $\frac{1}{n} \Rightarrow \frac{n}{n^2+1} \cdot n = \frac{n^2}{n^2+1} \Rightarrow \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$

So both converg or both diverge

and $\frac{1}{n}$ diverges $\Rightarrow$ p-Test

so the series diverges when $x = 4$

When $x = -4 \Rightarrow \sum_{1}^{\infty} \frac{-n\,\cancel{4^n}}{\cancel{4^n}(n^2+1)}$

$= \frac{-n}{n^2+1}$ by the root test

$$\lim_{n\to\infty} \frac{-n^{\frac{1}{n}}}{(n^2)^{\frac{1}{n}}+1^{\frac{1}{n}}} = \frac{-\cancel{1}}{\cancel{1}+1}$$

$= -\frac{1}{2}$ so converges by the root test when $\boxed{x=-4}$ because $\rho < 1$

7

$\frac{14}{25}$

Birzeit University-Mathematics Department
Math132-Calculus II

Second Exam
Name: Sandy Habash

Instructor of Discussion: Hasan Yousef

Second Semester 2013/2014
Number: 1122009
Section: Sat: 11:00-11:50.

Question 1. (19 points) Circle the correct answer:

$(2^3)^6 \quad 2^{18}$

(1) The sequence $a_n = e^{-n^{1/n}}$

(a) Converges to e.
(b) Converges to e^{-1}.
(c) Converges to 1.
(d) Diverges.

$(e^n)^{1/n} \quad \frac{1}{e^{n\cdot 1/n}} \quad \frac{1}{e}$

$e^{-n^{1/n}} \quad e^{-1} \quad \frac{1}{e}$

(2) $\int_2^\infty \frac{2dx}{x^2-1}$

(a) Converges to 1.
(b) Converges to ln 3.
(c) Converges to 0
(d) Diverges.

$\frac{2}{x^2-1}$

$\int_2^\infty \frac{2\,dx}{x^2-1}$

(3) The series $\sum_{n=0}^\infty \frac{1}{e^n+e^{-n}}$

(a) Converges by integral test.
(b) Diverges by integral test.
(c) Diverges by nth term test.
(d) None of the above.

$\frac{1}{1+1} = \frac{1}{2} + \cdots \quad (\cosh n)^{-1}$

$\int \frac{1}{e^n+e^{-\infty}} \quad \frac{1}{\infty+0}$

(4) The series $\sum_{n=1}^\infty \frac{n}{e^n}$

(a) Converges by nth term test.
(b) Diverges by nth term test.
(c) Diverges by ratio test.
(d) Converges by nth-root test.

$\frac{n}{e^n}$

$\sqrt[n]{\frac{n}{e^n}} \quad \frac{n^{\frac{1}{n}}}{e} \quad \frac{1}{e}$

$\frac{n+1}{e} \quad \frac{n+1}{en} \quad \frac{n}{en} + \frac{1}{en} \quad \frac{1}{e} + \frac{1}{\infty} \quad \frac{1}{e} + 0 < 1$

$n^{\frac{1}{n}} = 1$

$\sqrt[n]{n} = 1$

(5) The series $\sum_{n=0}^{\infty} \frac{3^n}{2^n+3^n}$

(a) Converges by integral test.
(b) Diverges by direct comparison with $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$.
(c) Converges by nth term test.
(d) Diverges by nth term test.

(6) The interval of convergence of the series $\sum_{n=2}^{\infty} \frac{(x-1)^n}{n \ln n}$ is

(a) $[0, 2]$.
(b) $[0, 2)$.
(c) $(0, 2)$.
(d) $(0, 2]$.

(7) One of the following series converges absolutely

(a) $\sum_{n=1}^{n} \frac{(-1)^n}{n}$.
(b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n^2-1}}$.
(c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$.
(d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n}$.

(8) $\sum_{n=0}^{\infty}(e^{-n} - e^{-(n+2)}) =$

(a) $1 + e^{-1}$.
(b) $e^{-1} + e^{-2}$.
(c) e^{-1}.
(d) None of the above.

(9) The Maclaurin series generated by $f(x) = 3^x$ is

(a) $\sum_{n=0}^{\infty} \frac{3^n}{n!}$.
(b) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$.
(c) $\sum_{n=0}^{\infty} \frac{(\ln 3)x^n}{n!}$.
(d) $\sum_{n=0}^{\infty} \frac{(\ln 3)^n x^n}{n!}$.

$\int \frac{\ln n}{\sqrt{n}}$ $\frac{1}{n^2}$ converge

(10) $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}} =$

(a) Converges by integral test.

(b) Converges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{n^2}$.

(c) Diverges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{n}$.

(d) Diverges by nth term test.

$\frac{\ln n}{\sqrt{n}}$ $\frac{\ln n}{\sqrt{n}} \cdot \frac{1}{\ln}$

(11) $\int_2^{\infty} \frac{2+\sin x}{x-1} dx$

(a) Converges to 0.

(b) Diverges by limit comparison with $\int_2^{\infty} \frac{dx}{x-1}$.

(c) Diverges by direct comparison with $\int_2^{\infty} \frac{dx}{x}$.

(d) None of the above.

$\ln(x-1)$

$\ln x$

$\int \frac{f}{g} = \int 2+\sin x$

$= (2-\cos x)\Big|_0^b$

(12) $\frac{\pi}{2} - \frac{\pi^3}{2^3(3!)} + \frac{\pi^5}{2^5(5!)} - \cdots + (-1)^n \frac{\pi^{2n+1}}{2^{2n+1}(2n+1)!} + \cdots =$

(a) 0.

(b) 1.

(c) -1.

(d) ∞.

(13) The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{n^3(x-2)^n}{2^n}$ is

(a) 0.

(b) 1.

(c) $\frac{1}{2}$.

(d) 2.

$n\left(e^{\left(\frac{-1}{n}\right)} - 1\right)$

(14) The sequence $a_n = n(e^{-1/n} - 1)$

(a) Diverges.

(b) Converges to -1.

(c) Converges to 1.

(d) Converges to e^{-1}.

$n(e^{-1/n} - 1)$

$n\left(\frac{1}{e^{1/n}} - 1\right)$

$\frac{n}{e^{1/n}} - n$

$\frac{1}{} - 1$

$$\frac{(n+1)!\,e^{n+1}}{(2n+1)!}\cdot\frac{(2n)!}{n!\,e^{n}}$$

$$\frac{e(n+1)}{2(n+1)} = \frac{e}{2} < 1$$

(15) $\sum_{n=0}^{\infty} \frac{n! e^n}{(2n)!}$

(a) Converges by ratio test.
(b) Converges by nth term test.
(c) Diverges by ratio test.
(d) Ratio test fails.

(16) The error in the approximation $\frac{1}{1+x^2} \approx 1 - x^2 + x^4 - x^6 - x^8$ in the interval $[-0.1, 0.1]$ is less than

(a) 1×10^{-10}.
(b) 1×10^{-9}.
(c) 1×10^{-8}.
(d) 1×10^{-7}.

(17) The Maclaurin series generated by $f(x) = \frac{x^2}{1+x}$ is

(a) $\sum_{n=0}^{\infty} (-1)^n x^n$.
(b) $\sum_{n=0}^{\infty} (-1)^n x^{n+1}$.
(c) $\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!}$.
(d) $\sum_{n=0}^{\infty} (-1)^n x^{n+2}$.

(18) Suppose that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = L$ then

(a) $\frac{3}{4} < L < 1$.
(b) $1 < L < \frac{5}{4}$.
(c) $\frac{1}{4} < L < \frac{3}{4}$.
(d) None of the above.

(19) The series $\sum_{n=1}^{\infty} \frac{\tanh n}{n^2}$

(a) Diverges.
(b) Converges by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.
(c) Diverges by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.
(d) Is an alternating series.

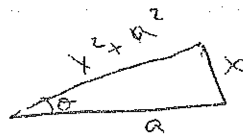
Question 2(6 points) Solve the integrals

(a) $\int \frac{x^2}{(x^2+1)^{3/2}}dx$ using trigonometric substitution.

$$\int \frac{x^2}{\sqrt{(x^2+1)^3}} =$$

$$\frac{x^2}{(x^2)^{3/2}+1} = \int \frac{x^2}{x^3+1}$$

$$= \int \frac{\cancel{x}}{x^3+1}$$



$x = \cancel{a \tan\theta}$

$dx = a \sec^2\theta$

$\sqrt{x^2+a^2} =$

(b) $\int \frac{x+1}{x(x^2+1)}dx$

$$\int \frac{x}{x^2+1} - \int \frac{1}{x^2+1}$$

$$\int \frac{x+1}{x(x^2+1)}dx = \frac{(x^2+1)A}{(x^2+1)x} + \frac{Bx+C}{x^2+1}\frac{(x)}{(x)} = \int \frac{1}{x} - \int \frac{x-1}{x^2+1} = \boxed{\ln x - \tfrac{1}{2}\ln(x^2+1) - \tan^{-1}x}$$

$2x(A+B)+0+c$

$$= \frac{A(x^2+1)+Bx^2+cx}{x(x^2+1)}$$

$$\frac{Ax^2+A+Bx^2+cx}{x(x^2+1)} = \frac{x^2(A+B)+A+cx}{x(x^2+1)}$$

$1 = A+B+A+c = 2A+B$

$1 = 2(A+B)+\cancel{A}+c = 2A+2B+\cancel{A}+c = 2A+2B+c$

$0 = 2A+2B$

$-2A = 2B$

$\boxed{-A=B}$

$1 = 2A-A = A \longrightarrow \boxed{1=A}$

$\boxed{B=-1}$

$1 = 2(1)+2(-1)+c$

$1 = 2+-2+c$

$\boxed{c=1}$

5

$\Rightarrow \ln x - \frac{1}{2}\ln(x^2+1) - \tan^{-1}x$

$\frac{84}{100}$

Birzeit University

Department of Mathematics

Second Hour Exam | **Math 132** | **Summer 2015**

Student name: [illegible] **Section..** 10:00 **Student no.:** [illegible]

Q#1 ~~60%~~ 72 circle the correct answer.

1. $\sum_{2}^{\infty} \frac{1}{n(n-1)}$ equals

(a) 4/3
(b) 1
(d) 3/4
(d) Diverges

$\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\cdots =$

$\frac{1}{n-1}-\frac{1}{n} = \left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)$

2) Which of the following series converges conditionally?

(a) 3 - 1 + 1/9 - 1/27 + ...

(b) $\frac{1}{1\times2}-\frac{1}{2\times3}+\frac{1}{3\times4}-\frac{1}{4\times5}+\ldots$

(c) $1/2^2 - 1/3^2 + 1/4^2 - \ldots$

56 (d) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+1)}$ $\qquad \sum \frac{(-1)^n}{n}$ $\qquad \sum_{n=2}^{\infty} \frac{(-1)^n}{n^2}$

3) If $\{s_n\} = \{1+\frac{(-1)^n}{n}\}$,then

(a) $\{s_n\}$ diverges
(b) $\{s_n\}$ converges to zero
(c) $\{s_n\}$ converges to e^{-1}
(d) $\{s_n\}$ converges to 1

$(1-1)+\left(1+\frac{1}{2}\right)+\left(1-\frac{1}{3}\right)$

$\sum$ converge + $\sum$ diverge = diverge

4.The sequence $(a_n) = (1-\frac{1}{n^2})^n$

a) Converges to e^{-1}
b) Converges to e
c) Converges to 1
d) diverges

$= \left(1-\left(\frac{1}{n}\right)^2\right)^n$

$\left(1-\frac{1}{n}\right)^n\left(1+\frac{1}{n}\right)^n$

$e^{-1}\, e^{1} = 1$

5) The Series $\left(1-\frac{1}{2}+\frac{1}{4}\right)\frac{1}{6}+\cdots$

a) Convrges to L Where $0.46 \le L \le 0.66$
b) Convrges to L Where $0.50 \le L \le 0.75$
c) Convrges to L Where $1 \le L \le 1.5$
d) Diverges

$\frac{3}{4}-\frac{1}{6}=\frac{18-1}{24}=\frac{17}{24}$

6)Which of the following series converges?

(a) $\Sigma\frac{1}{n}$ $\Rightarrow$ diverge $\Rightarrow$ P-test

(b) $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}$ $\Rightarrow$ diverge $\Rightarrow$ P-test

(c) $\sum_{n=1}^{\infty}\frac{1}{10n^2+1}$ $\Rightarrow$ converge by D.C.T

(d) $\sum_{n=2}^{\infty}\frac{1}{\ln n}$ $\Rightarrow$ diverge by D.C.T

7) The series $\sum_{n=1}^{\infty}\frac{1}{e^n+\sqrt{n}}$

$\left(\frac{1}{e}\right)^n$ converge $\frac{1}{e^n+\sqrt{n}} < \left(\frac{1}{e}\right)^n$ by D.C.T with $\frac{1}{e^n}$

a) Converges by limit comparison test compared with $\sum_{n=1}^{\infty}\frac{1}{2n}$.

b) diverges by direct comparison test compared with $\sum_{n=1}^{\infty}\frac{1}{2\sqrt{n}}$.

c) Converges by direct comparison test compared with $\sum_{n=1}^{\infty}\frac{1}{e^n}$.

d) diverges by *n*th term test

8) $\sum_{1}^{\infty}(\ln(x))^n$ Converges If

a) $-1<x<1$

b) $0<x<e$

c) $0<x<1$

d) $e^{-1}<x<e$

$|\ln x| < 1$

$-1 < \ln x < 1$

$\frac{1}{e} < x < e$

9. The series $\sum_{n=1}^{\infty} n\tan\frac{1}{n}$

a) Converges conditionaly

b) Converges absolutely

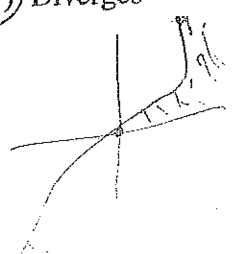
c) Converges by Integral Text

d) Diverges

$\lim_{n\to\infty} n\tan\frac{1}{n}$

$= \lim_{n\to 0}\frac{\tan n}{n} = 1 \neq 0$

$\Rightarrow$ converge



$\lim_{a\to\infty}\int_1^a n\tan\frac{1}{n}$

u
1
0

tan
ln cos

$$\frac{\frac{2}{5}}{1-\frac{2}{5}} = \qquad \frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{1}{5}\cdot\frac{5}{4} = \frac{1}{4}$$

$$\frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12}$$

10. The Series $\sum_{1}^{\infty}\frac{2^n-1}{5^n}$

$$\left(\frac{2}{5}\right)^n - \left(\frac{1}{5}\right)^n$$

a) Converges to $\frac{11}{12}$

b) Converges to $\frac{9}{12}$

c) Converges to 0

d) Converges to $\frac{5}{12}$

$$\cancel{\left[\left(\frac{2}{5}\right)-\left(\frac{1}{5}\right)\right]+\left[\frac{4}{10}-\frac{1}{10}\right]+\left[\frac{8}{25}-\frac{1}{25}\right]}$$

$$\cancel{\frac{1}{5}+\frac{3}{10}+\frac{7}{25}}$$

$$\cancel{\frac{5}{10}+\frac{7}{25} = \frac{19}{25} = \frac{4}{5}}$$

11) The radius of convergence of the series $\sum_{1}^{\infty}\frac{x^n}{2^n}$ is

a) R=1

b) R=2

c) R=0

d) R=∞

$|x| < 1$

$-1 < x < 1$

12) The series $\sum_{1}^{\infty}\frac{n^2}{e^n}$

$$\frac{n^2}{e^n}$$

a) Converges By n^{th} term Test

b) Converges By Ratio Test

c) Diverges By Integral Test

d) Diverges by ratio test

$$\lim \frac{(n+1)^2}{e^{n+1}}\cdot\frac{e^n}{n^2} = \frac{n^2+2n+1}{en^2} = \frac{1}{e} < 1$$

converges

13) The sum of the series (2 - 1 + 1/2 - 1/4 + 1/8 - ...) is

(a) 4/3

(b) 5/4

(c) 3/2

(d) ¾

$$\sum_{n=0}^{\infty} 2\left(\frac{-1}{2}\right)^n \qquad \left(\frac{2}{1-\frac{-1}{2}}\right) = \frac{2}{\frac{3}{2}} = \frac{4}{3}$$

14) The series $\sum_{1}^{\infty}\frac{2^n}{n^n}$

$$\lim \sqrt[n]{\left(\frac{2}{n}\right)^n} = \infty > 1$$

a) Diverges By n^{th} root Test

diverge by n^{th} root test

b) Diverges By Direct Comparison Test With $\sum_{1}^{\infty}\frac{1}{n^n}$

c) Converges By Integral Test

d) Converges By n^{th} root Test

15. Consider $\sum_{1}^{\infty} a_n$ Where $a_n \geq 0$ Then

a) If $\lim_{x\to\infty} a_n = 0$ then $\sum_{1}^{\infty} a_n$ converges.

b) If $\sum_{1}^{\infty} a_n$ diverges then $\lim_{x\to\infty} a_n \neq 0$

c) If $\lim_{x\to\infty} a_n \neq 0$ then $\sum_{1}^{\infty} a_n$ diverges

d) If $\sum_{1}^{\infty} a_n$ converges then $\lim_{x\to\infty} a_n \neq 0$

16. Consider $I_1 = \int_2^{\infty} \frac{dx}{x^2}$ and $I_2 = \int_0^{\frac{1}{2}} \frac{dx}{\sqrt{x}}$ Then

converge

$\lim_{a\to 0^+} \int_a^{\frac{1}{2}} \frac{1}{\sqrt{x}}$

$= 2\sqrt{x} \Big]_a^{\frac{1}{2}}$

$= 2\sqrt{1/2} - 2\sqrt{a}$

a) Both Integrals Converge
b) Both Integrals Diverge
c) I_1 converges and I_2 diverges
d) I_2 converges and I_1 diverges

17.Which of the following sequences diverges?

(a) $\{\frac{(-1)^n}{n}\}$

(b) $\{\frac{5^n}{4^n + \sin n}\}$

(c) $\{n^2/e^n\}$

(d) $\{\sqrt[n]{10n}\}$

$\lim \frac{n^2}{e^n} = \lim \frac{2n}{e^n} = \lim \frac{2}{e^n} = \infty \neq 0$

diverge

18. $\sum_{1}^{\infty} (-1)^n \frac{\ln(n)}{n^3}$

$\frac{\ln n}{n^3} \leq \frac{n}{n^3}$

$\frac{\ln n}{n^3} < \frac{1}{n^2}$

$\frac{1}{n^2}$ converge p-test

$\Rightarrow \frac{\ln n}{n^3}$ converges

a) Is a geometric series
b) Converges conditionaly
c) Converges abslutely
d) Diverges

18

1.(~~24%~~) Test for Convergence

a) $\int_1^{\infty} \frac{dx}{\sqrt{x^5+x}}$

$$\frac{1}{\sqrt{x^5+x}} < \frac{1}{\sqrt{x^3+x}}$$

since

$$\frac{1}{\sqrt{x^5+x}} < \frac{1}{\sqrt{x^4}} = \frac{1}{x^2}$$

$\int_1^{\infty} \frac{1}{x^2} \cdot dx$ converge by p-test as $p = 2 > 1$

and

$$\frac{1}{\sqrt{x^5+x}} < \frac{1}{\sqrt{x^3+x}}$$

$\Rightarrow \int_1^{\infty} \frac{1}{\sqrt{x^5+x}} \cdot dx$ converge by D.C.T

(Direct comparison test)

b) $\int_0^{\infty} \frac{\tan^{-1}x}{1+x^2} dx$

let $\tan^{-1} x = u$

$du = \frac{1}{1+x^2} \cdot dx$

$dx = 1+x^2 \cdot du$

$$\int_0^{\infty} u \cdot du$$

$$= \lim_{a\to\infty} \int_0^{a} u \cdot du = \lim_{a\to\infty} \frac{u^2}{2} \Big]_0^a$$

$$= \lim_{a\to\infty} \frac{(\tan^{-1}x)^2}{2} \Big]_0^a = \lim_{a\to\infty} \frac{(\tan^{-1}a)^2}{2}$$

$$= \cancel{\tan} \left(\frac{\pi}{2}\right)^2 = \frac{\pi^2}{4} \Rightarrow \text{converge}$$

للتأكد فقط

الحل خلف الورقة

10 ~~16~~

3. (~~16~~%) Consider the power series $\sum_{1}^{\infty} \frac{(-1)^n (x-2)^n}{n6^n}$ Find

a) Interval and radius of convergence

b)For what values of x does the series converges

1) absolutely 2) coditionally 3) diverge

$\left|\frac{x-2}{6}\right| < 1$

$-1 < \frac{x-2}{6} < 1$

$-4 < x < 8$

ⓐ The interval of convergence $[-4, 8]$

the radius of convergence $= 6$

-4 8

for $x = -4$

$\sum_{1}^{\infty} \frac{(-1)^n (-1)^n}{n\, 6^n} \Rightarrow \lim_{n\to\infty} \sqrt[n]{\frac{(-1)^n(-1)^n}{n\,6^n}} = \lim \frac{+1}{n^{\frac{1}{n}}\cdot 6} = \frac{+1}{6}$

$\Rightarrow$ converge by n^{th} root test $\left|\frac{+1}{6}\right| < 1$

$\lim_{n\to\infty} \sqrt[n]{\left|\frac{(-1)^n (-1)^n}{n\,6^n}\right|} = \frac{-1}{6} \Rightarrow \left|\frac{-1}{6}\right| < 1 \Rightarrow$ converge absolutely by n^{th} root test

for $x = 3$

$\sum_{1}^{\infty} \frac{(-1)^n (1)^n}{n\,6^n} \Rightarrow \lim_{n\to\infty} \sqrt[n]{\frac{(-1)^n(1)^n}{n\,6^n}} = \lim_{n\to\infty} \frac{-1}{n^{\frac{1}{n}}\cdot 6} = \left|\frac{-1}{6}\right| < 1$

$\sum_{1}^{\infty} \frac{(-1)^n (1)^n}{n\,6^n} \Rightarrow \lim \sqrt[n]{\frac{(-1)^n(1)^n}{n\,6^n}} = \lim_{n\to\infty} \frac{1}{n^{\frac{1}{n}}\, 6} = \left|\frac{1}{6}\right| < 1$

converge absolutely by n^{th} root test

ⓑ ① Converge absolutely $[-4, 8)$

② converge conditionally $\Rightarrow \{8\}$

③ diverge $(-\infty, -4) \cup (8, \infty)$

10

60/60 Excellent

Birzeit University-Mathematics Department
Math 1321-Calculus II

Second Hour Exam
Name:...Maha Dorgham...
Instructor:....Areej Awawdeh.........

Spring 2015/2016
Number:...1156494
Section:....12 D...

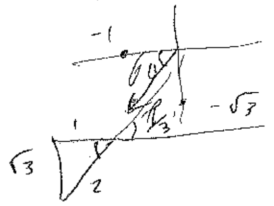
$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$
$\cos = \frac{1}{2}$

Question 1.(44 points) Circle the correct answer:

(1) One of the following is the point $(x, y) = (-1, -\sqrt{3})$ in polar coordinates

(a) $(2, \frac{\pi}{3})$ X
(b) $(-2, \frac{\pi}{3})$
(c) $(-2, \frac{4\pi}{3})$
(d) $(-2, \frac{\pi}{6})$

$\cancel{1 = r}$ $\quad r^2 = 4 \quad r = 2$ $\quad \tan\theta = \sqrt{3} \quad \theta = \frac{\pi}{6}$



(2) The slope of the polar curve $r = \frac{1}{2} + \cos\theta$ at $\theta = \frac{2\pi}{3}$ is

$y = r\sin\theta$
$x = r\cos\theta$

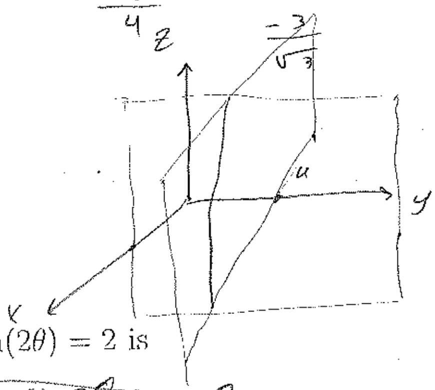
$\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$
$\cos = -\frac{1}{2}$

(a) $\sqrt{3}$
(b) $-\sqrt{3}$
(c) $\frac{1}{\sqrt{3}}$
(d) $-\frac{1}{\sqrt{3}}$

$$\frac{dy}{dx} = \frac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta} = \frac{-\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}}{-\frac{\sqrt{3}}{2} \times -\frac{1}{2}} = \frac{-\frac{3}{4}}{\frac{\sqrt{3}}{4}} \times \frac{4}{\sqrt{3}} = \frac{-3}{\sqrt{3}}$$

(3) The equations $x = 3, y = 4$ in space represent

(a) The point $(3, 4)$
(b) A line parallel to the $z-$axis
(c) A line perpendicular to the $z-$axis
(d) A line in the $xy-$plane.



(4) The cartesian equation of the polar curve $r^2\sin(2\theta) = 2$ is

(a) $x + y = 1$
(b) $xy = 1$
(c) $y^2 = 2$
(d) $xy = 2$

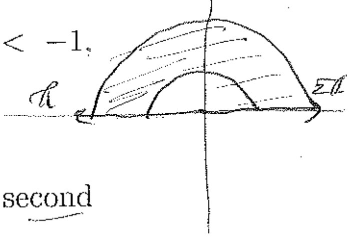
$\cancel{r^2(2\sin 2\theta\cos 2\theta}$
$r^2(2\sin\theta\cos\theta) = 2$
$xy = 1$

(5) The center and radius of the sphere $x^2 + y^2 + z^2 - 2x + 2y = 2$ are

(a) $(1, 1, 0)$, 4
(b) $(1, -1, 0)$, 2
(c) $(-1, 1, 0)$, 2
(d) $(-1, -1, 0)$, 2

$(x^2 - 2x + 1) + (y^2 + 2y + 1) + z^2 = \underbrace{2 + 1 + 1}_{4}$
$(x-1)^2 + (y+1)^2 + z^2$
$(1, -1, 0)$

(6) The set of points in the plane that satisfy the inequalities $-2 < r < -1$, $\pi \le \theta \le 2\pi$



(a) the region between the circles $r = -1$ and $r = -2$.

(b) the region between the circles $r = -1$ and $r = -2$ in the first and second quadrant.

(c) the region between the circles $r = -1$ and $r = -2$ in the second and third quadrant.

(d) the region between the circles $r = -1$ and $r = -2$ in the third and fourth quadrant.

(7) The vector projection of $\mathbf{u} = \mathbf{i} + \mathbf{k}$ onto $\mathbf{v} = \mathbf{j} + \mathbf{k}$ is

(a) $2\mathbf{v}$

(b) $\frac{1}{2}\mathbf{v}$

(c) $\frac{1}{\sqrt{2}}\mathbf{v}$

(d) $\mathbf{v}$

$\left(\frac{u \cdot v}{|v|^2}\right) v = \frac{1}{2} v$

(8) Let θ be the angle between $\mathbf{u} = \mathbf{i} + \mathbf{k}$ and $\mathbf{v} = \mathbf{j} + \mathbf{k}$. Then $\theta =$

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{6}$

$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|u||v|} = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

(9) The set of points equidistant (at the same distance) from the points $(0, 1, 0)$ and $(0, -1, 0)$ is

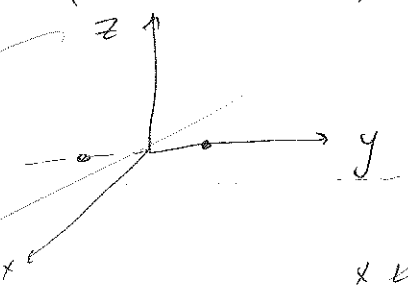
(a) the $z-$axis

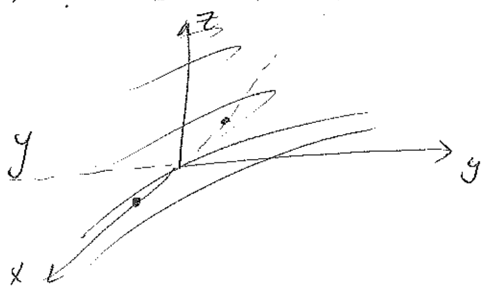
(b) the $xy-$plane

this is the answer

(c) the $xz-$plane

(d) the $yz-$plane





(10) The volume of the box determined by the vectors $\mathbf{u} = \mathbf{i} - \mathbf{j} - \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, $\mathbf{w} = \mathbf{i} - \mathbf{j}$ is

(a) 5

(b) 7

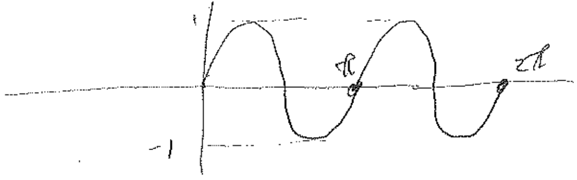
(c) 6

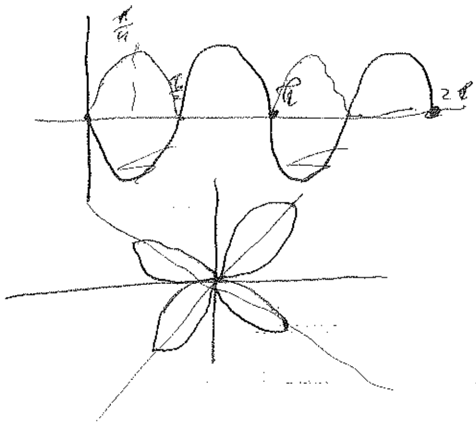
(d) 4

$V = \begin{vmatrix} 1 & -1 & -1 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{vmatrix}$ $\quad 1 + -1$

$= 1 \times 1 - (-1)(-1)$

$= 1 - 1 + (-1)(-5)$





(11) The polar curve $r = -\sin^2(2\theta)$ is symmetric about

(a) the x–axis.

(b) the y–axis.

(c) the origin.

(d) all of the above.

(12) The vectors $\mathbf{u} = a\mathbf{i} + b\mathbf{j} + \mathbf{k}$, $\mathbf{v} = a\mathbf{i} - \mathbf{k}$ are perpendicular if

$a^2 - 1 = 0$

(a) $a = b = 1$

(b) $a = 1, b = 0$

(c) $a = 1$ or $a = -1$ and $b = 0$

(d) $a = 1$ or $a = -1$

(13) One of the following statements is true

(a) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{u} = 0$ X

(b) $(\mathbf{u} \times \mathbf{v}).\mathbf{u} = 0$

(c) $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$ X

(d) $\mathbf{u}.\mathbf{u} = |\mathbf{u}|$ X

(14) The area of the triangle formed by the vectors $\mathbf{u} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = \mathbf{i} - \mathbf{k}$ is

$$A = \frac{1}{2} \times \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 1 & 0 & -1 \end{vmatrix}$$

$$= \frac{1}{2}\left(-2\hat{i} + 2\hat{j} - 2\hat{k}\right) \quad \frac{\sqrt{12}}{2} = \sqrt{3}$$

(a) $\sqrt{12}$

(b) $\sqrt{3}$

(c) $\frac{1}{2}\sqrt{3}$

(d) 6

(15) The angle between the lines $3x + y = 5$ and $2x - y = 4$ is

$3\hat{i} + \hat{j} = \vec{V} \quad 2\hat{i} - \hat{j} = \vec{U}$

$\cos\theta = \frac{V \cdot U}{|V||U|} = \frac{8}{\sqrt{10 \times 10}} \quad \frac{8}{10}$

$\cos\theta = \frac{5}{\sqrt{5}\sqrt{10}} = \frac{\sqrt{5}}{\sqrt{10}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{6}$

(16) The length of the polar curve $r = e^{-\theta}$, $0 \le \theta \le 1$ is

$$\ell = \int_0^1 \sqrt{e^{-2\theta} + e^{-2\theta}}$$

$$= \int \sqrt{2}\, e^{-\theta}$$

$$= -\sqrt{2}\, e^{-\theta} \Big|_0^1$$

$$= -\sqrt{2}\left(e^{-1} - 1\right)$$

$$= \sqrt{2}\left(1 - e^{-1}\right)$$

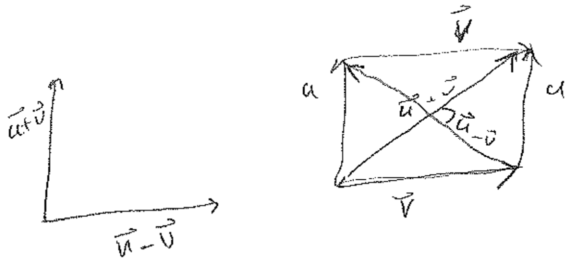
(a) $1 - e^{-1}$

(b) $2(1 - e^{-1})$

(c) $\sqrt{2}(1 - e)$

(d) $\sqrt{2}(1 - e^{-1})$

(17) If $(\mathbf{u}+\mathbf{v}).(\mathbf{u}-\mathbf{v})=0$ then



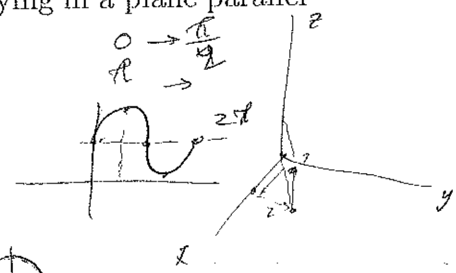
(a) $\mathbf{u}=\mathbf{v}$

(b) $\mathbf{u}.\mathbf{v}=0$

(c) $|\mathbf{u}|=|\mathbf{v}|$

(d) $\mathbf{u}=\mathbf{v}=0$

(18) The circle of radius 1 centered at the point $(1,2,3)$ and lying in a plane parallel to the xy-plane is



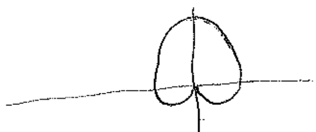
(a) $(x-1)^2+(y-2)^2+(z-3)^2=1.$ sphere

(b) $(x-1)^2+(y-2)^2=1.$ → cylinder

(c) $(x-1)^2+(y-2)^2=1,\ z=3.$

(d) $(x-1)^2+(y-2)^2=1,\ z=0.$

(19) The area inside the curve $r=1+\sin\theta$ is



(a) 3π $\quad 2\times\frac{1}{2}\int_{-\pi/2}^{\pi/2} 1+2\sin\theta+\sin^2\theta$

(b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{2}$ $\quad = \int 1+2\sin\theta+\frac{1}{2}-\frac{1}{2}\cos 2\theta$

(d) 2π $\quad = \frac{3}{2}\theta-2\cos\theta-\frac{1}{4}\sin 2\theta \Big|_{-\pi/2}^{\pi/2}$

$=\left(\frac{3\pi}{4}\right)-\left(-\frac{3\pi}{4}\right)$

$\frac{6\pi}{4}$ $\quad \frac{3\pi}{2}$ $\quad \frac{\sqrt{3}}{2}$

(20) If $\mathbf{u}.\mathbf{v}=\sqrt{3}$ and $\mathbf{u}\times\mathbf{v}=\mathbf{i}+2\mathbf{j}+2\mathbf{k}$. The angle between $\mathbf{u}$ and $\mathbf{v}$ is

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$

(d) $\frac{2\pi}{3}$.

$|u||v|\cos\theta=\sqrt{3}$ $\quad \tan\theta=\frac{3}{\sqrt{3}}=\sqrt{3}$

$|u||v|\sin\theta=3$

(21) Let $\mathbf{u},\mathbf{v}$ and $\mathbf{w}$ be vectors in space. One of the following operations is undefined

(a) $(\mathbf{u}\times\mathbf{v}).\mathbf{w}$

(b) $(\mathbf{u}.\mathbf{v})\times\mathbf{w}$

(c) $(\mathbf{u}+\mathbf{v}).\mathbf{w}$

(d) $(\mathbf{u}+\mathbf{v})\times\mathbf{w}$

(22) The set of points that satisfy the equations $x+y+z=1,\ x+y-z=1$ is

(a) the z-axis

(b) the xy-plane

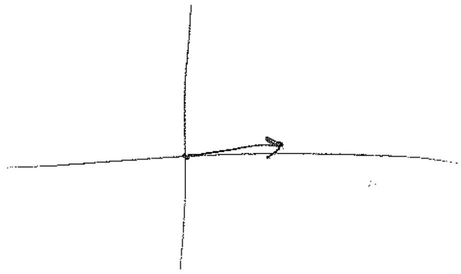
(c) the plane $x+y=1$

(d) the line $x+y=1$ in the xy-plane.

49

$\cancel{u.v = \sqrt{3}}$

$\cancel{u \times v = \hat{i} + 2\hat{j} + 2k}$



Question 2.(10 points) Consider the circle $r = 2\cos\theta$ and vertical line $r = \frac{1}{2}\sec\theta$.

(a) Write the curves in Cartesian coordinates and graph them.

$r = 2\cos\theta$

$r^2 = 2r\cos\theta$

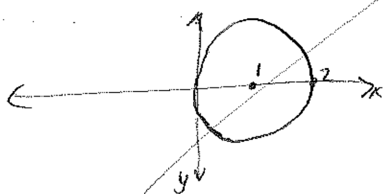
$x^2 + y^2 = 2x$

$(x^2 - 2x + 1) + y^2 = 1$

$(x-1)^2 + y^2 = 1$

center $(1, 0)$

$r = 1$



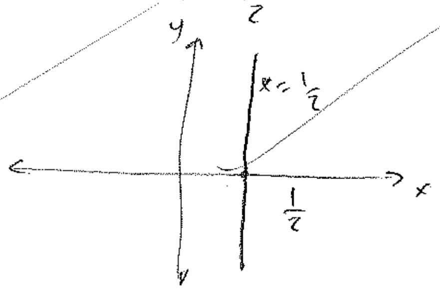
$r = \frac{1}{2}\sec\theta$

$r = \frac{1}{2\cos\theta}$

$2r\cos\theta = 1$

$2x = 1$

$x = \frac{1}{2}$



(b) Find the area inside the circle $r = 2\cos\theta$ and to the right of the line $r = \frac{1}{2}\sec\theta$.

the intersection

$2\cos\theta = \frac{1}{2}\sec\theta$

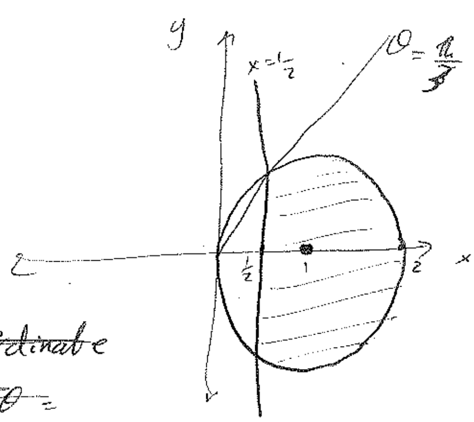
$2\cos\theta = \frac{1}{2\cos\theta}$

$4(\cos\theta)^2 = 1$

$(\cos\theta)^2 = \frac{1}{4}$

$\cos\theta = \pm\frac{1}{2}$ ~~first quardinate~~ ~~so $\cos\theta =$~~

$\theta = \frac{\pi}{3}, -\frac{\pi}{3}$



$A = \frac{1}{2} \times 2 \int_0^{\pi/3} (2\cos\theta)^2 - \left(\frac{1}{2}\sec\theta\right)^2 \, d\theta$

$= \int_0^{\pi/3} \left(4\cos^2\theta - \frac{1}{4}\sec^2\theta\right) d\theta$

$= \int_0^{\pi/3} \left(2 + 2\cos 2\theta - \frac{1}{4}\sec^2\theta\right) d\theta$

$= 2\theta + \sin\theta - \frac{1}{4}\tan\theta \Big|_0^{\pi/3}$

$= \left(\frac{2\pi}{3} + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{4}\right) - (0)$

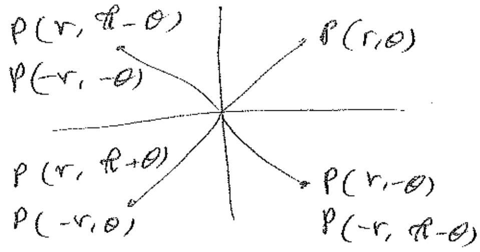
$= \frac{2\pi}{3} + \frac{\sqrt{3}}{4}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos = \frac{1}{2}$

$\tan = \sqrt{3}$

10



Question 3.(6 points) Consider the polar curve $r = \cos\left(\frac{\theta}{3}\right)$.

(a) Show that the curve is symmetric about the x−axis.

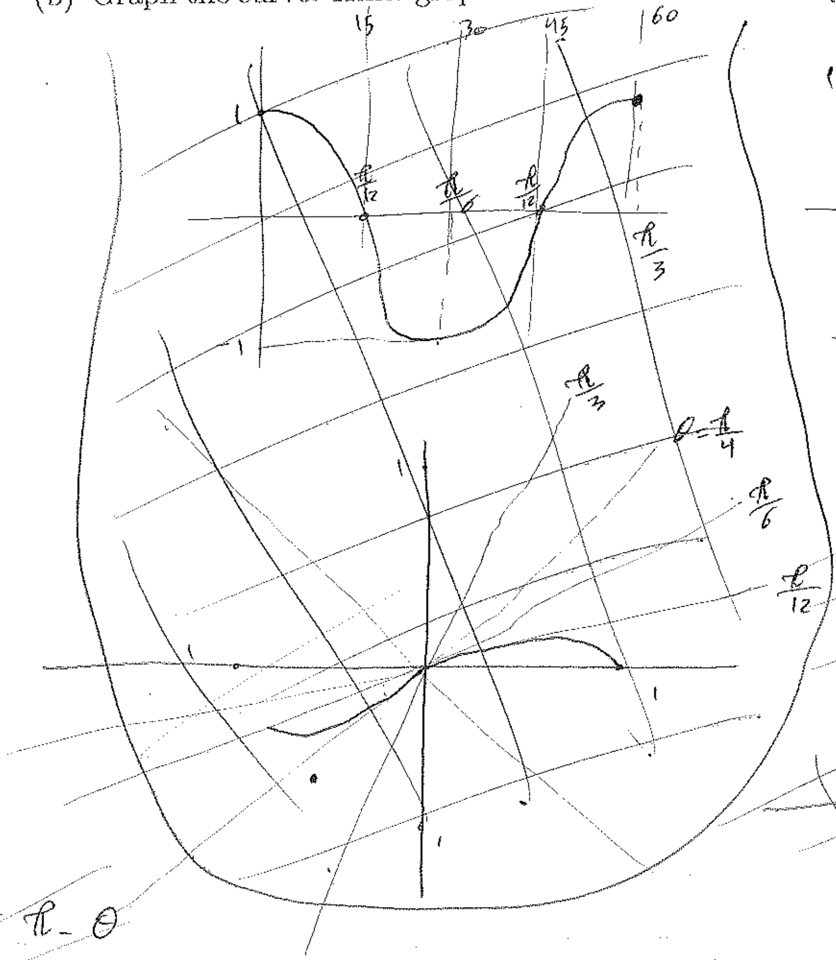
when $\theta_0 = -\theta$

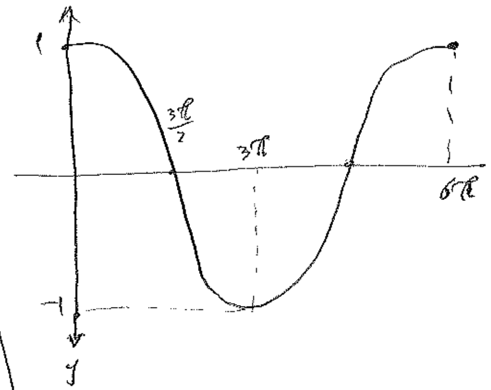
$r_0 = \cos\left(\frac{-\theta}{3}\right)$ but $\cos(-\theta) = \cos\theta$

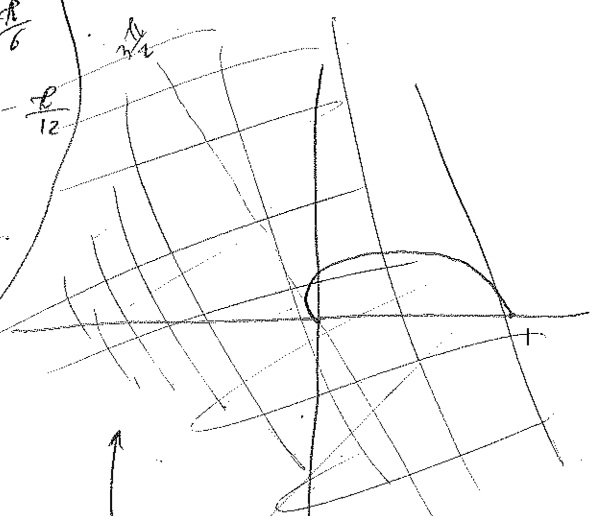
$= \cos\frac{\theta}{3} = r$

so $P(r, \theta)$ and $P(r, -\theta)$ are on the curve then it's symmetric about the x-axis

(b) Graph the curve. Hint: graph the curve in the interval $[0, \frac{3\pi}{2}]$ then use symmetry.





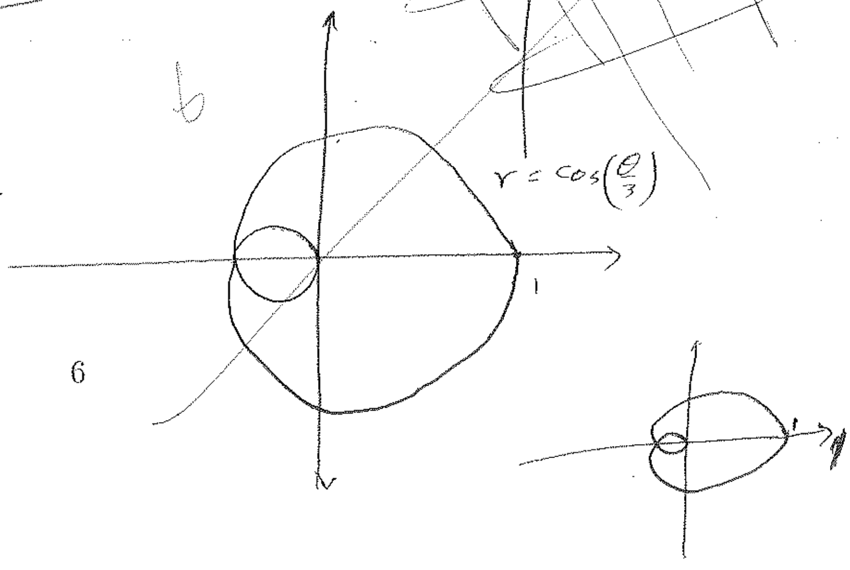


$\theta_0 = \pi - \theta$

$r_0 = \cos\left(\frac{\pi}{3} - \frac{\theta}{3}\right)$

$= \cos\frac{\pi}{3}\cos\frac{\theta}{3} + \sin\frac{\pi}{3}\sin\frac{\theta}{3}$

$= \frac{1}{2}\cos\frac{\theta}{3} + \frac{\sqrt{3}}{2}\sin\frac{\theta}{3}$



4