Birzeit University-Mathematics Department
Math 1321-Calculus II

Second Hour Exam Spring 2015/2016
Name:.. Number:...............
Instructor:..............KEY.................... Section:...............

Question 1.(44 points) Circle the correct answer:

(1) The polar curve $r = -\sin^2(2\theta)$ is symmetric about

- (a) the $x-$axis.
- (b) the $y-$axis.
- (c) the origin.
- [✓ (d)] all of the above.

(2) Let θ be the angle between $\mathbf{u} = \mathbf{i} + \mathbf{k}$ and $\mathbf{v} = \mathbf{j} + \mathbf{k}$. Then $\theta =$

- (a) $\frac{\pi}{2}$
- [✓ (b)] $\frac{\pi}{3}$
- (c) $\frac{\pi}{4}$
- (d) $\frac{\pi}{6}$

(3) The vector projection of $\mathbf{u} = \mathbf{i} + \mathbf{k}$ onto $\mathbf{v} = \mathbf{j} + \mathbf{k}$ is

- (a) $2\mathbf{v}$
- [✓ (b)] $\frac{1}{2}\mathbf{v}$
- (c) $\frac{1}{\sqrt{2}}\mathbf{v}$
- (d) $\mathbf{v}$

(4) The vectors $\mathbf{u} = a\mathbf{i} + b\mathbf{j} + \mathbf{k}$, $\mathbf{v} = a\mathbf{i} - \mathbf{k}$ are perpendicular if

- (a) $a = b = 1$
- (b) $a = 1, b = 0$
- (c) $a = 1$ or $a = -1$ and $b = 0$
- [✓ (d)] $a = 1$ or $a = -1$

(5) One of the following statements is true

- (a) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{u} = \mathbf{0}$
- [✓ (b)] $(\mathbf{u} \times \mathbf{v}).\mathbf{u} = 0$
- (c) $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$
- (d) $\mathbf{u}.\mathbf{u} = |\mathbf{u}|$

(6) The area of the triangle formed by the vectors $\mathbf{u} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = \mathbf{i} - \mathbf{k}$ is

(a) $\sqrt{12}$
[✓ (b)] $\sqrt{3}$
(c) $\frac{1}{2}\sqrt{3}$
(d) 6

(7) The angle between the lines $3x + y = 5$ and $2x - y = 4$ is

(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{3}$
[✓ (c)] $\frac{\pi}{4}$
(d) $\frac{\pi}{6}$

(8) The circle of radius 1 centered at the point $(1, 2, 3)$ and lying in a plane parallel to the $xy-$plane is

(a) $(x-1)^2 + (y-2)^2 + (z-3)^2 = 1$.
(b) $(x-1)^2 + (y-2)^2 = 1$.
[✓ (c)] $(x-1)^2 + (y-2)^2 = 1,\ z = 3$.
(d) $(x-1)^2 + (y-2)^2 = 1,\ z = 0$.

(9) The area inside the curve $r = 1 + \sin\theta$ is

(a) 3π
(b) $\frac{\pi}{2}$
[✓ (c)] $\frac{3\pi}{2}$
(d) 2π

(10) The length of the polar curve $r = e^{-\theta}$, $0 \leq \theta \leq 1$ is

(a) $1 - e^{-1}$
(b) $2(1 - e^{-1})$
(c) $\sqrt{2}(1 - e)$
[✓ (d)] $\sqrt{2}(1 - e^{-1})$

(11) One of the following is the point $(x, y) = (-1, -\sqrt{3})$ in polar coordinates

(a) $(2, \frac{\pi}{3})$
[✓ (b)] $(-2, \frac{\pi}{3})$
(c) $(-2, \frac{4\pi}{3})$
(d) $(-2, \frac{\pi}{6})$

(12) The cartesian equation of the polar curve $r^2 \sin(2\theta) = 2$ is

(a) $x + y = 1$
[✓ (b)] $xy = 1$
(c) $y^2 = 2$
(d) $xy = 2$

(13) The volume of the box determined by the vectors $\mathbf{u} = \mathbf{i} - \mathbf{j} - \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, $\mathbf{w} = \mathbf{i} - \mathbf{j}$ is

[✓ (a)] 5
(b) 7
(c) 6
(d) 4

(14) The equations $x = 3$, $y = 4$ in space represent

(a) The point $(3, 4)$
[✓ (b)] A line parallel to the $z-$axis
(c) A line perpendicular to the $z-$axis
(d) A line in the $xy-$plane.

(15) The slope of the polar curve $r = \frac{1}{2} + \cos\theta$ at $\theta = \frac{2\pi}{3}$ is

(a) $\sqrt{3}$
[✓ (b)] $-\sqrt{3}$
(c) $\frac{1}{\sqrt{3}}$
(d) $-\frac{1}{\sqrt{3}}$

(16) If $(\mathbf{u} + \mathbf{v}).(\mathbf{u} - \mathbf{v}) = 0$ then

(a) $\mathbf{u} = \mathbf{v}$
(b) $\mathbf{u}.\mathbf{v} = 0$
[✓ (c)] $|\mathbf{u}| = |\mathbf{v}|$
(d) $\mathbf{u} = \mathbf{v} = \mathbf{0}$

(17) Let $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ be vectors in space. One of the following operations is undefined

(a) $(\mathbf{u} \times \mathbf{v}).\mathbf{w}$
[✓ (b)] $(\mathbf{u}.\mathbf{v}) \times \mathbf{w}$
(c) $(\mathbf{u} + \mathbf{v}).\mathbf{w}$
(d) $(\mathbf{u} + \mathbf{v}) \times \mathbf{w}$

(18) The set of points that satisfy the equations $x + y + z = 1$, $x + y - z = 1$ is

(a) the $z-$axis

(b) the $xy-$plane

(c) the plane $x + y = 1$

[✓ (d)] the line $x + y = 1$ in the $xy-$plane.

(19) The set of points equidistant (at the same distance) from the points $(0, 1, 0)$ and $(0, -1, 0)$ is

(a) the $z-$axis

(b) the $xy-$plane

[✓ (c)] the $xz-$plane

(d) the $yz-$plane

(20) The center and radius of the sphere $x^2 + y^2 + z^2 - 2x + 2y = 2$ are

(a) $(1, 1, 0)$, 4

[✓ (b)] $(1, -1, 0)$, 2

(c) $(-1, 1, 0)$, 2

(d) $(-1, -1, 0)$, 2

(21) If $\mathbf{u}.\mathbf{v} = \sqrt{3}$ and $\mathbf{u} \times \mathbf{v} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. The angle between $\mathbf{u}$ and $\mathbf{v}$ is

[✓ (a)] $\frac{\pi}{3}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$

(d) $\frac{2\pi}{3}$.

(22) The set of points in the plane that satisfy the inequalities $-2 < r < -1$, $\pi \leq \theta \leq 2\pi$

(a) the region between the circles $r = -1$ and $r = -2$.

[✓ (b)] the region between the circles $r = -1$ and $r = -2$ in the first and second quadrant.

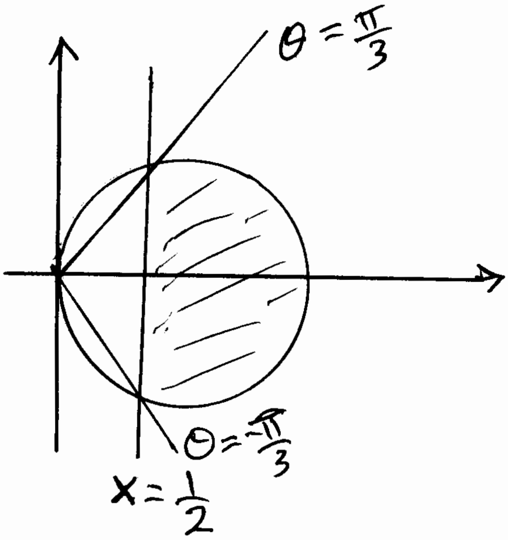
(c) the region between the circles $r = -1$ and $r = -2$ in the second and third quadrant.

(d) the region between the circles $r = -1$ and $r = -2$ in the third and fourth quadrant.

Question 2.(10 points) Consider the circle $r = 2\cos\theta$ and vertical line $r = \frac{1}{2}\sec\theta$.

(a) Write the curves in Cartesian coordinates and graph them.

$$r = 2\cos\theta$$
$$r^2 = 2r\cos\theta$$
$$x^2+y^2 = 2x$$
$$(x-1)^2 + y^2 = 1$$
$$r = \frac{1}{2}\sec\theta$$
$$r\cos\theta = \frac{1}{2}$$
$$x = \frac{1}{2}$$



(b) Find the area inside the circle $r = 2\cos\theta$ and to the right of the line $r = \frac{1}{2}\sec\theta$.

$$A = \frac{1}{2}\int_{-\frac{\pi}{3}}^{\pi/3}\left(4\cos^2\theta - \frac{1}{4}\sec^2\theta\right)d\theta$$

$$= \int_0^{\pi/3} 2(1+\cos 2\theta)\,d\theta - \frac{1}{4}\int_0^{\pi/3}\sec^2\theta\,d\theta$$

$$= 2\left[\theta + \frac{\sin 2\theta}{2}\right|_0^{\pi/3} - \frac{1}{4}\tan\theta\Big|_0^{\pi/3}$$

$$= 2\left[\frac{\pi}{3} + \frac{\sqrt{3}}{4}\right] - \frac{\sqrt{3}}{4}$$

$$= \frac{2\pi}{3} + \frac{\sqrt{3}}{4}$$

Question 3.(6 points) Consider the polar curve $r = \cos\left(\frac{\theta}{3}\right)$.

(a) Show that the curve is symmetric about the $x-$axis.

$r = \cos\left(\frac{\theta}{3}\right) \quad (r, \theta)$

$r = \cos\left(\frac{-\theta}{3}\right) : (r, -\theta)$

(b) Graph the curve. Hint: graph the curve in the interval $[0, \frac{3\pi}{2}]$ then use symmetry.